Advanced Soil Mechanics
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Lecture - 38
Stress Path for Undrained Triaxial Testing

Welcome back to all of you. So, we were discussing about stress path and in the last lecture we have seen stress path for drained triaxial testing. So, in today’s lecture, we will see the stress path corresponding to un-drained condition or un-drained test. So, what is the essential difference? We know that from the shear strength discussion the essential difference would be in terms of pore water pressure generation. So, we will see how that is going to impact the stress path.

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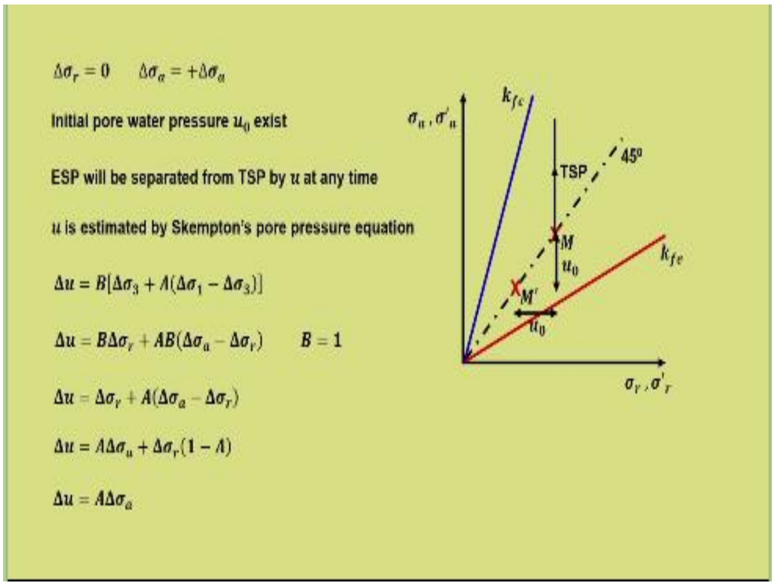

So, just like what we have done for drained compression test, there are four cases that we discussed. Here the first case is un-drained compression test with constant radial total stress. Now, when we discussed about drained stress path, we did not have to stress too much on effective or total, but now, it is important. So, you can see here that it is with constant radial total stress. And that is designated by U1, earlier it was D1.

But the conditions were same. The first condition we dealt was constant radial stress, but here we have specified total stress. So, it is very clear that the TSP and ESP will not be seen for the case of untrained test. Just like in normal triaxial testing, we always have the initial condition to be isotropic. So, let us start with isotropic condition M. And again we will discuss what three stress paths.

One force in actual radial, the other one is what t-s, s' and other one q-p, p'. So, this is the starting point the isotropic condition, so, M is the starting point. Like that, we have t-s, s' and q-p, p'. So, this is the starting point. Now, for the first condition, we know the radial stress is constant, so, $\Delta\sigma_r = 0$. And for creating compression, we know that the actual stress should increase.

So, $\Delta\sigma_a = + \Delta\sigma_a$. Now, you are more or less conversant with these usage. So, when there is release it is negative and when there is loading or increase in load it was positive. So, this is the starting point. Now, for this particular country condition, we know that TSP remains constant with radial and it increases. That is actual load increases. So, this is increasing in this direction.

Now, this is very much similar to the drained triaxial stress path for the same condition. So, the total stress path in both the cases for the drained and the untrained is not going to change. But, you may be wondering, why did I stop this total stress path at this particular point? Why it is so? And there is a discontinuity here. In fact, when it comes to effect and during testing, you will see that total stress path will not represent what it actually has to represent about failure.

So, it is all governed by effective stress path. So, whether it is complete or not whether the total stress path is meeting or whether it is crossing the failure line, it hardly matters. Now, let us say that for this particular condition, initial pore water pressure u_0 exists. What is the implication of this statement? It means that the total stress path and the effective stress path will not start at the same point. So, there will be a difference of u_0.

So, let us find. So, there is a difference of u_0 and that remains isotropic. So, the point M will get shifted to M' for effective stress paths. So, that is why It is called M', for effective stress path. Now is it straightforward to plot effective stress path with the given information? But ESP will be separated from TSP by pore water pressure at any time. So, it is an un-drained test, for every instant the pore water pressure changes during shearing.

So, we need to understand what we need to get what is the value of pore water pressure with loading. How do we do that? Either we measure it, get the results and you plot the effective stress parameters the variation or u is estimated by Skempton’s pore pressure equation. Now, you will again appreciate why we have studied Skempton’s pore water pressure equation in detail.

So, in the absence of measurement, one can also have some idea about pore water pressure by estimating it corresponding to the change in stresses. So, $\Delta u = B\ [\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$. So, here replacing σ_3, σ_1 by σ_r and σ_a, we have $\Delta u = B\ [\Delta\ \sigma_r + A(\Delta\ \sigma_a - \Delta\ \sigma_r)]$. It is a saturated case. So, B is equal to 1.

$\Delta u = \Delta\ \sigma_r + A(\Delta\ \sigma_a - \Delta\ \sigma_r)$. Now, this is a typical case of $\Delta\ \sigma_r = 0$ and $\Delta\ \sigma_a = +\ \Delta\ \sigma_a$. Substitute this into pore water pressure equation, we will get $\Delta u = A\Delta\ \sigma_a + \Delta\ \sigma_r\ (1-A)$. That is by rearranging and for this specific case $\Delta\ \sigma_r = 0$.

So, now, we have $\Delta u = A\Delta\ \sigma_a$. So, as σ_a changes, the pore water pressure will change. Once we know the pore water pressure parameter A, then we can always determine what is the change in pore water pressure.

