Advanced Soil Mechanics
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Lecture - 36
Stress path-some common cases – 2

Okay, welcome back all of you. In the last lecture, we were discussing about stress paths and some common cases. We have seen isotropic consolidation, one dimensional consolidation, the unloading of one-dimensional consolidation. That means the kind of OC state that gets generated. And when an OC soil getting again reloaded, what happens? So, these stress paths we were discussing in the last lecture.

Now, this lecture is a continuation of that, we will discuss some more examples related to stress path. And this will help us understand how we need to plot or how we need to represent this stress path for different cases.

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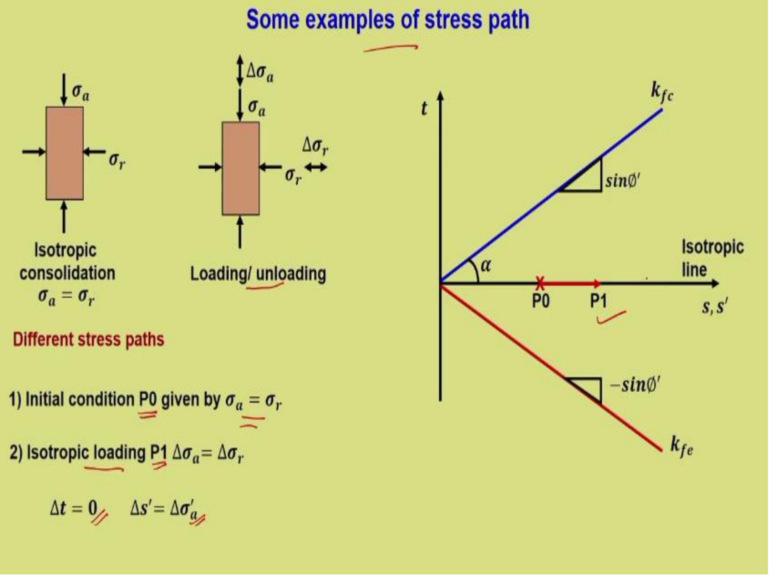

$$\Delta\sigma_a = \Delta\sigma_r$$
$$\Delta s' = \Delta\sigma'_a$$

So, we will just see the first one. Again some examples of stress path, let us start the initial condition. It is an isotropic consolidation, where $\sigma_a = \sigma_r$. And then we will discuss some specific cases of loading or unloading. So, it can be either loading unloading or loading unloading in the radial direction. So, different stress paths, we will just consider t, s, s' stress path plot.

All the procedure remains same. But, for convenience, we are just referring to only one stress path now. So, initial condition is given by P_o which is the isotropic consolidation condition. Now, when I say this it is very easy now for all of you to identify where the point would be on the isotropic line. So, let this be the initial point P_o. Now, the second one is P_1 which represents isotropic loading.

Please take this as a revision, because we have been discussing this quite a number of times. So, you will be knowing now without my help like in what direction it should move. It should definitely move along the isotropic line. So, P_1 is fairly easy and why it is so, because $\Delta t = 0$ and $\Delta s' = \Delta\sigma'_a$. You will get this without any difficulty. So, P_1, how will we represent P_1?

P_1 is represented by the arrow along the isotropic line. So, you keep on increasing the isotropic stress condition, the arrow keeps on progressing.

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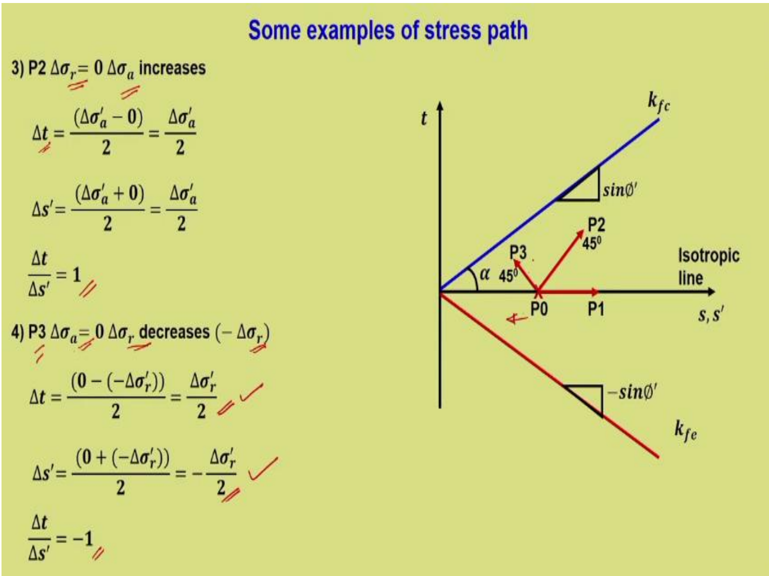

So, that is about 2 cases. The third case is P_2 which is given by there is no increase in radial stress $\Delta\sigma_r = 0$. And $\Delta\sigma_a$ that is axial stress increases. Now, if this is the case on σ_a σ_r plot it is very easy to plot. But when it comes to st plot and qp plot which are mandatory because they are invariants. So, identifying this stress path needs some sort of mathematical understanding.

So, let us see what this translate to in t, s, s' plot. You need to first understand what will be the increment in Δt.

$$\Delta t = \frac{\Delta\sigma_a' - 0}{2} = \frac{\Delta\sigma_a'}{2}$$

So, here $\Delta\sigma_a$ increases. $\Delta\ \sigma_a$' minus there is no increment in a radial stress so it is 0 by 2. That gives plus $\Delta\frac{\Delta\sigma_a'}{2}$. Now, you need to keep certain important things in mind. One is whether it is increasing or decreasing whether the radial stress or axial stress increases or decreases.

Now, if it is increasing then it is positive. And if it is decreasing it is negative. So, that is one part. Now, whether this increase, decrease or the relative increase, decrease of both axial and radial what it will result in terms of t and s. Whether t will be positive, s will be positive with whether there can be possibility of both being positive and negative. All these aspects need to be taken into account while understanding the stress path.

So, prima facie it looks a bit simple. But, there are several small small nitigrities that can hinder your understanding about stress path. So, we need to be careful about these small aspects as well. So, here Δt is known. Now, what is the next task? Δs'

$$\Delta s' = \frac{\Delta\sigma_a' + 0}{2} = \frac{\Delta\sigma_a'}{2}$$

Same thing you get $\frac{\Delta\sigma_a'}{2}$. So,

$$\frac{\Delta t}{\Delta s'} = 1$$

So, we know that it should be with a slope of 1. That means at a 45 degree.

So, how do we plot P_2? So, P_2 will be at an inclination of 45 degrees from the initial point. Next is P_3 which is given as $\Delta\sigma_a$ is = 0. There is no change in axial stress. And there is a decrease in radial stress. That means $\Delta\sigma_r$ will be - $\Delta\sigma_r$ (because this is decreasing). Now, when you substitute this in the same equation of Δt and Δs', what we will get.

$$\Delta t = \frac{(0 - (-\Delta\sigma_r'))}{2} = \frac{\Delta\sigma_r'}{2}$$

$$\Delta s' = \frac{(0 + (-\Delta\sigma_r'))}{2} = \frac{-\Delta\sigma_r'}{2}$$

$$\frac{\Delta t}{\Delta s'} = -1$$

So, that will give $\frac{\Delta t}{\Delta s'} = -1$. So, now, how P_3 will translate. So, P_3 is also with a slope of 1 or at an inclination of 45 degrees.

