Advanced Soil Mechanics
Prof. Sreedeep Sekharan
Department of Civil Engineering
Indian Institute of Technology – Guwahati

Lecture - 34
Failure line in stress path

Welcome back, all of you. In the last lecture, we have started a new module that is on stress path and we have seen what is the importance of stress path? Now, we will move on to another particular aspect of stress path. That is what how one can make the stress path plot meaningful.

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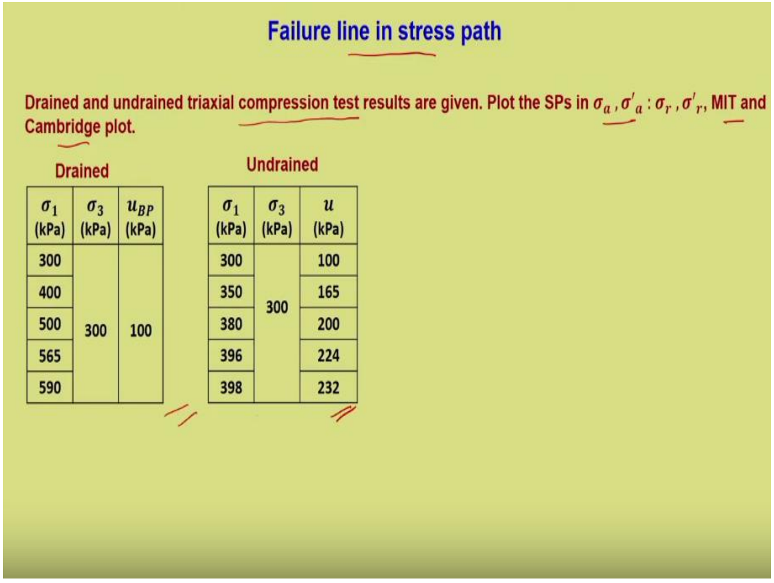

So, let us start with today's lecture and we will start with an example. So, before moving on to the actual aspect of failure line in stress path, we will just try to see a simple example of a drained and un-drained triaxial compression test, which are given. And, we have to plot this stress path in σ_a, σ_a', σ_r, σ_r' plot, MIT plot and Cambridge plot. So, we will see how this stress path will look like.

So, this is the data for the drained test, drained triaxial compression test where σ_1 is given 300, 400, 500, 565, 590 and σ_3 is constant, which is 300 and back pressure is given as 100. I hope you remember what is meant by back pressure. And the un-drained test results are given. Again, there is an increase in σ_1 and σ_3 is constant. And, at every instant of loading, we have the pore water pressure as well. So, this is how it looks like. So, this is the preliminary data which is given.

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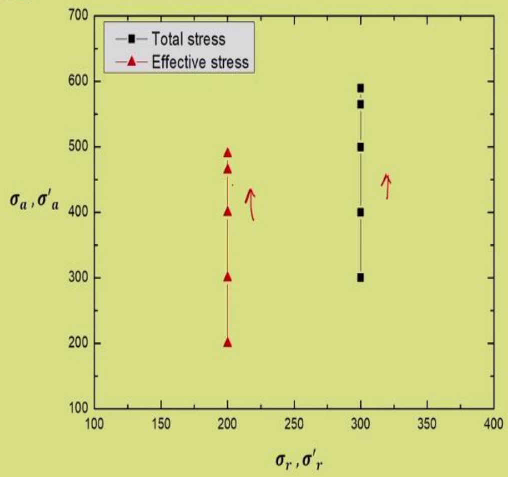
Triaxial compression $\sigma_a, \sigma'_a : \sigma_r, \sigma'_r$

Drained

σ_1 (kPa)	σ_3 (kPa)	u_{BP} (kPa)
300	300	100
400		
500		
565		
590		

σ_a (kPa)	σ_r (kPa)	σ'_a (kPa)	σ'_r (kPa)
300	300	200	200
400	300	300	200
500	300	400	200
565	300	465	200
590	300	490	200



So, let us first try with σa, σr, σa', σr' plot with a drained envelope. So, this is the drained data which is given. Now, is there anything to be changed in this? Yeah, first of all, we need to understand in the case of compression σ axial is equal to σ_1. So, we have written σ_a and σ_r now separately. σ_a is the same as σ_1. And σ_r is nothing but σ_3.

And for every σ_a, it is 300. Then, we have σ_a' and σ_r'. So, there will be total and effective stress path that hope you remember. For every loading, there can be total stress path and effective stress path. Those, which is plotted as per total stresses gives total stress path and the other one gives effective stress path. So, σ_a', σ_r' is given. How did we get?

