Advanced Soil Mechanics
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Lecture - 33
Stress path and representation

Welcome back, all of you. With last lecture, we have completed Module 2 which was basically on shear strength. We discussed various aspects which are important in terms of strength of the soil. And today we will be starting a new module and that is on stress path. So, we will see what is the importance of stress path vis-à-vis the shear strength of the soil and how this information becomes handy.

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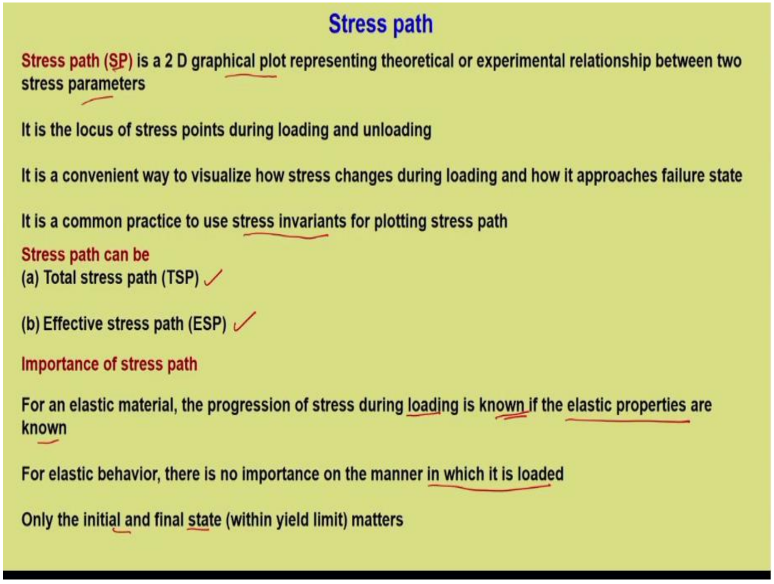

So, we will move on to what is known as stress path. So, stress path in short it is SP. I am calling it as SP is a 2-dimensional graphical plot representing theoretical or experimental relationship between 2 stress parameters. If this is the discussion if this is the definition of stress path I think we have already seen a stress very relevant stress path in terms of Mohr Coulomb failure envelope and in terms of τ and σ' that is shear stress versus normal stress because these are also 2 stress parameters and the plot indicates the relationship between these two τ and σ. That is shear stress and normal stress but relevant to the failure condition. So, that is what we have seen in terms of Mohr Coulomb. So, it is a similar kind of thing that it relates the variation of 2 different stress parameters.

So, that is what we understand by stress path and it is a 2-dimensional graphical representation. It is the locus of stress points during loading and unloading. So, whatever is happening to the soil during loading, what is the variations in the stresses? During loading as well as during unloading can be captured and when it is plotted it becomes a stress path. It is a convenient way to visualize how stress changes during loading and how it approaches failure state.

So, this is the difference what we will find generally when we discuss about stress path and the Mohr Coulomb failure space that we have discussed before. We never talked about what is the manner in which these stress changes and then how it approaches failure. We have simply defined the bound or the failure state within which the soil mass would operate. So, that is how we have conceived before.

But in the case of stress path, it clearly shows how the soil changes in terms of its stresses during loading and how it reaches its failure state. This is a very important information for non-linear materials like soils. So, it is a common practice to use stress invariants for plotting stress path. Now, you should understand the relevance of why we have discussed about stress invariants before.

So, it is very important to understand what is the why we one should go for stress invariants. Now, that those aspects we have already discussed. So, I am just go just highlighting that like it is very important that certain things are plotted in a particular manner. For example, stress path it is quite handy when you use stress parameters like stress invariants than the normal stresses.

Normal stress means not σ_n in general terms like if you use shear stress and normal stress rather than that it is always convenient to use stress invariants. Now, stress path can be total stress path and effective stress path. If it is drained or un-drained response in the case of drained response, total stress response and the effective stress response will be same. But if it is un-drained response, we will have both because there will be pore water pressure. Now, what is the implication of both? All these things, we will see in the subsequent lectures. Importance of stress path, for an elastic material, the progression of stress during loading is very well defined and it is known if the elastic properties are known. How do you generally represent this the mechanical behavior of elastic materials?

We have the stress strain response what is happening. The moment you know what is the initial state and the elastic properties more or less everything is defined for an elastic material. So, there it may not be that relevant. The concept of stress path may not be that relevant. For elastic behavior, there is no importance on the manner in which it is loaded. Because once the slope is fixed that is the modulus is fixed then everything is known.

We are not bothered how a particular stress rate changes from beginning of the loading to the end of elastic limit. So, this is valid only between initial and final state. Now, final state here for an elastic material is not in terms of failure it is up to its yield limit. So, it does not matter how these stress changes. Ultimately, it has to be within the elastic limit or the yield point or the yield limit.