But here we need to understand that this inclination is negative. So, that is why it is towards this direction. Now, another aspect is whether Δt is positive Δt increment is positive. Again an important thing which we need to understand is the actual value of t and s'. That is t s' and the increment now increment is Δt $\Delta s'$. Now, we are talking about increment because we are just talking about the slope.

But, what can be the actual value? That also matters. That will be t $\pm\Delta t$ or s, $s'\pm\Delta s'$. So, what will be the net? Only the final computation we will be able to understand. So, here $\Delta s'$ is going towards in the negative direction whereas, Δt is moving towards in the positive direction. So, if you can see here from this particular point s' negative direction is in this direction.

So, that is why it is moving in this direction whereas Δt is in the positive direction that is in the upward direction. It can be like this as well. From P_o, it can come down. But then accordingly then $\Delta s'$ will be positive, Δt will be negative. So, all these aspects we need to keep in mind. What is the direction of increment or decrement, whether it is increment or decrement in the respective stress parameter.

Now, increment decrement in $\Delta\sigma_a$, how it translates to these stress parameters like t and s, s'. Both aspects, we need to see. So, here it is clear it is - 1. So, slope has to be negative. That we know, but in what direction. Well it can in this whether it is downwards or upwards. So, that will information will get from here. So, here $\Delta s'$ is towards negative s direction and Δt is towards positive t direction.

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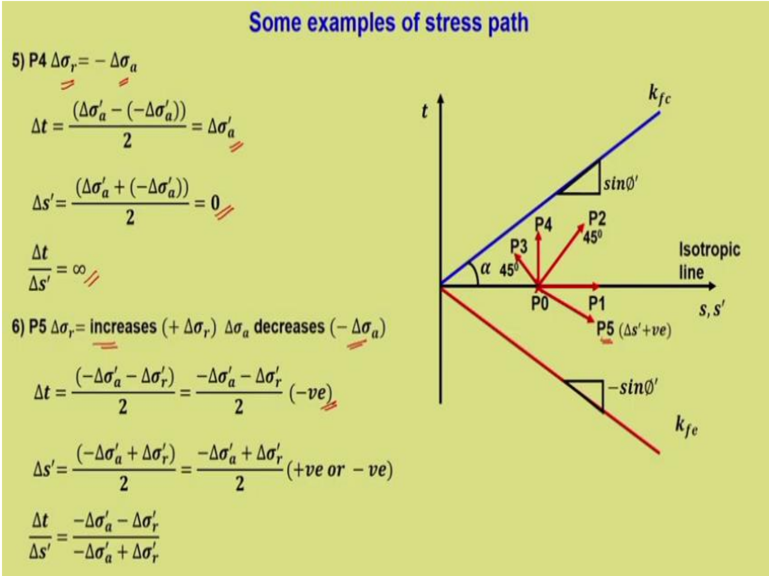

So, in the next example, it is P4 where $\Delta\sigma_r = -\Delta\sigma_a$. So, here again substituting this

$$\Delta t = \frac{\Delta\sigma'_a - (-\Delta\sigma'_a)}{2} = \Delta\sigma'_a$$

$$\Delta s' = \frac{\Delta\sigma'_a + (-\Delta\sigma'_a)}{2} = 0$$

$$\frac{\Delta t}{\Delta s'} = \infty$$

So, Δt upon Δs' is = infinity because there is no change in $\Delta\sigma_a$'. So, how this will move, there is no change in Δs', but an increment in Δt is happening. So, it will be the P_4 failure or this stress path is in this direction P_4. P_4 means it is vertically moving upwards. This means that there is no change in Δs, but only change in Δt. So, that is what is meant by Δt by Δs'.

That is infinity. That is keep on increasing. But this increase, please remember all these increases are restricted by the failure line on compression and extension part. That also we need to keep in mind. Those failure part, we will come a bit later. Now, the next one is P_5 where we have Δσr is increasing that is plus Δσr and Δσa decreases that is - Δσa. Now, how this will translate to Δt.

$$\Delta t = \frac{(-\Delta\sigma'_a - \Delta\sigma'_r)}{2} = \frac{-\Delta\sigma'_a - \Delta\sigma'_r}{2}\ (-ve)$$

$$\Delta s' = \frac{(-\Delta\sigma'_a + \Delta\sigma'_r)}{2} = \frac{-\Delta\sigma'_a + \Delta\sigma'_r}{2}\ (+ve\ or - ve)$$

$$\frac{\Delta t}{\Delta s'} = \frac{-\Delta\sigma'_a - \Delta\sigma'_r}{-\Delta\sigma'_a + \Delta\sigma'_r}$$

Δs' can be positive or negative depending upon the relative magnitudes of σ_a' and σ_r'. So, accordingly this can be positive or negative. Now, this is always negative. This can be positive or negative. So, Δt/ Δs', the expression is known and how this is going to affect our stress path. So, P5, how do we plot? One important aspect is Δt is always in the negative direction. So, from this particular point, t negative is in this direction. So, that is moving towards this, the arrow is moving towards this. Why?

Because t is negative and that is in this downward direction whereas in this particular case, Δs' is considered to be positive. So, this has to move towards the positive s direction. So, that is how it is written. Now, what will be the actual inclination of P_5 that will depend upon what is the type of magnitude of these stresses? So, that will determine in what variation what is the actual magnitude of the increment that will determine the slope?

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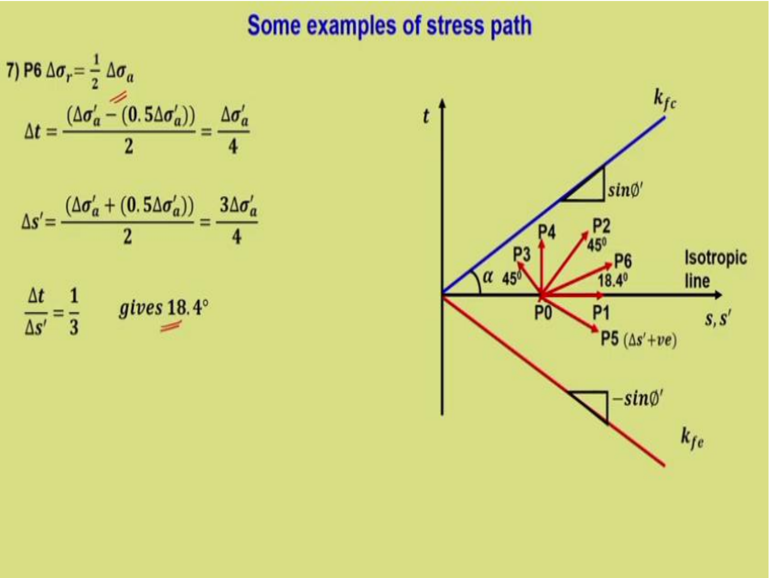

So, the next one is P_6 where $\Delta\sigma_r = \frac{1}{2}\Delta\sigma_a$. And this is another aspect. There is another way in which the stress increments are changing

$$\Delta t = \frac{(\Delta\sigma'_a - (0.5\Delta\sigma'_a))}{2} = \frac{\Delta\sigma'_a}{4}$$

$$\Delta s' = \frac{\Delta\sigma'_a + (0.5\Delta\sigma'_a)}{2} = \frac{3\Delta\sigma'_a}{4}$$

$$\frac{\Delta t}{\Delta s'} = \frac{1}{3}\ gives\ 18.4^0$$

So, accordingly P_6 can be plotted. Both are in the positive direction. That is upwards and towards this, so here P_6 with an inclination of 18.4 degrees.