Because there is a back pressure of 100 kilopascal. So, we need to minus it from all these minus 100 will give σ_a'. And σ_r minus 100 that is 200 gives σ_r'. So, how the plot will look like. So, the plot will look like this. So, σ_a σ_a' on y axis and σ_r σ_r' on x axis. So, here the total stress is plotted. We can see from the figure that σ_a is increasing σ_r is constant. So, that is what is given.

And σ_r' is constant at 200 and σ_a' keeps increasing. So, this is increasing in this direction for both effective stress and this total stress as you can see here and this is effective stress. So, now what we can make out from this plot. Yes, we have plotted the variation in stresses as it progresses towards failure. But, from this particular plot, what is that vital information that we are getting.

In fact, we are missing something in this particular plot. So, we will sum up in the end what is that we are missing.

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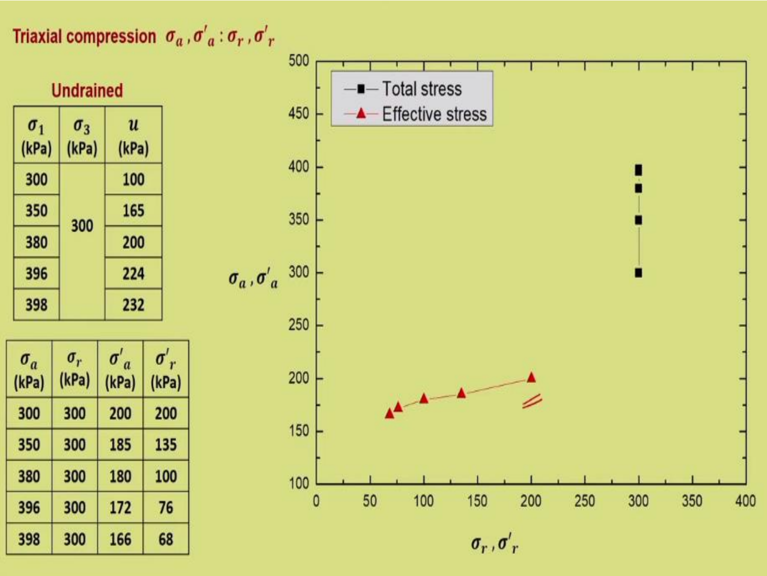

So, now for the same triaxial compression for the un-drained test results, so, we have un-drained test results and again, we need to find out what is σ_a, σ_r that remains same, same data and 300 is the σ_r. But σ_a' will change corresponding pore water pressure has to be detected. So, that is how it looks like 300 minus 100, 200 like 398 minus 232, 166. So, σ_r' here you can see it is no longer constant.

So, in initial case, it was constant. Now, even though σ_r is constant, σ_r' keeps changing so depending upon the pore water pressure. So, you can see that 300 minus 100, 200 whereas 300 minus 232, 68. So, how this particular stress path will look like. Now, the stress path has changed corresponding to drained and un-drained. Now, things have changed. But you can note that the pattern of total stress path is more or less same.

But total stress path is same then the effective stress how it changes in un-drained test is totally different from what we have seen in the previous slide. Like both were vertically rising but in this particular case you can see that it shifts towards left. Still, we are not able to make out much from this particular plot. Let us go ahead because let us see. We will be as I told in the beginning we will be considering 3 stress path plots.

Basically, σ_a σ_a' then st plot and qp plot. So, we will see for all the 3, how it looks like. Then, we will come back and summarize.

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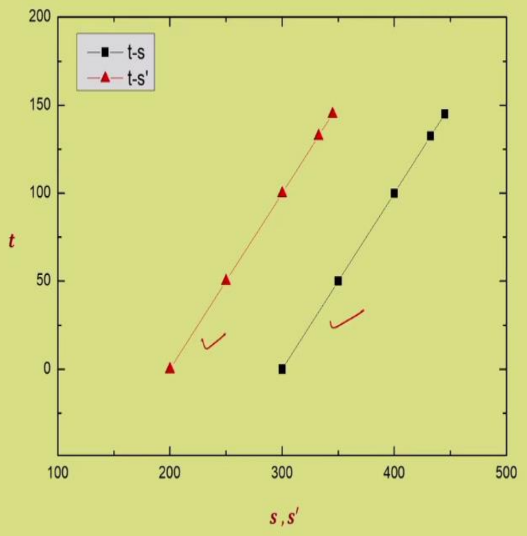
$t : s, s'$

Drained

s (kPa)	t (kPa)	s' (kPa)	t (kPa)
300	0	200	0
350	50	250	50
400	100	300	100
432.5	132.5	332.5	132.5
445	145	345	145



Next is t, s, s' plot with the same data. So, drained data remains same but whatever drain data we have seen in the previous slide, so, this one. So, this is the drain data. Now, from this drain data, we will have to find out what is the data for t, s, s' plot and we have the relationship for t and s, s'. So, that is given here. So, s and t is given for drained and s' will be this minus the back pressure is there. So, that will get changed.

