**Advanced Soil Mechanics**
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**Lecture - 25**
**Pore Water Pressure Plane Strain Effect of Sampling**

Welcome back all of you, in the last lecture we have seen the importance of pore water pressure in soils and the need for its estimation we discussed about Skempton pore water pressure equations and A and B parameters which are known as Skempton pore pressure parameters. So this lectures continuation of the last lecture and we will basically deal with pore water pressure for plane strain conditions, Henkels pore water pressure equation and the effect of sampling on pore water pressure which occurs in the field. So these 3 aspects we will cover in today's lecture.

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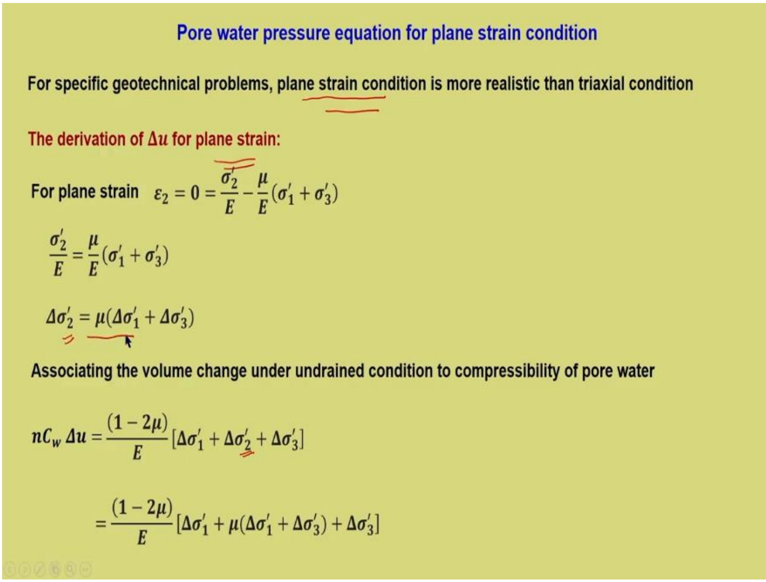


So for specific geotechnical problems we have already seen plane strain condition is more realistic than triaxial condition, we have already seen this. So we will try to have the derivation for Δu corresponding to plain strain. Now the condition for plane strain let us say we have if you deal with a principle stresses and principle strains. Let us consider intermediate principle strain $\epsilon_2 = 0$ and that can be expressed as

$$\varepsilon_2 = 0 = \frac{\sigma'_2}{E} - \frac{\mu}{E}(\sigma'_1 + \sigma'_3)$$

This gives

$$\frac{\sigma_2'}{E} = \frac{\mu}{E}(\sigma_1' + \sigma_3')$$

In terms of incremental stresses, we can write

$$\Delta\sigma_2' = \mu(\Delta\sigma_1' + \Delta\sigma_3')$$

Now how did we derive the pore water pressure equation we have associated or we have already allocated. We saying that there will be some volume change that can occur in undrained condition then we associated this volume change to the compressibility of water. Then we have derived the equation for pore water pressure doing the same thing for plane strain. So we will have to start from the same point whatever is the volume change that is due to compressibility of water doing that associating the volume change under undrained condition to compressibility of pore water. We can write this is the starting

$$nC_w\,\Delta u = \frac{(1-2\mu)}{E}[\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3']$$

associating the undrained volume change that could have happened due to Δu to the actual volume change that would have happened under undrained condition.

Then delta $\Delta\sigma_2'$ can be replaced by this equation $\Delta\sigma_2' = \mu(\Delta\sigma_1' + \Delta\sigma_3')$. So, when you replace that you get this equation.

$$nC_w\,\Delta u = \frac{(1-2\mu)}{E}[\Delta\sigma_1' + \mu(\Delta\sigma_1' + \Delta\sigma_3') + \Delta\sigma_3']$$

