Advanced Soil Mechanics
Prof. Sreedeep Sekharan
Department of Civil Engineering
Indian Institute of Technology, Guwahati, Assam

Lecture - 02
Stress Acting at a Point - Cauchy Stress

So, welcome back in this lecture, we will start the first module introduction to continuum mechanics.

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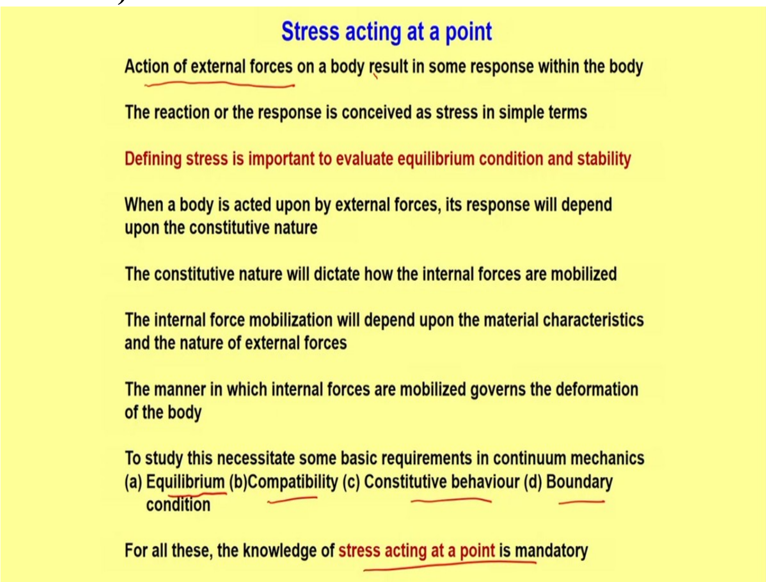

And the first lecture is stress acting at a point. So, if you see, stress is a very common terminology, which all of us understand and we all appreciate that this particular terminology is important in mechanics. Now, what is our conception or visualization of a stress and why we need to discuss this particular aspect stress at a point. What is that concept all about, why do we need it, so that is what we will see in this particular lecture and why this is important that also will be discussed?

Now, action of any external force on a body what is its reaction the reaction is in the form of some sort of stress that build up. It is same for all like, when we say we are stressed, it gives an indication of some internal mechanism. It is something with respect to reaction not action. So, when we are stressed it means, there are certain external factors which is happening because of which our body is responding. So, there is something which is developed internally the same with all materials.

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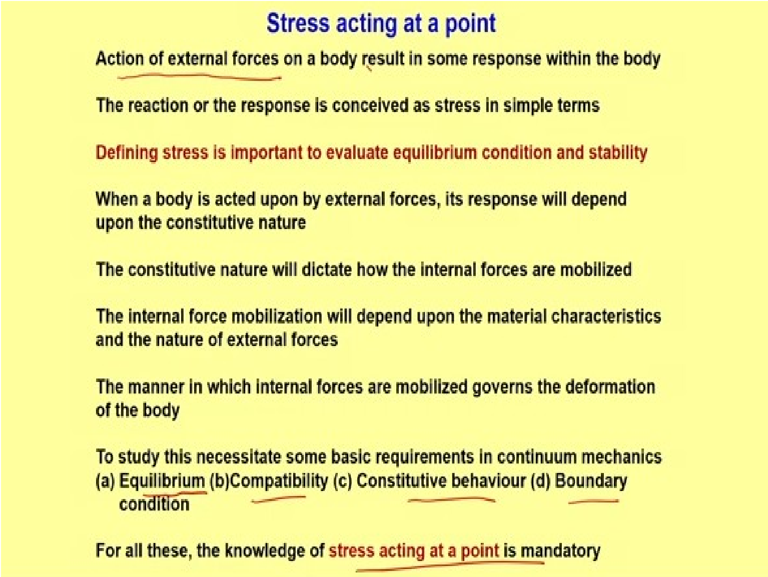

Which means to say when a material or a body is acted upon by external stresses. So, for example, in this particular case, it is action of external forces on a body it results in some response within the body. So, that is what it is within the reaction or the response is conceived as stress in very simple terms. So, defining stress is important to evaluate equilibrium condition.

