Advanced Soil Mechanics
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Lecture - 37
Stress Path for Drained Triaxial Testing

Welcome back all of you. So, we are dealing with stress paths in these lectures. So, in the last lecture, we have discussed about some common cases and to some extent now, you are conversant with the plotting of stress path. Now, let us be a bit focussed on the use of stress path. First let us see the laboratory stress paths mostly for triaxial testing. So, today's lecture will be about stress path for triaxial testing and very specifically for drained triaxial test.

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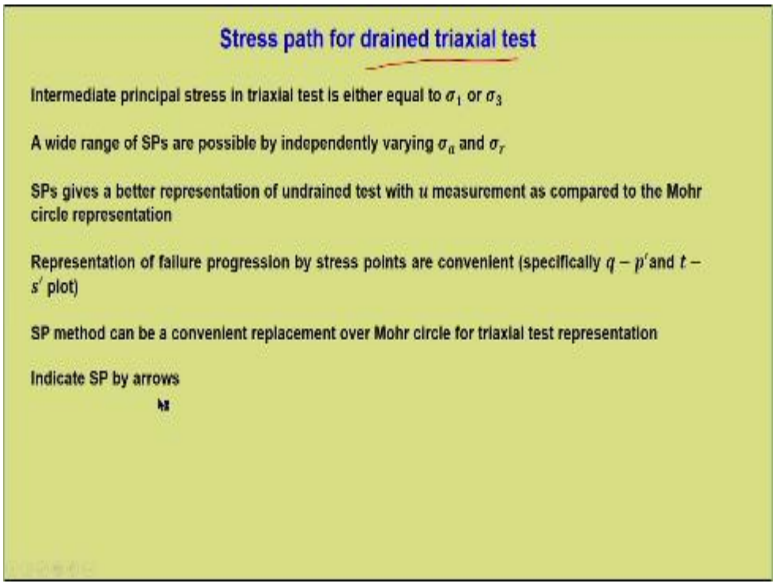

So, today's lecture is exclusively for drained triaxial test. So, let us start with the statement that all of these most of us know, but for completeness we need to understand this. Intermediate principal stress interaction test is either equal to σ_1 or σ_3. The effect of σ_2 is not taken into account explicitly. A wide range of stress paths are possible by independently varying σ_a and σ_r.

This we know that in triaxial testing, we have better flexibility in modulating these stress paths or for very specifically, when we have stress path triaxial system, it is very much possible to vary the stresses to suit the requirements of whatever is the scenario existing in the field. A stress path gives a better representation of un-drained test specifically, when we have u measurement as compared to Mohr circle representation.

Otherwise, also if you want to understand clearly how the failure is taking place, at what instant the failure would happen, and why it is happening at that particular instant, these informations are not available, when we plot the stresses in terms of Mohr circle. So, stress path is much more handy in understanding this and to some extent you have already appreciated this point in the last few lectures.

And it is very specifically for un-drained test that also we need to keep in mind because it clearly shows how the un-drained stress path it routes to its failure. Representation of failure progression by stress points are convenient, specifically when we do it on q-p' and t-s' plot. Hence, stress path method can be a convenient replacement over Mohr circle for triaxial test representation.

But, having said that, we need to keep in mind that if our primary objective is to obtain shear strength parameters, even plotting Mohr circle or t-s, s'plot also would do. If you want to understand further if not failure mechanism, the progression of failure then stress part would be much more handy. Another aspect before we start seriously into the stress path plotting we need to keep in mind that the stress path has to be indicated by arrows because we have to specify the direction in which the progression is happening. So, arrows are a must for stress path.

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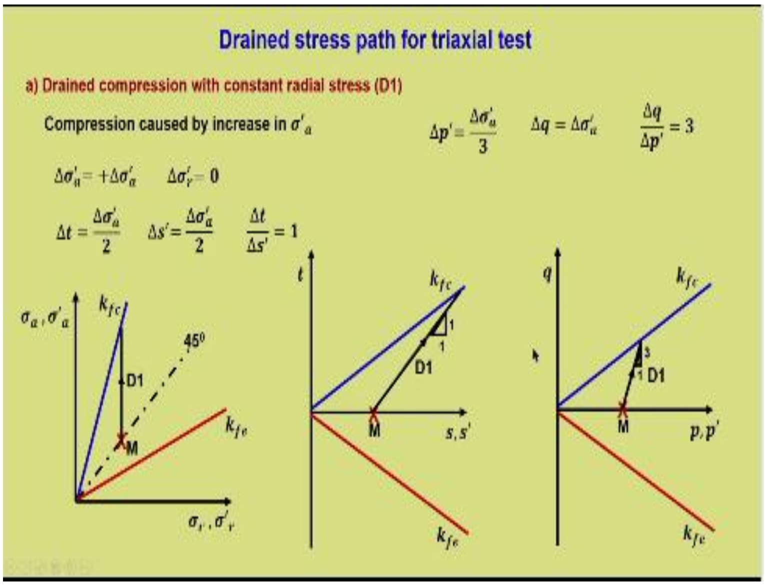

So, let us start with different cases of drained stress path for triaxial test. So, the first case is drained compression with constant radial stress and it is designated as D1. So, it is the conditions are given already. It is constant radial stress means σ_r is not changing. So, compression is brought out by σ_a. Now this is a very typical common triaxial compression test.

