Advanced Soil Mechanics
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Lecture – 39
Stress Path-Additional Undrained Case

Welcome back all of you in the last lecture, we have seen the stress path for un-drained test. Now, there are a few more aspects which we need to discuss related to un-drained stress path. So, we will see those in the today's lecture.

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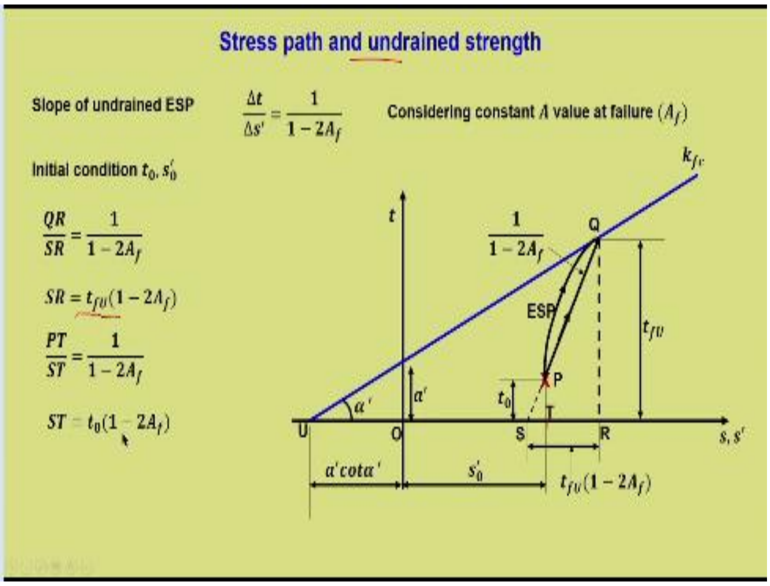

So, stress path and un-drained strength, so, this is the first one where we will try to obtain the expression for un-drained shear strength based on the stress path and this we have already done this for drained stress path in the towards the end of those lectures we have already derived it for drained stress path. So, similarly for un-drained stress path, let us try to derive it. Slope of un-drained ESP, let us say, we are taking t-s, s' plot is 1/ 1 - 2 A_f.

Now, these are familiar to all of you now, considering constant A value at failure. So, that is how we obtained this expression. Let the initial condition t_0, s'_0. So, let us consider t-s, s' plot and this is a general case where even the cohesion is also accounted for this intercept is nothing but the cohesion in t-s, s' plot. This will be a'. So, this will be a'cotα' and α'is the inclination of the failure line and all these things are familiar to us now.

So, the starting point or the initial condition is t_0, s'_0. This P is the initial condition. So, this is s'_0 and it is at t_0. So, this accounts for all the conditions like the influence of cohesion, the influence of k_0 value and we have already seen how k_0 influences the initial starting point, it is not starting from the isotropic line. So, that is what we need to keep in mind. This is a general case.

Now, for TSP or we do not have much confusion, but presently we are discussing about ESP. Now, ESP, this slope is governed by this. So, this is a typical value of A_f, let us say that it is a positive slope. Now, what is the un-drained strength? This is the slope 1/ 1 - 2 A_f. Let the point where the effective stress path meets the failure line P. And this is the case when A is considered to be constant, let us say take the value of A_f.

And here this one is when a changes with shearing. Now, depending upon A value, we will have different slopes of the effective stress path. We have seen one particular case where a is equal to when A_f is substituted by 0.5, we see that it is vertically upwards. Similarly, this some value of A_f , what we need to see is whether this is a positive slope or a negative slope. So, the moment A_f takes the value of 1, for example, we will have 1 – 2, this – 1.

So, there will be a negative slope in this particular direction at. So, here let us say that A_f is such a value that it gives a positive slope whatever it is going to be the same. Whatever we are going to study or whatever we are going to explain here that remains more or less the same procedure. So, now this is t_{fU}. Now, what is the implication of t_{fU}. t_{fU}, we know that it is the radius of Mohr circle and radius of Mohr circle gives the un-drained shear strength s_U.

I hope you remember this point. And that is the reason why we have spent a lot of time describing shear strength characteristics. So, if you do not understand that, it is very difficult to correlate how t_f represents un-drained shear strength. So, it is one of the same t_f is $\sigma_1 - \sigma_3$/ 2, which is nothing but the radius of the Mohr circle and radius of the Mohr circle gives the un-drained shear strength as per trescas criterion.

So, please refer back to the lectures if you have not followed this particular point. So, let us say QR is what it is represented by that is t_{fU} is given by QR. Now, if you extend this particular stress path from P downwards, it will meet s, s' axis at capital S. Now, we know from this particular geometry that QR / SR, it gives the slope 1 / 1 – 2 A_f. Why? Because it is the slope of this. So, 1 / 1 – 2 A_f.