So, these are some of the examples wherein the starting point was isotropic consolidation that is P_o Now, let us see a few more cases wherein the starting point is not isotropic consolidation.

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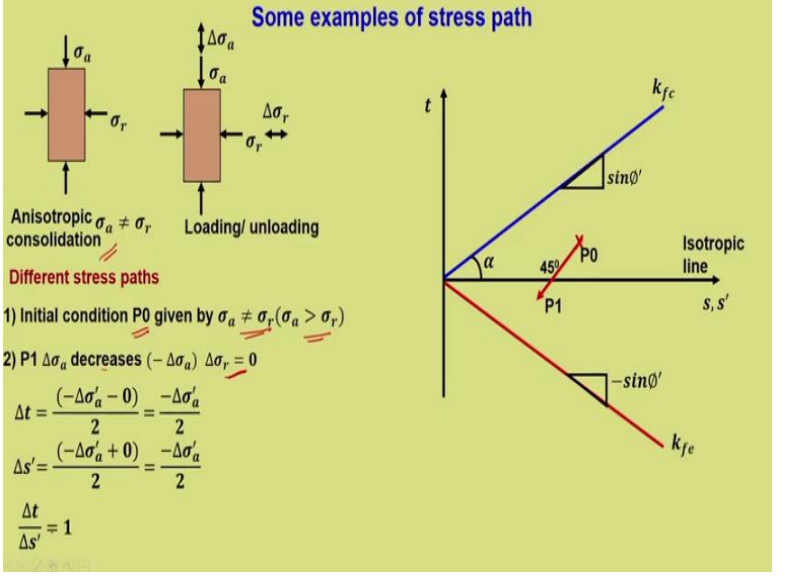

So, here we have anisotropic consolidation where $\sigma_a \neq \sigma_r$, again loading unloading. The same t, s, s' plot is considered. So, the first initial condition is P_o which is given by $\sigma_a \neq \sigma_r$ $(\sigma_a > \sigma_r)$, a typical triaxial compression case. If that is the case, then the point will be in the upper half, where Po is the starting point, so this P_o.

Now, why it remains in the upper half? Again this will become even more clearer when we discuss the next slide on k value how it is changing. So, here let us say that this is in this particular point Po. Now, P1 is Δσa decreases. That is $(-\Delta\sigma_a)\Delta\sigma_r = 0$. So, how this will look like.

$$\Delta t = \frac{(-\Delta\sigma'_a - 0)}{2} = \frac{-\Delta\sigma'_a}{2}$$

$$\Delta s' = \frac{(-\Delta\sigma'_a + 0)}{2} = \frac{-\Delta\sigma'_a}{2}$$

$$\frac{\Delta t}{\Delta s'} = 1$$

So, how P_1 will look like. Now, please note here, $\frac{\Delta t}{\Delta s'} = 1$ and it is positive. So, our general tendency is to draw it in the upward direction. For example, I may draw like this, because it is a positive slope.

So, here, we also need to see, it is negative Δt is pulling it down because it is negative $\Delta\sigma_a'/2$. Δs', that is also in the negative direction. So, that qualifies this P_1 stress path. So, t is also coming down, s is also moving in the negative direction. So, you need to keep in mind both this variation and the final slope as well.

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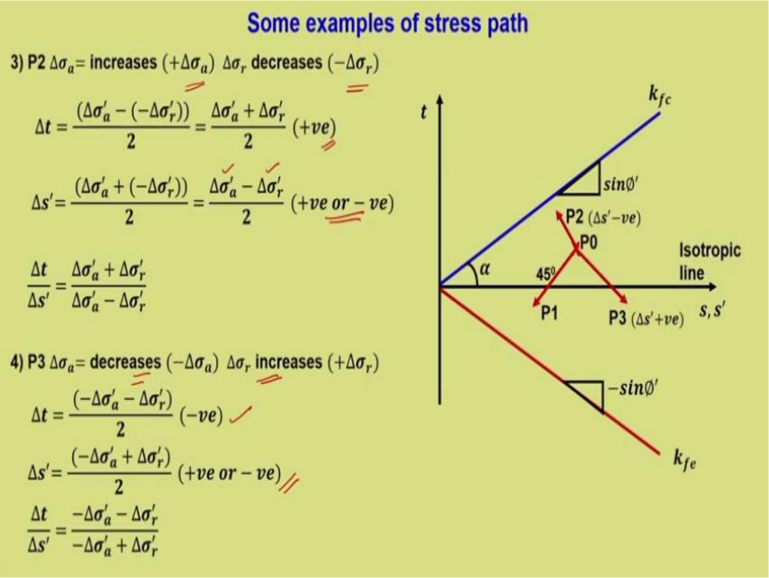

The next stress path is $\Delta\sigma_a$ increases that is positive $\Delta\sigma_a$ and $\Delta\sigma_r$ decreases - $\Delta\sigma_r$. So, if you substitute this

$$\Delta t = \frac{\Delta\sigma_a' - (-\Delta\sigma_r')}{2} = \frac{\Delta\sigma_a' + \Delta\sigma_r'}{2} (+ve)$$

$$\Delta s' = \frac{\Delta\sigma_a' + (-\Delta\sigma_r')}{2} = \frac{\Delta\sigma_a' - \Delta\sigma_r'}{2} (+ve\ or - ve)$$

Now depending upon the relative magnitude of these two it can be positive or negative. So we have

$$\frac{\Delta t}{\Delta s'} = \frac{\Delta\sigma_a' + \Delta\sigma_r'}{\Delta\sigma_a' - \Delta\sigma_r'}$$

So, how P_2 will look like in the stress path plot. So, here we are considering P_2 that is t it is in the upward direction because it is always positive. And here Δs' is considered negative. So, that is why it is going towards the negative direction. So, both ways it is possible. It is possible in the positive direction as well. It is positive in it is possible in the negative direction as well. And, what will be the actual slope of P_2? That will depend upon the relative magnitude.