In drained test, total and effective stresses are same and hence TSP is total stress path is same as effective stress path. u is assumed to be zero. Now, there can be possibility of initial border pressure within the sample, because of back pressure. Even if it is there, then it acts like a static pore pressure as we have seen in the previous lectures, then the total stress path and effective stress path will not be same.

And this we will be discussing towards the last part of today's lecture. Initial stress conditions are considered to be isotropic. So, that is the starting point which is designated as M. So, we keep everything same as that of the triaxial testing conditions, simple triaxial testing. Now, if we want to create anisotropic consolidation that is also possible, but we are going ahead with simple initial isotropic stress condition.

So, now, we are plotting in terms of all the three stress paths, but later, you will see that we will adopt any one of these three stress paths as and when we depending upon the convenience So, it can be t-s, s' or q-p, p' plots. So, now, let us first start with σ_a σ'_a, σ_r σ'_r plot. It is given the isotropic is the starting point and we should not have any difficulty in identifying this, because it will be on the 45-degree line.

So, this is the starting point M and similarly, for t-s, s' it is M on the isotropic line and q-p, p' M. So, let us start now, how the stress path will move. It is constant radial stress. So, compression is caused by increasing sigma a dash. So, it is clear now, these things have to be kept in mind. So, it is increase in σ'_a and hence $\Delta\sigma'_a = + \Delta\sigma'_a$. This aspect again we have discussed in the previous lecture.

So, depending upon whether it is increase or decrease whether it is compressing or loading or unloading accordingly the sign of the increase would change or increase or decrease would change. So, here it is brought about by increase in σ'_a. So, $\Delta\sigma'_a$ in constant radial stresses means it is not changing. $\Delta\sigma'_r$ is equal to 0. Now, for σ_a σ'_a, σ_r σ'_r plot there is nothing great because it is a direct replication of what this stress path would be and it is very easy to plot.

So, this will be the designated stress path D1 for in this particular plot. Now, how it will look like in t-s, s’ plot? So, you just need to substitute this and that is where we have to be careful about. While substituting do not forget to put the sign. So, here that is $\Delta\sigma'_a$ - 0 / 2 that is $\Delta\sigma'_a$ / 2. Here plus again $\Delta\sigma'_a$ / 2 so, it will be the slope will be Δt / Δs dash equal to 1.

So, D1 is one is to one. Now, this again you have to keep in mind like in t-s, s’plot, if it is triaxial compression, the stress path and for specifically for drained stress path it will be at an inclination of 1 is to 1 or 45 degrees. That aspect we need to keep in mind. Now, what will happen in q-p, p’ plot? $\Delta p'$ is equal to $\Delta\sigma'_a$ /3, q is $\Delta\sigma'_a$, So, Δq / $\Delta p'$ is equal to 3. So, this is the next understanding in q-p, p’ plot, if it is drained triaxial compression then the slope will be 3.

So, this is D1 and this represented by 3 so, all the three has been done. So, this is the first case.

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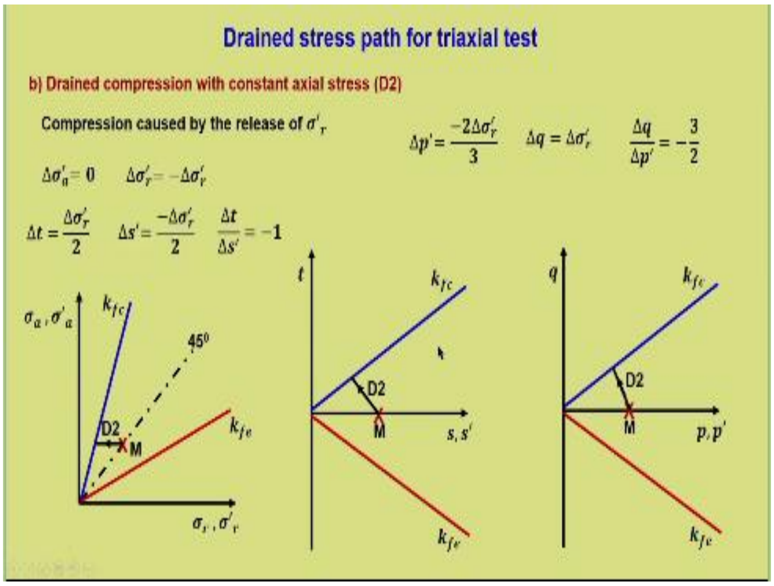

Now, the second case is drained compression with constant axial stress designated by D2. Now, axial stress there is a triaxial sample, there is an axial stress, there is a radial stress how do we account for or how do we bring about compression in the sample? It is told that σ'_a remains constant. Now, what is the possibility of compression? A released σ'_r.

So, when you release it, there is a tendency to undergo compression under constant σ'_a. So, compression caused by release of σ'_r. Now, when there is a release and here it is constant axial stress so, it is zero and just because it is released it will be minus of $\Delta\sigma'_r$. So, in this plot, there is a starting point a' remains constant and $\Delta\sigma'_r$ = - $\Delta\sigma'_r$ and so, it has to be in this manner.