So, we can write SR. What is QR? QR is nothing but t_{fU}. So, this QR is substituted here. So, t_{fU} / SR is equal to 1 / 1 – 2 A_f. So, SR = t_{fU} *(1 – 2 A_f) So, that is what has been given here. That is what is marked in this figure SR. Now, let us say if you drop a line from P vertically downwards, it will meet at point t on the s, s' this particular axis.

So, there is a dropping of the vertical it meets here. Similarly, the way we have discussed for QR/ SR, we can also write PT/ ST and that is also equal to 1 / 1 – 2 A_f, because it is an extension. If that is the case, ST can be written as this distance is nothing but t_0; t_0 *(1 – 2 A_f). So, ST also we got.

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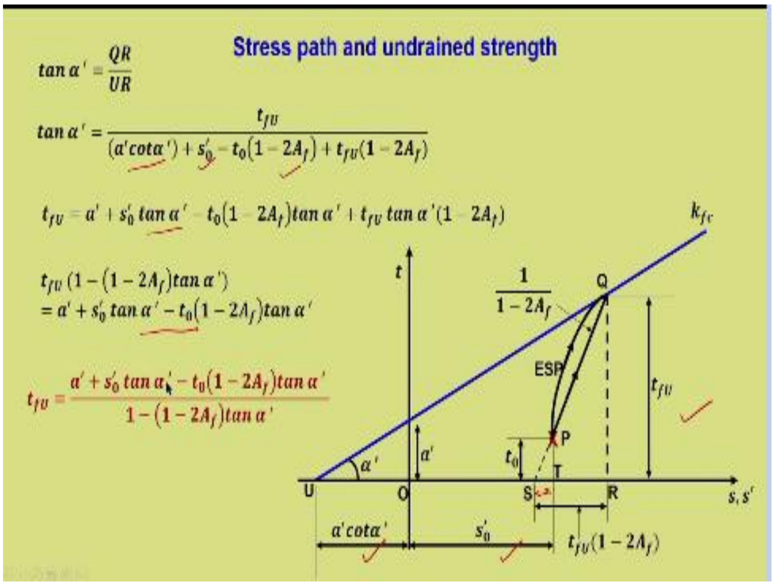

So, we have got now SR and ST. Now, we can also write tan α' = QR/ UR again from the geometry. Now, if you substitute it, QR is t_{fU} this and so, tan α'= t_{fU} / UR. Now, what is UR? UR, you can see that this is, this distance UO + OS then + SR. So, if you submit, you will get this whole distance UR. Now, this is known a' cot α', but the next distance is up to s'_0 that is up to the point t.

So, s'_0 is known and this ST is also we have got the expression. So, that – ST will give you the point where this s + SR will give you UR. So, that is what is done here. So, a' cot α' comes from here plus s'_0, it reaches here minus this distance t_0 into (1 – 2 A_f)+ SR that is t_{fU}

into (1 – 2 A_f). So, this distance is also known. Now, if you multiply denominator to tan α’, we get t_{fU} is equal to a’ into cot α’in tan α’is 1.

So, that will give you a’ + s'_0 tan α’- t_0 (1 – 2 A_f) tan α’+ t_{fU} (1 – 2 A_f) tan α’. If you rearrange again, we can take t_{fU} to one side. So, that will give you t_{fU} into, if you take it outside, one will come from here minus when you take this on the other side, it will be – (1 – 2 A_f) tan α’ that is equal to this particular term which comes here.

So, we will get the expression for t_{fU} which is what we want and that t_{fU} is then equal to a’ + s'_0 tan α’- t_0 (1 – 2 A_f) tan α’divided by this term come on the denominator that is 1 – (1 – 2 A_f) tan α’. So, this is the required expression for t_{fU}. Now, if you substitute for, we already know that a’ and α’is correlated to Mohr coulomb shear strength parameters.

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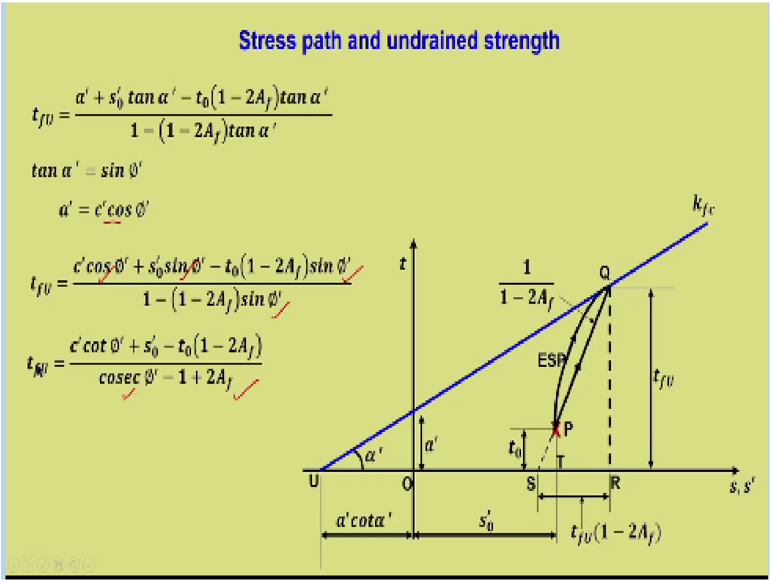