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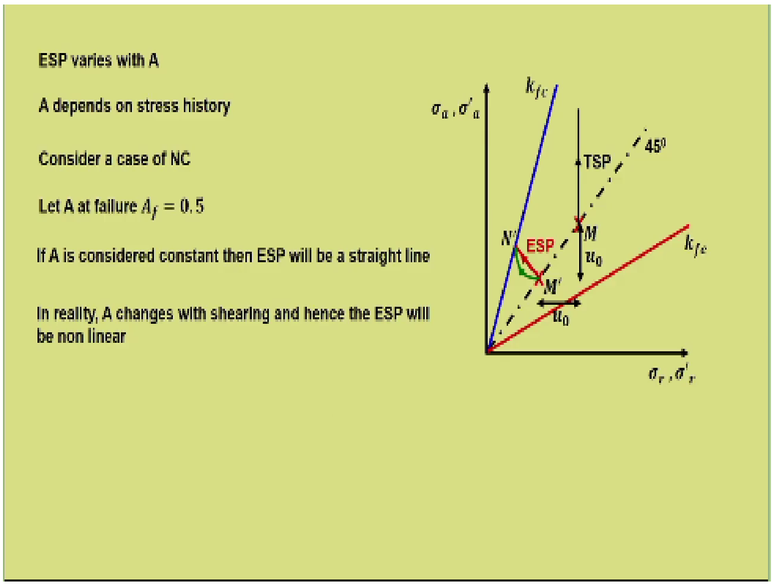

So, now, it is easy to plot ESP, but is it that easy? One, we have got the pore water pressure estimated. So, now, you can find out what is the effective stress. $\Delta\sigma'_a = \Delta\ \sigma_a - \Delta u$. We have the expression for Δu. Substitute that we will get $\Delta\sigma'_a = \Delta\sigma_a(1-A)$. So, $\Delta\sigma'_r = \Delta\sigma_r - \Delta u$. $\Delta\sigma'_r = 0 - A\,\Delta\sigma_a$.

So, $\Delta\sigma'_r = -A\,\Delta\sigma_a$. So, the slope $\Delta\sigma'_a / \Delta\sigma'_r = -(1-A) / A$. So, you divide this by this, $-(1-A) / A$. So, now, we have we are left with an equation for the slope earlier it was a number so, it is not changing here it changes the effective stress path keeps changing with shearing. And that change will be with respect to the parameter A.

So, ESP varies with A. So, and A is dependent on stress history. This much we have already seen in our previous lectures. Now, consider a case of normally consolidated because once we have stress history coming into picture, let us first see what happens for NC. Now, let A at failure A_f is equal to 0.5 as approximate value for NC, this we know. If A is considered constant, then ESP will be a straight line.

So, we have an expression in terms of A. Now let us say that we assume that A is constant and we take the final A value. This also we have seen. A value at failure that is A_f. Now let us say A_f is equal to 0.5 for NC. Then we approximate it by a straight line. But if we presume that let us first discuss what happens when A is equal to 0.5. So it will be always negative because it is - A upon, in the previous slide we have seen the slope which is $-(1-A) / A$.

So if you substitute 0.5, this will be positive quantity with a negative sign so the slope will be negative. So this is the negative slope, which you can see and M' N' represents the effective stress path. For a value of let us say A_f equal to 0.5. And in reality, that is for a case this straight line is a case when A_f is equal to 0.5 that is considered to be constant, but A changes with shearing.

And hence, the effective stress path will be nonlinear. This every time it keeps changing. So, if you consider the variation of A with respect to shearing, then it will be more like a nonlinear change or nonlinear effective stress path. But most of the practical cases that we discussed, we consider A value to be a constant and we take the value of A_f and hence, we can consider the effective stress path for un-drained condition also to be a straight line.

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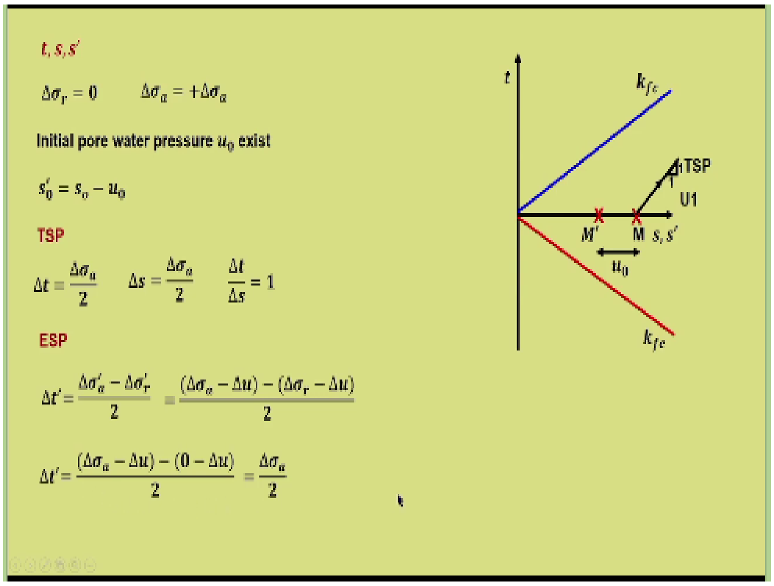

So, the same thing that we try to do in t-s, s' plot. Starting point is there. Δ σ_R =0, Δ σ_a= + Δ σ_a. Everything remains same, u naught exist. So then it will be shifted. Now, this also we have already seen in our initial stress path discussions like $s'_o = s_o - u_o$. So, in case if you have any doubt please refer back to the earlier lectures.