So, this is constant σ'_a and this is decreasing and finally, it reaches the failure line. And that is given as D2. What about t-s, s' plot? Δt is equal to remember $\Delta\sigma'_a$ is zero, minus of minus so, that will become plus , so plus $\Delta\sigma'_r$ / 2. Here it will be zero minus $\Delta\sigma'_r$ / 2 and hence the slope will be Δt / Δs equal to minus 1. So, this will be minus 1 slope.

And remember here again your Δt is positive Δs is negative So, you should be able to understand in which direction it will move, minus 1 can be in this direction as well. But why it is specifically in this direction because s is going towards negative, t is going towards positive as you can see here. So, that is why it is negative slope in this upward direction and it meets the compression failure line.

What about q-p, p' plot? So, $\Delta p'$ = $-2\Delta\sigma'_r$ / 3 and Δq = $\Delta\sigma'_r$ because minus of minus that will be plus $\Delta\sigma'_r$. Δq / $\Delta p'$= -3/2. So, it will be in this direction at a slope of 3 upon 2. So, this is the second case.

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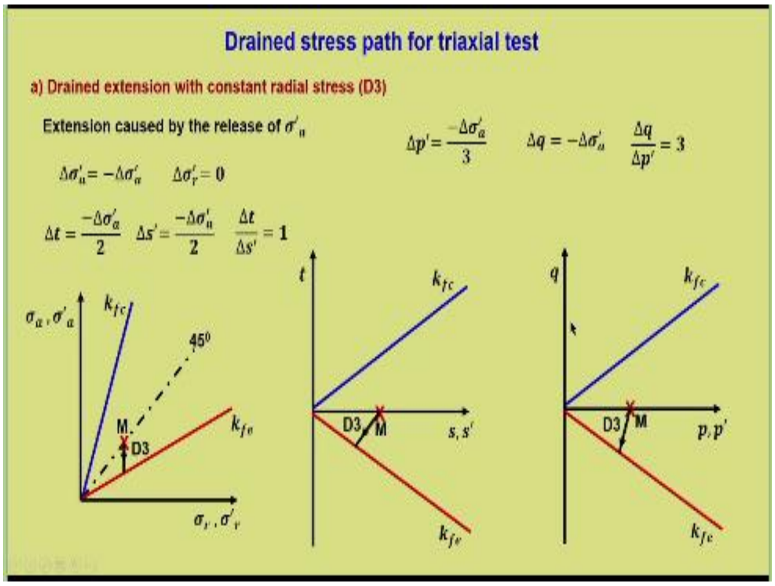

Now, the third case is drained extension with constant radial stress. Now, if it has to create a sort of extension based on the compressive stresses, so, how it will happen? It is told that radial stress is constant. Now, if extension has to happen, then we need to release the axial stress. That is why it is like kind of squeezing effect. You are squeezing something so, it extend. So, that is the same philosophy that is adopted. So, here it is constant.

So, the only possible way of extending is release σ'_a. So, extension caused by release of σ'_a, that is because of that, it will be minus $\Delta\sigma'_a$, $\Delta\sigma'_r = 0$. So, a is moving in the negative direction. So, a is changing a is changing in the negative direction. So, that will be D3 and that will be downwards. How it will be in t-s, s' plot?

So, $\Delta t = -\Delta\sigma'_a /2$, $\Delta s' = -\Delta\sigma'_a /2$. So, both Δt and Δs is changing towards negative direction, but the slope is 1. Both are moving towards but then slope is 1. So this slope, what is the sign of this slope and what is the sign of individual stress parameter also need to be considered. So, here it will be in D3 direction. So, this is positive 1 slope but both are moving towards negative.

So, this is negative s direction and negative t direction. And the same will be there for q-p, p' plot. We have Δq upon Δp' equal to 3, but in the downward direction.

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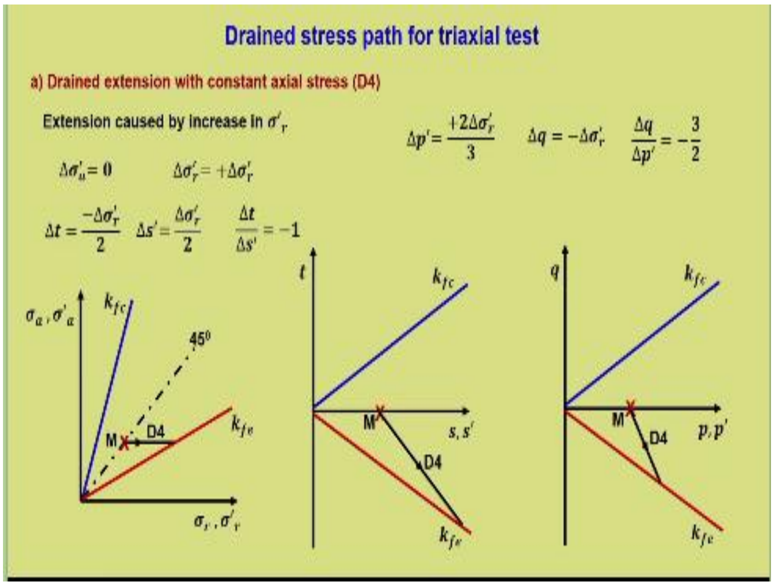

Now, the fourth case is drained extension with constant axial stress. Now, axial stress is remaining constant. Now, how do we bring about extension? If you want to bring about extension you need to squeeze it. So, σ'_a constant so, σ_r is increasing. So, extension caused by increase in σ'_r. So, $\Delta\sigma'_a = 0$; $\Delta\sigma'_r = + \Delta\sigma'_r$.