So, let us see what expression we get. Now, this is the same expression for t_{fU}. We know tan α’= sin φ’ and a’ = c’ cos φ’. So, this is known to us. So, then t_{fU} is equal to, let us substitute for a’ which is c’ cos φ’ + s'_0 sin φ’. So, tan α’gets replaced by sin φ’. This is what -t_0 (1 – 2 A_f) instead of tan α’ sin φ’ divided by 1 – (1 – 2 A_f) instead of tan α’ sin φ’.

Now, if you divide this the numerator and denominator by sin φ’, because that is a common term, so, then we will get t_{fU} is equal to c’ cot φ’ that is cos φ’/ sin φ’+ s'_0 – t_0 (1 – 2 A_f). 1 by sin φ’is cosec φ’ – 1, if you expand the bracket, + 2 A_f. So, this is the expression for t_{fU} in terms of Mohr coulomb failure envelop parameters that is what is the expression for un-drained shear strength t_{fU}.

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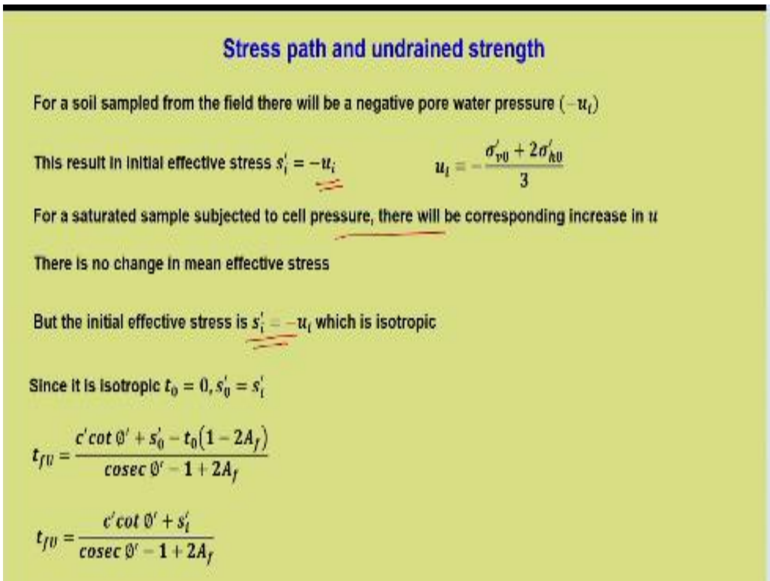

Now, let us take a case of a sample which is sampled from the field soil which is sampled from the field, we know that we have already seen that there will be a negative pore water pressure of $- u_i$. This we have already seen when we discussed about sampling of soil from the field. Now, this initial pore of pressure, what is the effect of this? This result in initial effective stress $s'_i = - u_i$.

What is the implication of this? And this initial pore water pressure helps the soil to be intact. So, - s'_i that is the initial effective stress is equal to $- u_i$ and u_i is generally taken as the mean effective stress when it was sampled. When in the field, what is the mean effective stress that is equal to u_i. So, $u_i = - (\sigma'_{vo} + 2\ \sigma'_{ho})/ 3$.

So, there is a mean effective stress. So, u_i expression is equal to mean effective stress. For a saturated sample, which is subjected to cell pressure, there will be a corresponding increase in use, so, that what it means is that. We mount the sample. So, there is an initial effective pressure which is attributed to the sampling in the field and when you raise the deviator pressure under un-drained condition, there will be an equivalent increase in the pore water pressure.

And hence, it is not going to change the initial effective stress conditions. So, that is what this particular statement means. So, there is no change in mean effective stress. Why? Because it is an un-drained condition, but the initial effective stress can be taken equal to $s'_i = - u_i$, which is isotropic. Now, this we need to understand a bit carefully, because there is an initial effective stress of $s'_i = -u_i$, this we have seen.

Now, since it is initial pore water pressure, this will act equally in all direction and hence the condition is considered to be isotropic. So, but in the previous slide, we have seen that it is at a different starting point. So, t_0 is also there; s'_0 is also there. Now, it is not on the isotropic line. But since in this particular case, when we are sampling the soil from the field, we know that it is at an initial negative pore water pressure and hence the condition has to be enforced to isotropic condition.

Now, if it is isotropic condition, then it has to start from s, s' axis, which we know that it represents the isotropic condition. We have seen this all these concepts by this time should be very clear to all of you. Now, if it starts from the isotropic stress condition, we have to take $t_0 = 0$ and that is what since it is isotropic $t_0 = 0$ and $s'_0 = s'_i$ that is the starting point.

