Advanced Soil Mechanics
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Lecture - 35
Stress path - some common cases

Welcome all of you. In the last 2 lectures, we have seen, what is a stress path? And, we have defined the failure line. And, for all the 3 cases of stress path plotting, we have defined the failure line and its slope. And, we have told that bounds are very important in understanding the stress path variation. So, from today's lecture onwards we will see some common cases for stress path plotting first.

Then that will be followed by specifically for the triaxial testing in the lab and some examples relevant to field conditions. So, that will be the progression of this module further. So, in today's lecture, we will see the stress path for some common situations. Obviously, you will be having some knowledge now based on our previous lectures like how things would move or how the stress path would move for some specific conditions.

We need to keep in mind that whenever we are doing the stress path what we have to understand is we need to determine how the 2 different stress parameters changes with respect to each other. And for that we may need some sort of computations. For example, in the case of σ_a σ_r variation, it is straight forward. Because that is the external stress that we are imposing onto the soil mass.

So, it is very clear how σ_a and how σ_r if the variations are given it is very easy for us to plot. But if it comes to st plot or qp plot the issue is now this becomes a function of this σ_a and σ_r. So, we do not know exactly how this stress path would move and plotting in terms of st plot and qp plot becomes very important because they are more like stress invariants. And there are several failure criterion which are based on this.

So, it is necessary for us to actually determine the variation even though the variation is known to us we may have to derive. For example, the slope in which the stress path move. Because unless we know the slope we will not be able to know when it will it is going to reach the failure. So, all these aspects we will see in our few lectures from now. So, with that we will start our first case.

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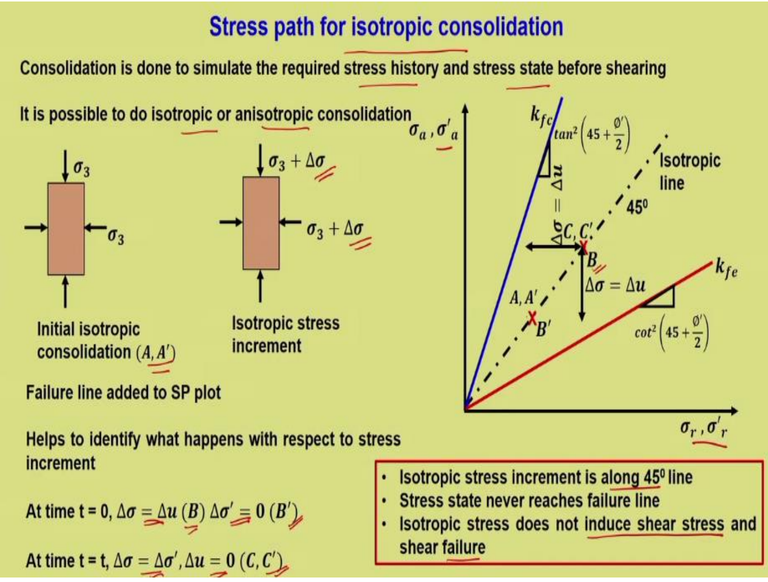

That is stress path for isotropic consolidation. Now, without explaining much in this lecture itself you will be in a position now to know how this stress path will move in a isotropic consolidation case. Because we have started off with some examples in the beginning. But for completeness let us again go through these because these forms some of the very important and very unavoidable situations in geotechnical testing and in the field situations.

So, consolidation is done to simulate to the required stress history and stress state before shearing. All of us know if it is a triaxial testing we try to simulate some condition initially with respect to that is what is available in the field. But in many cases when the testing is done in the lab this particular aspect of how much the initial condition relates to field is generally not checked to that extent. But that is invariably necessary.

We should have an overall understanding of what was the overburden pressure the soil was subjected to in the past. I mean at least the overburden pressure because whether it has been subjected to more stresses in the past. Sometimes, we may not be knowing. The reason is if a hill is cut and the engineer in charge he does not know whether there was a hill initially then he could not he will not be in a position to anticipate the possibility of over consolidation.

But at least looking at the subsurface investigation plot we can always understand what could be the possible overburden pressure which the soil is subjected to from a particular sampling point. And any stress range that we simulate in the lab the initial state which is less than this would definitely give an over consolidated behavior of the soil. So, it is very important that we should know what is the overburden and at least we should know how much more stresses the soil will be subjected to in future. So, based on this we make a judicial selection of initial state simulation in the lab. So, that is what we have to do. Now, consolidation helps us to attain this particular state of soil. For example, the whatever stress history we want to simulate, maybe we can load it and then unload it to create a certain amount of over consolidation.

Or, maybe we can consolidate and then start the test on that without any unloading. So, what sort of stress history we want to simulate or what sort of initial stress condition we want to simulate depending upon that we need to decide upon the consolidation. It is possible to do isotropic or anisotropic consolidation. Even though both are possible, majority of the cases we prefer isotropic consolidation because it is fairly simple and it can be executed well without much error. Whereas it is also possible in a specific triaxial testing system that we can also induce anisotropic consolidation and remember we have told this in the previous lecture. When we have anisotropic consolidation, we should also expect some amount of shear stress also developed within the soil.