And stability of a body or a material when it is acted upon by external stresses, now why it is stress at a point so when a body is acted upon by external forces, it is response will depend upon the constitutive nature of that particular body and constitutive nature means, what is the built of that body that will influence what will be its response and response here is stress. The constitutive nature will dictate how the internal forces are mobilized.

So, now, it is not a time that you discuss about stress, when it is acted upon by some external loads, what develops is the internal force. Now, internal forces which are mobilized within the body. It will depend upon the constitutive nature of that particular body. So, the internal force mobilization it will depend upon the material characteristics and the nature of external forces.

So, what type of external forces are acting, how they are acting, what are the combinations of external forces which are acting and also the material characteristics. So, these two factors together determine what will be the response. And that response is the internal force which gets developed, probably you will understand this, we have when we learned about analysis of stresses, we have done this we were asked to find out what is the internal force.

Within the each of the stress member is similar to that. So, here we are making it more clear and we are generalizing it. So, the internal force mobilization it will depend upon what sort of material it is, what is its built as well as the nature of external forces. The manner in which the internal forces are mobilized governs the deformation of the body. Now, what is the net effect?

Now, how the internal forces are generated within the body will determine what type of deformation will it undergo and that is very important for us. So, when a body deforms, so, what causes its deformation. So, the way in which the internal forces gets developed that will indicate what sort of deformation the body will undergo. Now, to study these necessitate some basic requirements in continuum mechanics.

What are those basic requirements? For example, the first 1 is the equilibrium any body has to be in equilibrium. So, the equilibrium conditions are very well necessary. Then comes the compatibility condition. Then the constitutive behaviour constitutive behaviour has something to do with the material properties itself. Then, the boundary condition boundary condition in the sense what type of external loads are acting and how.

So, to understand or to have these basic requirements, one need to understand what is the stress that gets developed within the body and to be very specific stress acting at a point. So, stress acting at a point is mandatory in order to address the basic requirement which has been discussed above.

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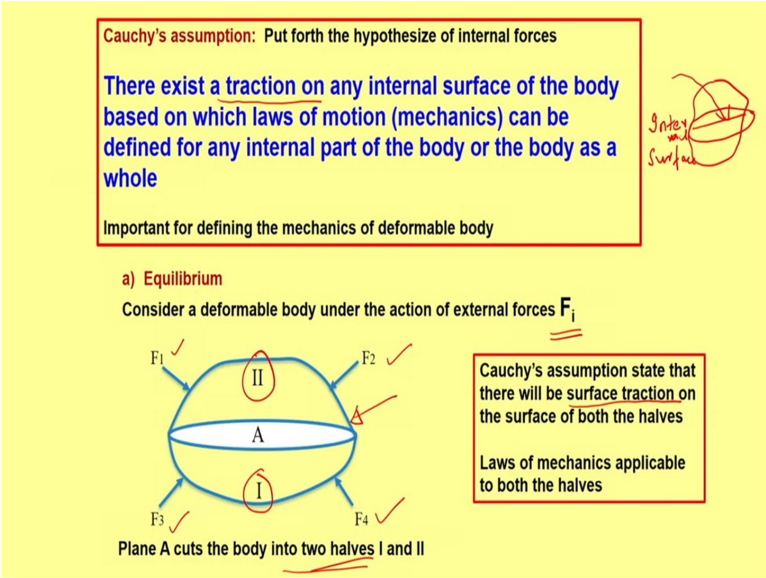

Now, how do we do this, so before getting into the actual stress at a point, we need to discuss about Cauchy's assumption, he put forth the hypothesize of internal forces, how do we know that when a body is acted upon by some forces, there is an internal force which gets developed. Now, this is based on Cauchy's assumption or rather hypothesis of internal forces, the entire mechanics of stress at a point is built on this.

To be very specific, what is the assumption stating? There exist, this is very important, there exist traction on any internal surface of the body based on which laws of motion or other mechanics can be defined for any internal part of the body or the body as a whole. For example, if you have a body it is not 3 dimensional, this 2 dimensional, what it means is that, if there is an internal surface of the body, let us say this is the internal surface.