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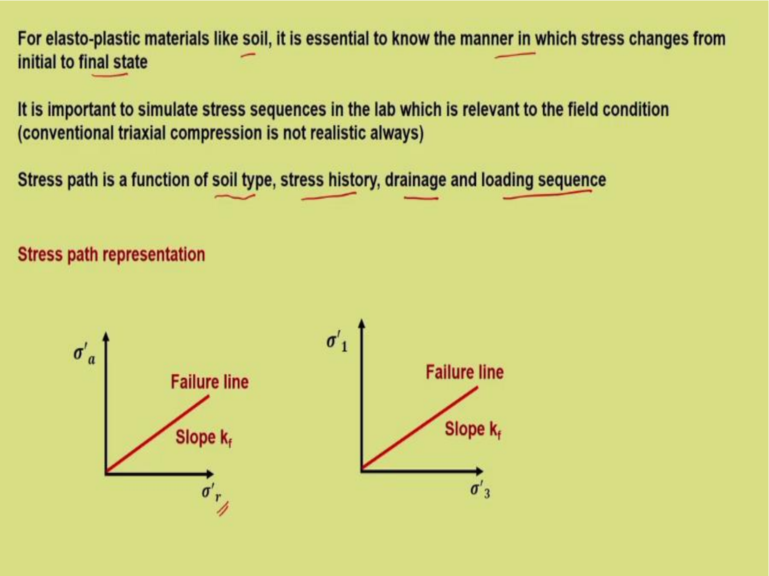

Now, for elasto-plastic materials like soil, it is essential to know the manner in which stress changes from initial to final state. This is very important because depending upon the initial state we have already got a clue from our previous lecture. Especially with respect to Module 2 like depending upon the initial state there can be different possibilities for the same soil and that can be further dictated by the kind of drainage it possess and also the kind of stress history's.

Initial state means it can be a kind of how it is packed or what disturbance it has occurred. But, on top of it, it is also dictated by stress history as well. So, here in the case of soil, it is not enough like if you know, what is the initial state and, what is the final state. Final state here refers to the failure state. But it is equally important how the stress states in the soil changes during its course from initial to its failure state.

So, it is important to simulate stress sequences in the lab which is relevant to the field condition. The conventional triaxial compression is not realistic always. So, if you are really serious about the kind of performance of the soil in the field. Especially for very important structures where a lot of load is going to get imparted then we also need to simulate the actual field condition.

For example, if it is building load, we know that it is going to be compressive most of the time. But if it is an excavation that need to be made then we know that there will be a kind of release that is happening. If it is for a retaining wall then the things may be different. If you are looking at a slope then the stress conditions may be different. Now if we want to actually understand the performance of the soil under different sequences.

Sequences means all these problems what I just mentioned. It all induces a different type of stress response within the soil. If you want to actually understand that with respect to its failure then we need to simulate different kind of conditions in the lab. But in general practice for finding out the design parameters it is a common practice that we adopt only triaxial compression test and conventional triaxial compression test.

So, that may not be realistic all the time. And it is also important even in a conventional triaxial compression test. It is also important like what is the initial state of the soil and how the state changes during loading. So, stress path is essentially a function of soil type, stress history, drainage and loading sequence. All of these factors we have discussed in detail when we discussed in our Module 2. So, all these things have been touched upon.

So, I am not getting into the details. All those factors which affected shear strength is going to affect the evolution of stress path as well. Now if we want to adopt stress path for our further understanding we also need to know how these are represented. So, this is one way of representation. That is actual stress as a function of radial stress and there is a failure line which is represented by k_f.

So, if σ_a and σ_r is a more general representation now I would always prefer and recommend that we always adopt axial versus radial than in terms of principal stresses. Because, depending upon conditions, axial and radial major minor principal stresses would change. And this we are going to see in this particular module. So, it is better in general represent σ_a' on y axis and σ_r' on x axis.

So, that is what we will be following in terms of stress path representation for this module. So, this is another way of representing σ_a and σ_r'.