Now, this is a typical case of initial isotropic consolidation and which is represented by point A, A'. We will have all the stress path plots. First, we will start with σ_a, σ_a', σ_r, σ_r' Now, we can draw this isotropic line. So, this is this part of stress path plotting is fairly simple because we know that for isotropic consolidation all the points would lie on the isotropic line.

And this we have seen initially also in while we started this lecture. So, the first point the initial isotropic consolidation point is A, A'. Now, this is a saturated soil sample initially isotropic consolidated. Now, we will increment this stress by $\Delta\sigma$. So, the from the initial point A, A', we apply an incremental stress of $\Delta\sigma$ to it. So, what happens? Failure line is added to stress path plot.

So, that is first and foremost thing that we have to do because we know that without that the stress path plot is not complete. So, we have added. This we have already discussed in the previous lecture. Helps to identify what happens with respect to stress increment that we have already seen. At time t = 0, so, now we will consider both situations. An instantaneous un-drained condition that will get generated which corresponds to t time t = 0.

That is at a very small time immediately after the application of $\Delta\sigma$ there will be a total stress condition. So, we will just try to understand that even though it may not be relevant. But we will try to understand how it will look like and then at time t = t when this pore water pressure aspect is not there it will be a drained or an effective stress path. So, when time t = 0 $\Delta\sigma$ is taken by the pore water pressure.

And $\Delta\sigma'$ that is gain in effective stress 0 at time t = 0. That means nothing has happened to effective stress but there is an increment of $\Delta\sigma$ in the total stress condition. So, that is what again it is on the isotropic line because it is a hydrostatic stress increment. So, the point B will be here. And, where will be point B'? Since, there is no change it will be at its initial point itself.

So, B' will be here. And this, please remember the point B' and B may not be relevant. It is relevant only with respect to time t = 0, condition that is an instantaneous loading condition. So, that means this will increase by $\Delta\sigma = u$, $\Delta\sigma = u$. That is why it is on the isotropic line. Now, time t equal to t that means a lot of time has gone.

So, whatever we have to expect, we have to expect in terms of effective stress only. So, in that case all the stress increment is now in terms of effective stress, pore water pressure is 0. And this will be indicated by point C, C' because there is no total stress condition here. If there is total stress condition at time t equal to t then the only change is point C and C' will not coincide so, here in this case point C and point C' are the same. So, if you actually look at the given load increment of $\Delta\sigma$ there is only a case of A' and C'. But we have expanded it and shown both in terms of effective stress path and total stress path. Now, if you see the total stress path will be A B and the effective stress path will be A' C'. Now, imagine that there is sort of pore water pressure also happening then because of pore water pressure the points C and C' will not be same. So, that is the only difference in the case of effective stress path and total stress path. So, isotopic stress increment is along 45 degrees line as it is visible. Now, imagine so this is another important point which we have to keep in mind. We know this but in the form of stress path we need to understand this.

Initially itself we have discussed that for failure in soil to happen and when I say failure it is shear failure we need to have shear stress developed. If we want shear stress to develop then the soil should be subjected to unequal stresses. This means that for isotropic stress there is no failure possibility. Now that is also visible here now if from A you keep on increasing Δσ what will happen is it will simply move along the isotropic line.

Now, for failure to take place, the stress path should deviate and move towards either of these failure line either in compression or in extension. Now, this is not a possibility in the case of isotropic consolidation case. And hence isotropic stress does not induce shear stress and shear failure. This we have to keep in mind. So, that is all about isotropic consolidation. We will quickly see this on t, s, s' and q, p, p' plot as well.

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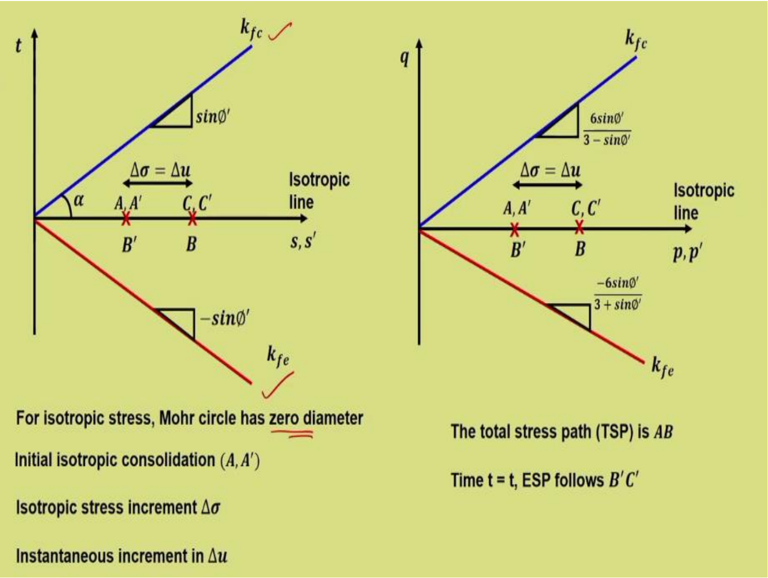

So, in t, s, s', we have given k f compression and k f extension. For isotropic stress, Mohr circle has 0 diameter. There is no Mohr circle for isotropic case because σ_1' and σ_3' is same. Initial isotropic consolidation is A, A'. Isotropic stress increment of Δσ same case instantaneous increment in Δis u. So, that is what is given B, B'. And then the total stress path is AB.