Now, what it means is that, there is a traction that gets developed on any internal surface of the body, it can be any reference plane based on which the laws of motion can be defined for any internal part of the body and any internal part of the body itself would confirm to the whole body as a whole. So, the further discussion what we are going to see will entirely depend upon this assumption.

Now, this is very important for defining the mechanics of deformable bodies. So, any body which undergoes some load will try to deform. So, this particular aspect put forth by Cauchy’s assumption is important for defining the mechanics of deformable body. Let us say the equilibrium condition consider a deformable body under the action of external forces Fi let us say why it is Fi because you have F_1, F_2, F_3 and F_4.

Now, this is a typical deformable body. Now, there is a plane which this deformable body is cut into, that is plane A cuts the body into 2 halves 1 and 2. So, you have a deformable body and a plane A cuts the body into 2 halves that is 1 and 2. Now, according to Cauchy's assumption, it states that there will be a surface traction, traction means a kind of stress or a force there will be a surface traction on the surface of both the halves.

So, Cauchy's assumption state; that there will be a surface traction on the surface of both the halves there. So, if you consider half 1 or half 2, there will be a surface traction which gets developed and law of mechanics applicable to both the halves. So, if you consider half 1 or half 2, it is one of the same the overall response of the body can be defined if you consider half 1 or half 2. And what is more important and what need to be stressed here is there is a surface traction, there is a traction that gets developed on the surface which is cut by the plane A so, that is what we need to focus.

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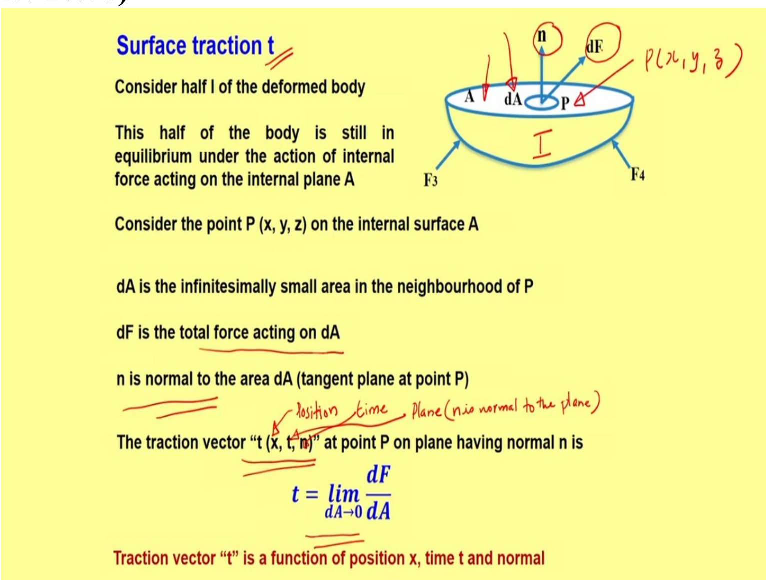

So, now, we will discuss more about surface traction which is represented by small t and now, we are considering only half 1, so, this is half 1 F_3 F_4 is there and the plane A is there. So, this half of the body it is still in equilibrium under the action of internal force acting on the plane A. So, you have external forces F3 and F4 which has indicated it also has some internal force that gets developed which keeps the body in equilibrium.

The same is identical to half 2 as well. So, when it joins, it becomes the whole body it comes consider the point x, y, z on the internal surface now, on the plane A we can consider any point P which is denoted by P x, y, z and that is on the internal surface A. Now, consider dA this is dA which is a small infinitesimal area in the neighbourhood of P so it is very close to P.

And you consider a small area dA or infinitesimally small area dA in the neighbourhood of P now, dF is the total force which is acting on dA let us say this dF is the total force which is acting on dA and the most important thing and which all of you know is how to define a plane now, we have plane A with us now, how will you define or how will you identify this plane, we identify the plane using normal acting to that plane.

So, n is normal to the area dA and what is dA? It is the tangent plane at point P why tangent because you are considering a point P so, normal should be with respect to that tangent. So, that is why it is called tangent plane. So, n is the normal to the area dA, which we are considering and there is dA is in the neighbourhood of P so, that is what we understand now. Now, the traction vector t, is defined at point P on plane having normal n by

$$t = \lim_{dA \to 0} \frac{dF}{dA}$$

Now, this is similar to the definition of stress. Now, if you see here, traction vector t is expressed as a function of x t and n what does that mean, traction vector t it is the function of position, it is a function of time and it is the function of plane n is normal to the plane.