So, this is in the positive direction that is represented by D4. What about t-s, s', $\Delta t = -\Delta\sigma'_r /2$. Now, you can see that t should move towards negative direction, $\Delta s = \Delta\sigma'_r /2$ and hence Δt/

Δs = -1. So, you will have from here it moves towards you can see that s is in the positive direction, whereas, t is in the negative direction.

So, that is all about t-s, s', what about q-p, p'? So, $\Delta p' = 2\Delta\sigma'_r /3$, that is $\Delta q = -\Delta\sigma'_r$. Hence, $\Delta q/ \Delta p' = -3/2$, so slope is minus 3/ 2. It will move in the downward direction that is for D4. So, these are the important possible 4 cases one kept constant at a time, so, both compression and extension.

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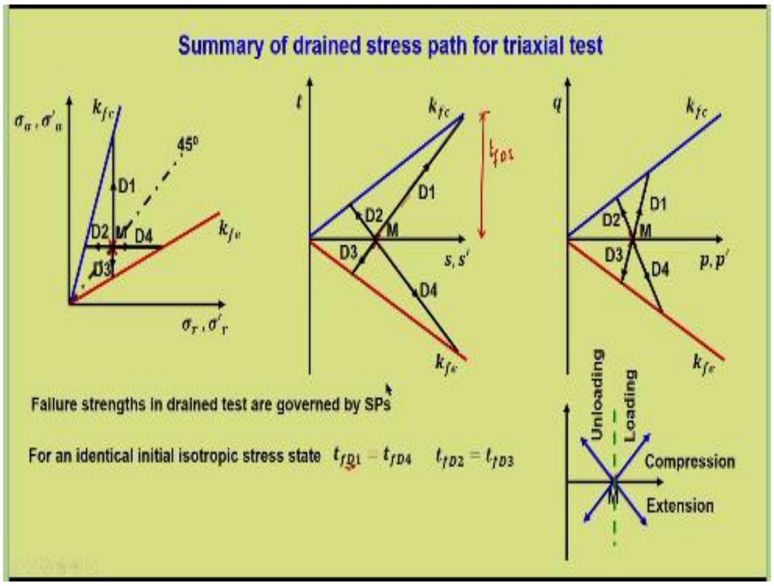

So, this is a summary of drained stress path for triaxial tests. Now, another important aspect which we need to understand is that, depending upon the stress path, you can see that the failure when it is going to happen, that is also dictated by the manner in which the stress path changes. Let us say for the case of D1, you can see that so, much of allowance is possible. So, the strength available is this. So, when it reaches here, it fails.

Whereas, for D3, you can see that the failure occurs quite fast, even though the starting point is same and that is true for all other stress path representation. So, this stress path representation also gives such insight as to same initial point, but depending upon the manner in which the stress changes, the failure condition also changes. To some extent we have seen this when we discussed about shear strength tests.

So, depending upon the manner in which the loading changes the it is it influences the failure conditions as well. So, now, you are just summarising this. These are the four different stress path directions. If I draw a line here, then because this is the starting point above portion, this particular portion it represents compression, because k it is dictated by what k you are talking about. Here it is σ'_a is more than σ'_r. So, it is compression.

This region is basically an extension from here towards this direction we talk about unloading and towards this direction it is loading. So, all the cases have been summarised in this particular plot and this x and y can be any of these stress parameters either t-s, s' or q-p, p'. So, failure strengths in drain tests are governed by stress paths. For an identical initial isotropic stress state, we have t_{fD1} equals t_{fD4}.

Now, we are talking in terms of t here, you can see that t_{fD1}, t_{fD1} means the strength which is given by the failure stress path D1. So, this is what is meant by t_{fD1}. So, if I draw here t_{fD1}, that means, it is the strength of the soil corresponding to the stress path D1. D1 this stress path strength is same as t_{fD4} it is more like a mirror image. Similarly, t_{fD2} is equal to t_{fD3}.

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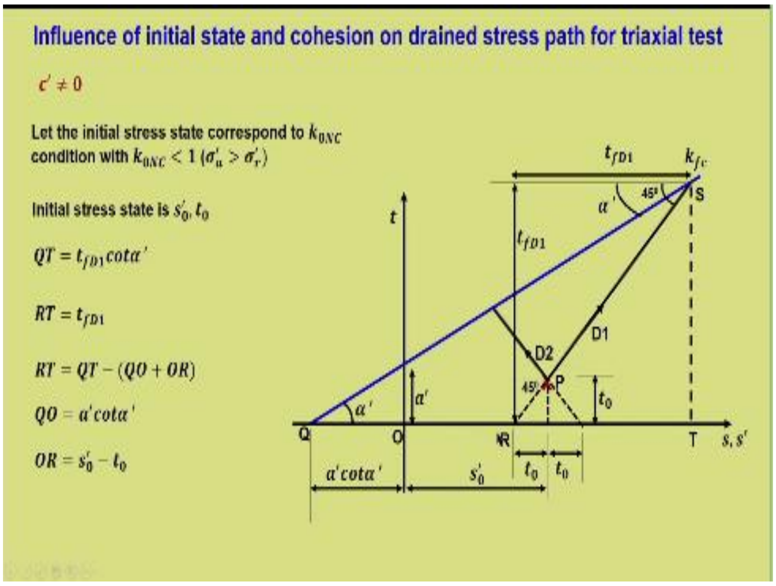

Now, let us see the influence of initial state and cohesion on drained spot for triaxial test. Now, here till now we have been considering c' equal to zero. Now, let us see that what will happen if cohesion is there? Is there significant difference from what we have learned or how it changes? And the other aspect is the initial state. Initial state means isotropic consolidation or whether it is a normally consolidated soil or any other k value which we tend to use.