So, this is the point M'. So, it is separated by initial pore water pressure u_o. So, a total stress path we need to compute. It is fairly straightforward and it will be same as that of the drained stress path. So, Δt/ Δs = 1. This we have already seen. So, it is a it is a straight line at an inclination of 1 is to 1. And this is U1. So, this is the total stress path.

Now, for effective stress path, we need to compute the pore water pressure expression and that we have already seen, We have already got the pore water pressure expression. So, Δt' now, we already know that Δt' or Δt = Δt'. It is not getting influenced, but then still in this particular case, I am just representing it as dash only to make you understand.

So, Δt' = Δ σ'_a - Δ σ'_r / 2. Substituting so, (Δ σ_a - Δ u) - (Δ σ_r - Δ u)/ 2. So, now, we have the expression for Δ u. So, substituting for Δ σ_r = 0, because that is the condition we have it is Δ σ_a /2. Because - Δ u minus of minus is + Δ u, so, that gets cancelled. So, it will be σ_a /2.

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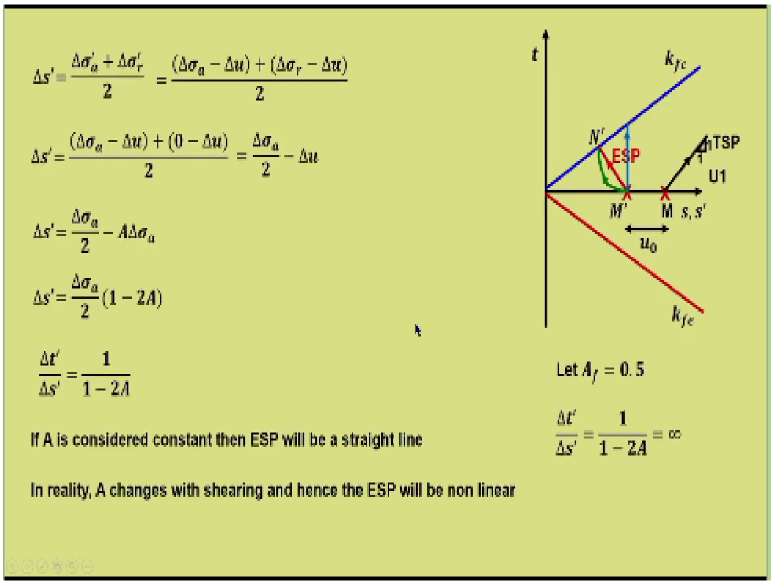

$\Delta s' = \Delta \sigma'_a + \Delta \sigma'_r / 2$. Substituting so, in terms of total stress and then you will get $\Delta s' = \Delta \sigma_a /2 - \Delta u$. Why? Because here it is minus of minus. So, it is minus 2 Δ u by 2. That is minus Δ u. So, $\Delta s' = \Delta \sigma_a /2 - A \Delta \sigma_a$. Because $\Delta u = A \Delta \sigma_a$ which we have already found out.

Now, $\Delta s' = \Delta \sigma_a /2 *(1-2A)$. So, rearranging this so, $\Delta t'/ \Delta s' = 1/ (1-2A)$. Everything remains same as that of the previous derivation the same we need to find out what is the slope. So, slope is 1/ (1-2A) and it is again a function of A. So, if A is considered constant it will be a straight line.

Now, you need to see that this particular line can vary with A and it can be of any slope in this direction. So, N' M' N' gives the effective stress path when A is considered to be constant and if A changes, then it is a nonlinear stress path. For a specific case in this particular exercise, if A is equal to 0.5, you will see that what happens it becomes infinite. $\Delta t'/ \Delta s'$ is equal to infinity. So, it will go in the upward direction.

The slope of the effective stress path will be in the upward direction. In fact, these informations are very important, when we try to understand the critical state soil mechanics. For NC a typical A value will be 0.5, if you are representing the stress part in t-s, s' plot, we know that it is moving in the upward direction.

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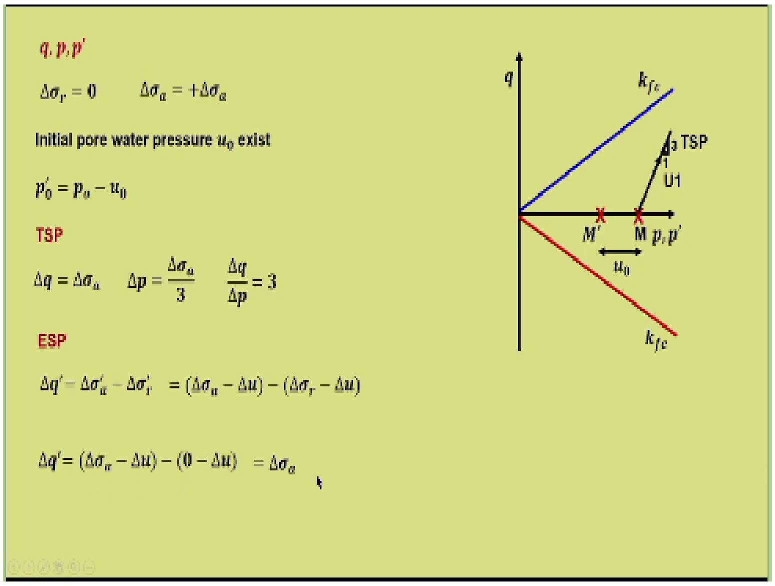

Similarly, for compression in q-p, p' plot it is one at the same, only thing is the expression changes. So, we are just trying to find out the expression all other things remain same. So, $p'_0 = p_0 - u_0$. So, this is the starting point, total stress path remains same. So, we can say that it is at an inclination of 3 because it is a compression stress path. $\Delta q = \Delta \sigma_a$ and $\Delta p = \Delta \sigma_a /3$.