So, in a sense the traction vector t is a function of position. It is a function of time and plane. So, it is always identified by some point in this space. So it is not as it is some position, it is also defined by time, because you are considering a deformable body. So, what deformation does this mechanics correspond too so, that is why time becomes important and it is also dependent on which plane it is acting. So, this plane is identified by n. So, traction vector t is a function of position x time t and normal.

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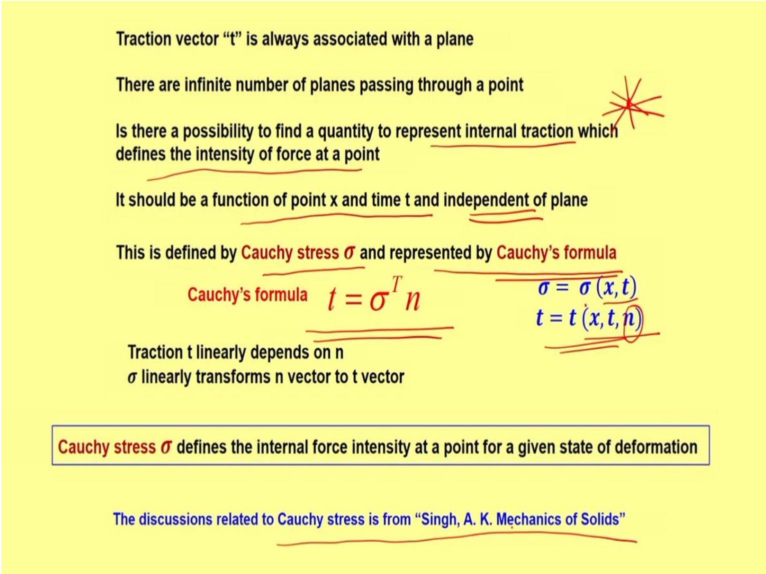

Now, we can see that the traction vector t is always associated with a plane problem is there are infinite number of planes passing through the point. So, what happens is, if you consider point here you have different possibilities of planes. Now, if traction vector t is associated with a plane, so which plane are we going to consider, so this is not a proper preposition. So, is there a possibility of finding out a particular quantity which is a substitute or which represents the traction vector?

At the same time, it defines the intensity of force at a point because traction does it, but traction is associated with a given plane. Now, there are different planes now, can you find a quantity which is not dependent on plane, but at the same time it would represent the internal force. So, it should be a function of point x and time t and very specifically, it should be independent of plane.

Is there any quantity as such this is very much possible this is defined by Cauchy's stress σ and represented by what is known as Cauchy's formula and what is Cauchy's formula?
It is in this manner,

$$t = \sigma^T n$$

Where, t is traction vector, σ is known as Cauchy's stress, because the whole of the formulation is based on Cauchy's hypothesis and assumption. So, because of that, σ is known as Cauchy's stress and $t = \sigma^T n$, this is very important and one should keep in mind for all further discussion, so, this Cauchy's formula and Cauchy's stress these are the two terms which we need to keep in mind further, so, what is the idea the idea is about finding a quantity, which is not dependent on plane but it still represents the internal force.

Now, σ or Cauchy's stress is such a term now, what is that so, σ is a function of x and t whereas, traction is a function of all the three. So, we have eliminated the contribution of plane here. So, traction t in this it linearly depends on A you can see in the expression and σ transforms n vector to t vector. So, that is what it is according to Cauchy's formula. So, Cauchy's stress σ defines the internal force intensity at a point for a given state of deformation please keeps this mind for a given state of deformation.

That is why the time factor becomes important. So, this σ defines the internal force intensity and that is what we want. And according to Cauchy's formula, it ensures that the σ defines the internal force intensity which traction also did at the same time it is independent of the plane. The discussions related to Cauchy's stress is taken from Singh, A.K mechanics of solids book, this book I have listed in the reference.

It is a very good book for beginners in continuum mechanics. And it explains well so, most of the discussions which I am following in module 1, especially in the initial half is from this particular book you can refer to it.