So, let the initial stress state correspond to k_{0NC}. Now first case, let us consider k_{0NC} condition with k_{0NC} is less than 1. That means σ'_a is greater than σ'_r a typical triaxial test that we normally do in the lab. So, we are talking now, in terms of t-s, s' as I indicated before, we will not do for all the three now, as in when the case comes, we will generally use any one and typically we will be dealing with t-s, s', q-p, p' plot.

So, now, let us try to understand what will be the influence on stress path because of the initial state and cohesion. Now, initial stress state is given by s'_0 and t_0. So, the initial point corresponds to k_{0NC} state. So, if you draw a line here k_{0NC}, this initial stress state falls on that and it is clear that this k value is less than 1. So, we have already seen in the previous lecture like if it is k value less than 1 which corresponds to σ'_a greater than σ'_r, it will be in the upper part or in this particular region.

So, this is our starting point P. Now, to add value to this stress path plot, let us first draw the failure line. Now, you can see that it is no longer through the origin. There is an intercept here and in t s s dash plot what is the intercept? Definitely it is not c', it is a'. And the relationship between a' and c' is already known to us. So, this is the inclination α' or tan α', which is equal to sine φ' which we have already seen.

So, this is α' and this is the point Q. This is a' which is the intercept in t-s, s' plot. Now, if this is a' we can write this to be a' cot α'. Now, for drained stress path in compression, you can see that the one which we have already seen before let this point be S. Now what has happened? Initially, this stress path started off from here, because it is normally consolidated.

So, the point has shifted upwards, but at the same time the failure line also got shifted upwards which means that strength because of C' has been incorporated. This is the other spot so, we will be generally dealing with D1 and D2 both refers to a case of compression. Now, this particular point P is at s'_0, the initial point and this is t_0. We have also seen that if you know this particular point P one can also find out what are the stresses it is subjected to.

So, this is σ'_a, this is σ'_r, what we have to do? Draw a horizontal line draw 45 degrees to both sides, wherever it is intercepting the S line so, that will give us σ'_a, and σ'_r. This is already discussed. In that case this will be t_0 and this will be t_0. Now, what we are trying to do is we are trying to understand or determine what will be the expression for the strength represented by D1 and D2.

Now, what is the strength? It is t_f. That is a failure point in T. Let us assume this point to be S and T. So, you are dropping a vertical from this particular point. So, let it be ST. Now, we know that this angle is 45, this angle is 45. So, this angle is 45 degrees. And this is α and hence this is also α’. This is t_{fD1}, that is the strength corresponding to the stress path D1 is given by t_{fD1}.

Since this is 45 degrees, take this triangle, this particular triangle this is 45 degrees. So, here it is t_{fD1}. So, definitely this has to be t_{fD1}. Now, referring to this figure QT is equal to this is t_{fD1} divided by QT is equal to tan α’. So, QT will be equal to t_{fD1} divided by tan α’. That is QT equal to t_{fD1} cot α’. So, what we are trying to do we are trying to get an expression for t_{fD1}.

Now, what is RT? RT is equal to t_{fD1}. That is already clearly marked on the figure. We can also write RT = QT – (Q_0 + OR). So, that will give RT. So, we have the expression for RT, t_{fD1} substituting that, we also have the expression for t_{fD1}. QT that is t_{fD1} cot α’. QRO is already marked here a’ cot α’and OR that is OR = $s'_0 - t_0$. So, that will be OR.

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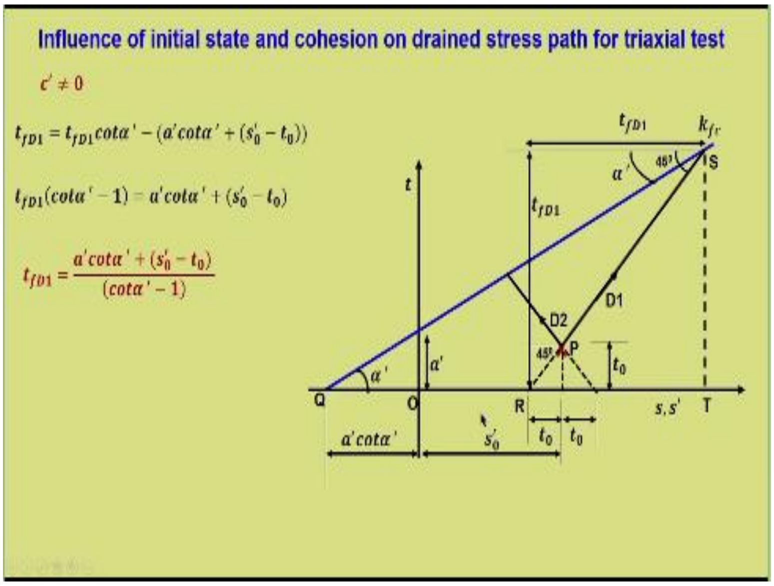