Time t equal to t, ESP follows B' C' as is given. The same thing marked on q, p, p', A, A', B, B', C, C'. So, that is all with respect to isotropic consolidation, a very simple case to understand.

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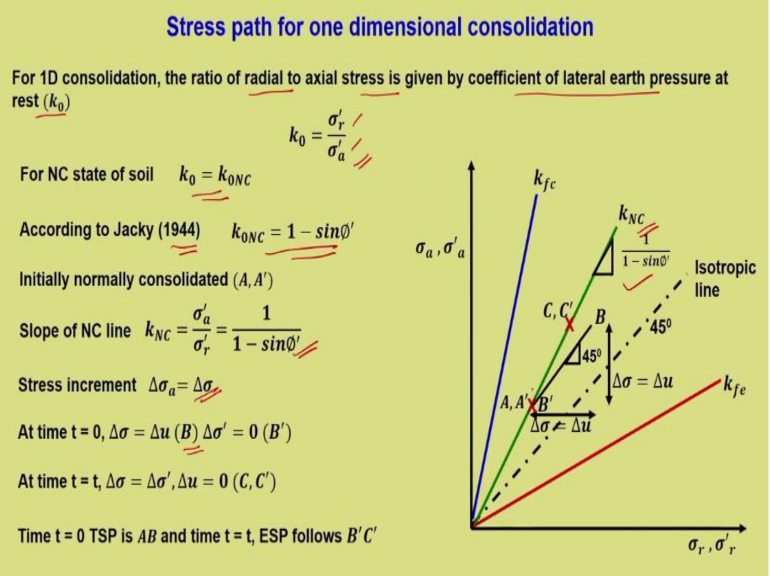

So, now let us move on to the next set of consolidation where we have one dimensional consolidation. Now, the stresses are not same in the radial as well as in the axial direction. So, that is the most important difference as we do one dimensional consolidation. So, in the case of one-dimensional consolidation a special case you can call it as isotropic consolidation. This is a more general case.

So, 1D consolidation, the ratio of radial to axial stress is given by coefficient of lateral earth pressure at rest. Now, we presume that there is no lateral strain that is happening. And that is exactly what is simulated in an oedometer. It has got an infinite confinement because of these metal container or metal cylinder in which these metal ring to be very specific metal ring in which this soil is placed.

So, we do not allow any lateral strain to happen. And, where do you find such a situation? If the loading is happening over a large area then we presume that the amount of confinement is good enough and there is no lateral strain that is happening in the field and that closely relates to one dimensional consolidation. So, that is how we take it. Now, for one dimensional consolidation as it is already told.

The ratio of radial to axial stress is defined by coefficient of lateral earth pressure at rest because there is no strain that is happening. Had it been a passive active then there is a moment. So, then it will deviate. So, presently for this particular lecture and for subsequent analysis we will have the use of coefficient of earth pressure at rest which is k_0. And $k_0 = \sigma_r'/\sigma_a'$.

Please note it always has to be in terms of effective stresses. I think we have already discussed this during while explaining the sampling of soil. For NC state of the soil, k_0 is specifically equal to k_{0NC}. Now, for both NC and OC, the k_0 parameter would change. Now, according to Jacky 1944 which is a very popular equation $k_{0NC} = 1 - \sin \phi'$

You will find different forms of this equation in the literature. For me, in this particular lecture that is not important. What is important is how the radial and the axial stress varies. Now, for that I have taken a very simple case of Jacky's equation which is $k_{0NC} = 1 - \sin \phi'$. And this is applicable for a certain range of ϕ'. I am not going into those details. Even you will find different forms of this equation in the textbooks.

But we will just consider this particular one. So, we need some sort of value for k_{0NC}. And this is very prominent equation that people use. So, now, let us say that the sample is initially normally consolidated. Now, let us get on to the stress path. So, what change will it have? So, now this is an isotropic consolidation line. Initially our point was located here. Now, what will be the change in A, A'? Should we plot it here?

No, because it is governed by this ratio k_{0NC}. So, let us put the failure line. And now this is k NC which is the line corresponding to normally consolidation, so that k $_{NC}$. So, where should A, A' fall? Definitely it is normally consolidated initial point. So, it has to fall on k NC. So, A, A' is on k NC line. Now, slope of k_{NC} is σ_a' upon σ_r' which is the reciprocal of this reciprocal of k_0. So, it will be

$$k_{NC} = \frac{\sigma_a'}{\sigma_r{}'} = \frac{1}{1 - sin\phi'}$$

So, this will be the slope of k_{NC} line. Stress increment, let us say that it is $\Delta\sigma_a$ is incremented by $\Delta\sigma$. So, what happens at time t = 0 again the concept of instantaneous pore water pressure happens and hence there will be point B and B'. Now, remember this is hydrostatic stress condition when it comes to pore water pressure at time t = 0 this you have to keep in mind because it is not relevant but we are discussing it. So, it will help us to understand. When there is an immediate or for an un-drained condition when you are in incrementing the stress if you have an un-drained condition how this stress path would move. So, this is very important. So, that is why time t = 0 is discussed. Now, what would be the possibility of the point A, B.