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$$nC_w\,\Delta u = \frac{(1-2\mu)}{E}[(1+\mu)\Delta\sigma_1' + (1+\mu)\Delta\sigma_3']$$

$$= \frac{(1+\mu)(1-2\mu)}{E}[\Delta\sigma_1' + \Delta\sigma_3']$$

$$= \frac{(1+\mu)(1-2\mu)}{E}[\Delta\sigma_1' + \Delta\sigma_3' + \Delta\sigma_3' - \Delta\sigma_3']$$

$$= \frac{(1+\mu)(1-2\mu)}{E}[2\Delta\sigma_3' + (\Delta\sigma_1' - \Delta\sigma_3')]$$

$$= \frac{2(1+\mu)(1-2\mu)}{E}\left[\Delta\sigma_3 + \frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_3) - \Delta u\right]$$

$$\left[nC_w + \frac{2(1+\mu)(1-2\mu)}{E}\right]\Delta u = \frac{2(1+\mu)(1-2\mu)}{E}\left[\Delta\sigma_3 + \frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

So rearranging the terms you get

$$nC_w\,\Delta u = \frac{(1-2\mu)}{E}\left[(1+\mu)\Delta\sigma_1' + (1+\mu)\Delta\sigma_3'\right]$$

$$= \frac{(1+\mu)(1-2\mu)}{E}\left[\Delta\sigma_1' + \Delta\sigma_3'\right]$$

$$= \frac{(1+\mu)(1-2\mu)}{E}\left[\Delta\sigma_1' + \Delta\sigma_3' + \Delta\sigma_3' - \Delta\sigma_3'\right]$$

$$= \frac{(1-2\mu)(1+\mu)}{E}\left[2\Delta\sigma_3' + (\Delta\sigma_1' - \Delta\sigma_3')\right]$$

Now this equation is similar to the equation that we have seen during the derivation of Skempton pore water pressure equation, so this has become very similar to that. Now if we take 2 outside. And we have we are expressing this effective stress in total stress.

$$nC_w\,\Delta u = \frac{2(1-2\mu)(1+\mu)}{E}\left[\Delta\sigma_3 + \frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_3) - \Delta u\right]$$

Now if we take this Δu on the other side.

$$\left[nC_w + \frac{2(1-2\mu)(1+\mu)}{E}\right]\Delta u = \frac{2(1-2\mu)(1+\mu)}{E}\left[\Delta\sigma_3 + \frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

Now if you notice this is now very much similar to the Skempton pore water pressure equation except the fact that there it was 1 / 3 and now it is half.

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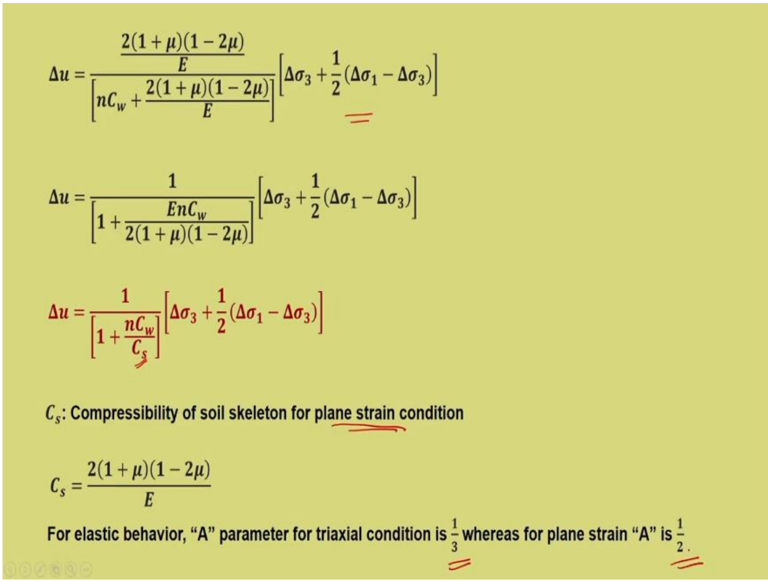

Again rearranging.