So, all these are known. Substituted, we will get $t_{fD1} = t_{fD1}$ cot α’ – (a’ cot α’ + ($s'_0 - t_0$)). So, rearranging similar terms then we will get t_{fD1} cot α’ – 1. What we are taking is you we are bringing this to this and t_{fD1} on to this side. That will give t_{fD1} = (a’ cot α’ + ($s'_0 - t_0$))/ (cot α’ – 1). So, this is about t_{fD1}.

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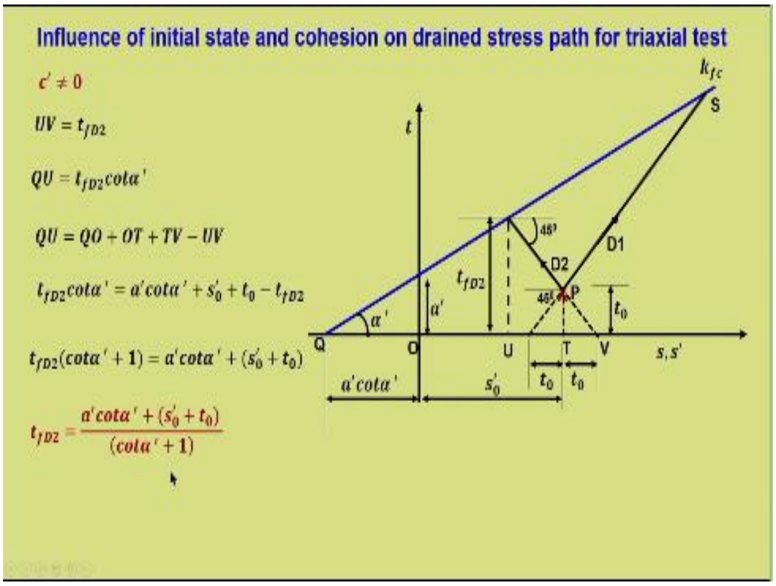

So, we need to also find expression for t_{fD2}. Let us see how. Let us drop a vertical at U. This is t_{fD2}. This is at an angle of 45 degrees we can write UV = t_{fD2} because this is 45 degrees. QU = t_{fD2} cot α'. This QU is equal to this one. So, that is very clear. We can also write QU = Qo+ OT + TV -UV. That will give you QU. So, all these are known to us.

QU is t_{fD2} cot α', Qo a' cot α' + OT. This is s'$_0$ - t_0 UV that is t_{fD2}. Again rearranging we will get t_{fD2} (cot α' + 1) = a' cot α' + (s'$_0$ - t_0). So, we will get the expression for t_{fD2} similar to t_{fD1} = a' cot α' + (s'$_0$ + t_0))/ (cot α' + 1). So, this is about the expression for both t_{fD1} and t_{fD2}.

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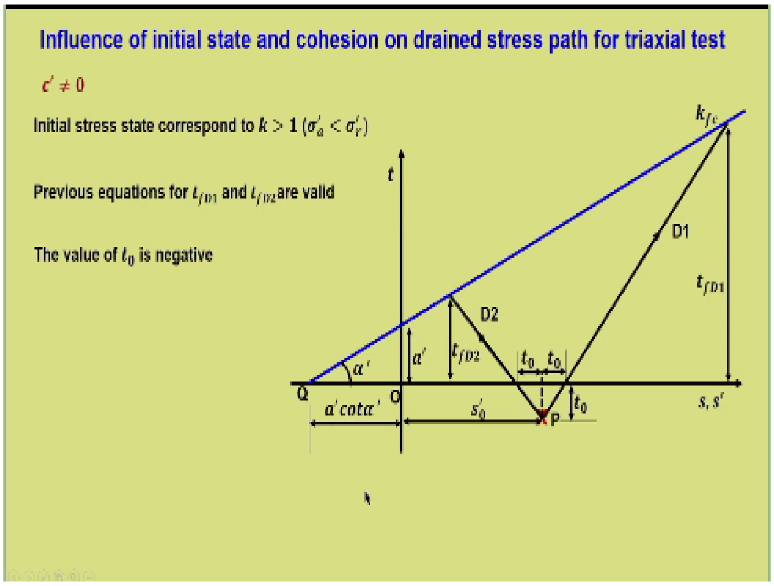

Now, let us see what happens if k is greater than 1. So, earlier case was k less than 1. That is σ'$_a$ less than σ'$_r$. So, where will the points start from? It has to be in the bottom portion. So, the same for the previous case. So, now, you can see that point P is in the bottom half. Every all other aspects remain same. Now, D1, it starts from here. Then this is t_{fD1} same and this is D2. And this is the strength t_{fD2}.

You can see that there are more allowance now, because the it is starting from this particular point it takes a while reaching the compression failure line. So, what is the effect? It has got shifted. Now, here this will be t_0, t_0 because why it is t_0 , t_0 here? P

Because this is at an inclination of 1 is to 1, or 45 degrees. Previous equations for t_{fD1} and t_{fD2} are valid for this case as well.