So, the slope will be 3 and this remains same as that of drained stress path. Now, for effective stress path $\Delta q' = \Delta \sigma'_a - \Delta \sigma'_r$. Substituting for total stress in terms of total stress, so, we have $\Delta q' = \Delta \sigma_a - \Delta u$. Minus of this becomes $- \Delta u$ plus Δu zero. So, that will be equal to $\Delta \sigma_a$.

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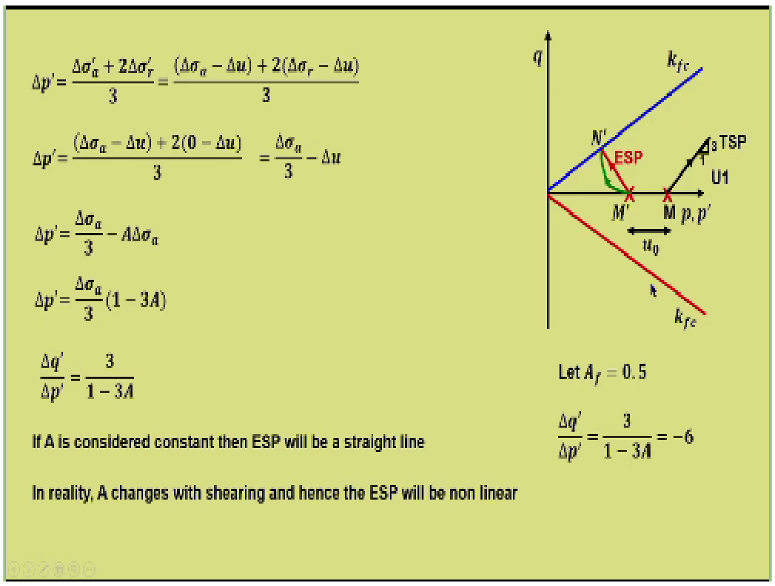

So, $\Delta p'$ is equal to, again substitute in terms of pore water pressure. $\Delta p'$ is equal to $\Delta \sigma_a - \Delta u + 2 (- \Delta u)$. So, that will become $- 3\Delta u / 3$, that goes. So, $\Delta \sigma_a / 3 - \Delta u$. So, $\Delta p'$ is equal to $\Delta \sigma_a / 3$ - substitute for Δu, $A \Delta \sigma_a$. So, $\Delta p' = \Delta \sigma_a / 3 * (1-3A)$.

So, Δq’/Δ p’ = 3/ (1-3A). So, again if A is considered constant, it will be in this. This is only an example direction it can be in any direction depending on A. So, M’ N’ is the effective stress path and if it is considered to be A is considered to be varying, so, then it becomes a nonlinear stress path. Now, for example, if A_f is equal to 0.5, then in this case you will have an inclination of - 6. So, it will be even more steep.

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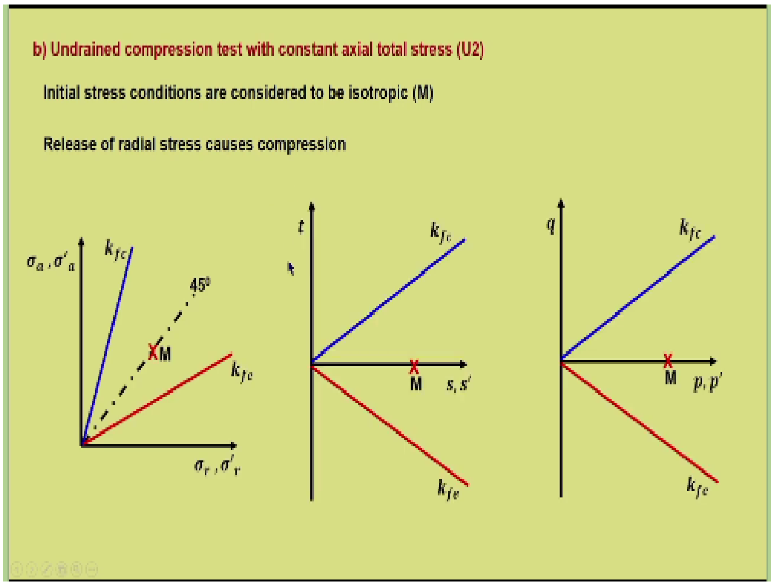

So, you will have the effective stress path. Now, what we have done for un-drained compression, we have determined the effective stress path for all the three cases. Now, the compression it will just like we discussed for drained stress path, the compression can also be brought about in another sequence. So, un-drained compression test with constant actual total stress, so, that is represented by U2. Initial stress conditions are considered to be isotropic.

So, it is all remain same, release of radial stress causes compression. So, this we know that, when actual stress is kept constant, we need to release the radial stress so, that there is a compression. So, that is what we are discussing now. So, rate release of radial stress causes compression. Again the starting point remains same.