$$\Delta u = \frac{\frac{2(1-2\mu)(1+\mu)}{E}}{\left[nC_w + \frac{2(1-2\mu)(1+\mu)}{E}\right]}\left[\Delta\sigma_3 + \frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

$$\Delta u = \frac{1}{\left[1 + \frac{EnC_w}{2(1-2\mu)(1+\mu)}\right]}\left[\Delta\sigma_3 + \frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

So the final pore at a appreciate equation will be

$$\Delta u = \frac{1}{\left[1 + \frac{nC_w}{C_s}\right]}\left[\Delta\sigma_3 + \frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

Again rearranging you can get is the same way we have done composability of soil skeleton the same thing we have done before also and E comes down. So $C_s$ is the compressibility of soil skeleton for specifically form plane strain condition and that is

$$C_s = \frac{2(1-2\mu)(1+\mu)}{E}$$

So what is the essential difference between plane strain and triaxial condition. So A parameter for triaxial actual condition is $\frac{1}{3}$ whereas for plane strain condition $A = \frac{1}{2}$ remember this values $\frac{1}{3}$ and $\frac{1}{2}$ corresponds to elastic behaviour only.

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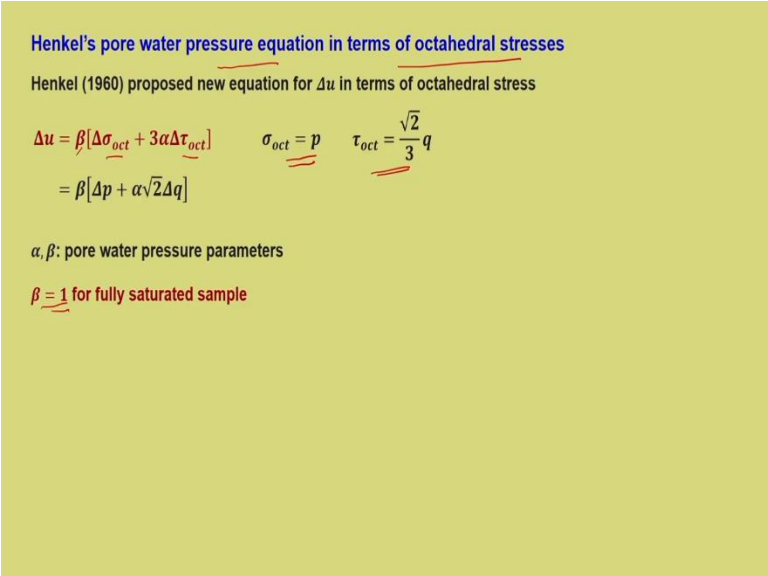

So that is about the derivation of equation for plane strain condition. Now let us see Henkel pore pressure equation in terms of octahedral stresses. Henkel 1960 proposed new equation for Δu in terms of octahedral stresses I hope you remember what is octahedral stresses? Which, we have seen in the earlier lecture. So according to Henkel it is

$$\Delta u = \beta[\Delta\sigma_{oct} + 3\alpha\tau_{oct}]$$

So both $\sigma_{oct}$ and $\tau_{oct}$ has been accounted this is very much similar to our previous 6 Skempton equation b is replaced by $\beta$ a is replaced by 3 into $\sigma$ because $\sigma_{oct}$ mostly it represents condition similar to that of deviatoric stress which is $\sigma_1 - \sigma_3$ for triaxial condition so it is one at the same. Now $\sigma_{oct}$ just to revise let us write $\sigma_{oct}$ is equal to mean principle stresses or mean stress p.

I am not intentionally putting prime because now it corresponds to total stress condition because pored pressure is there it is an undrained conditions. So I am referring it to t mean stress and t $\tau_{oct} = \frac{\sqrt{2}}{3}\Delta q$ all these expressions we have seen in the previous lectures. Now if you rearrange in terms of p and q one can write

$$\Delta u = \beta\left[\Delta p + \alpha\sqrt{2}\Delta q\right]$$

So we are just replacing it in terms of p and q. Here $\alpha$ and $\beta$ are called pore water pressure parameters just like a and b and here also $\beta = 1$ for fully saturated samples. So this is about Henkels pore water pressure equation we need to understand that this equation is in terms of octahedral stresses. So in those models which adopt octahedral stresses it is quite handy to see how the pore water pressure changes with corresponding changes in the octahedral stresses.