Whereas in the case of t, there is no change as we have seen earlier that t is not affected by pore water pressure. So, then how the stress path will look like. Again, now the pattern of stress path has changed in ts plot. So, this is total stress and this is effective stress. But the pattern of both effective and total stress remains same in the case of drained stress path.

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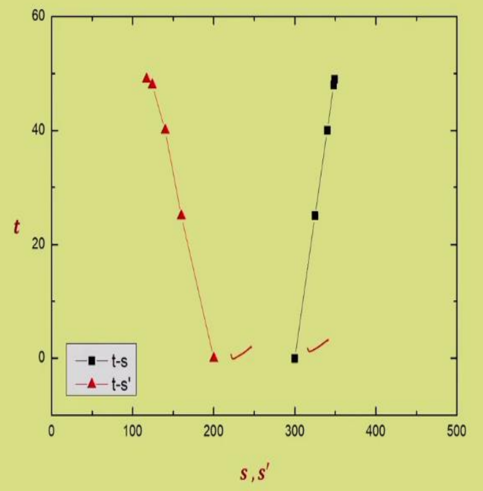
$t : s, s'$

Undrained

s (kPa)	t (kPa)	s' (kPa)	t (kPa)
300	0	200	0
325	25	160	25
340	40	140	40
348	48	124	48
349	49	117	49



So, corresponding un-drained, so, we have pore water pressure. So, accordingly s will change and we know that s’ is nothing but s - u. So, that relationship we have seen. So, then 300 325, this is s, t is 0, this up to 49. That remains same even for effective stress whereas total stress s effective stress s’ will change. So, this is the data for s t and s’ t. So, how it will look like. Again, the patterns have changed.

So, in this total stress path, the pattern is more or less same whereas in the case of effective stress it moves leftwards. So, this how it looks like for t, s, s’ plot.

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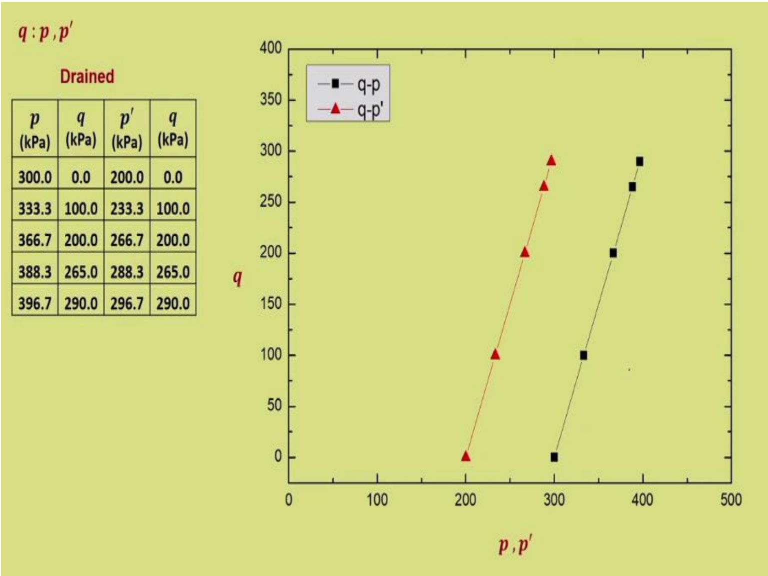

Now, for q, p, p’, it is more or less same as that of st plot. Again, we need to find out p and q. I hope you remember, what is p and q? If not, please refer back. And corresponding p’ is obtained and q is obtained. Note that q remains same. q is equal to q’. So, in this particular case for the drained envelope, it is very much similar to st plot and it the pattern of both total stress and effective stress paths are same.