Since, it is hydrostatic, definitely the stress path should be parallel to isotropic line because that is the hydrostatic line. Hence, this from AB will be at 45 degrees with whatever is the stress increment. Now, here both = $\Delta\sigma = \Delta u$. Now, at time t equal to t that is when there is no issue of pore water pressure effective and total stress path is same, what will happen?

The point will lie on the normally consolidated line itself and that points will be C, C'. So, what happens is there is no point which is joining from here to here because that is not what we are simulating. It is not a stress path which is going from A to B and B to C. AB is only a concept which has to be described here. Actual stress path is A, A', C'. So, that is what it means and then at time t equal to 0 TSP is AB and time t equal to t, ESP follows B' C' or A' C'.

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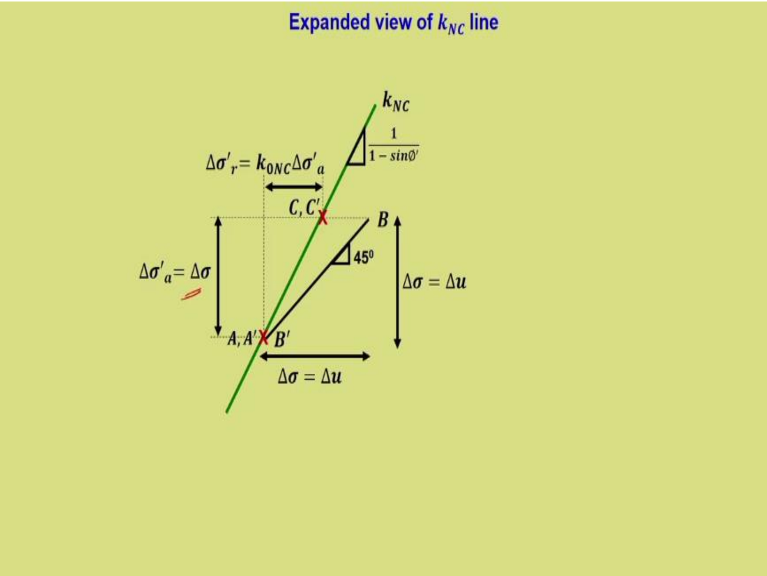

So, I am just showing an enlarged version of k_{NC} line A, A', AB and C, C' is falling on this. Just want to make clear this particular point here AC or A' C' this vertical distance will be $\Delta\sigma_a$' = $\Delta\sigma$. Because we have incremented $\Delta\sigma$ but what will be the horizontal distance I mean this. So, that will be k naught NC times $\Delta\sigma_a$'.

So, that is governed by the slope of k NC. So, here $\Delta\sigma_r$' = k_{ONC} $\Delta\sigma_a$'. So, that we have to keep in mind. So, what ultimately we learned from this exercise is that in stress path we need to determine the slope with which this stress path will progress and how that slope compares with the slope of failure line. So, then we will be able to understand in what manner and in what the distance of this stress path the failure is going to take place.

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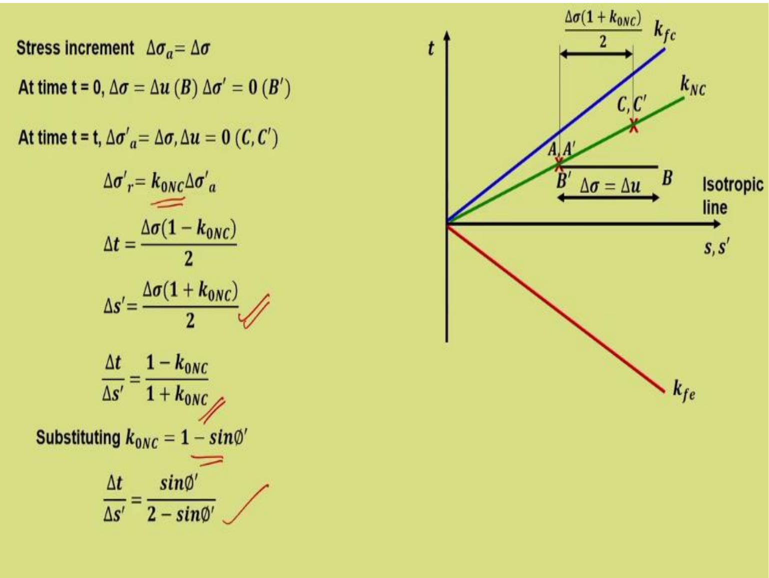