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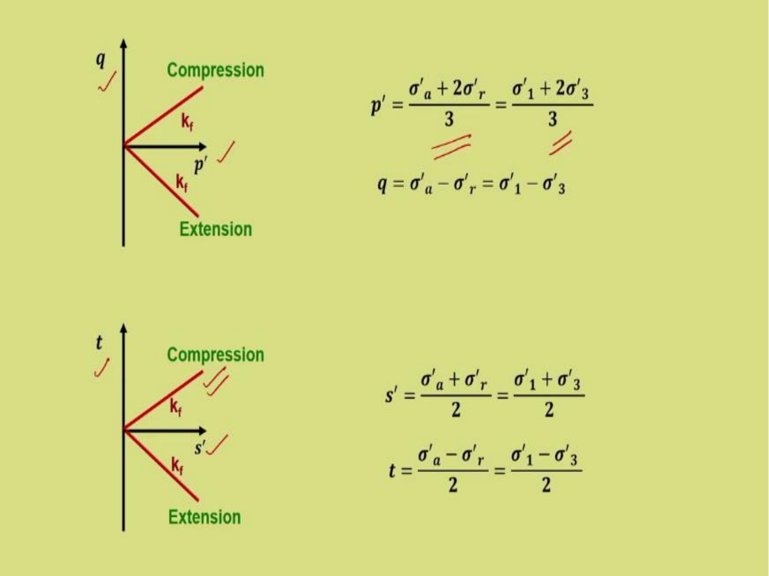

We have also understood in detail q versus p' plot. And that essentially came from Cambridge plot and what is q and what is p' is all well explained. Now the terms stress invariants comes into picture where q and p' can be treated as stress invariants. And p' just to revise because we will be using extensively these symbols in this particular module. It is just to summarize.

$$p' = \frac{(\sigma_a' + 2\sigma_r')}{3} = \frac{(\sigma_1' + 2\sigma_3')}{3}$$
$$q = \sigma_a' - \sigma_r' = \sigma_1' - \sigma_3'$$

It can also be represented in terms of t and s' which is essentially MIT plot or maximum shear stress points. So, here t versus s' we have also dealt these aspects of stress representation in our earlier lectures and we have already seen that.

So, it is just to revise in this particular module because we will be using it for stress path plotting. So, s' $= \frac{\sigma_a' + \sigma_r'}{2}$. And t = $\frac{\sigma_a' - \sigma_r'}{2}$. So, t is plotted on y axis, s' is plotted on x axis.

$$t = \frac{\sigma_a' - \sigma_r'}{2} = \frac{(\sigma_1' - \sigma_3')}{2}$$

Similarly, q is plotted on y axis and pp' is plotted on x axis. That is what we generally use. Normally, we talk mostly about compression.

But now in this particular module to cover the entire aspect for holistic discussion we will be discussing the cases of extension as well. Extension not necessarily mean the soil is sample is just held on both ends and it is extended. It is not that. We can based on the sequence in which we apply stresses we have already seen in the previous module that a kind of compression or extension can be simulated based on how we change the compressive stresses.

We only talk about compressive stresses σ_1 in the axial and σ_3 radial for a typical triaxial sample. Now, controlling these two , we can even simulate extension conditions. So, I am talking about that particular extension and not the realistic extension of soil. We know that soil cannot tolerate such tensile stresses. Now plotting stress path for given stress variation. So, we will slowly get into the plotting of stress path for different conditions.

Now in this particular module, we will be dealing only with this like how to plot stress path for different cases. Now this manner in which how you plot stress path is important for the fourth module. We need to know the inside out of how stress path emerges and how we will define the stress path for different cases.

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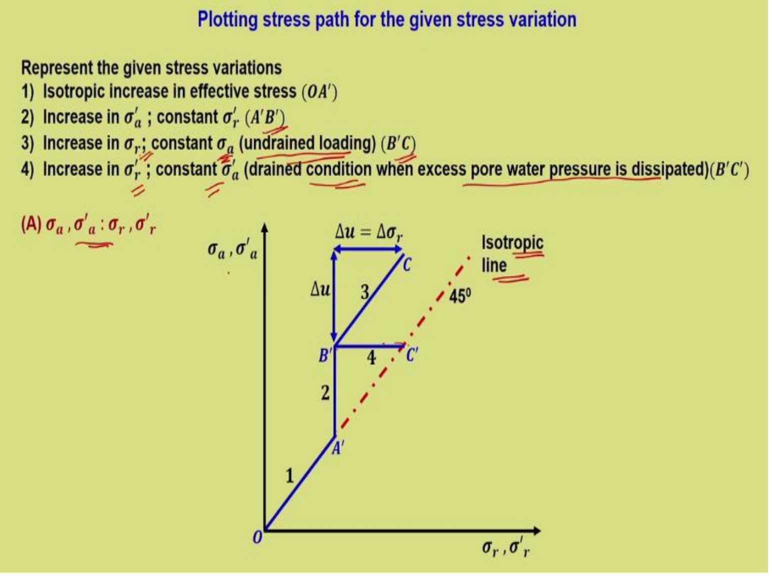

So, it is basically to represent the given stress variations. First let us talk about there is an isotropic increase in effective stress and which is represented by OA'. Now we will be essentially dealing with 3 stress path plots. One is σ_a, σ_a' versus σ_r, σ_r'. Second one is s or t versus s s’. I hope you understand why I am not telling t t’ and s s’ because deviatoric component does not get influenced by pore water pressure.