Substituting this in the previous expression, that is t_{fU} equal to c' cot φ'. This expression what we just discussed, we substitute these values put $t_0 = 0$ and $s'_0 = s'_i$. So, then it will give the expression $t_{fU} = (c' \cot \varphi' + s'_i) / (\text{cosec } \varphi' - 1 + 2 A_f)$. Now, this is a condition which is specifically useful for soil sample from the field and also for an un-drained condition where the initial effective stress condition is not altered.

Now, if it is subjected to further consolidation before shearing, the condition would alter again. So, this you have to understand that it is mostly prominent for UU condition where the initial effective stress is not altered. It is a purely a case of un-drained shear strength.

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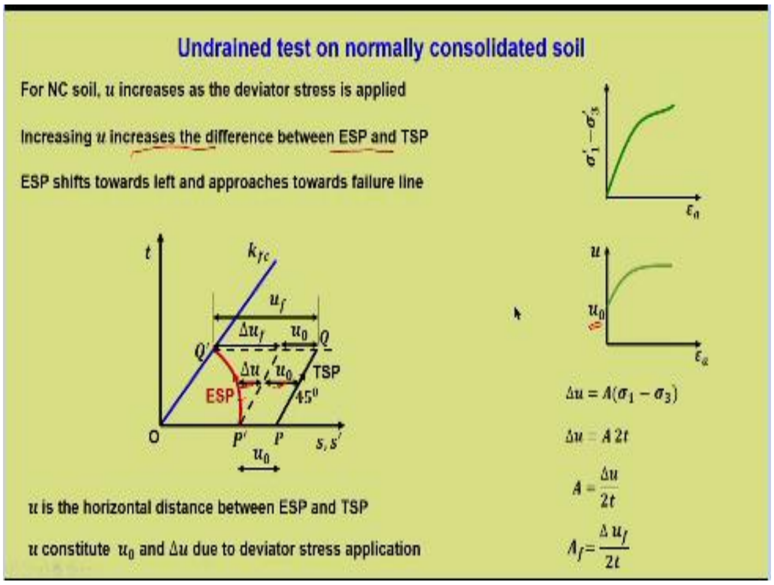

Now, let us discuss specifically on the stress path for normally consolidated soil under un-drained conditions. For NC soil, we know that u increases as the deviator stress is applied. Again, it does not need any sort of explanation for this. This is a typical stress strain variation for NC soil, you can note that it is strain hardening. Now, if you see that there is a initial pore water pressure of u_0 in the sample during sharing, we know that it is a saturated sample and hence pore water pressure would increase.

Now, increasing u, increases the difference between ESP and TSP. There is nothing great in this particular sentence, because we know that as pore water pressure keeps increasing; ESP keeps reducing from the total stress. So, let us again understand this in terms of t-s, s' plot, let us say P is the starting point. Now, P and P'; it is one as total stress, the other one is effective stress; is separated by u_0 because that is the initial pore water pressure as you can see in this particular plot.

Now, for total stress path, that is if you draw a total stress path from P, now, what will be the slope in t-s, s'. This is a case of conventional compression test and we know that in conventional compression test, the slope is going to be positive one that is 45 degrees. So, that is what is shown here and this is TSP that is the total stress path. PQ represents TSP. Effective stress path we know that because of 1 / 1 – 2 A_f; this is the typical condition where it is moving towards the k_f line.

For example, in the case of if A_f is equal to 1 it is equal to negative one; will be the slope. So, P' Q' represents the effective stress path. Now, let us try to understand this particular figure more in detail. Now, let us say like this line is parallel to PQ and this is separated by u_0. Now, then only we will be able to know like there is an initial pore water pressure of u_0. During shearing, what will be the kind of pore water pressure that gets generated?

Now, u is the horizontal distance between ESP and TSP that is the total pore water pressure that is always it will be u_0 which remains constant. You can see here plus this particular whatever is these differences that will be the Δu. Now, why did I stop? The total stress path here, why it is not keep on increasing? Why? Because the failure has already happened when the effective stress path has just meets the K_{fc} line.

So, that is this particular point and there is no soil beyond this. So, why? There is no point in increasing or extending the TSP beyond this point. So, that is why this dashed line has been drawn. Now, u constitute that is the total pore water pressure constitute u_0 and Δu, which is due to the deviatoric stress application. Let us see how it looks like. Now, this parallelogram, this one, it represents u_0.

For any instance of shearing, it is going to be this u_0 as this going to be there and when we start the shearing from here, when it reaches here, so, this particular, this one, is the Δu. So, you can see here at any particular point, it is $\Delta u + u_0$. So, this part of Δu comes from shearing and u_0 is the initial pore water pressure. At failure still that u_0 is there; you can see that at failure the pore water pressure is more and that is given by Δu_f.