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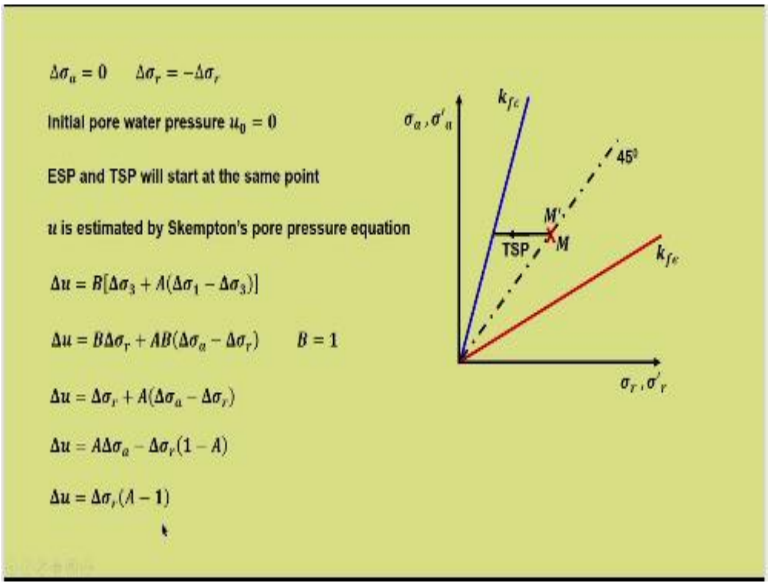

So, $\Delta\sigma_a = 0$ and $\Delta\sigma_r = -\Delta\sigma_r$. why because it is release. So, we know that in TSP is same as that of the drained stress path and that is for constant σ_a, σ_r is reducing So, it goes and meets the failure line. Let us presume in this particular case that there is no initial pore water pressure, so, $u_0 = 0$.

So, what is the implication of this statement? The total stress path and the effective stress path it starts at the same point. So, M = M' So, that is what it means. So, ESP and TSP will start at the same point. So, we have M'. Now, what is the next procedure we know that is un-drained. So, we need to have the pore water pressure measured or estimated. So, let us estimate u based on Skempton's pore water pressure equation.

Is it going to be same or different? It will definitely be different because it will be a corresponding to radial stress variation. Δu again the same procedure. It is written in terms of $\Delta\sigma_r$, $\Delta\sigma_a$, B equal to 1. So, Δu is equal to $\Delta\sigma_r$. Rearranging this plus A into $\Delta\sigma_a - \Delta\sigma_r$. So, $\Delta u = A\,\Delta\sigma_a - \Delta\sigma_r\,(1-A)$. So, now, $\Delta u = \Delta\sigma_r\,(A-1)$.

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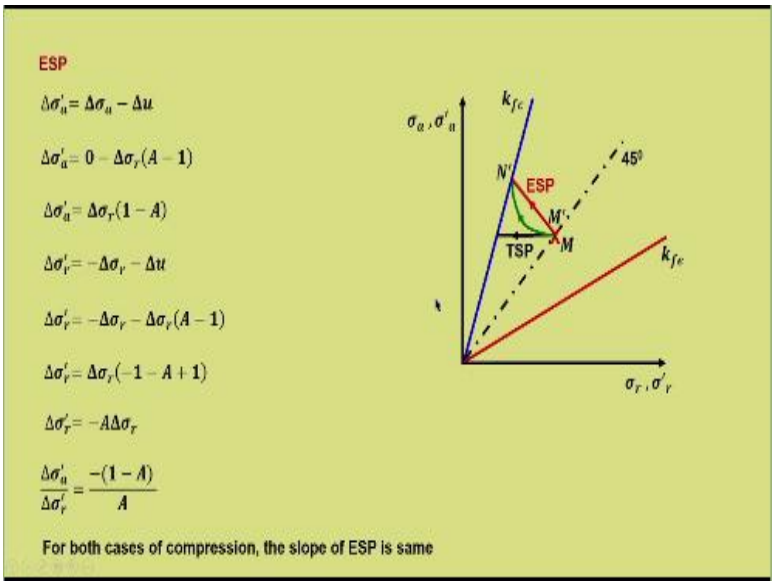

So, earlier the expression was different. Now, we need to find out what is the effective stress path. So, effective stress $\Delta\ \sigma'_a = \Delta\ \sigma_a - \Delta\ u$. Substitute for Δ u. So, that will be $\Delta\ \sigma_a$ is already zero because that is the condition what we are dealing with. So, - $\Delta\ \sigma_r$ (A-1). $\Delta\ \sigma'_a = \Delta\ \sigma_r$ (1-A).

And $\Delta\ \sigma'_r = -\ \Delta\ \sigma_r - \Delta$ u. So, $\Delta\ \sigma'_r = -\ \Delta\ \sigma_r - -\ \Delta\ \sigma_r$ (A-1). So, $\Delta\ \sigma'_r = \Delta\ \sigma_r$ (-1 –A+1). So, this minus plus 1 goes away. So, it gives - A $\Delta\ \sigma_r$. So, we will take the slope $\Delta\ \sigma'_a / \Delta\ \sigma'_r$ = -(1-A)/ A.

Now, have you seen this expression before? Yes, we have seen because the slope of the effective stress path remains same as in the previous case even though the sequence of loading has changed. Definitely it has impacted the determination of pore water pressure. The expressions were different, but the slope is going to be same. So, whatever be the manner in which the compression has been done or performed the effective stress path slope is same.

So, that is a typical example where in there is a slope and if it is a is varying then it will be nonlinear. So, M, M', N' is the effective stress path. For both cases of compression, the slope of effective stress path, this expression remains the same.