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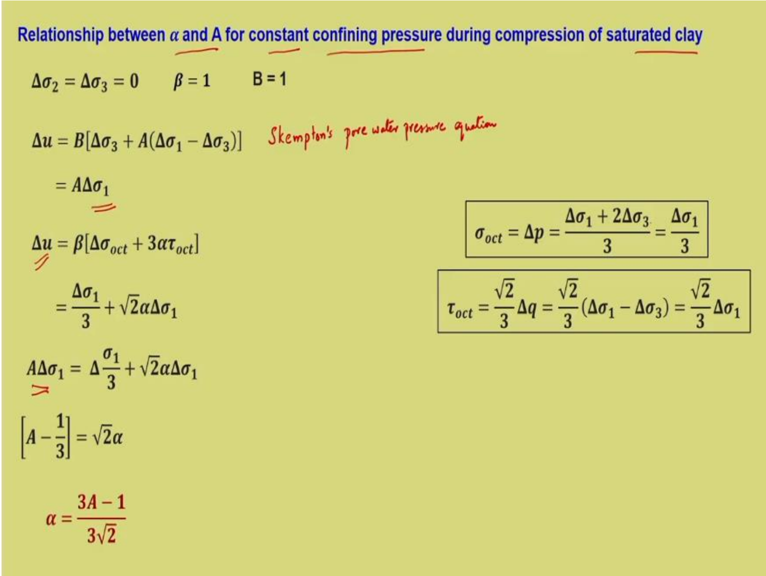


B = 1

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$
$$= A\Delta\sigma_1$$

Now let us try to find out the relationship between $\alpha$ and A, because these 2 parameters are similar A one is in henkel the other one is in skempton pore pressure equation for constant confining pressure. So there is there are conditions given for constant confining pressure during compression of saturated clay. So there are certain thing which comes out from here there is no change in confining pressure that is constant confining pressure.

So $\Delta\sigma_2 = \Delta\sigma_3 = 0$ since it is saturated $\beta$ = 1 and B =1. So let us first start with Skempton's pore pressure equation. So this is skempton's pore water pressure equation.

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

So here we know that $\Delta\sigma_2 = \Delta\sigma_3 = 0$ substituting that we will get

$$\Delta u = A\Delta\sigma_1$$

Now this is $\Delta u = \beta[\Delta\sigma_{oct} + 3\alpha\tau_{oct}]$ the Henckel's pore pressure equation which we have just seen,

We know

$$\sigma_{oct} = \Delta p = \frac{\Delta\sigma_1 + 2\,\Delta\sigma_3}{3} = \frac{\Delta\sigma_1}{3}$$

And

$$\tau_{oct} = \frac{\sqrt{2}}{3}\Delta q = \frac{\sqrt{2}}{3}(\Delta\sigma_1 - \Delta\sigma_3) = \frac{\sqrt{2}}{3}\Delta\sigma_1$$

For corresponding to a typical triaxial condition we know that $q = \sigma_1 - \sigma_3$. So $\Delta q = (\Delta\sigma_1 - \Delta\sigma_3)$ so finally $\tau_{oct} = \frac{\sqrt{2}}{3}\Delta\sigma_1$. We will substitute this back into the equation $\Delta u = \beta[\Delta\sigma_{oct} + 3\alpha\tau_{oct}]$, this will give

$$\Delta u = \frac{\Delta\sigma_1}{3} + \sqrt{2}\alpha\Delta\sigma_1$$

Now $\Delta u$ we already know that $\Delta u = A\Delta\sigma_1$. So substituting that

$$A\Delta\sigma_1 = \Delta\frac{\sigma_1}{3} + \sqrt{2}\alpha\Delta\sigma_1$$

now you will take this Δσ1 on the other side so that will give us

$$\left[A - \frac{1}{3}\right] = \sqrt{2}\alpha$$

So this gives the relationship between alpha and A where after rearranging you can get

$$\alpha = \frac{3A - 1}{3\sqrt{2}}$$

This is the relationship between alpha and A but for specific condition where the confining pressure remains constant and the soil is fully saturated. So this about henkel's pore water pressure equation and its relationship with skempton's pore pressure parameter A.