Now, $\Delta u_f + u_0$ gives this particular term u. So, that is u_f at failure. So, ESP, it shifts towards left and approaches towards failure line. And why? Because this is a case of normally consolidated soil and we can always take A value equal to 1 which gives a negative slope. Now, we know that $\Delta u = A\ (\sigma_1 - \sigma_3)$. Please note that if you refer to the expression for Skempton's pore water pressure we have used $\Delta\sigma_1 - \Delta\sigma_3$.

Here, we are just writing it in a general term just to make us make understand like where t is going to come here. So, $\Delta u = A2t$. So, this will be 2 of t. So, $A = \Delta u / 2t$. So, it is just for our understanding like $A_f = \Delta u_f / 2\ t$ so, where t is going to come because when we discussed Skempton's pore water pressure equation, we did not have to actually tell about t parameter there.

So, here we are just writing for our understanding and $\Delta\sigma_1 - \Delta\sigma_3$ still holds good. Here, it is only to make you understand how the t comes into picture.

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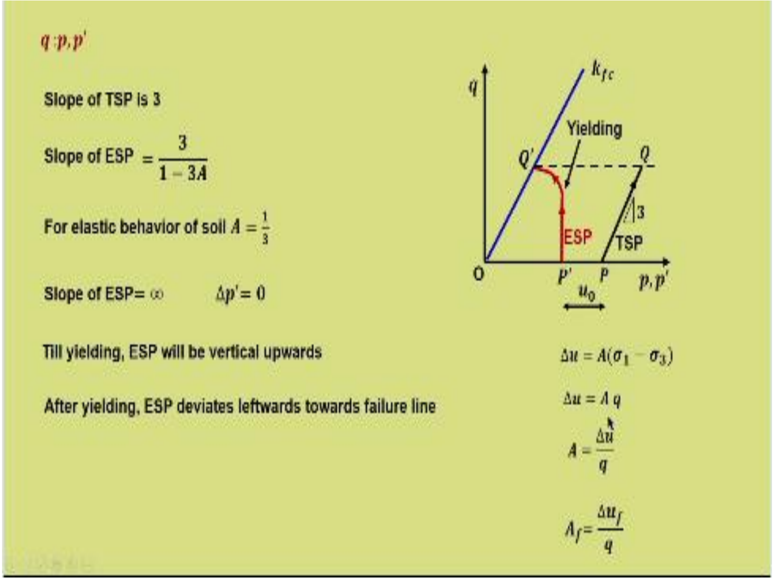

Now, if you see this in q-p, p' plot, I am telling about un-drained stress path for normally consolidated soil. Slope of TSP is 3, which we know which we have already seen. And that is not going to change. This is the TSP which is given by PQ with an inclination of 3. Why it is 3? Because it is conventional triaxial compression test. And slope of ESP is equal to 3 / (1 – 3 A). We know this.

Now, there is something interesting fact which comes out of this expression. Now, for elastic behaviour of soil A is equal to 1 / 3, this we have already seen. Now, if you substitute A equal to 1 / 3 in this expression, we know that that is going to be infinity that means Δ p' is not going to change during shearing. For what condition we need to understand this very carefully, 3 / 1 – 3 A_f or 3 A.

In general is the slope of effective stress path in q-p, p' because this particular understanding is useful when we discuss critical state soil mechanics. So, 3 / 1 – 3 A is the slope of effective stress path when it is drawn in q-p, p' plot. A is equal to 1 / 3, we know for elastic behaviour that comes from the derivation of Skempton's pore water pressure equation.

If you substitute A equal to 1 / 3, we know that the slope of effective stress path is going to be vertical. Is it going to be vertical till the failure line? No. It is going to be vertical, so, long as the soil behaves elastically that is the initial portion of shearing. At some point of time, soil would yield. So, the behaviour it changes from elastic to plastic behaviour after yielding. So, the value of A equal to 1 / 3 yield is there.

So, it is vertical only till the point the soil yields. So, that particular aspect we will see here. From p', so, this till yielding ESP will be vertically upwards. This information is very important for critical state. After yielding, ESP deviates leftwards towards failure lines. So, once it yields, then the response is the same. So, here now, how to define the yield point when it is going to shift leftwards. This things will be very clear when we discuss 4th module.

So, here this is the ESP. You can see here it is vertical till this particular point and here the yielding has occurred and then it moves towards the left, towards the failure line. So, P' Q' is slightly different in q-p, p' plot and this P' Q' gives the effective stress path. Now, this is the point of a yielding and up to yielding, it is elastic and hence it is vertically upwards which comes from this again $\Delta u = A(\sigma_1 - \sigma_3)$.

You can replace A into q. So, $A = \Delta u/q$. $A_f = \Delta u_f /q$. So, this is again where the q comes into picture or where t comes into picture which we have already discussed.