Next is P_3 where $\Delta\sigma_a$ decreases and $\Delta\sigma_r$ increases.

$$\Delta t = \frac{(-\Delta\sigma'_a - \Delta\sigma'_r)}{2} (-ve)$$

$$\Delta s' = \frac{(-\Delta\sigma'_a + \Delta\sigma'_r)}{2} (+ve\ or - ve)$$

The slope expression is

$$\frac{\Delta t}{\Delta s'} = \frac{-\Delta\sigma'_a - \Delta\sigma'_r}{-\Delta\sigma'_a + \Delta\sigma'_r}$$

So, how P_3 will look like. So, P_3 will look like it moves in the downward direction because t tends to negative value. So, that is why it is coming down and here it can be positive or negative. So, here we have considered Δs' to be in the positive direction. So, that is why it is moving towards rightwards. So, and the actual slope will depend upon the relative magnitudes of the stress increment.

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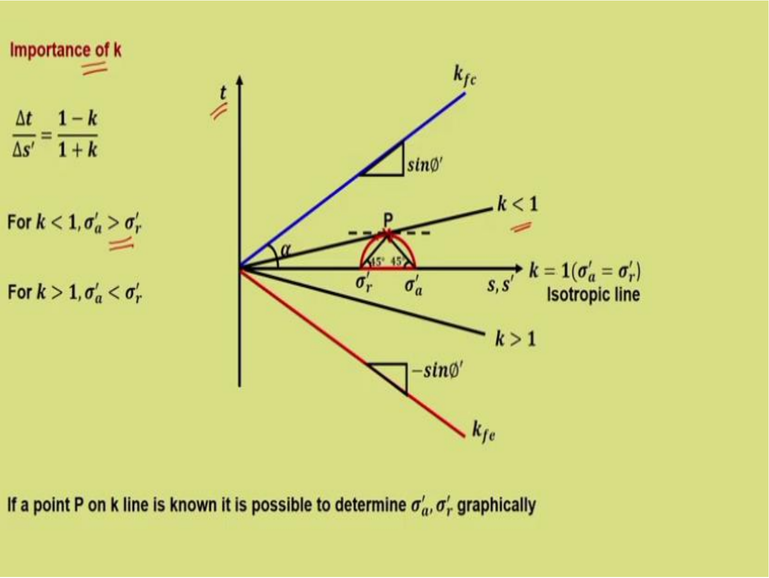

$$\frac{\Delta t}{\Delta s'} = \frac{1-k}{1+k}$$

$$k < 1, \sigma'_a > \sigma'_r$$

$$k > 1, \sigma'_a < \sigma'_r$$

So, that is about some examples to make us understand how we need to plot the stress path direction. And probably depending upon what direction the stress path is, we will be able to understand when this stress path is going to touch the failure line. If at all that kind of stress increment decrement is going to fail, all these information becomes quite handy from the stress path plot.

More of that we will follow because the actual application of when it is going to fail and how it is going to fail is in the Module 4, when we discuss about critical state. So, here we understand the philosophy of stress path plotting for Module 4. Now, as I told you in the previous slide that what is the importance of k. Now, we have seen different types of k. The k for isotropic is 1. Then we have seen k for normally consolidated condition, k for failure condition. So, all these things we have seen now. k for unloading case. All these things we have seen. Now, what will what is the actual importance of k? So, $\Delta t / \Delta s'$, again I am referring to this. We can do this with any other stress parameter plots. So, this is k = 1 which is isotropic line. Now, for k < 1, what is k?

k is $\sigma r'$ upon $\sigma a'$. So, k < 1 means $\sigma a'$ is > $\sigma r'$. So, k > 1 that is a case of maybe triaxial compression where $\sigma a' > \sigma r'$. It comes in this particular plot. So, this is k = 1. All conditions where k is < 1 that means $\sigma a' > \sigma r'$ will come in this particular domain. So, it is in the compression domain. So, that is k < 1.

And please remember, this is initial condition mostly because where it falls in the anisotropic consolidation. We are referring to that. Now, for k > 1 that means $\sigma_a' < \sigma_r'$ we obviously know it has to be < the bottom domain. So, this is k > 1. Now, if we know any point on the k line, let us say we know any point on the k line it is possible to determine σ_a' and σ_r' graphically. Why?

Because each of these points in t, s, s’ plot is maximum shear stress point. I hope you remember that. So, let us say that we know the point P. We want to determine, what is the σ_a' and σ_r' corresponding to this particular point? Now, for us in the triaxial sample, σ_a' and σ_r' corresponds to major and minor principal stress conditions.

So, here if I draw a horizontal line like this, and then draw 2 lines which is at 45 degrees or maybe 45 degrees with this particular line, either way here and at the meeting point considering these 2 meeting points, if we draw a circle, then these points will represent σ_a' and σ_r'. So, that is how we determine. If you know a point you can easily draw a circle there.

Both ends will give you σ_1' σ_3' which is nothing but our axial and radial stress condition in a triaxial sample.

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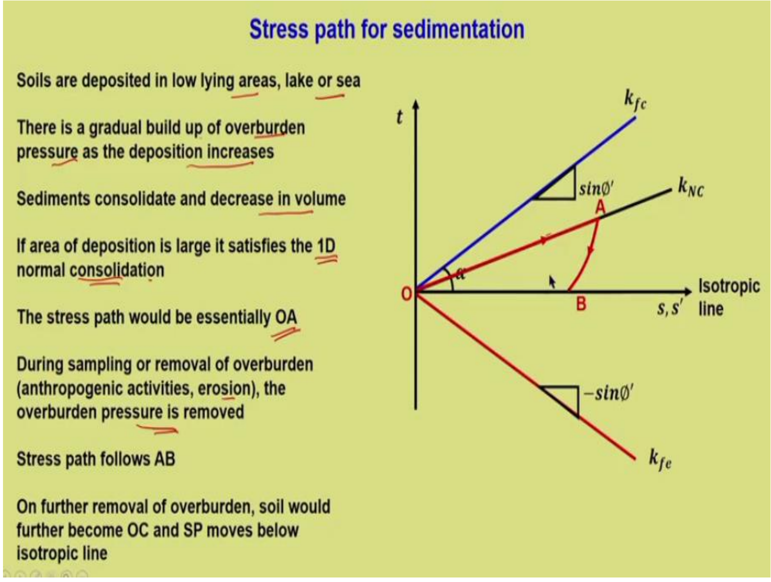

Now, there is a specific case of stress path for sedimentation. We have seen this. We have already discussed this in our previous lecture. I am just trying to map that understanding with the actual condition that can happen in the field. Soils are deposited in low lying areas, lake or sea. That is how the evolution of soil takes place. The river carries it deposits at some particular place, then this deposition keeps on going.

Now, this is a virgin deposition. And hence, any sort of consolidation we know that is going to be normally consolidated sample. So, there is a gradual buildup of overburden as the deposition keeps on going. And because of this the overburden pressure increases as the deposition increases. Sediments consolidate and decrease in volume because as the deposition goes on the overburden keeps on increasing.

So, self-weight consolidation happens densification happens, the volume decreases. So, this process keeps on going and that is how the sedimentary soils are produced. If the area of deposition is large, it satisfies the one dimensional criteria. This also we have seen. As the area is large then the one dimensional consolidation condition is satisfied. It also is a normal consolidation. Why?