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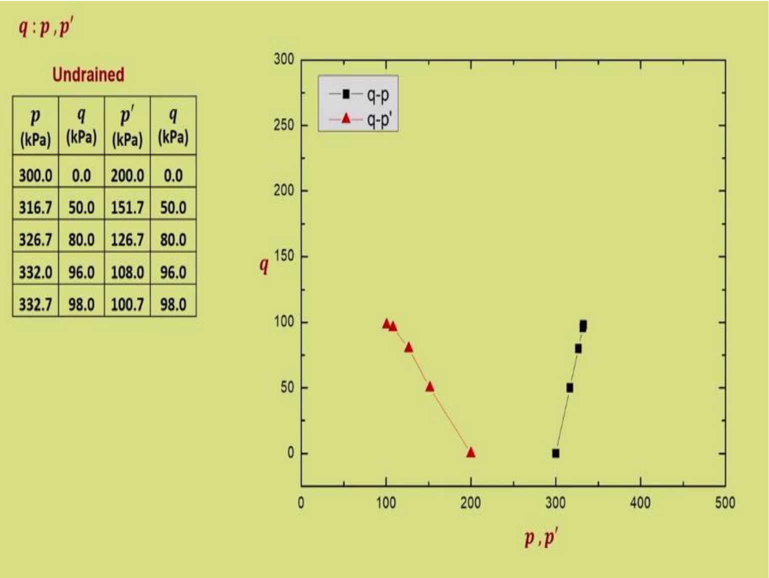

Now, for the corresponding un-drained case for p q and p' q. So, that will give you again the total stress path maintains the same kind of pattern as that of drained but in the un-drained effective stress path moves towards left. So, we have now presented a very simple example how the total stress total stress path and the effective stress path looks like in different stress path plots.

So, now what we have understood from these stress path plots. Why we are plotting this and why we are studying this. We want to understand when the soil will fail or and from the beginning how it traverses how this stress changes up to its failure state. So, definitely that information is missing from whatever we have discussed till now. Now, there has to be a bound which gets attached to each of these stress path plots.

Without which this information is missing. So, that is the importance of today's lecture, failure lines in stress path plots.

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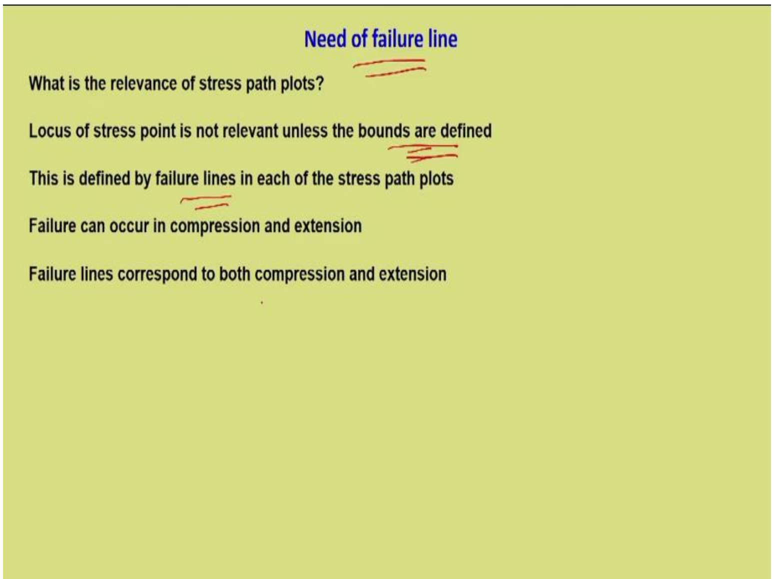

So, we will now get on to that what is meant by failure line. So, it is need of failure line in stress path plot. What is the relevance of stress path plots? So, to make you understand this, we have explained the given examples so in all the 3 plots we have seen. So, what is the relevance of stress path plots? Locus of stress point, stress point means variation of any 2 stress parameters. So, that gives you a particular point. So, how this point changes?

So, locus of stress path point is not relevant unless the bounds are defined. We should know like there is a space given to us that is st plot or qp plot or whatever in that domain what are the possibilities for this stresses to change for a given soil. This is very important. So, that is why it is told that we need to know the bounds within which this stress path points can change. So, that bound is defined by failure lines in each of the stress path plots.

Failure can occur both in compression and extension. Normally, it is a general tendency that we discuss only about compression and we do not really talk about extension. But, we should also understand that extension is not like you are holding soil and pulling it apart. That is not the only case of extension. Extension can be created even by compressive stresses the manner in which you are applying. I think some examples we have seen in Module 2.

So, here when we are discussing for stress path, we will discuss both. That is one is the mirror image of the other. So, we will discuss both. So, failure can occur both in compression and extension. Failure lines correspond to both compression and extension because of this.