So, t = t’. So, t versus s s’. Similarly, for q versus p p’. So, these are the 3 different plots which will be adopting whole throughout this module. So, first let us consider σ_a, σ_a’, σ_r, σ_r’. So, this σ_a, σ_a’ is floated on y axis and σ_r, σ_r’ is plotted on x axis. So, first is isotropic increase. Now $\sigma_a = \sigma_r$ or $\sigma_a' = \sigma_r'$ means it is a 45 degrees line.

And that 45 degrees line represents the isotropic condition or hydrostatic condition, isotropic line. So, this is another important representation in a given stress path. So, first our condition is isotropic increase in effective stress. So, it is OA’. So, we start from O and hence the first aspect is OA’. It is along the isotropic line. So, you may be wondering what is so great in doing this.

In fact, there is nothing great. What we are doing is we are simply following what are the given increments of stresses. That is all. But it may look very simple. But as you get into the details of it even the simple plotting can sometimes create a bit of confusion. Now with that confusion it will be difficult to learn critical state soil mechanics chapter. So, that is why even the very simple things are described in this particular module.

So, the first one is it is this particular exercise what we are discussing is extremely simple. There is nothing great but then there are some simple aspects which becomes clear from this discussion. So, OA’ is the isotropic increase. So, we know that this is the isotropic line. So, it goes along OA’. Now forget about the magnitude. We are only understanding it notionally. That means it is only conceptually we are trying to understand.

Forget about all the magnitude now. The second one is increase in σ_a’ and σ_r’ is constant. So, that is represented by to be represented by A’ B’. So, the starting point is here. So, here it is at constant σ_r’ its σ_a’ is increasing. Now third condition is increase in σ_r. So, σ_r is increased and σ_a is constant. Now please remember here. I have represented it in terms of total stress.

Because, total stress is what actually happens onto the soil, the external stress which gets imposed onto the soil. So, what is done is σ_a is now kept constant, σ_r is increased. So, now there is an additional condition which is stated there during the increase in σ_r, the valve is closed or the drainage is not permitted or it is an un-drained loading. So, we need to highlight.

How the stress path moves during the un-drained loading which is to be represented by B’ C. Now this is only for your conceptual understanding may not be actually realistic. Now when at this particular point B’ it is a saturated condition at a particular B’ if we raise the σ r component without allowing drainage to happen then obviously there will be pore water pressure development and it is a saturated condition. So, what happens?

So, then this is represented by B’ C. Now, what is happening here? We have not increased σ_a. We have increased only σ_r. But remember it is an un-drained condition and hence it is a hydrostatic pressure condition. So, when it is an un-drained condition when stresses are increased there is an all-round increase in pore water pressure. And that means it is a hydrostatic condition. σ_a also gets increased σ_r also gets increased within the soil mass or it is in both the direction the rise in pressure will be same. So, pore water pressure is all round. So, it will have in increment in both the direction provided it is un-drained. So, definitely the stress path BC will be parallel to isotropic line because it is equal increment in both the directions. So, $\Delta u = \Delta\sigma_r$. So, equal Δu will be available for σ_a as well.

This happens only under un-drained condition. So, the fourth condition is increase in σ_r’. Now if we are allowing drainage to happen. So, drainage condition when excess pore water pressure is dissipated. So, it is increase in σ_r’ and constant σ_a’. Now how this point is going to get shifted. This is going to get shifted on the line C’. So, it does in fact it need not be at C’.

It is again for your understanding the if it is not specifically mentioned that it is going to be at C’. It only states that σ_r’ increases and σ_a’ remains constant. So, it has to be horizontal. So, it can be anywhere in between. Now we are just for convenience we are just showing with respect to C’ because from here it just falls down. So, B’ C’ represents the fourth condition.

Now this is very simple because it is in terms of σ_a, σ_r, σ_a’, σ_r’ whatever happening we can plot it. Now let us plot the same exercise on st plot and qp plot. Now there we need to determine how the stress path moves here it is straightforward.