Because, these are freshly laid deposits of soil and hence the consolidation will be will adhere to the normally consolidation. So, this stress path would be essentially OA. So, now we know that this is normal consolidation. So, we have to have a normally consolidation line. So, this is k NC line which we have already discussed in the previous lecture. So, how will be OA? So, OA will be it is starting from some point.

And then it is increasing and this follows the normally consolidation line as the sedimentation keeps on increasing overburden keeps on increasing the sedimentation keeps on happening. So, that will reach to OA. Now, during sampling or removal now, we have reached the point A. Now, what we do, if there is any subsurface investigation, we need to sample it.

So, during sampling, what we are doing essentially we are removing the overburden. Now, this removal of already deposited sample can also be due to anthropogenic reasons, because of clearing of that particular area for construction or it can be natural reasons because of again erosion happening. So, any of these factors can ease out the deposit. So, during sampling or removal of overburden, because of anthropogenic activities erosion, the overburden pressure is removed.

So, what will happen? This results in a stress path which follows AB. So, from A, there is a release which is happening because of which there is a downward trend. So, AB represents the removal of the overburden. So, on further removal of overburden, soil would further become OC. So, now, the moment it is unloaded this portion represents over consolidation. Now if further removal happens then it may further go down.

And soil would become OC and the stress path moves even below the isotropic line. So, which I am not showing we have just stopped here up to isotropic line.

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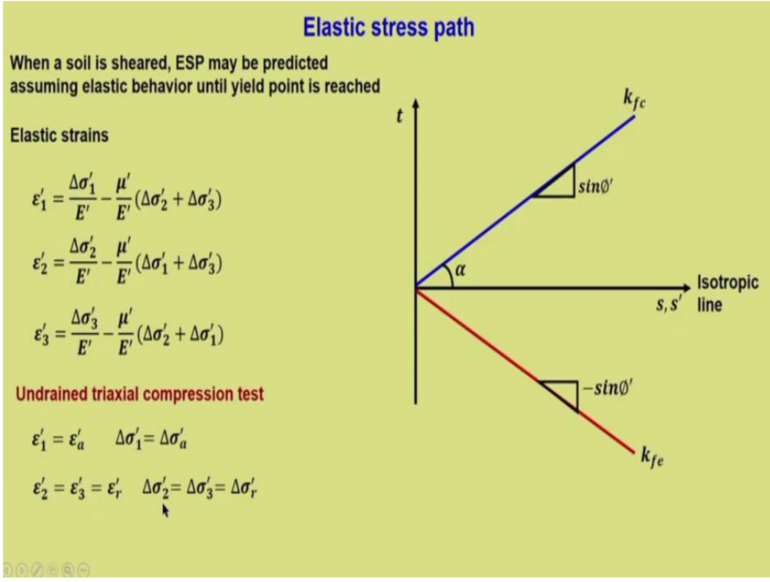

Now, we will see something what is known as elastic stress path essentially which confirms to the elastic behaviour. So, here again the plot is same t, s, s'. When a soil is sheared, effective stress path, ESP may be predicted assuming elastic behaviour until the yield point is reached. This is possible, very much possible. So, we tend to have the elastic behaviour up to the yield point. And this aspect will be again discussing in the next module.

We know that what are the elastic strains where

$$\epsilon_1' = \frac{\Delta\sigma_1'}{E'} - \frac{\mu'}{E}(\Delta\sigma_2' + \Delta\sigma_3')$$

$$\epsilon_2' = \frac{\Delta\sigma_2'}{E'} - \frac{\mu'}{E}(\Delta\sigma_1' + \Delta\sigma_3')$$

$$\epsilon_3' = \frac{\Delta\sigma_3'}{E'} - \frac{\mu'}{E}(\Delta\sigma_2' + \Delta\sigma_1')$$

Let us say a typical case of un-drained triaxial compression test where because of triaxial compression we know that

$$\epsilon_1' = \epsilon_a' \quad \Delta\sigma_1' = \Delta\sigma_a'$$

$$\epsilon_2' = \epsilon_3' = \epsilon_r'$$

Again because of axisymmetric condition $\Delta\sigma_2' = \Delta\sigma_3' = \Delta\sigma_r'$

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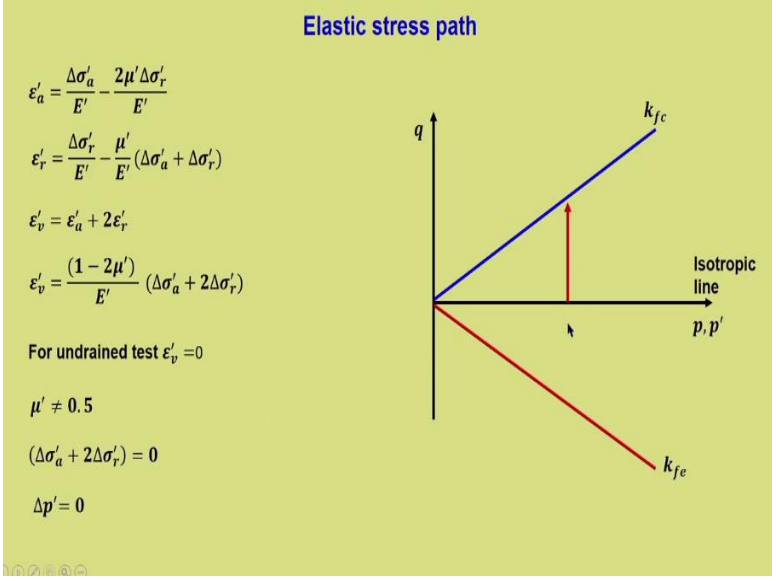

$$\Delta p' = 0$$

ϵ_a' can be written as

$$\epsilon_a' = \frac{\Delta\sigma_a'}{E} - \frac{2\mu'\Delta\sigma_r'}{E}$$

$$\epsilon_r' = \frac{\Delta\sigma_r'}{E} - \frac{\mu'}{E}(\Delta\sigma_a' + \Delta\sigma_r')$$

Now, what will be volumetric strain? The volumetric strain is $\epsilon_v' = \epsilon_a' + 2\epsilon_r'$. This is for triaxial sample. Now, we know for un-drained condition ϵ_a' = 0. So, the final expression is

$$\epsilon_v' = \frac{1-2\mu'}{E}(\Delta\sigma_a' + 2\Delta\sigma_r')$$

For un-drained condition, we know that there is no volumetric strain, so ϵ_v'= 0. Now, $\mu' \neq 0.5$. Please remember, in our earlier lecture, we have already discussed that μ corresponding to un-drained condition is = 0.5 because it is an incompressible state. Now, please remember that mu corresponds to μ un-drained. So, there it is = 0.5. You can conveniently take = 0.5 because a it is a un-drained condition. Here, even though the condition is un-drained we are talking about μ'.

That is the effective Poisson's ratio which is not supposed to be 0.5. Now, in this expression, we know ϵ_v'= 0, but that can happen only when μ' is = 0.5. Under effective condition, $\mu' \neq 0.5$. So, then what is the possibility? The only possibility is $(\Delta\sigma_a' + 2\Delta\sigma_r') = 0$. So, this is a particular condition. This means Δp' is = 0.