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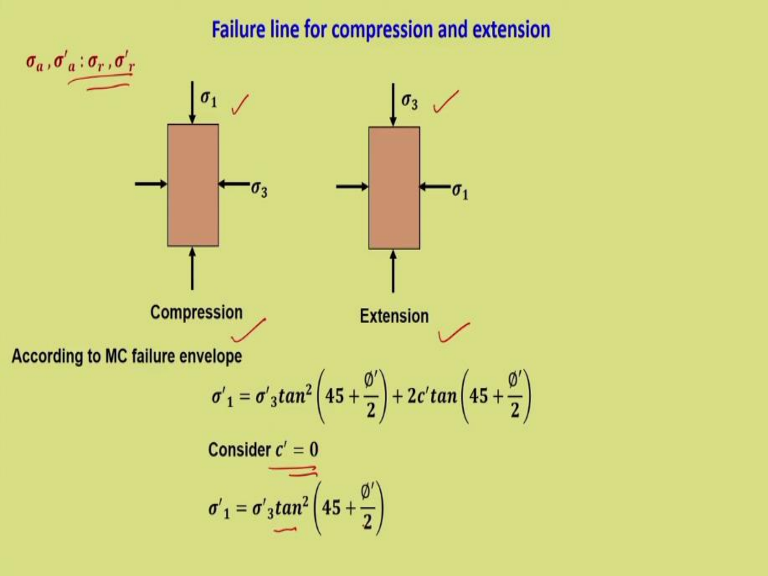

So, first let us see σ_a, σ_a', σ_r, σ_r'. Now, I have already told you before that let us identify all these stresses in terms of axial stress and radial stress. The reason you will understand in this particular slide. So, here is a typical case of compression where the triaxial compression what we generally do in the lab. So, this σ1 is the major principal stress, σ3 is the minor principal stress, the confining stress.

Now, here let us see that both are compression but here it is a case of extension. Why? Here you can see that σ1 is major and σ3 is minor. So, please understand that when there is a larger stress which is acting readily it has a tendency to squeeze the soil apart. So, squeezing is just like an extension. So, same compression but in a different manner of stress application can result in extension.

So, that is what it is not the direct extension that we are talking about. So, these are 2 cases. Now, you can see that the axial stress in this. And the axial stress in this is changing. So, depending upon whether it is compression or extension σ_a, σ_a' will change, σ_r, σ_r' will change. So, according to Mohr Coulomb failure envelope now that is the only failure criterion that we know.

That is Coulomb's envelope and the extended version of the modified version which is the Mohr Coulomb failure envelope. These are the failure criteria we know. How the soil will fail based on this. So, according to Mohr Coulomb failure criterion, we know that σ 1' is equal to

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

Now, for convenience, let us assume c' to be 0. So, that it becomes a bit simplified. Otherwise also, we know that c' is a parameter that can happen only if there is cementation or if there is dilation. So, for the time being, let us consider c' equal to 0. So, we can write

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

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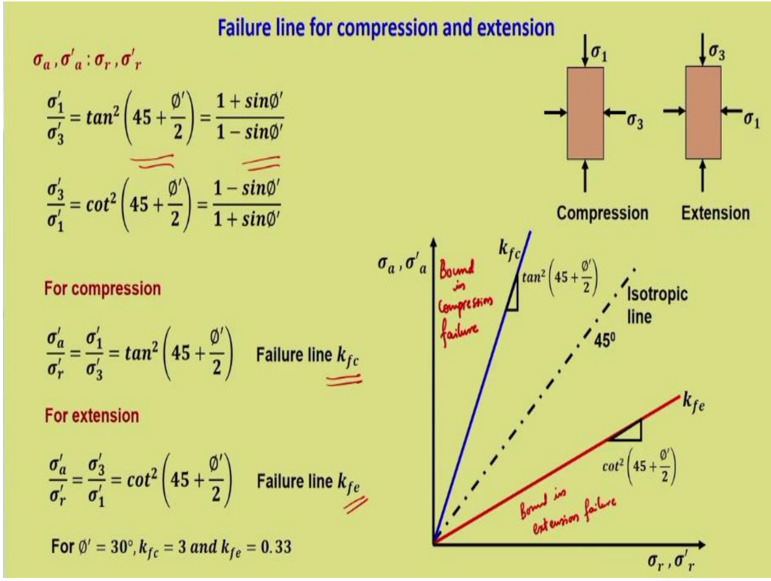

So,

$$\frac{\sigma_1'}{\sigma_3'} = \tan^2\left(45 + \frac{\phi'}{2}\right) = \frac{1 + \sin\phi'}{1 - sin\phi'}$$

$$\frac{\sigma_3'}{\sigma_1'} = \cot^2\left(45 + \frac{\phi'}{2}\right) = \frac{1 - \sin\phi'}{1 + sin\phi'}$$

So, in a way, we have now defined the failure lines in the σ_a, σ_a' plot.