But the only difference is the starting point is below. So, the value of t_0 will be negative.

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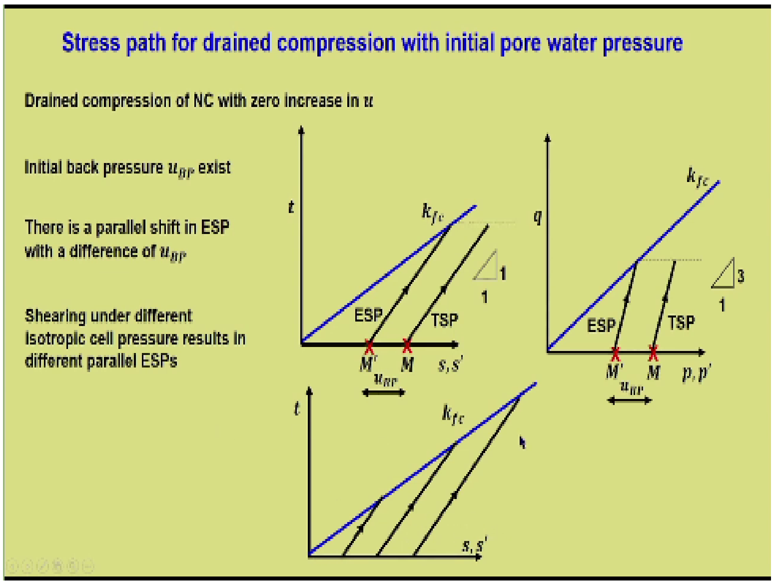

Now, let us see that what will happen if there is some initial pore water pressure? We know that this is a drained test we are discussing about and we do not expect any pore water pressure development during shearing. But what if it happens that there is some pore water pressure in the beginning? And that we have seen there are different possibilities why there should be pore water pressure.

A very good example is the presence of backpressure because of saturation. We know that it is a static pore water pressure, but then still that acts as pore water pressure and hence, the stress path will be different even though we know that total stress path and effective stress path are same in drain triaxial testing. You should not confuse why there is a contradiction here, there is no contradiction. The pore water pressure referring to is only the initial pore water pressure.

Drain compression for a normally consolidated sample with zero increase in pore water pressure. Here you can see that you are discussing about initial not during sharing so that is why It is zero increase in u means u is pore water pressure. But there is an initial back pressure which exists in the sample during saturation. So, again t-s, s' plot k_{fc} has been drawn, the initial isotropic consolidation M is stated here. So, there is a back pressure.

So, now, the initial point, total stress point and the effective stress points are different because of this difference in back pressure. So, now, here it is M, here it is M'. So, here the effective stress path is in this direction. Now, what is the slope of effective stress path here, we have already understood that in a drained triaxial compression in t-s, s' plot it is 1 is to 1. So, 45 degrees, so, this is 45 degrees or 1 is to 1.

Now, total stress path will be parallel to effective stress path. Now, there is something interesting, which is coming out here, why did I stop this total stress path here? You have the failure line it should have extended. Now, this could be an immediate response for some of you. Please note that failure is always brought about by effective stress. So, the failure is dictated by effective stress path.

So, this is the effective stress path. It fails at this point. So, total stress path beyond this point is not meaningful, because the soil does not exist that you have to keep in mind. This is 1 is to 1, I have already told. In the same q same condition for q-p, p' is the same thing. Only the slope changes all other details remains the same. There is a parallel shift in effective stress path with a difference of back pressure.

Shearing under different isotropic cell pressure results in different parallel ESPs. Again, it is better that we understand this point right now, because what it means is that now, this is this corresponds to one particular effective cell pressure. Now, when we share at different cell pressure in triaxial drained condition, we will have parallel ESPs. This is what it is, t-s, s' and this is the failure line you can see that this corresponds to the first cell pressure.

This corresponds to the second cell pressure and this corresponds to the third cell pressure. So, these are different cell pressures, we can see that how the failure is happening. So, it is just a parallel shift in effective stress path.

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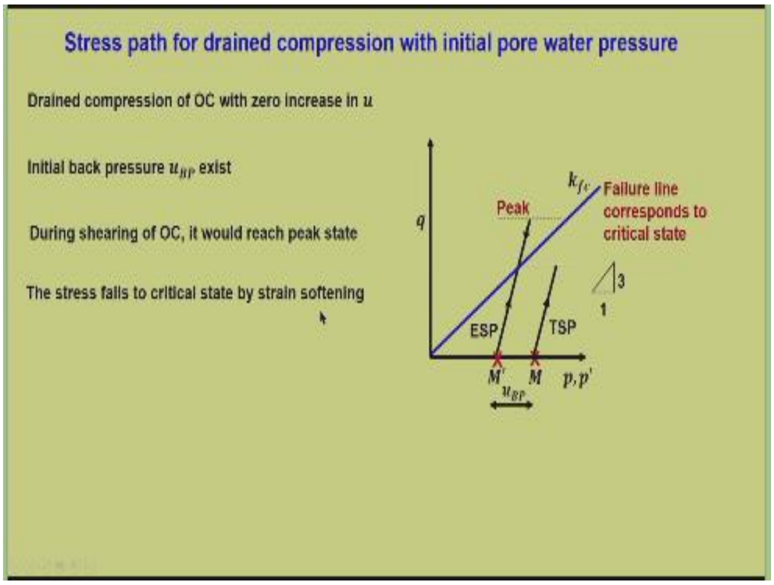

Now, let us deal with drained compression of over consolidated soil with zero increase in pore water pressure. Earlier it is normally consolidated. Now, it is over consolidated. Initial back pressure exist. Now, remember, this failure line corresponds to critical state friction angle. All those things we have already discussed. Now with problem with over consolidated sample is that it exhibits peak and then fails at critical state or ultimate state.