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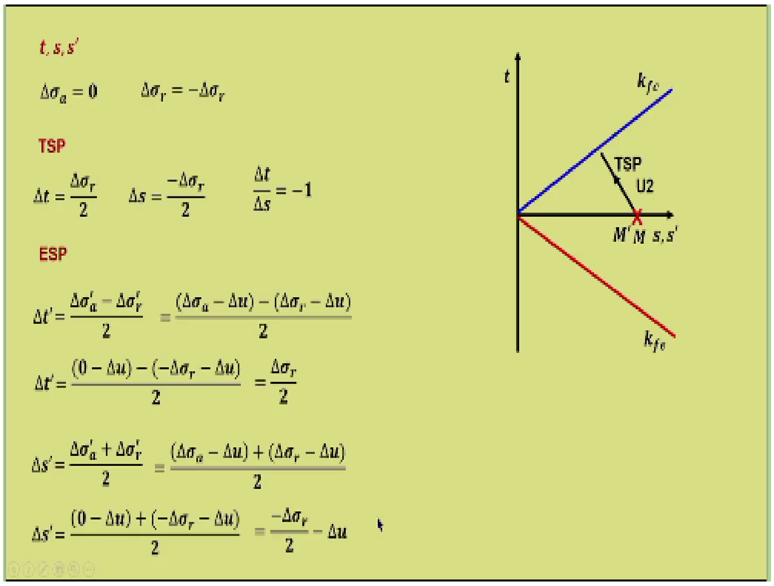

Same thing we need to repeat with t-s, s'. So, it is clear now, for a since the slope for σ_a –σ'_r plot effective stress path was same in both the cases of compression. We know that it is going to be same for t-s, s' and q-p, p'. So, let us see how and whether it is or not. So, TSP is $\Delta t = \Delta \sigma_r /2$, $\Delta s = - \Delta \sigma_r /2$ because that is the condition.

So, $\Delta t / \Delta s = -1$. This remains same as that of our drained stress path. So, U2 is in this direction. So, this is the total stress path. Effective stress path the initial point is same, because $u_0 = 0$. $\Delta t' = \Delta \sigma'_a - \Delta \sigma'_r /2$. Substitute in terms of total stress. Yes, Δu the expression is known. So, we have substituting for $\Delta \sigma_a = 0$, we get minus Δu minus minus.

So, that will become plus $\Delta \sigma_r$ and this will become plus Δu that gets cancelled off. So, $\Delta t' = \Delta \sigma'_r /2$ and $\Delta s' = \Delta \sigma'_a + \Delta \sigma'_r /2$. Substituting it for in terms of Δu, we have $\Delta s' = [(0- \Delta u) + (\Delta \sigma_r - \Delta u)] /2$. So, that will give – $(\Delta \sigma_r / 2) - \Delta u$.

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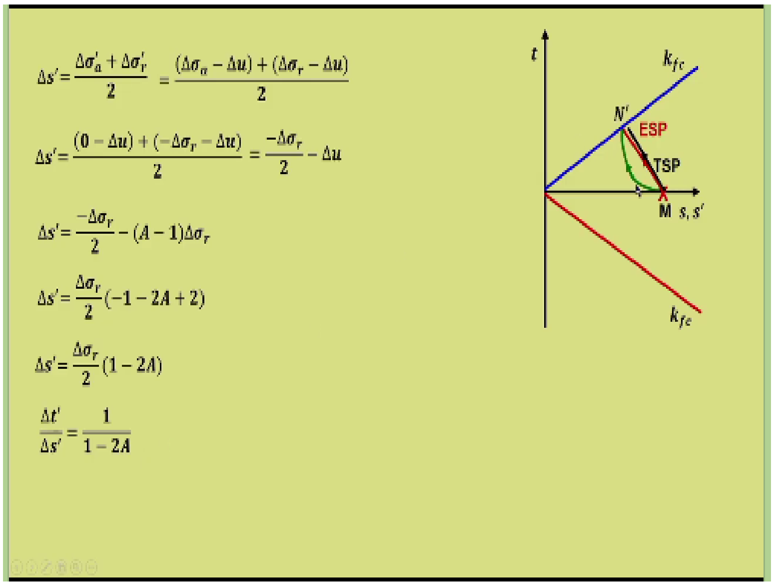

So, if we substitute for Δu, so, then we will get Δs' = – (Δ σ_r / 2) - Δu. So, Δs' = – (Δ σ_r / 2) – (A-1) Δ σ_r. So, this is the expression for Δu. So, Δs' = (Δ σ_r / 2)(-1 – 2A+ 2). So, that is 1-2A. Δt'/ Δs' = 1/(1-2A) is the same expression. This is the effective stress path and if it is varying then nonlinear.

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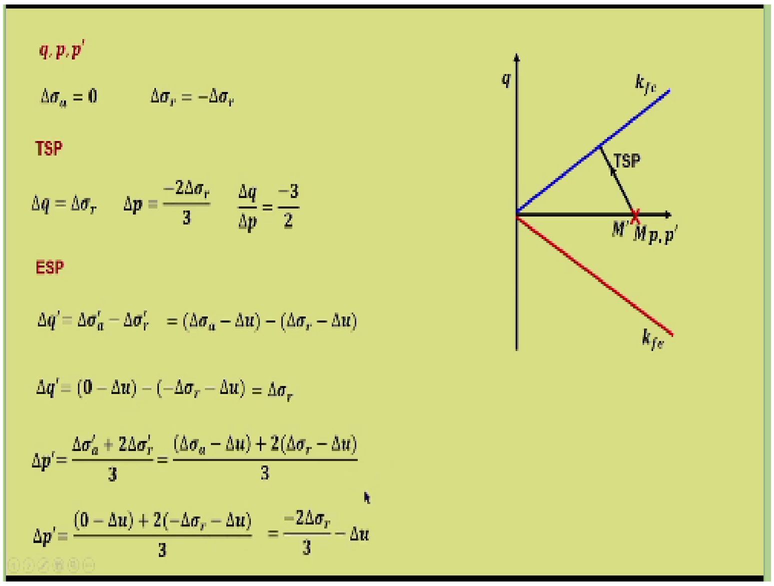

So, similarly, with q-p, p' we will quickly glance through the all the steps remains same. TSP is equal to Δq = Δ σ_r. So, we have Δp = -2 Δ σ_r / 3. So, that gives the slope of – 3/ 2 is the total stress path. And effective stress path starting same in the same procedure, we get Δq' = Δ σ_r and Δp' = Δ σ'_a + 2 Δ σ'_r / 3.