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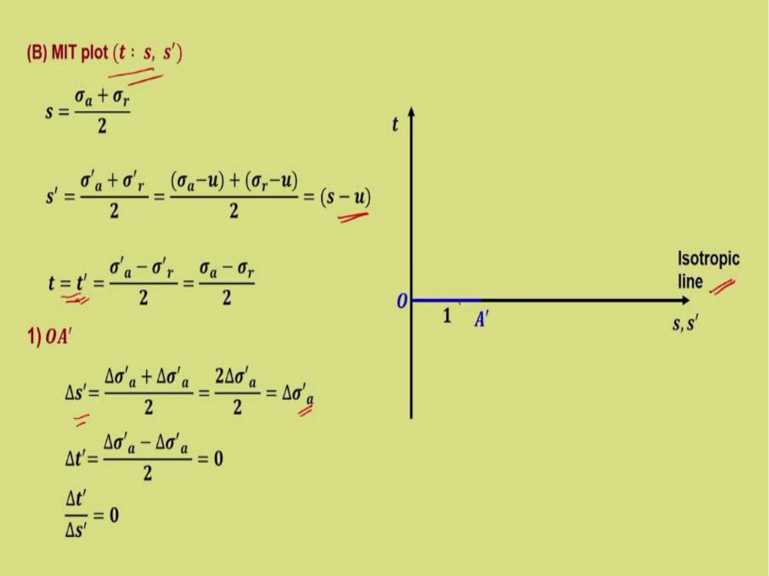

So, next is the plot on MIT. That is t, s, s' plot. So, we know that

$$s = \frac{\sigma_a + \sigma_r}{2}$$

$$s' = \frac{\sigma_a' + \sigma_r'}{2} = \frac{(\sigma_a - u) + (\sigma_r - u)}{2} = (\mathrm{s} - \mathrm{u})$$

So, t = t'. Both are same.

$$t = t' = \frac{\sigma_a' - \sigma_r'}{2} = \frac{\sigma_a - \sigma_r}{2}$$

$$\frac{\Delta t'}{\Delta s'} = 0$$

So, this is deviatoric component and hence not affected by pore water pressure. It gets cancelled off from both ends. So, the same representation OA' that is on an isotropic line. So, this is t on y axis, s, s' on x axis. So, now here it is already written it is isotropic line. Why? Why this x axis is isotropic line. You can refer to here.

That means if you take this isotropic condition means these two are same. So, σ_a'- σ_a'=0. So, t is always 0 for isotropic conditions. So, that is why this particular x axis is an isotropic line in t s' plot. So, now OA' we cannot plot simply directly for in this case it is possible but it is not direct. So, we will find out how we have to do that.

$$\Delta s' = \frac{\Delta\sigma_a' + \Delta\sigma_a'}{2} = \frac{2\Delta\sigma_a'}{2} = \Delta\sigma_a'$$

So, Δs' that is in this direction it will be $\Delta\sigma_a$'. And Δt' = $\Delta\sigma_a$'- $\Delta\sigma_a$' = 0. So, it has to be on this particular line.

And hence

$$\Delta t' = \frac{\Delta\sigma_a' - \Delta\sigma_r'}{2} = 0$$

So, it will give OA' that is on the isotropic line and this is isotropic condition.

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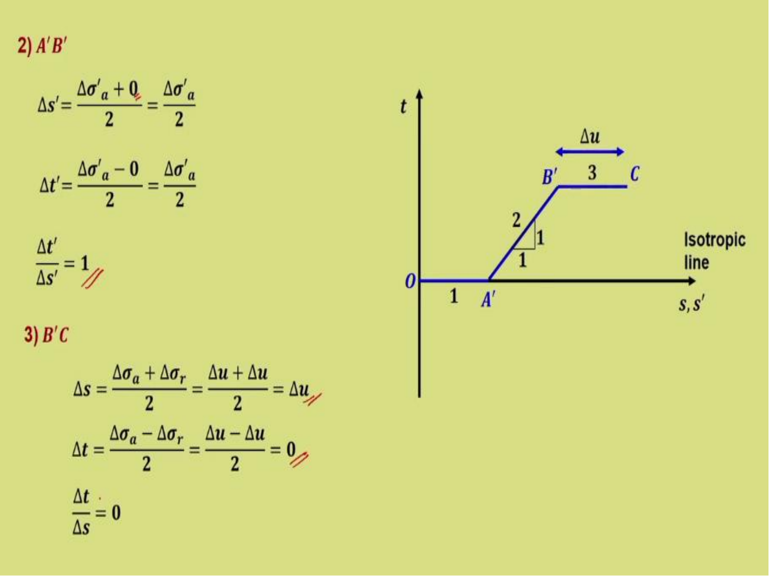

3)B'C

$$\Delta t = \frac{\Delta\sigma_a - \Delta\sigma_r}{2} = \frac{\Delta u - \Delta u}{2} = 0$$

$$\frac{\Delta t'}{\Delta s'} = 1$$

We will be dealing more with isotropic loading in the subsequent lecture. Here it is only for our notional understanding. So, here it is A'B'. What is A'B'? A'B' represents σ_a' is increasing whereas σ_r' is constant. So, that is $\Delta s' = \frac{\Delta\sigma_a'+0}{2} = \frac{\Delta\sigma_a'}{2}$ and then $\Delta t' = \frac{\Delta\sigma_a'-0}{2} = \frac{\Delta\sigma_a'}{2}$