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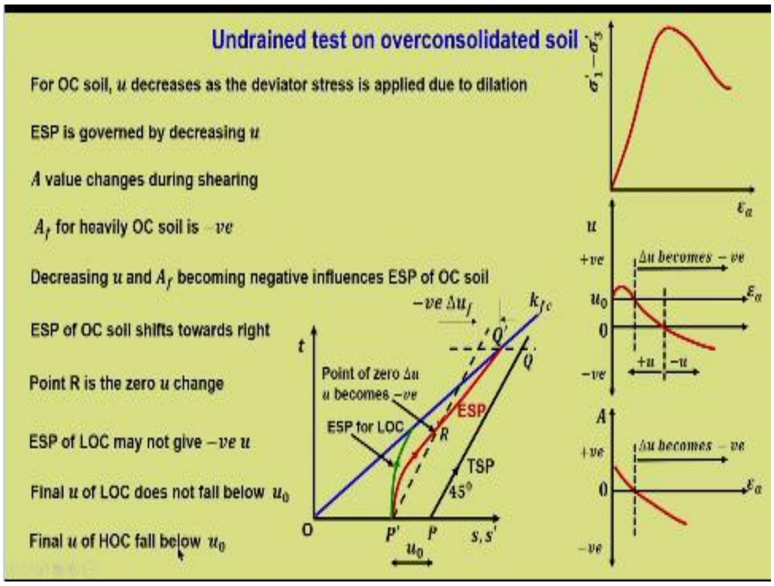

Now, let us see un-drained test on over consolidated soil. Now, we have discussed normally consolidated and we know where the difference is going to come, it is basically dictated by the value of A parameter and this we already have seen. So, let us see this in more detail. So, this is the typical stress strain response you can see that it is peaking and then exhibits a strain softening behaviour.

For OC, what happens because of this particular aspect? u decreases, now, NC, u increased. Here, it is decreasing as the shearing happens why, because of dilation. ESP is governed by decreasing u. Now, this decreasing u will have its effect on effective stress path. Let us say that u_0 is the initial pore water pressure and this is u equal to 0. Now, this is positive. Here, upwards is positive and here, it is negative. Now, from u, we can see that it initially increases and then starts reducing.

Now, this particular point at this particular point Δu start becoming negative that means then you can see that it keeps on reducing drastically to a final value. And if you see this particular point, this is the point where pore water pressure actually becomes from a positive value to negative value. So here from here, this is the zero, so, where it crosses; it crosses from here.

So, this particular portion exhibits negative pore water pressure even though there are 2 things here. One is, disagreement increment is positive or negative and here, the actual value of u is negative or positive. Now, from here onwards, it starts becoming negative. And from here, you can see that u actually becomes negative, the magnitude and towards this will be plus u. A value changes during shearing. Now, because of this, we know that A value is not constant.

So, at some point, A value becomes negative. So, it basically from here onwards, A will start becoming negative and finally, it will reach a negative value for OC. So, A_f for heavily OC soil is negative. Final A_f will always be negative. So, decreasing u and A_f becoming negative. This would influence the ESP of OC soil. So, both because of decreasing u, there will be a condition of A_f becoming negative; both these influences the effective stress path of OC soil.

Now, we again consider t-s, s’ plot. This is the TSP which is PQ as like in the previous case and that is not going to change. The starting point is because there is a initial pore water pressure, it is at p’. ESP of OC soil shifts towards right. Why? Because of decreasing u. So, let us say, this parallel line P P’, this line is dashed line it is extending and from P’, we can see that there is a ESP curve that is shifting towards right.

Initially, it was for NC it is towards left. Now, it is towards right. This is the ESP. You can see that at one particular point, this ESP crosses this particular point. So, this is the initial pore water pressure which is there in the sample. Now, what is actually happening the final pore water pressure is becoming less than the initial pore water pressure. So, point R is the zero u change.

So, here this particular point R means, there is no change in pore water pressure because it comes from here and it reaches here and then comes down. So, there is the change in pore water pressure. This particular point is zero and then it becomes negative. So, here, it will be positive. From here, it crosses and then here it will be negative. So, this particular portion, this final u is less than the u_0. So, point of zero Δu or where u start becoming negative.

So, that is a point R. P' Q' represents the effective stress path. So, this particular value is the Δu_f and that is negative Δu_f. In the case of NC, what has happened? Since, this is joining on the other side, we have $u_0 + \Delta u_f$ whereas, in the case of OC, we have $u_0 - \Delta u_f$. So, this u_0 minus this value. So, that is the final point Q'. So, let us say, this particular curve is for heavily over consolidated soil. Now, for ESP likely over consolidated soil; may not give a negative view.