And [(Δ σ_a – Δu) + 2(Δ σ_r – Δu)]/3. So, Δp' = [(0 – Δu) + 2(-Δ σ_r – Δu)]/3. And that is equal to (-2Δ σ_r /3) – Δu.

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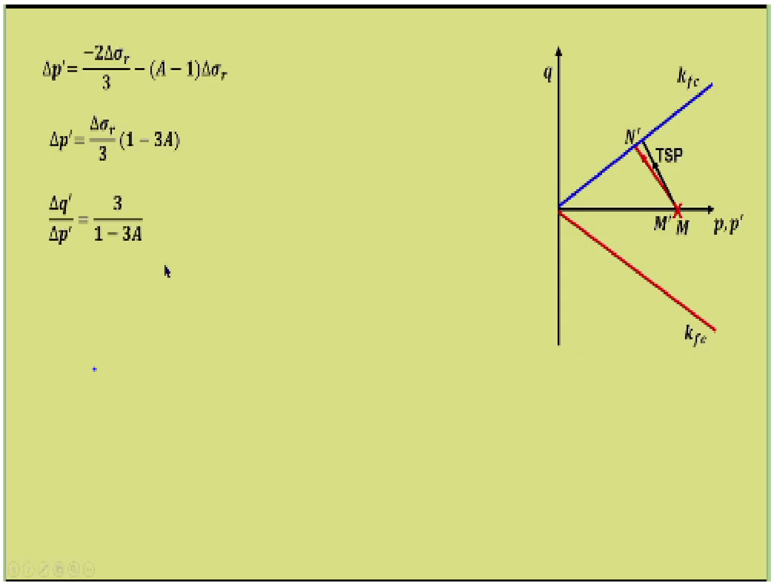

So, Δu gets substituted. So, $\Delta p' = (\Delta \sigma_r /3)(1- 3A)$. And the slope is $3/(1-3A)$ and that remains same. There is the M' N' is the ESP and the nonlinear variation.

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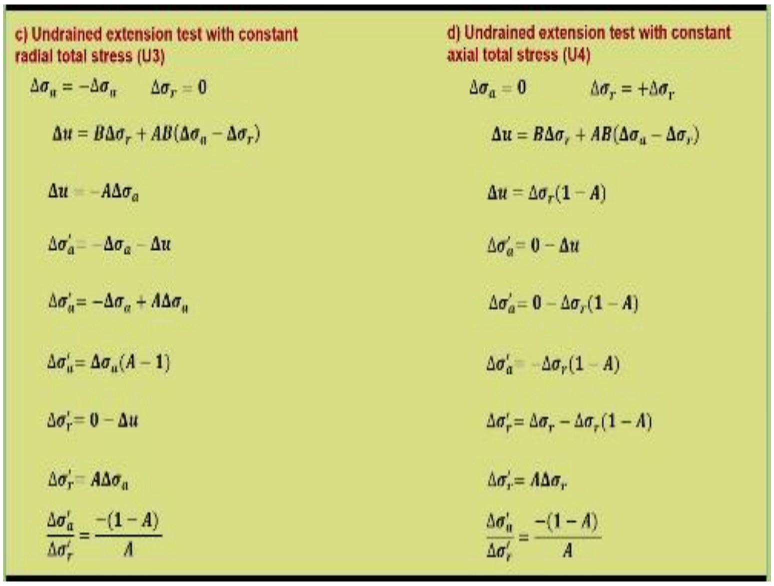

So, in all the 3 cases, we have got these slope of the effective stress paths same in both the cases of compression. In both ways of compression it is same. Now, obviously, the curiosity arises whether this will be true for extension, because we have discussed 4 cases in un-drained as in the case of drained stress path. So, is it going to be same? So, let us see the case for just the plot of $\sigma_a - \sigma'_r$, what will be the slope of un-drained effective stress path for both the cases of extension.

So, un-drained extension test with constant radial total stress. So, we know that, if it has to be extension, then the actual stress has to be released. So, - $\Delta \sigma_a$ and $\Delta \sigma_r = 0$. So, $\Delta u = B \Delta \sigma_r + AB(\Delta \sigma_a - \Delta \sigma_r)$. So, that gives substituting $\Delta \sigma_r = 0$. We have $\Delta u = -A \Delta \sigma_a$. Now, we can find out $\Delta \sigma'_a$.

Substituting for Δu, we get $\Delta \sigma'_a = \Delta \sigma_a (A-1)$. And $\Delta \sigma'_r = 0 - \Delta u$ that gives $\Delta \sigma'_r = A \Delta \sigma_a$. So, we are substituting it for the pore water pressure and that gives again $\Delta \sigma'_a / \Delta \sigma'_r$. If you rearrange it will give $-(1-A)/A$. So, the same slope as in the case of compression.

So, even the extension case the slope is going to be same and it is unique. Now, the fourth case is un-drained extension test with constant axial total stress. So, we need to create extension if it is axial total stress. Then $\Delta \sigma_{a=0}$. Then if extension has to be created then it has to be increased in the radial stress. So, that is the kind of squeezing effect.