So, that gives the slope of $\frac{\Delta t'}{\Delta s'} = 1$. So, how this point is going to move? Now probably you will understand why we need to pay a lot of attention when we are plotting stress path. It is the information to us is only in terms of σ_a', σ_r' or in terms of major minor principal stresses. Now for us to plot in terms of stress invariants, we also need to make this calculation and we also need to know, what is the slope at which the stress path would move? So, entire exercise of stress path plotting is all about finding out the slope in which it moves. So, here $\frac{\Delta t'}{\Delta s'} = 1$. So, we know that A' B' should be moving at an inclination of with a slope 1. So, that is what is shown there A' B'. Second part it is with a slope of 1 is to 1.

Now B'C is that momentary un-drained condition at point B'. So, we will see that how it happens. $\Delta s = \frac{\Delta\sigma_a + \Delta\sigma_r}{2}$. Now the pore water pressure is equal. So, $\Delta s = \frac{\Delta u + \Delta u}{2} = \Delta u$. So, increment in along this direction is Δ u. That is what it means. But Δ t since both are same it is =0. So, that means the slope is 0 but Δ s has an increment of Δ u.

How do we plot it? We plot it like this. B'C with an increment in Δu. And it is horizontal because Δ t is 0.

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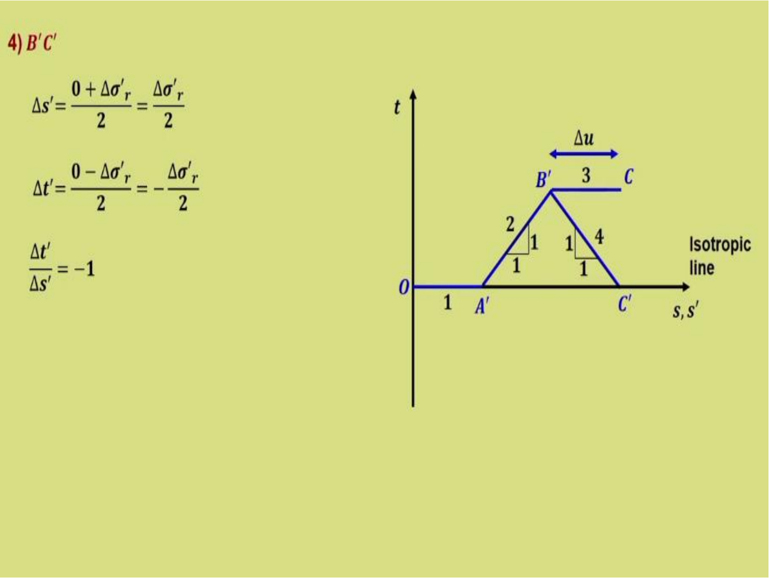

Now let us see the last one. That is σ_a' is kept constant and σ_r' is increasing.

$$s = \frac{0 + \Delta\sigma_r'}{2} = \frac{\Delta\sigma_r'}{2}$$

So, here $\Delta\sigma_r$' is increasing. That is $s = \frac{\Delta\sigma_r'}{2}$.

$$\Delta t' = \frac{0 - \Delta\sigma_r'}{2} = \frac{-\Delta\sigma_r'}{2}$$

So,

$$\frac{\Delta t'}{\Delta s'} = -1$$

Δt'/Δs' is -1. That means it has to drop in the stress path.

As you can see is B' C' is =1 is to 1 slope but with a negative slope. So, this is how we plot for st plot.

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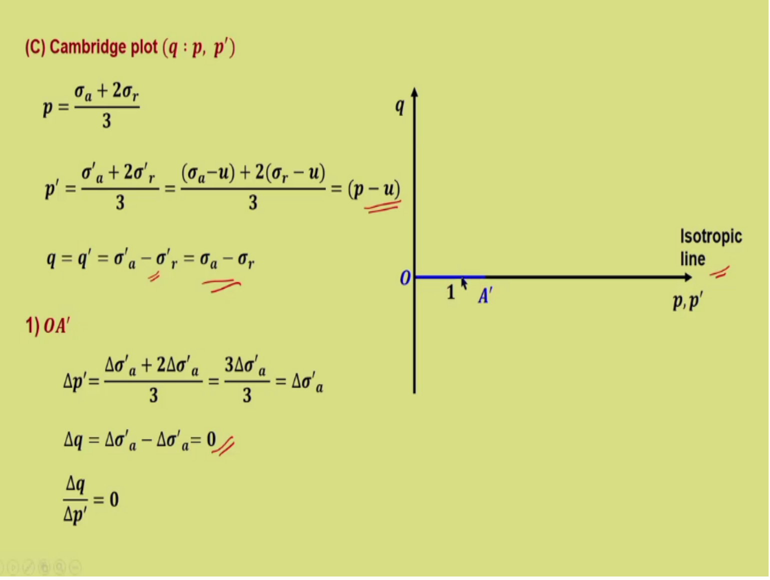