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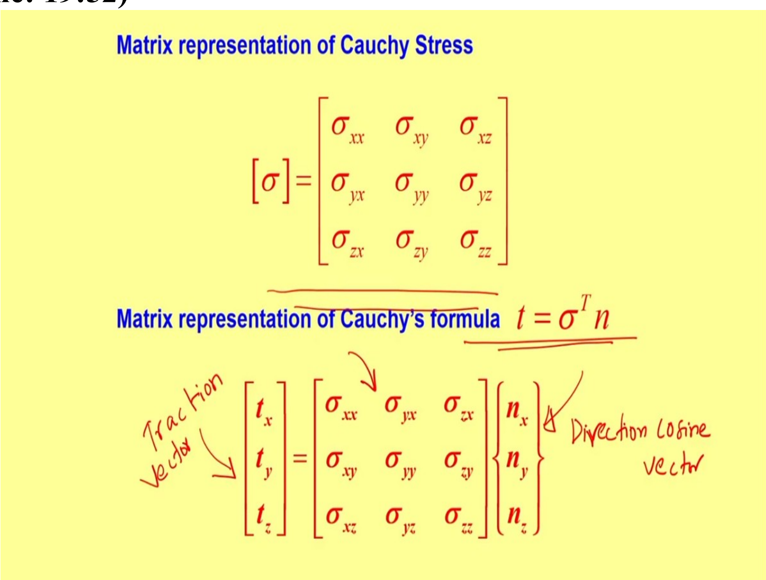

So, now, we will come to the matrix representation of Cauchy's stress which is given as follows now in the matrix form the stress σ, which is known as Cauchy's stress,

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$

The matrix representation of Cauchy's formula is in this manner.

$$\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

So, you have traction vector then you have these stress Cauchy's stress tensor or Cauchy's stress and this is direction cosine vector. So, direction cosines these are always associated with the plane. So, you have $t = \sigma^T n$. So, this is what the matrix representation of Cauchy's stress and Cauchy's formula.

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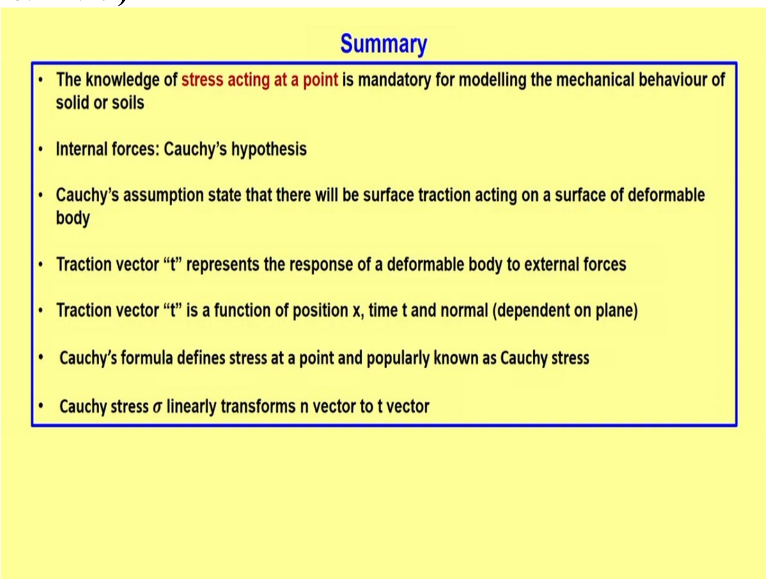

So, in summary, the knowledge of stress acting at a point is mandatory for modelling the mechanical behaviour of solid or soils. In our case it is advanced soil mechanics. So, that is why I have specified the term soils as well internal forces, it is entirely dependent on Cauchy's hypothesis. And according to Cauchy's assumption, it is stated that there will be a surface traction acting on a surface of deformable body.

And the entire definition of stress acting at a point is dependent on this the traction vector t represents the response of a deformable body to external forces, the traction vector t is a function of position x time t and normal dependent on plane and Cauchy's formula defines stress at a point and popularly known as Cauchy's stress. Cauchy's stress σ linearly transforms n vector to t vector. So, this is the summary of whatever we have seen till now.