The same thing is repeated for the t, s, s' plot. So, I will not spend much time here. We will just quickly glance through it, same the A, A' B. That is $\Delta\sigma_a = \Delta\sigma$. And the final stress path is A' C'. Now, here we need to understand that this relationship is known and it was fairly simple for us to determine in σ_a, σ_r plot. But here the slope will be slightly different in the case of k_{ONC} line. So,

$$At\ time\ t = 0, \Delta\sigma = \Delta u(B)\Delta\sigma' = 0(B')$$

$$At\ time\ t = t, \Delta\sigma'_a = \Delta\sigma, \Delta u = 0(C, C')$$

$$\Delta\sigma'_r = k_{ONC}\Delta\sigma'_a$$

$$\Delta t = \frac{\Delta\sigma(1 - k_{ONC})}{2}$$

$$\Delta s' = \frac{\Delta\sigma(1 + k_{ONC})}{2}$$

$$\frac{\Delta t}{\Delta s'} = \frac{(1 - k_{ONC})}{(1 + k_{ONC})}$$

The slope is given by $\Delta t/\Delta s'$. So, determining the slope is important in stress path. So, we need to understand what exactly is happening when I say isotropic consolidation one dimensional consolidation and so on. Substituting $k_{ONC} = 1 - \sin\phi'$, the slope can also be written as

$$\frac{\Delta t}{\Delta s'} = \frac{sin\phi'}{2 - sin\phi'}$$

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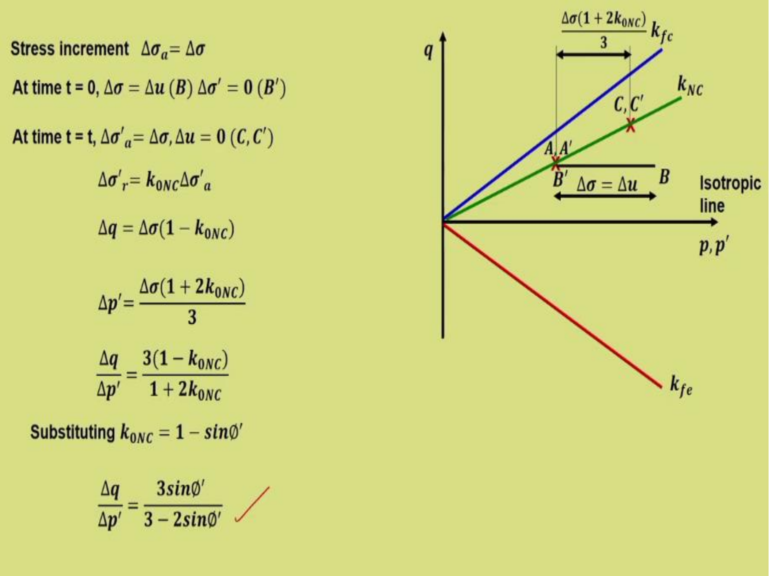

The same is repeated for q, p, p'. The points are not important because now we are converse in with that. We will just see, what is the slope of k_{NC}?

$$Stress\ increment\ \Delta\sigma_a = \Delta\sigma$$

$$At\ time\ t = 0, \Delta\sigma = \Delta u(B)\Delta\sigma' = 0(B')$$

$$At\ time\ t = t, \Delta\sigma'_a = \Delta\sigma, \Delta u = 0(C, C')$$

$$\Delta\sigma'_r = k_{ONC}\Delta\sigma'_a$$

$$\Delta q = \Delta\sigma(1 - k_{ONC})$$

$$\Delta p' = \frac{\Delta\sigma(1 + k_{ONC})}{3}$$

$$\frac{\Delta q}{\Delta p'} = \frac{3(1 - k_{ONC})}{(1 + 2k_{ONC})}$$

$$Substituting\ k_{ONC} = 1 - \sin\phi'$$

$$\frac{\Delta q}{\Delta p'} = \frac{3sin\phi'}{3 - 2sin\phi'}$$

So, all these things are repetitive. So, you can just go through it and you can understand it.

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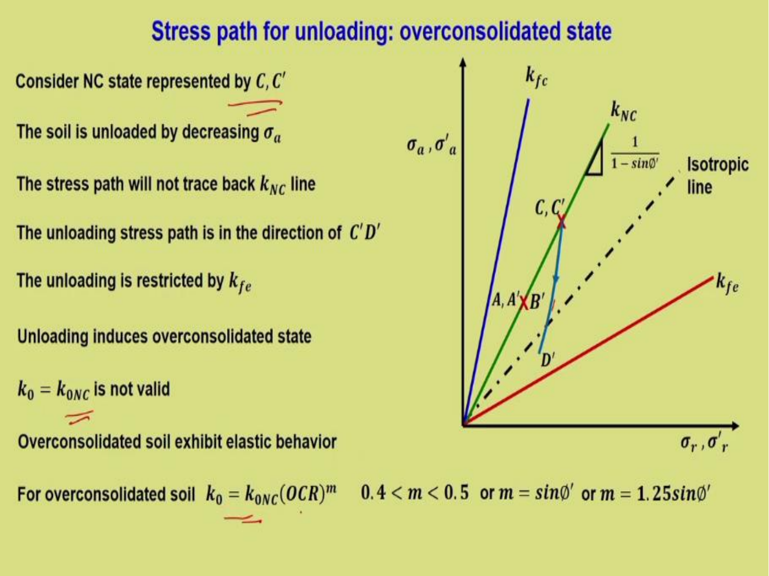

we have completed isotropic consolidation one dimensional consolidation. Now, we will see the next case which is stress path for unloading. What is the relevance of unloading? The relevance of unloading that is that it creates an over consolidated state. I hope if you recall the oedometer test, we do the loading and then we will do staged unloading. What is the relevance of staged unloading?