Now we will move on to the third plotting. That is Cambridge plot which is in terms of q, p, p'. So, once we have done with st plot, it is quite easy to understand even q, p, p' plot. More or less is the same procedure where

$$p = \frac{\sigma_a + 2\sigma_r}{2}$$

$$p = \frac{\sigma'_a + 2\sigma'_r}{3} = \frac{(\sigma_a - u) + 2(\sigma_r - u)}{3} = (p - u)$$

$$q = q' = \sigma'_a - \sigma'_r = \sigma_a - \sigma_r$$

Because this plus this becomes p minus u minus 2u becomes 3u by 3 that is u. And q is =q'. This we have already seen. So, first is OA'. That is isotropic stress path and here also in q, p, p', similar to t; $q = \sigma_a - \sigma_r$. So, if they are same that is σ_a' =σ_r' then it will be 0. The slope will be 0. So, this becomes the isotropic line. That is x axis is the isotropic line.

So, Δp' for OA' now both are it is an isotropic increase. So, σ_a' and σ_a' this becomes same.

$$\Delta p = \frac{\Delta\sigma'_a + 2\Delta\sigma'_a}{3} = \frac{3\Delta\sigma'_a}{3} = \Delta\sigma'_a$$

And Δ q is =

$$\Delta q = \Delta\sigma'_a - \Delta\sigma'_r = 0$$

$$\frac{\Delta q}{\Delta p'} = 0$$

So, definitely it is slope with 0. So, that OA' is on the x axis.

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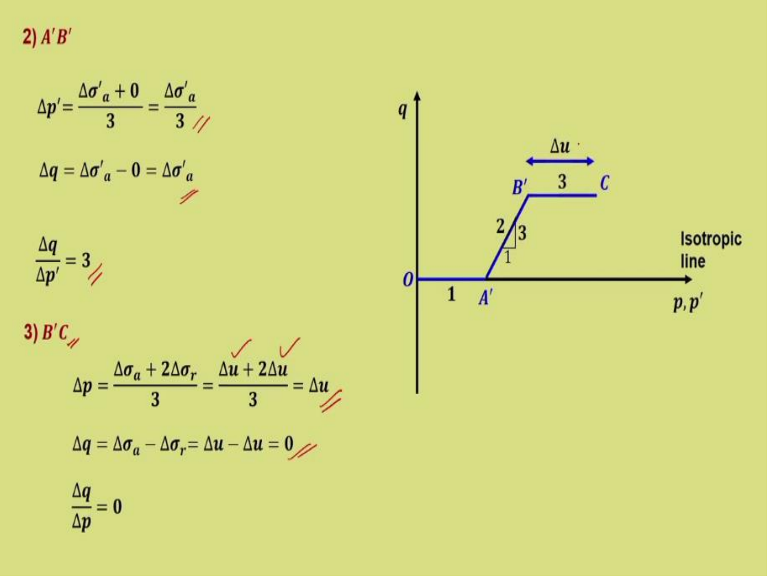

3) B'C

Next is A' B'. So, A' B' is σ_a' is increasing σ_r' is 0 which is given

$$\Delta p = \frac{\Delta\sigma'_a + 0}{3} = \frac{\Delta\sigma'_a}{3}$$

$$\Delta q = \Delta\sigma'_a - 0 = \Delta\sigma'_a$$

$$\frac{\Delta q}{\Delta p'} = 3$$

So, it has a slope of 3. So, that is plotted in s 2. That is A' B' is with a slope of 3. Now we need to understand for such a particular increase the slope is 3. Now there can be different ways by which σ_a', σ_r' are changing. Here it is a simple case where σ_r' is kept constant. So, that is why it is giving a slope of 3. Now B'C now that intermittent un-drained condition. How it will look like.

$$\Delta p = \frac{\Delta\sigma_a + 2\Delta\sigma_r}{3} = \frac{\Delta u + 2\Delta u}{3} = \Delta u$$

There is an increment of Δu.

$$\Delta q = \Delta\sigma_a - \Delta\sigma_r = \Delta u - \Delta u = 0$$

$$\frac{\Delta q}{\Delta p} = 0$$

So, slope is 0 for B'C where the increment is Δ u in an un-drained condition.