So, that is what why we have to specifically discuss it for over consolidated soil. So back pressure exists all other condition remains same. Now, you may ask here, we have already learned that in over consolidated state pore water pressure will be negative. You are correct. But that corresponds to shearing state. Here we are referring to initial state. You have not started shearing. Negative pore water pressure happens because of dilation.

Dilation happens during shearing, so, do not have to confuse that. So, here other things remaining same now, here the effective stress path is crossing the failure line. So, what is happening? How can this be possible? There is no state of this soil that is possible beyond the failure state. Now, here we have to be very clear about our understanding, in fact, the same aspect is going to be very clear, when we discuss about yielding.

Here, what has happened is the soil would fail only after exhibiting peak in the case of over consolidated. That means, the yielding has to happen first then only it will result in failure. But, unfortunately this line corresponds to the critical state lines. So, soil has not yielded, so, first it will go to the yield point and then it will come back to failure. So, here it has reached the peak, because it is always greater than the critical state.

So, it reaches the peak and then it comes down. You can see this arrow coming down and then it will fail at this point of critical state. So, it goes up and then come down. And that is just because, when you understand this, viza viz..., the stress strain response of over consolidated soil which we have already discussed, this will this aspect will become very clear.

And your total stress path will be up to this particular point. Now, if you want to find out what is the pore water pressure corresponding to peak, then the total stress path will go here. So, this is the difference. In fact, total stress path does not come into picture when we talk about failure. So, this again the difference is backpressure. Here it is q-p, p' so, the slope is 3.

So, during during sharing of OC, it would reach peak state first the stress then falls to critical state by strain softening.

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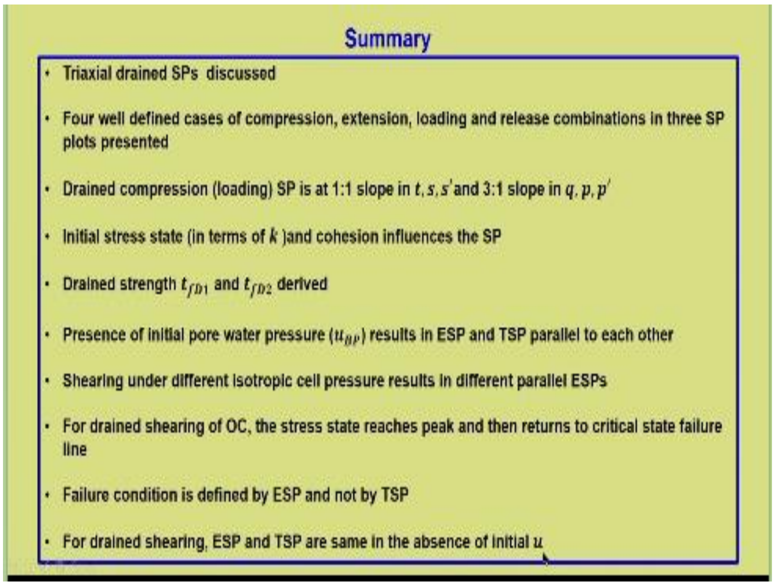

So, that is all about triaxial testing drained case specificall. We have not talked about un-drained now. So, what we have learned in this lecture, let us summarise. Triaxial drained SPs discussed for well defined cases of compression, extension, loading and release combinations is discussed in three stress path plots. Drained compression loading, that means, we are talking about increasing of the load is at 1 is to 1 slope in t-s, s' and 3 is to 1 slope in q-p, p'.

This aspect you need to keep in mind. That is for triaxial case, compression and it is loading that is increase in stresses. So, it is 1, positive 1 and positive 3. Initial stress state that is which is defined by the k value and cohesion influences this response. We have clearly seen that. So, here we have already derived drained strength t_{fD1} and t_{fD2}.

Presence of initial pore water pressure, how this could impact the stress part we have discussed, even though in drained triaxial condition. We do not expect ESP and TSP but just because of the presence of initial back pressure both gets shifted parallel to each other, but with a difference of u maybe that is back pressure. Shearing under different isotropic cell pressure results in different parallel ESPs.

For drained sharing of over consolidated sample, the stress state reaches peak and then returns to critical state failure line. A failure condition is totally defined by ESP and not by TSP. This is a very important point that we have to keep in mind. For drained sharing, ESP and TSP are same in the absence of initial pore water pressure. So some of these concepts are very relevant. And you may be able to understand it very well. But then these points we need to keep in mind so that we understand it better.

So that is all for today’s lecture. We will see few more cases that is exclusively triaxial un-drained test and its stress path in the next lecture. Thank you