$$\sigma_1 = \sigma_v$$

$$\sigma_2 = \sigma_3 = \sigma_h$$

$$\sigma_v = \gamma h + \gamma_{sat}(Z - h)$$

$$u_i = (Z - h)\gamma_w$$

$$\sigma_v' = \sigma_v - u_i$$

So next let us see the effect of sampling in the field on pore water pressure. Many a times we need to sample soils from the field specifically undisturbed soils sample this is very much important for studying the compressibility of the soil as well as the shear strength characteristics. If you want to plan a foundation at a particular level we need to know what is the bearing capacity of that particular soil So we need to have the undisturbed sample with us.

Now during sampling what happens let us say it is a saturated soil mass and when you do the sampling whether there is any change that can happen to the soil mass in terms of its pore water pressure. So that is what we will see just now and this part of the lecture I have taken from head manual of soil laboratory testing volume 3 effective stress tests published by john Wiley. Let us consider a soil element A in the field.

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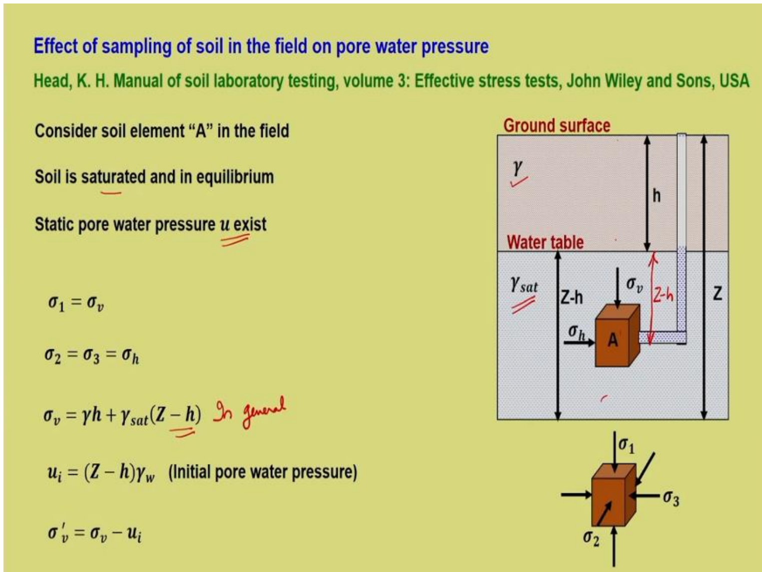


This is the depiction of a given soil you can see that the ground surface is here, the water table is given and we know the soil beneath the water table it is saturated. So accordingly the unit weight here will be bulk unit weight or total unit weight. So here please remember the soil above the water table is not a saturated soil mass it is a partially saturated. So we will consider this to be bulk unit weight.

We will consider a soil element A which is shown. Now if you put a piezometer what we will get the head causing what is the head of water. Here in this case it is a static pore water pressure static head. So here the water will rise in the piezometer up to the groundwater table it is an unconfined case. We have vertical stress and horizontal stress acting.

The thickness considered as z and the thickness of the partially saturated portion that is thickness about the water table is h. So here it is Z - h. So showing the kind of stresses which is acting on the element which is $\sigma_1, \sigma_2$ $and$ $\sigma_3$. Soil is saturated and it is in equilibrium. So this given soil it is in saturated state and it is also in equilibrium please note that we are considering the element A.

So with respect to that the soil is saturated it is in equilibrium means what? The pore water pressure has reached to its equilibrium value that is what it means so here we have static pore water pressure and that is denoted by u static pore pressure u. Let us see the total stress $\sigma_1 = \sigma_v$. So let us we are considering the vertical stress to be major principle stress and $\sigma_2 = \sigma_3 = \sigma_h$.