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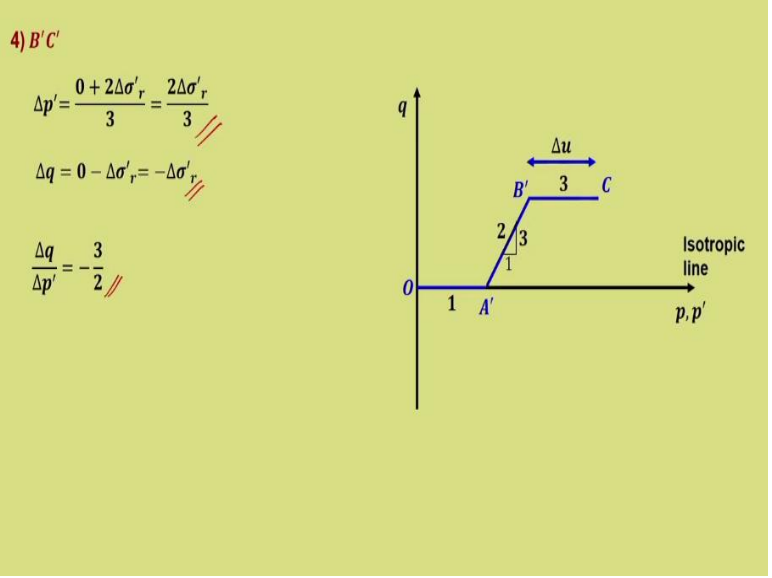

Now the last one is B' C'. So how the slope changes when you have σ_a' constant and σ_r' is increasing.

$$\Delta p' = \frac{0 + 2\Delta\sigma_r'}{3} = \frac{2\Delta\sigma_r'}{3}$$

$$\Delta q = 0 - \Delta\sigma_r' = \Delta\sigma_r'$$

$$\frac{\Delta q}{\Delta p'} = \frac{-3}{2}$$

So, you see here, slope should be known and the slope is specific to the kind of stress increment which we have imposed.

So, this may change again if the stress variations are same. So, we should understand how important it is the sequence and the manner in which the stress changes. You can plot this. Now this is negative slope of 3/ 2 which is plotted here. So, this is a quick example of how a stress path variations look like. Now what we will be doing in the subsequent lectures is to understand how these stress path need to be plotted for common conditions.

That we normally find in soil mechanics. So, that is what we will be do. So, we will summarize for today's lecture.

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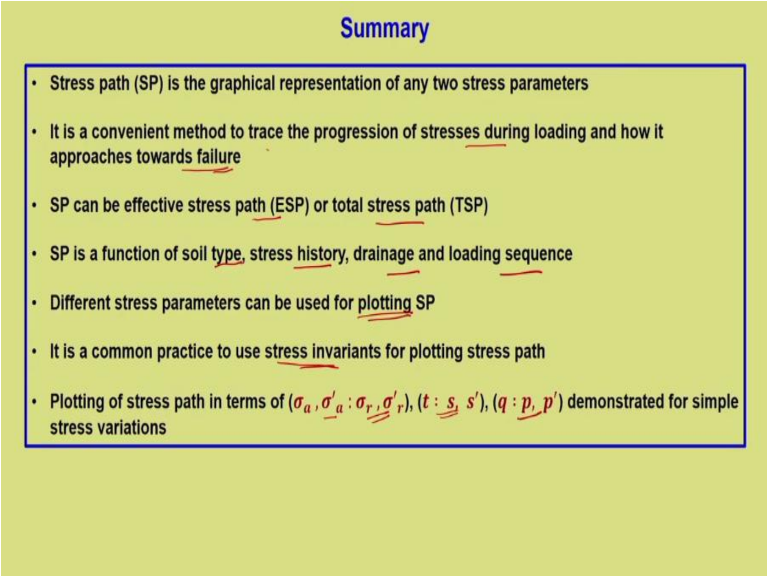

As stress path is the graphical representation of any 2 stress parameters. It is a convenient method to trace the progression of stresses during loading and how it approaches towards failure. Now this particular sentence will be very clear as we move forward and understand stress path much better. So, stress path can be effective stress path or total stress path. Now the relevance of these 2 will also be known in the subsequent lectures.

Stress path is a function of soil type, stress history, drainage and loading sequence. Different stress path parameters can be used for plotting stress path. It is a common practice to use stress invariants for plotting the stress path. Now plotting of stress path in terms of σ_a, σ_a', σ_r, σ_r', then t versus s, s', q versus p, p' is demonstrated for simple stress variations. So, that is all for today's lecture.

In the next lecture, we will add more meaning to this stress path plot. That is all for now, thank you.