What is p'? p' is the mean stress. What is mean stress? $(\Delta\sigma_a' + 2\sigma_r')/3$ which is = 0. Now, if you plot it in q p' plot, this particular stress path will look like something like this. So, starting point there is no change in Δp'. So, q/p' will be infinity. So, it moves vertically upwards. How this will be in t, s, s' plot.

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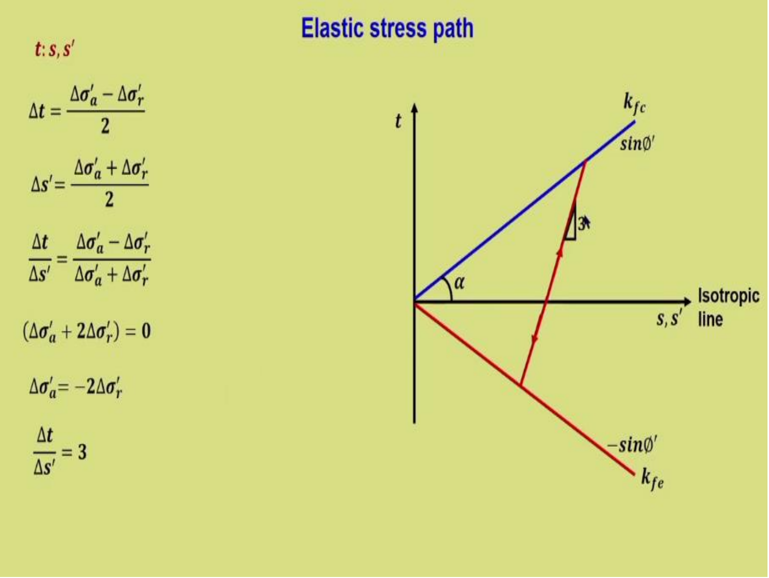

So, we will see in t, s, s',

$$\Delta t = \frac{\Delta\sigma_a' - \Delta\sigma_r'}{2}$$

$$\Delta s' = \frac{\Delta\sigma_a' + \Delta\sigma_r'}{2}$$

$$\frac{\Delta t}{\Delta s'} = \frac{\Delta\sigma_a' - \Delta\sigma_r'}{\Delta\sigma_a' + \Delta\sigma_r'}$$

So, Δt/Δs' is known. Now, we know this condition, $\Delta\sigma_a' + 2\Delta\sigma_r' = 0$. Substituting that in the particular expression for Δσa' putting $\Delta\sigma_a' = -2\Delta\sigma_r'$ We get the slope as

$$\frac{\Delta t}{\Delta s'} = 3$$

So, how does it look like in the t, s, s' plot. So, if this is the starting point it goes in this direction. So, one is in the upward direction the other one is towards the extension. So, this is the kind of stress path we get.

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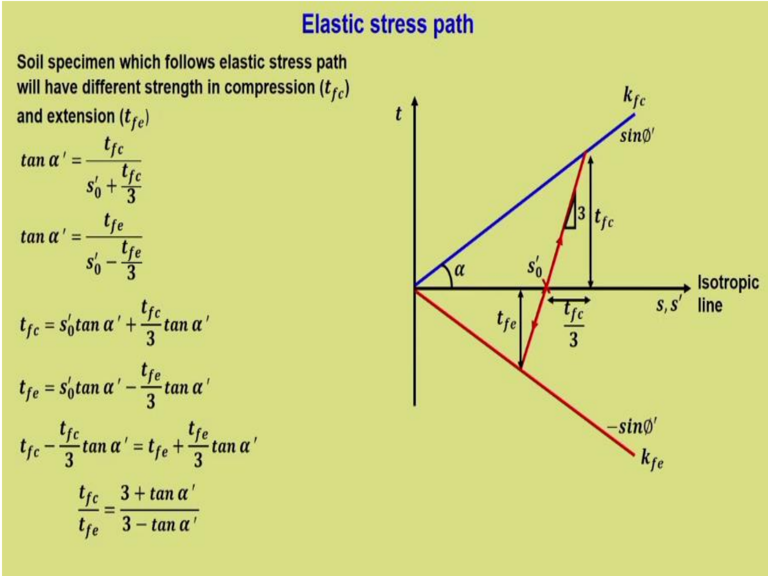

Now, the soil specimen which follows elastic stress path will have different strength in compression. That is t_{fc} and extension t_{fe}. So, the earliest stress path let us say this is the starting point. And we know that now, this is 3 that we have already derived, then this is the strength corresponding to compression which is given us t_{fc} and for extension it is t_{fe}. We can note here that this $t_{fc} \neq t_{fe}$ when we follow the elastic stress path.

So, $\tan\alpha'$ let us try to understand what is the ratio $\tan\alpha'$, it can you can write as t_{fc}. This particular triangle t_{fc} divided by $s_o' + t_{fc}/3$. $\tan\alpha'$ can also be written as

$$\tan\alpha' = \frac{t_{fc}}{s_o' + \frac{t_{fc}}{3}}$$

So, t_{fc} can be written as

$$t_{fc} = s_o' tan\alpha' + \frac{t_{fc}}{3} tan\alpha'$$

t_{fe} can be written in this manner.

$$t_{fe} = s_o' tan\alpha' - \frac{t_{fe}}{3} tan\alpha'$$

Substituting for we will get

$$t_{fc} - \frac{t_{fc}}{3} tan\alpha' = t_{fe} + \frac{t_{fe}}{3} tan\alpha'$$

So, rearranging, we can get

$$\frac{t_{fc}}{t_{fe}} = \frac{3 + tan\alpha'}{3 - tan\alpha'}$$

which is ±1. So, when we follow an un-drained triaxial compression following the elastic stress path, the strength in compression and the strength in extension follows this particular ratio.

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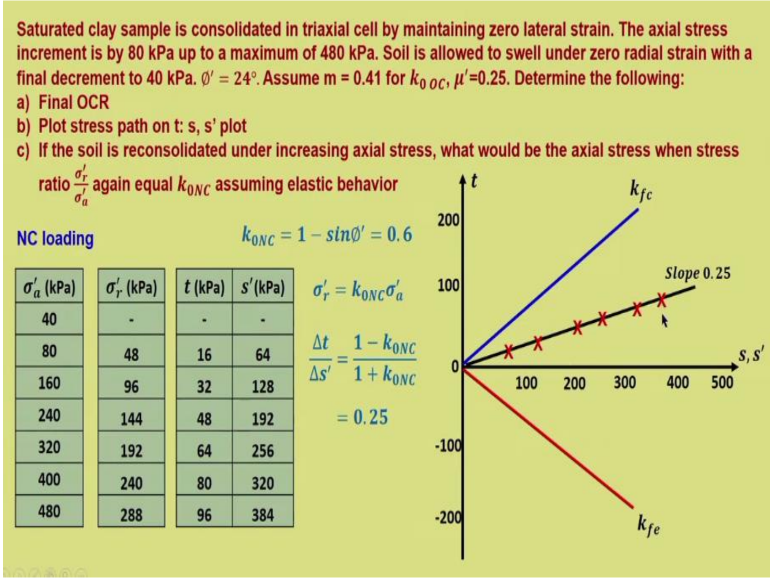

So, we have discussed quite a number of cases related to stress path. We will quickly see an example from what we have already learned. So, the given example is the saturated clay sample is consolidated in triaxial cell by maintaining 0 lateral strain. So, it is as good as simulating a one dimensional compression. The axial stress increment is by 80 kPa up to a maximum of 480 kPa.