Now $\sigma_v$ what will be $\sigma_v$ from the figure it will be

$$\sigma_v = \gamma h + \gamma_{sat}(Z - h)$$

Initial pore water pressure $u_i = (Z - h)\gamma_w$.

So one can write $\sigma_v' = \sigma_v - u_i$.

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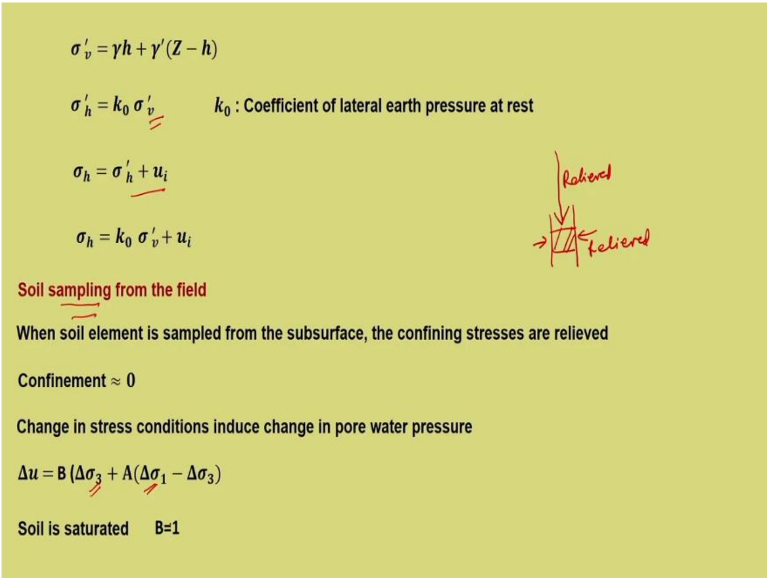


$$\sigma_h = \sigma_h' + u_i$$

$$\sigma_h = k_o\sigma_v' + u_i$$

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

$$B = 1$$

So that will give

$$\sigma_v' = \gamma h + \gamma'(Z - h)$$

and

$$\sigma_h' = k_o\sigma_v'$$

$k_o$coefficient of lateral earth pressure at rest. So $\sigma_h$ can be written as $\sigma_h = \sigma_h' + u_i$

Now why did we write this, why can we write? $\sigma_h = k_o\sigma_v$. Why can we write that? This is not possible because all these coefficient of lateral pressure active passive case all of them are in terms of failure condition and hence it is not only failure condition the lateral earth pressure is always in terms of effective stress why?

Because we are considering the effect of soil or the lateral pressure of soil acting on something and this is mostly dictated by the effective stress condition. So the equation $\sigma_h = k_o\sigma_v$. It is always in terms of effective stress that is why we have first found out what is the total vertical stress acting then found out affective component then applied $k_o$for getting $\sigma_h'$.

Now if you want to obtain $\sigma_h$ then we just need to add pore water pressure to it. So that is what has been done here which is $\sigma_h = \sigma_h' + u_i$and that gives k naught into $\sigma_h = k_o\sigma_v' + u_i$. Now let us consider the soil sample is taken from the field that is the process of sampling when soil element is sampled from the subsurface the confining stresses are relieved. So confinement now reduced to 0 that is true.

So when you are trying to take this particular soil maybe in a bore hole what we are doing is we are making a hole here. So this is relieved and when you take out the sample this is also relieved. So change in stress conditions induce change in pore water pressure now just because there is a release of stresses or the stresses are relieved, we will have changes in pore water pressure because it is under undrained condition and it is for a smaller period of time. So both combined there will be changes in pore water pressure.

So let us start with the Skempton pore water pressure equation Δu is equal and then we need to find out what is the changes in $\Delta\sigma_3\ and\ \Delta\sigma_1$?. Soil is saturated and mass, so B can be considered equal to 1.