For compression

$$\frac{\sigma_a'}{\sigma_r'} = \frac{\sigma_1'}{\sigma_3'} = \tan^2\left(45 + \frac{\phi'}{2}\right) \; Failure\ line\ k_{fc}$$

So, failure line is denoted as k_{fc} where c stands for compression. So, here σ_a, σ_a', σ_r, σ_r', this is the 45 degrees line and then the k_f compression line that is the bounds of compression failure is given by this line. So, this is nothing but the bound in compression failure. So, this line governs that any soil mass which is starting from any initial point in this domain has to fail here. So, there is no stress state which is possible for a given soil above this line. So, that is what it means. So, we have now defined one bound. Similarly, for the other bound that is for the extension case.

For extension

$$\frac{\sigma_a'}{\sigma_r'} = \frac{\sigma_3'}{\sigma_1'} = \cot^2\left(45 + \frac{\phi'}{2}\right) \; Failure\ line\ k_{fc}$$

So, in the figure the red line is the failure line for extension. So, k_f extension, this is denoted by k_f extension. And this is $\cot^2\left(45 + \frac{\phi'}{2}\right)$ is the bound in extension failure.

And for example, $For\ \phi' = 30^0,$ then you substitute this you will get k_f of compression will be equal to 3. To show that the relative position of k_f compression and k_f extension with respect to isotropic line. Now for isotropic line you have k is equal to 1. Now, here k_f of compression is equal to 3. So, the slope will increase and k_f of extension is 0.33. So, slope will be less than 1.

So, the relative position of k_f compression and k_f extension is known. Now, any stress state that we have discussed in the previous slides if you put we know that where it should fail. Since the friction angle for that particular example is not given. We can find out and then we can see where it falls like. So, we will not get into that. Just want to make sure that any initial point we know where the soil would fail. Now in what manner it will fail whether it is a straight upwards or moving towards left it depends upon the type of the test. So, this is about σ_a, σ_a', σ_r, σ_r' failure line.

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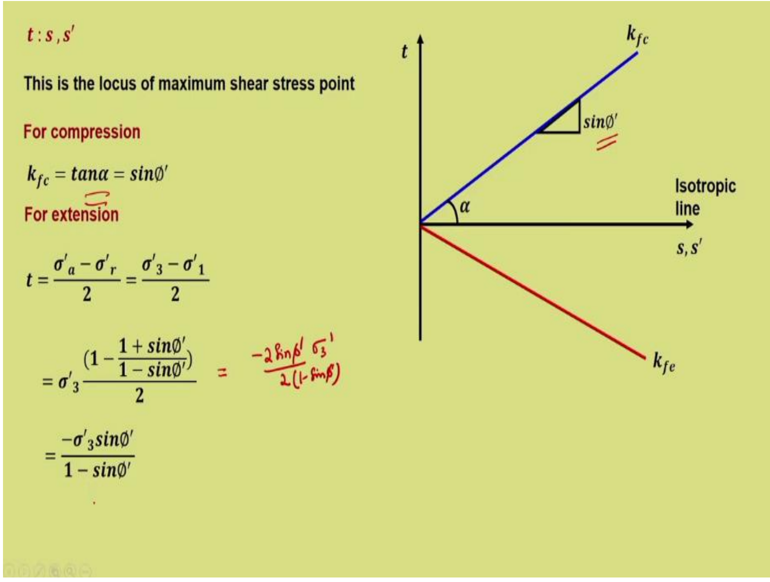

$$= \frac{-\sigma_3' sin\phi'}{1 - sin\phi'}$$

So, next we will come to t, s, s' plot. Now, we have already discussed this in our previous lecture. If you remember the stress representation in Module 2, we have touched upon MIT plot. And we have derived the relationship between MIT plot and Mohr Coulomb failure envelope parameters. So, I will not take much time in this particular lecture. For compression, if you remember, we have obtained $k_{fc} = tan\alpha = \sin\phi'$

Where alpha is the inclination of origin of the line that joins the origin to the maximum shear stress points because here c' is 0. If you join all the maximum shear stress point that line extended that will give a' is the intercept and α is the angle. So, if cohesion is present, so a will also be present. If cohesion is not present, a will not be present. But the inclination of the failure line is α and $tan\alpha = \sin\phi'$

For compression, we have already derived. So, this is how it looks like t, s, s' and we know that this is the isotropic line k_f compression. Here α is the angle. So, here the inclination is equal to $\sin\phi'$ or the slope is equal to sine phi'. Now, what about for extension? For extension, we have not derived. So, we will see that what will be the slope for extension. For that

$$t = \frac{\sigma_a' - \sigma_r'}{2}$$

Now, for the extension, σ_a'= σ_3'. Now, this is some important points or the minor points which we have to keep in mind. We need to know what stress is acting in axially or radially. So, in this case σ_3' is σ_a'. So,

$$t = \frac{\sigma_3' - \sigma_1'}{2}$$

If you take σ_3' outside, we have

$$t = \frac{\sigma_3'\left(1 - \frac{1 + \sin\phi'}{1 - sin\phi'}\right)}{2} = \frac{-2sin\phi'\sigma_3'}{2(1 - sin\phi')}$$