$\Delta \sigma_r$ substituting now, the same procedure $\Delta u = \Delta \sigma_r (1-A)$. $\Delta \sigma'_a$ we can find out that is $0 - \Delta \sigma_r(1-A)$. So, $\Delta \sigma'_a = -\Delta \sigma_r(1-A)$, $\Delta \sigma'_r = \Delta \sigma_r - \Delta \sigma_r(1-A)$. And that will give $\Delta \sigma'_r = A\Delta \sigma_r$.

So, $\Delta \sigma'_a / \Delta \sigma'_r = -(1-A)/A$. These are identical. So, whatever be the manner in which the stresses are varying, if it is compression or extension the effective stress path expression is unique.

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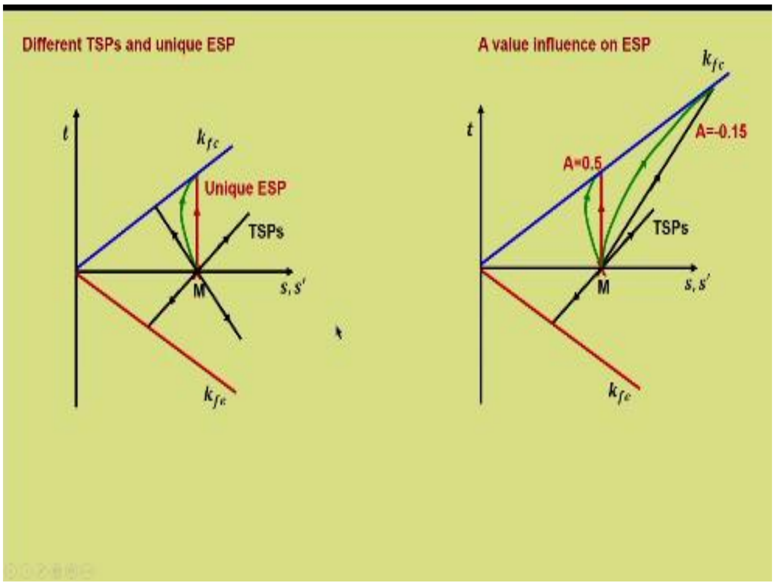

So, different TSPs and unique ESP. So, we have different TSP and this TSP variations for different cases of compression and extension. We have seen that this is very much similar to that of the drained stress path but for all these TSPs we have the unique ESP. Here is just an example, this particular slope, so it is unique ESP that is what we have understood from our previous discussion.

So, A value influence on ESP, how A is going to change the ESP. So, let us, this is a t-s, s' plot, the starting point M. So, this is the TSP for a typical case of compression and extension. Now, if A is equal to 0.5 in t-s, s', we know that it is infinity. So, it moves in the vertical direction. Now, for A equal to minus 0.15. That is this has a minus 0.15 means it refers to a heavily or consolidated soil.

So, in that case, the effective stress path moves towards in this direction. That is in the right ward direction and touches the failure envelope. Now, if it is different value other than A, then we will see that this effective stress path will move leftwards. So, if it is a kind of, so, these are the different cases of what how the A value would influence the effective stress path. So, more or less, we have discussed all the different aspects of stress paths for un-drained testing.

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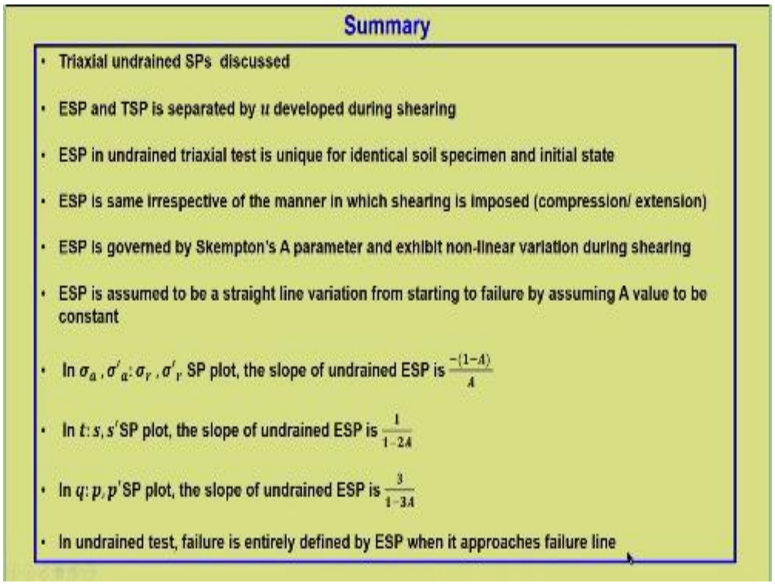

So, let us summarise today's lecture. So triaxial un-drained stress paths are discussed. ESP and TSP is separated by pore water pressure developed during shearing. ESP in un-drained triaxial test is unique for identical soil specimen and initial state. ESP is same irrespective of the manner in which shearing is imposed whether It is compression or extension mode. In both all the cases we have seen that the the expression for effective stress path remains the same.

ESP is governed by Skempton's A parameter and exhibit nonlinear variation during shearing. And ESP is assumed to be straight line variation from starting to failure by assuming A value to be constant. In σ_a, σ'_a, σ_r, σ'_r stress path plot the slope of un-drained effective stress path is –(1-A)/A. Now, in t-s, s' plot it is 1 /1- 2A and q-p, p'plot it is 3/ 1- 3A.

So, these values, these expressions are unique. So, in un-drained test failure is entirely defined by ESP when it approaches the failure line. So, we do not have anything to do with the total stress path and this particular aspect will become very clear as we move further. So, that is all for this lecture. Thank you.