Because we are also interested in getting the slope of swelling reloading line and that state represents over consolidated state. We have already discussed this in our previous lecture. So, how or what is the relevance of stress path with respect to unloading or over consolidated state. Now, let us again start with σ_{aa}'. The normally consolidated state is represented by C, C'. That is where we ended up with. So, all these things are there.

So, now C, C' is the normally consolidated point. Now, the soil is unloaded by decreasing σ_a. So, what we are doing we are releasing the axial stress σ_a is released. Now, what will happen if we release σ_a? So, we are starting with C, C' the point. When it is released because it has started from A to C', does it mean that when σ a is released that it will trace back k_{NC} line.

It will not and this information we already know because we know this from oedometer test as well. So, when we load soil to a particular point and then unload it, it is not going to trace back rather it takes a different route and that is what we known as the swelling line is the same aspect which is happening here as well. The unloading stress path is in the direction of C' D'. So, the unloading will be here.

Now, this unloading will be restricted by the presence of k_{fe}. So, it does not mean that you can unload the soil the way you want, there will be formation of tension cracks and the soil is not going to be existent. And that is defined by the bound is defined by k_{fe}. Now, unloading what have what does unloading do to the soil? Unloading induces over consolidated state into the soil. Now k naught equal to k_{ONC} is not valid.

And that is very relevant from this plot also. Only those points on k_{NC} line, it is valid. The moment it deviates it is not valid. So, $k_O = k_{ONC}$ is not valid for the unloading line. Now, the information that we have is that over consolidated state of the soil exhibits elastic behavior. Why? Because when you load to a certain point, we know what is the pre-consolidation pressure?

Now, pre-consolidation pressure, we have seen it is also stated as yield stress. Now, once it is unloaded from it then again reloading till the point it reaches pre-consolidation it is bound to have better stiffness and hence it will exhibit more like an elastic material elastic behavior. It will exhibit elastic behavior. So, you can safely assume that the over consolidated soil behaves elastically.

So, over consolidated soil, it is normally reported that $k_o = k_{ONC}(OCR)^m$. Again, there are several relationships which are reported in the literature. I am just selecting this which is generally found in literature. And the value of m is in the range of 0.4 to 0.5 or m is taken equal to $or\ m = sin\phi'\, or\ m = 1.25 sin\phi'$. So, different researchers have proposed different type of relationship for m and these are the ones which are generally considered. And hence k_O for OC soil will depend upon OCR.

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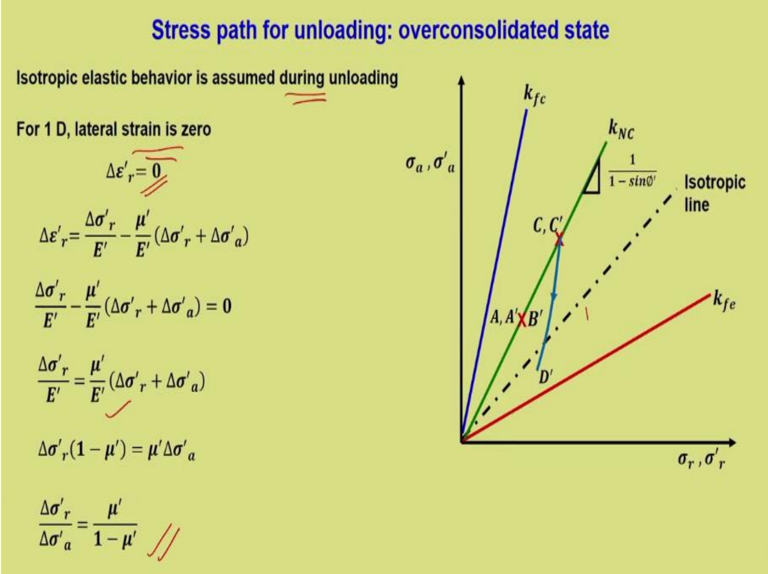

$$\frac{\Delta\sigma'_r}{\Delta\sigma'_a} = \frac{\mu'}{1 - \mu'}$$

So, we have understood that we need to understand in what manner this C' D' is going to move. It can have multiple possibilities but where exactly it is going to move that will depend upon the elastic behavior of the soil. So, that particular assumption is very important to understand the slope of C' D'. So, elastic isotropic behavior is assumed during unloading. The logic is because it has become an over consolidated state.

For one dimensional, lateral strain is 0. This we have already discussed. So, $\Delta\epsilon'_r = 0$. And we also have the relationship

$$\frac{\Delta\sigma'_r}{E'} - \frac{\mu'}{E'}(\Delta\sigma'_r + \Delta\sigma'_a) = 0$$

$$\frac{\Delta\sigma'_r}{E'} = \frac{\mu'}{E'}(\Delta\sigma'_r + \Delta\sigma'_a)$$

So, this gives the relationship of

$$\Delta\sigma'_r(1 - \mu') = \mu'\Delta\sigma'_a.$$

So, this will move with respect to this particular ratio defined by the Poisson's ratio of the soil.