So, in the case of HOC, it gives negative value whereas for LOC, it may not be negative, but it is still, it is inclining towards right but it is not crossing u_0 that means this particular value the final u value is u_0 plus this value, not minus. So, final u of LOC does not fall below u_0. So, it is. In this particular case, it is falling below u_0 whereas in lightly over consolidated, it is generally; it is u_0 plus this particular increment.

So, finally u of HOC fall below u_0. So, this is the comparison between HOC and LOC how it changes with respect to pore water pressure.

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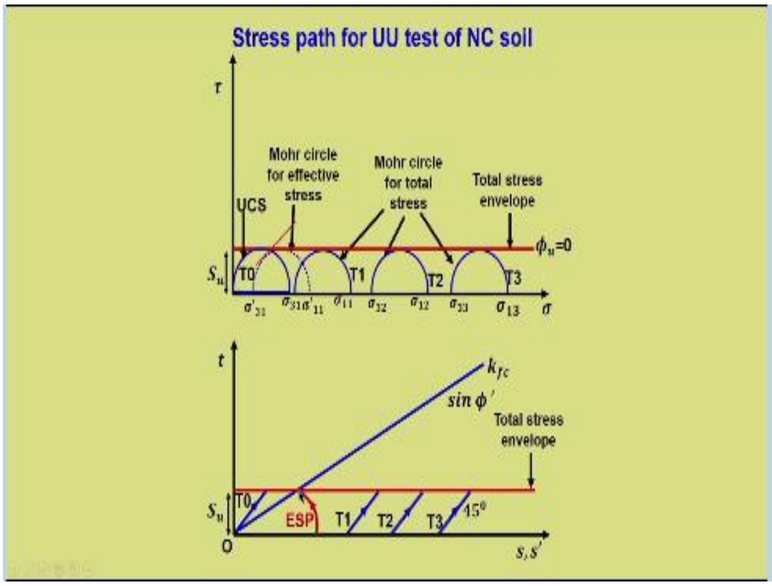

Now, we have discussed for a typical case of OC and NC specifically, I just want to highlight what is the stress path for UU test on NC soil. We have already discussed this. These are the total Mohr circle for 3 different combinations that is T 1, T 2, T 3. Mohr circle for total stress this we have seen. And along with that, I have just added UCS which is a special case of UU test. So, all the 4 Mohr circles are total Mohr circle.

Now, for all these we have seen that for UU test there will be only one effective stress circle and that is given by this dotted circle and that is the Mohr circle for effective stress and this gives the Mohr coulomb failure envelope which is $\varphi_u = 0$ this is the total stress envelope and S_u is the un-drained shear strength. Now, let us see in t-s, s' plot. Now, this particular radius of the circle is represented by t.

So, in k_{fc} is this one and sin φ' is the slope. You have total stress envelop. S_u is the same thing. So, it is S_u. This corresponds to the Mohr circle T1. So, this is the stress part, which is total stress path which is T 2 and T 3 and in the case of UCS, it has to start from zero and this is at 45 degrees because it is in t-s, s' plot and this gives T 0 which has to be starting from the origin.

And this is the effective stress path. You can see that the effective stress path is at the meeting point of un-drained strength and this drained strength that is k_{fc} = sin φ'. You remember when we discussed about UU test, we have shown that one cannot draw the effective stress envelope from UU test because we have only one effective stress circle, but if we know φ', then one can always draw a tangent to the effective stress circle.

For example, if you have it here, we can always draw a kind of failure envelope corresponding to this, but for this, we have to know what is the value of φ’. Now, if the φ’ is known, k_{fc} can be drawn and the effective stress path will meet at this particular point. This is the effective stress path corresponding to UU test, but that will be at the meeting point of k_{fc} and the total stress envelope. And this slope is 1 / 1 – 2 A that is all related to some additional cases of un-drained shear strength and the stress path representation.

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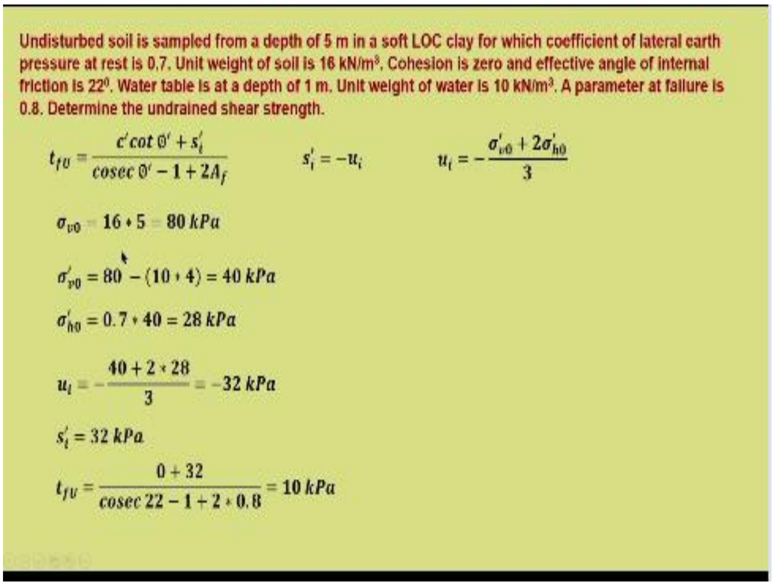