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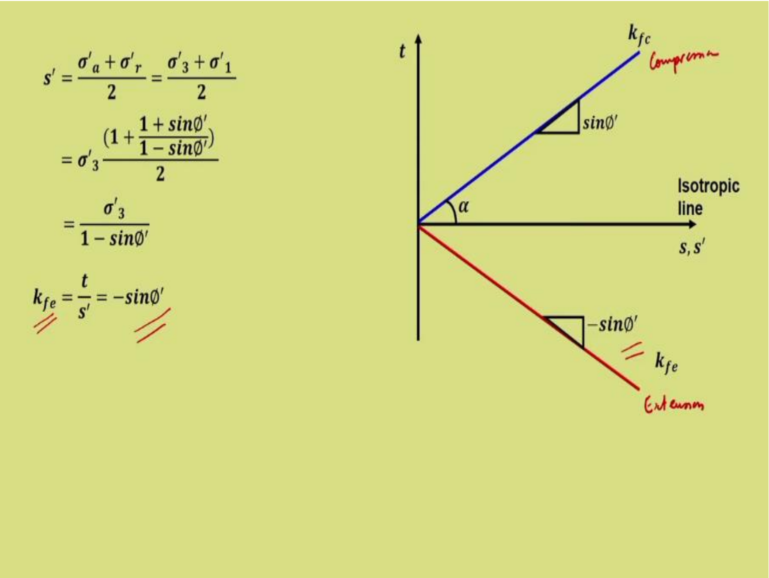

Similarly, s'

$$s' = \frac{\sigma_a' + \sigma_r'}{2} = \frac{\sigma_3' + \sigma_1'}{2}$$

Same procedure, only thing is here the sign changes.

$$= \frac{\sigma_3'\left(1 + \frac{1 + \sin\phi'}{1 - sin\phi'}\right)}{2}$$

$$= \frac{\sigma_3'}{1 - sin\phi'}$$

$$k_{fe} = \frac{t}{s'} = -sin\phi'$$

So, that will give $-sin\phi'$ as the slope. So, we have defined the failure line in both t, s, s' plot for both compression as well as extension.

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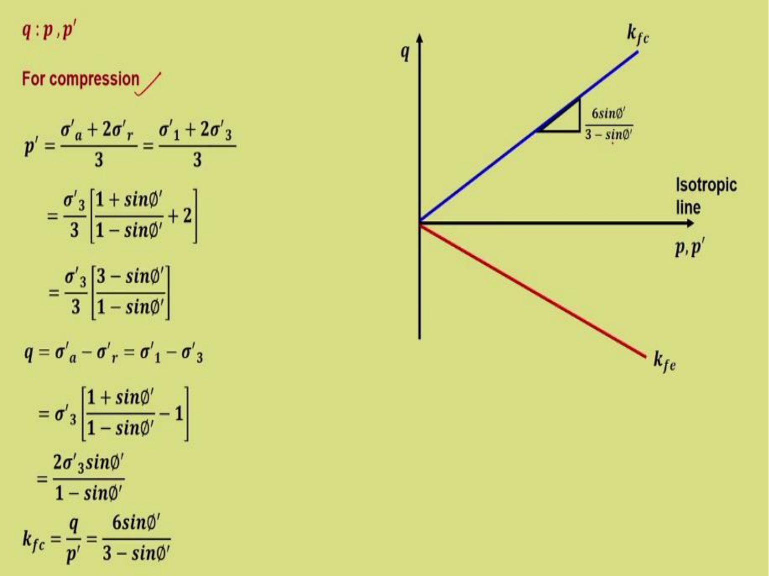

So, next is q, p, p’ plot for compression. Again, this particular slope, we have already defined in Module 2 during stress representation. So, we have