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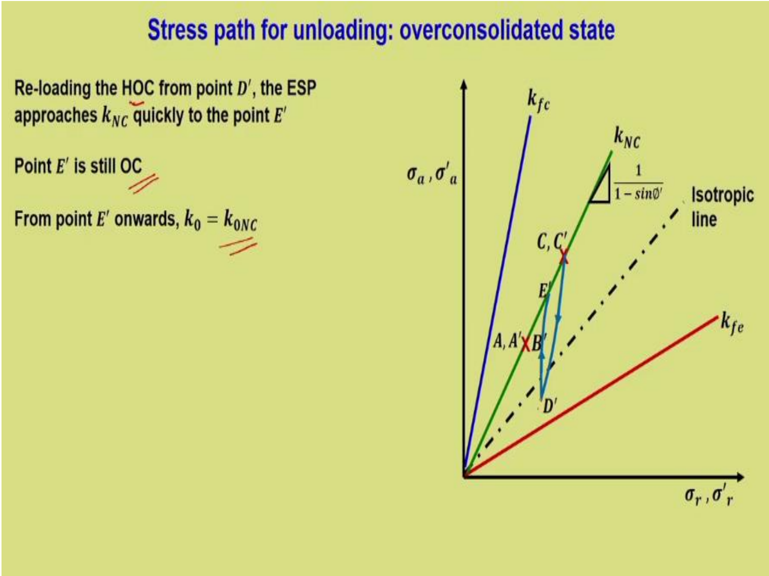

So, reloading the heavily over consolidated from point D’. Now, if you happen to reload from this particular point D’, now let us say that based on the location of point D’ the soil is heavily over consolidated. HOC means heavily over consolidated. Now, if you are reloading it from this point D’ then the effective stress path for reloading it quickly approaches the k_{NC} line. That is the ESP approaches k NC quickly to the point E’.

So, this is what it means. Now, in oedometer test, we have idealized it to be a straight line. So, loading, unloading and reloading it is traces the same path. But we know that in reality that is not going to happen it is not the same path. So, that difference is shown clearly in this stress path. So, when you reload it, it is going to approach the k_{NC} line very quickly. And that reaches at point E’. It is not joining at C, C’. Please remember that.

So, point E’ is still over consolidated. So, that means when it reaches E’ that point is still under over consolidation even though it has approached the normally consolidated line. But after E’ when it is subsequently loaded it will move along the NC line itself. So, that is the only difference. So, instead of joining C, C’, it joins at E, E’. It approaches the NC but then we have to remember that the point E’ is still OC. Why?

It has not reached the point C, C’. It is still less than the stresses at C, C’. Now, from point E’ onwards $k_O = k_{ONC}$.

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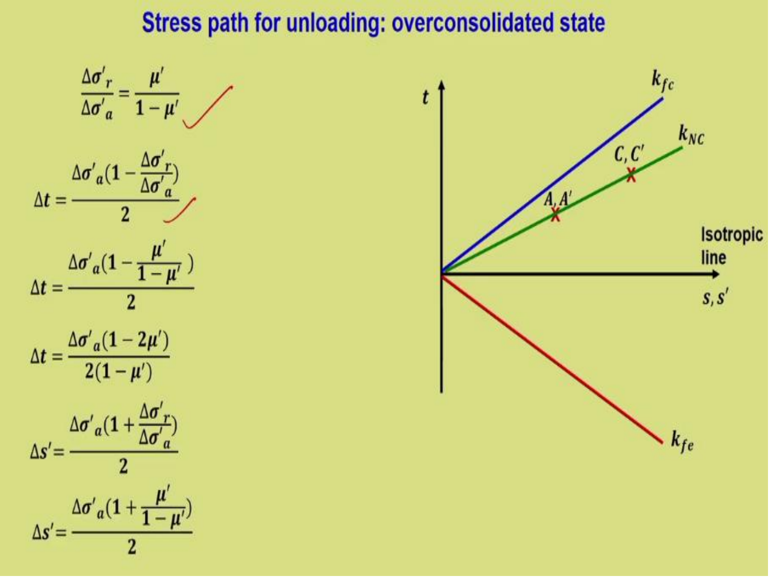

The same thing is plotted for t, s, s' plot. So, that we know this relationship now,

$$\frac{\Delta\sigma'_r}{\Delta\sigma'_a} = \frac{\mu'}{1-\mu'}$$

We have to use this for finding out what will be the slope in t, s, s' plot. So, this Δt = Δσ a'. This now by this time you are already converse in with this formulation. So, I will not spend much time here.

$$\Delta t = \frac{\Delta\sigma'_a\left(1-\frac{\Delta\sigma'_r}{\Delta\sigma'_a}\right)}{2}$$