So, maximum stress is 480 kPa with an increment of 80. Soil is then allowed to swell under 0 radial strain. Again maintaining one dimensional consolidation with a final decrement to 40 kPa. So, this starting from 40 to 80 it comes then further on it is an increment of 80. Once after reaching 480, it is unloaded in steps till it reaches the final value of 40 kPa. So, that is what it means to a final decrement to 40 kPa.

ϕ' = 24 degrees angle of internal friction of the sample is given. Assume m = 0.41. Where is this m coming from? This is for determining the coefficient of lateral earth pressure at rest corresponding to OC state. For k_{OC}, that is why it is written m = 0.41 for OC. The Poisson's ratio effective Poisson's ratio is given as 0.25. Now, we need to determine the following. What is that? What is the final OCR?

Plot this stress path for the given condition on t, s, s' plot. If the soil is reconsolidated under increasing axial stress, I mean to say after reaching 480 you are allowing it to unload. It reaches up to 40 kPa axial stress. Now, if you again reload it, if the soil is reconsolidated, reconsolidate means reloaded under increasing axial stress, what would be the axial stress when stress ratio of σ_r' by σ_a' again equal to k_{oNC}.

And you need to assume it to be elastic behaviour. Remember, we have loaded unloaded. And while reloading we have seen that this particular reloading curve quickly approaches the k_{NC} line. So, just to prove that particular point just to see that how it happens, we have given an example. And in that reloading curve, we need to assume that it behaves in an elastic manner. It is just giving you a hint to use that appropriate equation.

So, let us see how we will solve this. So, the normal consolidated loading is given σ_a' is increased. And we are adjusting the σ_r' such that it maintains a normally consolidated condition. No lateral strain is permitted. This is a one dimensional consolidation case. Now, you can see that the starting is not at 80. Why? Because the final decrement we have to reach to 40.

So, from 40 to 80, further, it is an increment of 80 up to 480. So, we are again seeing that in t, s, s' plot. Now, we know that $k_{ONC} = 1 - sin\phi'$. ϕ' is given 24 degrees. Substituting it, we will get ko NC is = 0.6. Now, all this increment corresponds to ko NC line and hence σ_r'. What will be σ_r'? Because, we do not know the value of t and s, s'.

So, that is possible only when we have σ_a' and σ_r'. So, σ_r' is ko NC of σ_a' because that is the coefficient of lateral earth pressure at rest condition. So, knowing this k_{ONC} is known, σ_a' is known. So, here it is not there because that is not where the loading starts. It starts from 80. So, σ_r'= k_o.

So, we can determine the value of σ_r' which confirms to one dimensional consolidation. And these are the values which we can determine using this expression. Once we know these 2 values, we can determine t and s' values. And these values can be plotted on the stress path. And Δt/Δs' the slope is known which is $\frac{\Delta t}{\Delta s}' = \frac{1 - k_{ONC}}{1 + k_{ONC}}$. And that is = 0.25.

So, we need to draw a line in this plot t, s, s' with a slope which is given by 0.25. That is what we have just drawn. Slope is = 0.25. And these points which are marked in red, these are the points which is 16, 64, 30, 128, up to 96, 384. So, this is one part of the exercise where we have plotted the given NC stress path. Now, what is the next condition? Now, the next condition is after reaching 480, σ_a' now this is released one by one.

And it will follow an OC state. So, there will be unloading which is happening. Now, how to determine that?

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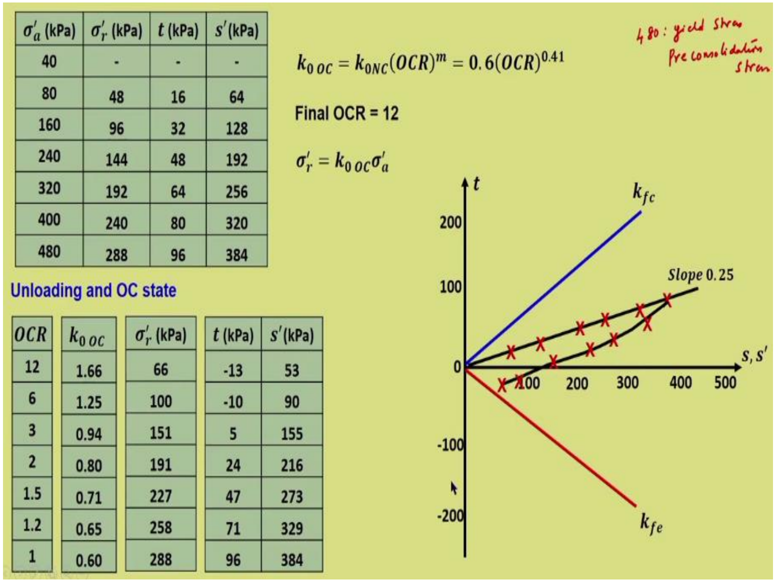

σ'_a (kPa)	σ'_r (kPa)	t (kPa)	s' (kPa)
40	-	-	-
80	48	16	64
160	96	32	128
240	144	48	192
320	192	64	256
400	240	80	320
480	288	96	384

Unloading and OC state

OCR	$k_{0\,oc}$	σ'_r (kPa)	t (kPa)	s' (kPa)
12	1.66	66	-13	53
6	1.25	100	-10	90
3	0.94	151	5	155
2	0.80	191	24	216
1.5	0.71	227	47	273
1.2	0.65	258	71	329
1	0.60	288	96	384

So, this is the initial condition what we have already determined and with this we have plotted. Now, unloading and OC state. Now, $k_{o\,OC} = k_{ONC}(OCR)^m$ Now, this m is given 0.41 k_{ONC} is known. So, knowing OCR we can determine $k_{o\,OC}$. Now OCR, how to determine OCR? Here we know that the maximum pressure the soil is subjected to is 480.

So, 480 can be considered as the preconsolidation pressure. So, here 480 is yield stress or preconsolidation stress. Now, we know how to determine OCR. The preconsolidation pressure divided by the pressure it is subjected to. Now, if during unloading next is 400. So, 480 up by 480 that is on the normally consolidated line. Now, when it is unloaded to 400 it is 480 divided by 400 is the OCR. That is 1.2.

Similarly, 320 by 480 that is 1.5 and if last value is 480 divided by 40 which is 12. So, one of the question is, what is the maximum OCR? So, the final OCR is = 12. So, this point of 40 corresponds to OCR of 12. Now, once OCR is known, we can substitute here and get the values of $k_{o\,OC}$ and that keeps changing depending upon the level of OCR. Now, the same thing can be written in terms of elastic conditions as well but here we are using ko OC. And here ko OC is = substituting each of the OCR values in this equation we will get 0.6, 0.65. Now, it does from down upwards. Because it is unloading, 0.6, 0.65, 71 up to 1.66 is the $k_{o\ OC}$. Now, once ko OC is obtained we can determine what is σ_r' because σ_a' is known. This is σ_a'.