$$p' = \frac{\sigma'_a + 2\sigma'_r}{3} = \frac{\sigma'_1 + 2\sigma'_3}{3}$$

$$= \frac{\sigma'_3}{3}\left[\frac{1+\sin\phi'}{1-sin\phi'}+2\right]$$

$$= \frac{\sigma'_3}{3}\left[\frac{3-\sin\phi'}{1-sin\phi'}\right]$$

$$q = \sigma'_a - \sigma'_r = \sigma'_1 - \sigma'_3$$

$$= \sigma'_3\left[\frac{1+\sin\phi'}{1-sin\phi'}-1\right]$$

$$= \frac{2\sigma'_3 sin\phi'}{1-sin\phi'}$$

$$k_{fe} = \frac{q}{p'} = \frac{6sin\phi'}{3-sin\phi'}$$

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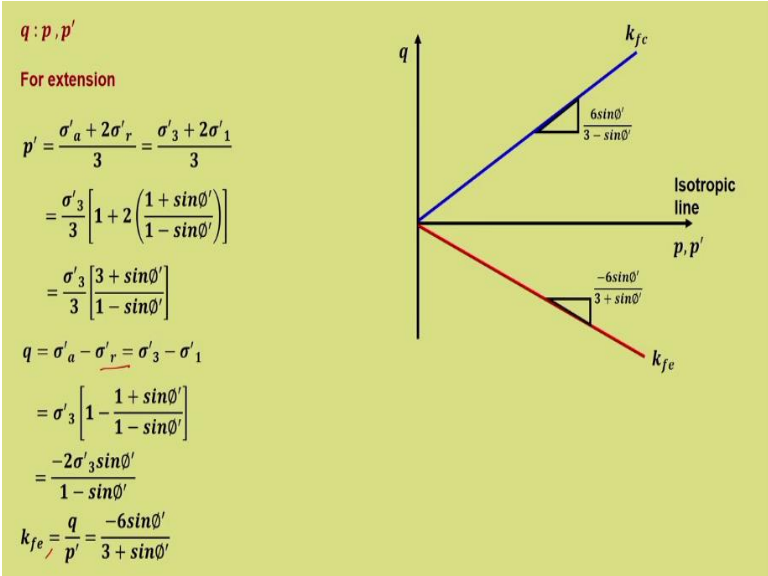

Now, for extension,

$$p' = \frac{\sigma'_a + 2\sigma'_r}{3} = \frac{\sigma'_3 + 2\sigma'_1}{3}$$

$$= \frac{\sigma'_3}{3}\left[1 + 2\left(\frac{1 + \sin\phi'}{1 - sin\phi'}\right)\right]$$

$$= \frac{\sigma'_3}{3}\left[\frac{3 + \sin\phi'}{1 - sin\phi'}\right]$$

$$q = \sigma'_a - \sigma'_r = \sigma'_3 - \sigma'_1$$

$$= \sigma'_3\left[1 - \frac{1 + \sin\phi'}{1 - sin\phi'}\right]$$

$$= \frac{-2\sigma'_3 sin\phi'}{1 - sin\phi'}$$

$$k_{fe} = \frac{q}{p'} = \frac{-6sin\phi'}{3 + sin\phi'}$$

So, what we have done? for all these stress path plots now the bounds are defined. Now, in whatever manner we need to study the stress path, we can do that.

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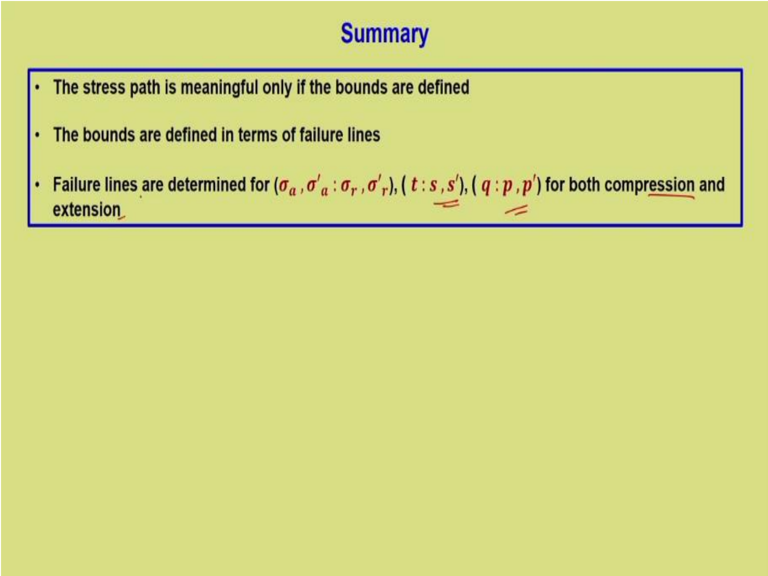

So, summary of today's lecture, this stress path is meaningful only if the bounds are defined. The bounds are defined in terms of failure lines. And in this lecture, the failure lines are determined for σ_a, σ_a', σ_r, σ_r' plot, t, s, s', q, p, p' for both compression and extension. So, that is all for today's lecture. In the next lecture, we will see, what are the stress paths for common cases that we encounter?

The common type of stress variations, we will try to see the stress path. So, that is all for now, thank you.