Substituting for $\Delta\sigma_r'$ this ratio here we have

$$\Delta t = \frac{\Delta\sigma'_a\left(1-\frac{\mu'}{1-\mu'}\right)}{2}$$

$$\Delta t = \frac{\Delta\sigma'_a(1-2\mu')}{2(1-\mu')}$$

$$\Delta s' = \frac{\Delta\sigma'_a\left(1+\frac{\Delta\sigma'_r}{\Delta\sigma'_a}\right)}{2}$$

$$\Delta s' = \frac{\Delta\sigma'_a\left(1+\frac{\mu'}{1-\mu'}\right)}{2}$$

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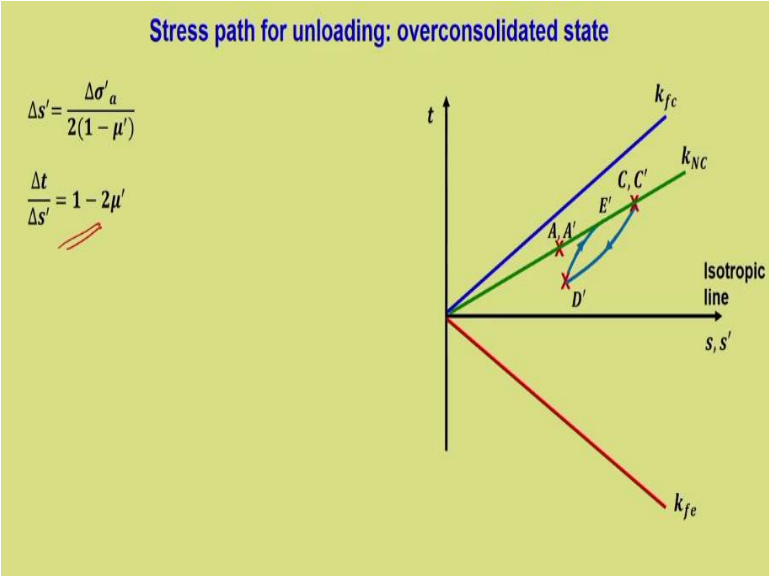

$$\Delta s' = \frac{\Delta\sigma'_a}{2(1-\mu)}$$

$$\frac{\Delta t}{\Delta s'} = 1 - 2\mu'$$

So $\Delta t / \Delta s'$ is the required slope. So, what turns out? Again, we are determining the slope in st plot. For any stress path plotting, this is what we have to do. Depending upon the variation, we need to determine the slope. So, this how it looks like is the same as that of σ_a σ_r' plot.

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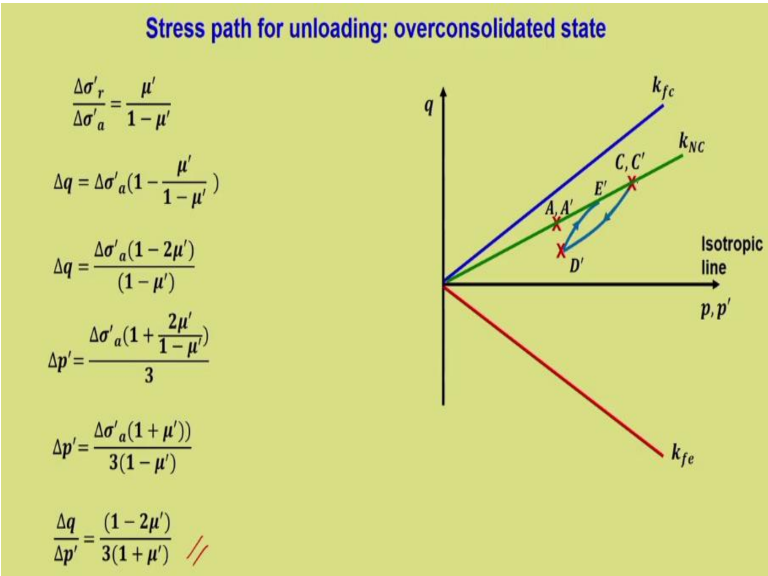

When we repeat this for q, p, p', we will have

$$\frac{\Delta\sigma'_r}{\Delta\sigma'_a} = \frac{\mu'}{1-\mu'}$$

$$\Delta q = \Delta\sigma'_a\left(1 - \frac{\mu'}{1-\mu'}\right)$$

$$\Delta q = \frac{\Delta\sigma_a'(1 - 2\mu')}{1 - \mu'}$$

$$\Delta p' = \frac{\Delta\sigma_a'\left(1 + \frac{2\mu'}{1 - \mu'}\right)}{3}$$

$$\Delta p' = \frac{\Delta\sigma_a'(1 + \mu')}{3(1 - \mu')}$$

So, this will give this slope

$$\frac{\Delta q}{\Delta p'} = \frac{(1 - 2\mu')}{3(1 + \mu')}$$

And the slope of the unloading line will look like C’ D’ and D E’. So, that is how it looks like. So, in all the cases we need to determine the slope of the stress path and that is exactly what we have to do. So, these are some of the common cases like isotropic one dimensional consolidation and unloading which is over consolidation. There are a few more cases that we need to discuss and that we will see in the next lecture.

For the time being, let us stop here. And we will see some more cases in the next one and then that will be followed by the triaxial testing. So, that is all for now, thank you.