We will just do a quick example of finding out the un-drained shear strength from whatever we have learned. What is given? Undisturbed soil is sampled from a depth of 5 metre in a soft lightly over consolidated clay for which coefficient of lateral earth pressure at rest is 0.7. Unit weight of soil is 17 kN/ m^3. Cohesion is 0. An effective angle of internal friction is given to be 22 degrees.

Water table is at a depth of 1 metre. Unit weight of water is 10 kN/ m^3. A parameter at failure that is Skempton’s A parameter is 0.8. We are asked to determine the un-drained shear strength. We know the expression for un-drained shear strength. We have derived it which is t_{fU} = (c’ cot φ’+ s'_i) / (cosec φ’– 1 + 2 A_f). It is sampled from the field undisturbed.

So, here, we need to use s'_i and we know that $s'_i = - u_i$. So, this relationship also we know. And $u_i = - (\sigma'_{v0} + 2\ \sigma'_{h0})$ /3. So, we need to find out σ'_{v0} and σ'_{h0}. σ'_{v0} is equal to 16 * 5 because it is sampled at a depth of 6 and the unit weight is given as 16. So, that gives 80 kPa.

σ'_{v0} is equal to 80 minus that is pore water pressure, static pore water pressure is 10 * 4 that will give 40 kPa.

You can see here, it is 10 and the groundwater table is at 1 metre. So, the remaining 4 metre will be the water table. So, that will give 40 kPa as σ'_{v0}. So, this is obtained. σ'_{h0} will be 0.7 into 40 kPa that gives 28 kPa. So, u_i can be determined minus 40 + 2 (28)/3 that gives minus 32 kPa. So, s'_i is now known. This is known.

φ' is given. c' is zero, because it has been already given and then all other A_f is also known. So, s'_i substituting it, we get t_{fU} is equal to 10 kPa. Just to give an example of how do we use t_{fU} expression.

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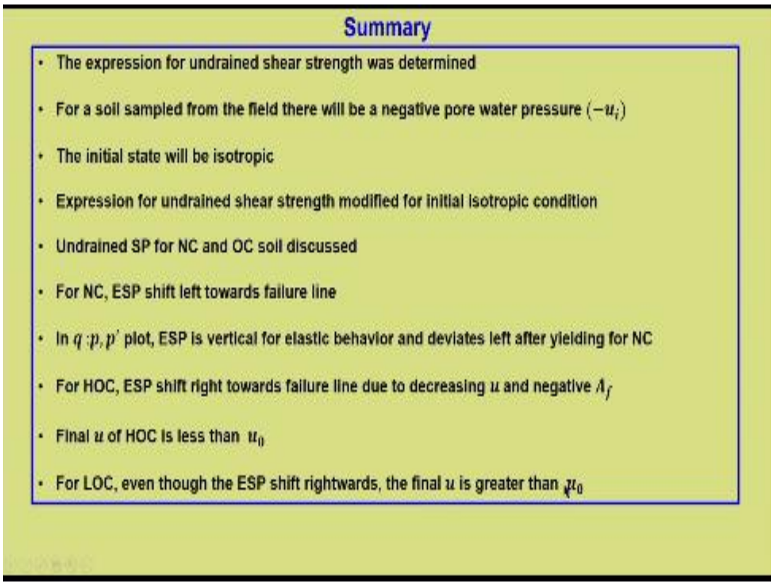

So, let us summarise today's lecture, the expression for un-drained shear strength was determined. For a soil sample from the field, there will be a negative pore water pressure of minus u_i. In fact, this we have already summarised before as well. The initial state will be isotropic because it is pore water pressure. Expression for un-drained shear strength modified for initial isotropic condition. So, this because of the sampling and minus u_i, the starting point will be at isotropic line.

Un-drained stress path for normally consolidated and over consolidated soil is discussed. For NC, ESP shift leftwards towards failure line. In q-p, p' plot, ESP is vertical for elastic behaviour and deviates left after yielding for normally consolidated soil. For heavily over consolidated, ESP shift rightwards towards the failure line due to decreasing u and negative

A_f; we have seen that. Final u of HOC is less than the initial pore water pressure because of this negative pore water pressure.

For likely over consolidated soil, even though the ESP shift rightwards that we have seen. The final u is greater than u_0. So, the final u always be $u_0 + \Delta u$. So, that is all for this lecture and we have completed the effective stress path for un-drained condition. Now, we are left with ESP for some specific field problems. So, that is all for now. Thank you.