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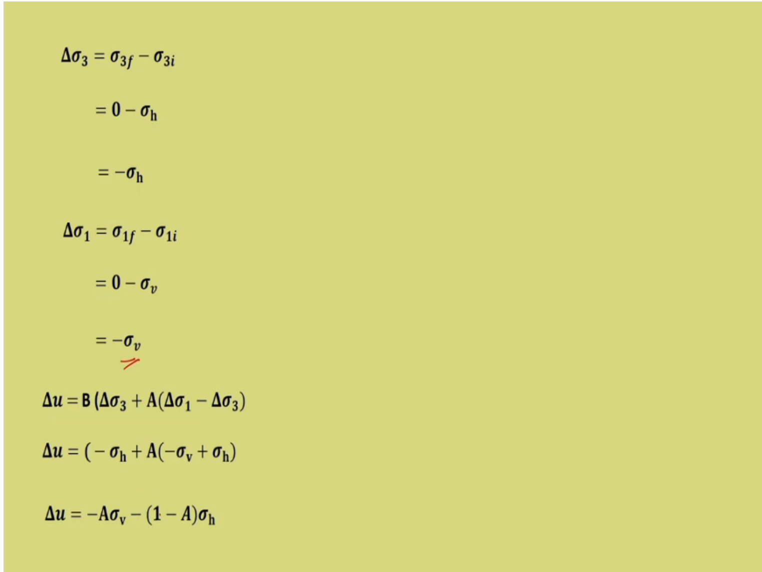


$$\Delta\sigma_3 = \sigma_{3f} - \sigma_{3i}$$
$$= 0 - \sigma_h$$
$$= -\sigma_h$$
$$\Delta\sigma_1 = \sigma_{1f} - \sigma_{1i}$$
$$= -\sigma_v$$
$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

Now $\Delta\sigma_3$ is what the changes in confining stress $\Delta\sigma_3 = \sigma_{3f} - \sigma_{3i}$ now $\sigma_{3f}$ we know it is 0 because the confinement is lost. And $\sigma_{3i}$ is nothing but the lateral pressure $\sigma_h$. So this will give $\Delta\sigma_3 = -\sigma_h$. Now what is $\Delta\sigma_1$? it is $\Delta\sigma_1 = \sigma_{1f} - \sigma_{1i}$ , again $\sigma_{1f}$ is also 0 because it is sampled $\Delta\sigma_1 = 0 - \sigma_v$.

If we substitute this in the equation

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

you will get

$$\Delta u = [-\sigma_h + A(\Delta\sigma_1 - \Delta\sigma_3)]$$
$$\Delta u = -A\sigma_v - (1 - A)\sigma_h$$

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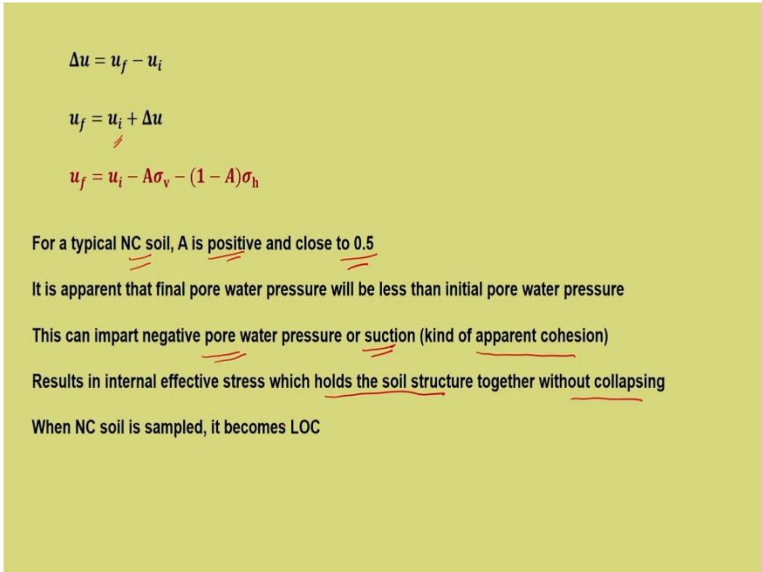


Now we have Δu what is the change in pore water pressure which is

$$\Delta u = u_f - u_i$$

So $u_f = u_i + \Delta u$. So we are interested to find out what is that particular pore water pressure after sampling that is nothing but $u_f$ which is equal to what was existing in the beginning $u_i$ + Δu.

$$u_f = u_i - A\sigma_v - (1 - A)\sigma_h$$

So this is what we get for $u_f$ when you substitute Δu that is a change in pore water pressure.