Hence, we can determine what is σ_r' which is 66 100 up to so, it is from 288 upwards up to 66. Now, once σ_a' σ_r' is known, we can find out what is t and s'. So, the final value is from 96 384 to - 13 53. So, how does it look like? Plot this stress path. It is keep on changing. You can see this because ko OC keeps on changing. So, this will give up to the last point is - 13 53. So, this is the unloading path.

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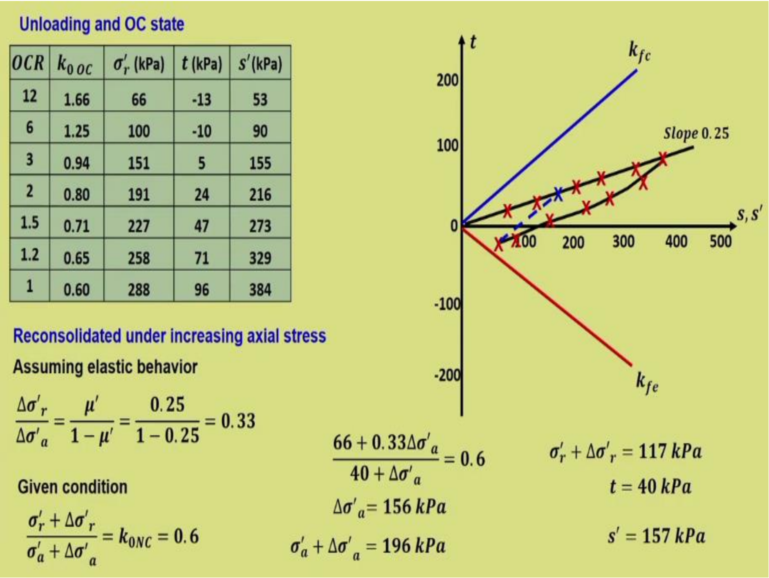

Now, we have one more aspect to be considered, which is the reloading and at what stress condition of t and s' it will reach the normally consolidated line. So, that is what we need to find out next. So, reconsolidated under increasing axial stress and we need to assume elastic behaviour. So, if that is the case then this equation is already available to us,

$$\frac{\Delta\sigma_r'}{\Delta\sigma_a'} = \frac{\mu'}{1-\mu'} = \frac{0.25}{1-0.25} = 0.33$$

Now, this is the slope with which it is go back to the ko NC line. So, from here reloading means it goes at some inclination and meets the ko NC line. That is what it happens. Now, the slope is known 0.33. Now, we need to find out at what stress condition it joins back the ko NC line. So, given condition is

$$\frac{\Delta\sigma_r' + \Delta\sigma_r'}{\Delta\sigma_a' + \Delta\sigma_a'} = k_{ONC} = 0.6\,.$$

Now, from here if $\Delta\sigma_r$' and $\Delta\sigma_a$' is given it should reach the ko NC line. So, that is what it means. So, this condition is already given. At what stress increment now, when you reload it what stress increment of $\Delta\sigma_a$' which is a which is again determine $\Delta\sigma_r$' under what increment this particular point will reach the k_{ONC} line. So, that is why it is equated to k_{ONC}. And its value is known which is 0.6.

Now, σ_r' the final point σ_r' is 66. So,

$$\frac{66 + 0.33\Delta\sigma_a'}{40 + \Delta\sigma_a'} = 0.6$$

Now, solving this, we will get $\Delta\sigma_a' = 156\ kPa$.And hence $\sigma_a' + \Delta\sigma_a' = 196\ kPa$

Similarly, you can find out

$$\sigma_r' + \Delta\sigma_r' = 117\ kPa$$

So, knowing this stress we can determine t and s, s'

$$t = 40\ kPa$$

$$s' = 157\ kPa$$

So, this is the point where the reloading line or the stress path is going to meet. So, that is given by this particular point. So, this particular Blue cross it represents t = 40 kPa and s' is = 157 kPa. So, the reloading happens to this particular point.

So, it is a clear demonstration of the loading k_{NC} loading unloading OC and again reloading which we have discussed in the previous lecture. So, this is all about some examples, let us summarize the last 2 lectures whatever we have learned till now.

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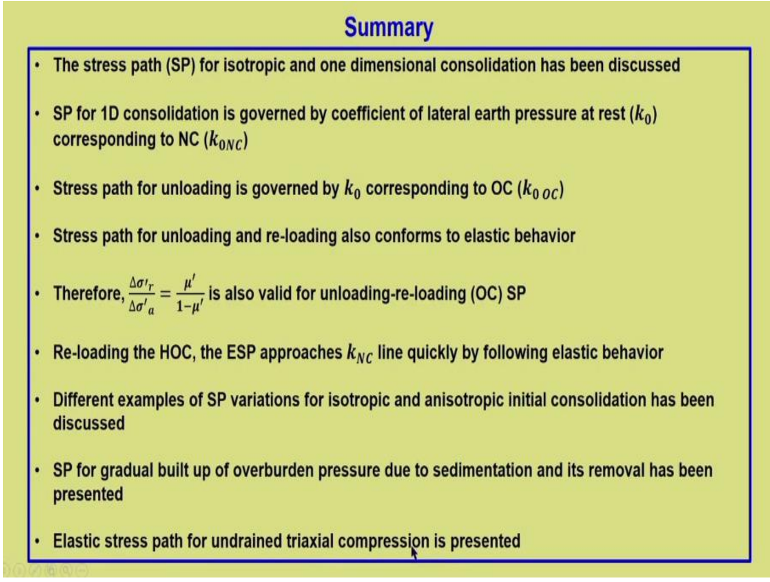

This stress path SP for isotropic and one dimensional consolidation has been discussed. Stress path for one dimensional consolidation we have seen that it is governed by coefficient of lateral earth pressure at rest k_o corresponding to NC which is k_{ONC}. Stress path for unloading is governed by k_o corresponding to OC. That is $k_{o\,OC}$. Stress path for unloading and reloading also confirms to elastic behaviour μ'/(1 - μ').

So, therefore, this $\frac{\Delta\sigma_r'}{\Delta\sigma_a'} = \frac{\mu'}{1-\mu'}$ particular expression is also valid for unloading reloading OC. So, you can always approximate by unloading reloading to be of the same path. That is also possible. If it is not then the unloading will confirm to $k_{o\,OC}$. And again reloading will confirm μ'/(1 - μ') as the example we have seen. Reloading the heavily over consolidated soil, the ESP approaches k_{NC} line quickly by following elastic behaviour.

We have seen this already and we have also demonstrated this in the last example. Different examples of SP variations for isotropic and anisotropic initial consolidation has been discussed. Stress path for gradual built up of overburden pressure due to sedimentation and its removal has been presented. And finally, the elastic stress path for un-drained triaxial compression is presented.

Now, these are some of the cases which will help us to draw this stress path better. Now, in the next lecture, we will see specifically stress path for triaxial testing in the lab. So, that is all for now, thank you.