Now let us say that the given soil element is a typical NC soil. So A will be positive and close to 0.5. So what is the implication A is positive and this close to 0.5 which means that this portion $A\sigma_v - (1 - A)\sigma_h$ in the equation $u_f = u_i - A\sigma_v - (1 - A)\sigma_h$ will be positive that means $u_i$ minus some quantity. So when it is u i is minus some quantity we know that the final border pressure will be always less than the initial pore water pressure.

So it is apparent that final pore water pressure will be less than initial pore water pressure only under the condition when A is positive. Now this can impart a negative pore water pressure because the pressure or the confinement is getting released. So the tendency of the soil is to suck more water and that is during volume change condition but that is during drained condition. But since the time is less here he has it we can consider this to be a typical undrained condition.

If that is the case then the pore water pressure tends to negative or suction. So this induces a kind of apparent cohesion in the soil. This results in an internal effective stress which is instrumental in holding the soil structure together without collapsing. So this apparent cohesion as a result of this negative pore water pressure helps to hold the soil and it will not collapse and fall. So when normally consolidated soil is sampled it becomes slightly over consolidated because of this release and that is the reason why it can stand on its own.

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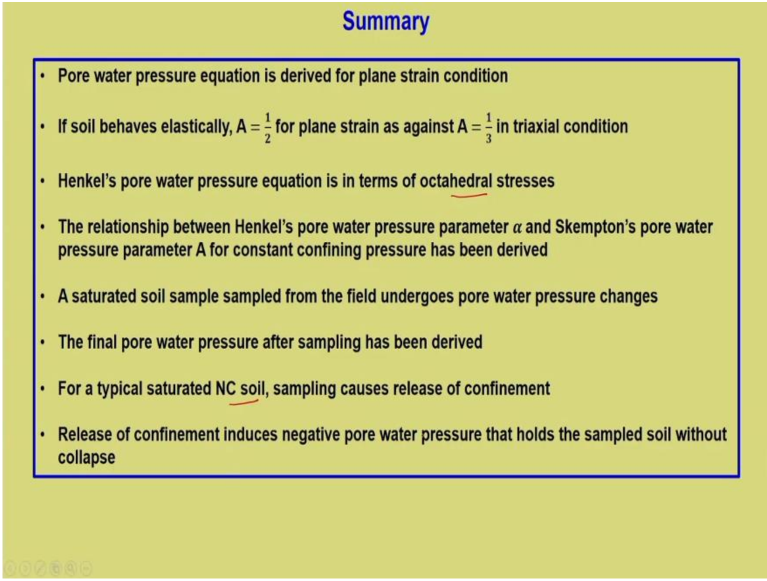


So that is all about the effect of sampling. So what happens new to release there will be a kind of a negative pore water pressure which helps to hold the soil intact. So that is why when the undisturbed sample is taken from the field it can stand on its own that is basically because of the effect of pore water pressure changes. So let us summarize this lecture pore water pressure equation is derived for plane strain condition.

If the soil behaves elastically $A = \frac{1}{2}$ for plane strain as against $A = \frac{1}{3}$ in triaxial condition which we have seen henckels pore pressure equation is in terms of octahedral stresses the relationship between henckels border pressure parameter $\alpha$ and Skempton's pore water pressure parameter A for constant confining pressure has been derived. A saturated soil sample from the field undergoes pour water pressure changes.

And we have estimated it the final pore water pressure after sampling has been derived. For a typical saturated NC soil sampling causes release of confinement. So release of confinement induces negative pore water pressure that holds the sampled soil without collapse. So these are all the things which we have seen as part of pore water pressure estimation. So in the next lecture we will just work out one small problem just to give us a feel of how pore water pressure is determined and how practically it is significant. So that is all for now. Thank you.