Advanced Soil Mechanics
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Lecture - 19
Shear Strength Granular Soil - 1

Welcome all of you, in the last lecture we have discussed about basics of shear strength and stress representation in different form. Now we will get into the shear strength of soils specifically, while doing so, we will divide the soil into 2, the first one will be for cohesionless soil and the other one is for cohesive soil. This is divided in this form because the shear strength behavior or shear strain behavior of these 2 classes of soils are entirely different.

While you know that for cohesive soil it is generally influenced by the drainage condition and pore water pressure we will see that in detail whereas for cohesionless soil the drainage it is almost draining soil, so the question of undrained condition does not come into effect except for some very specific problems like liquefaction. So, we will now first get on to the shear strength of granular soils.

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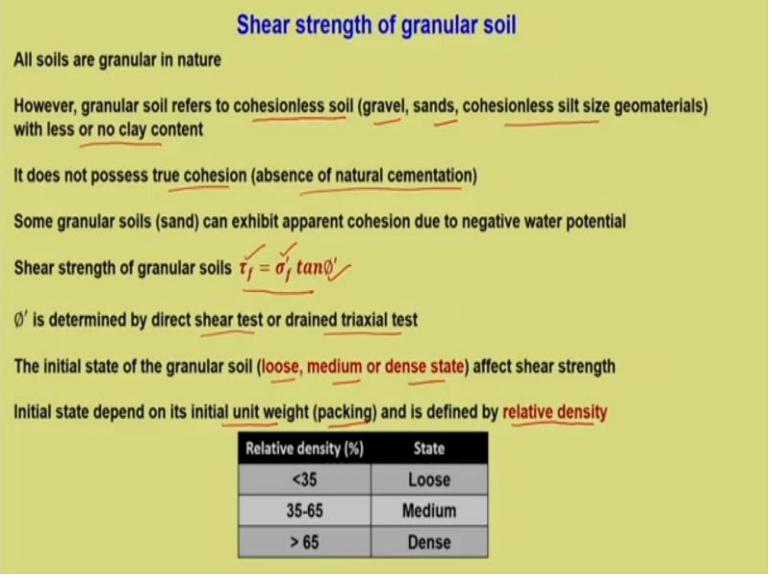

Now we can see that all soils are granular in nature. What is so specific about it? But here when we discuss about soil mechanics, we will refer granular soil to essentially cohesionless soil which will include gravel sands or cohesionless silt size geomaterials because this will include materials like fly ash which are essentially silt size and hence they can be considered as geomaterials with less or no clay content.

So, here when we say granular soil it essentially refers to cohesionless soil. It does not possess true cohesion that means, it is the absence of natural cementation. But some granular soils like sand, it can exhibit what is known as apparent cohesion which is conditions specific. It depends upon the water content or the degree of saturation of that particular sand. You might have noticed that it is fairly comfortable walking on beaches which are partially wet. Why this is so?

This is mainly because of the capillary action or the negative pore water pressure which we call it as soil suction which is present in partially saturated sands. Hence it imparts more strength and hence you are able to walk comfortably on beaches. But that is not the case with dry sand or fully wet sand. So, this is known as the contribution or this additional strength which the sand posses under partially saturated condition, this can be termed as apparent cohesion.

So, we are not considering about apparent cohesion right now, we, what we understand is cohesionless soil there is no true cohesion. So, since we understand that for cohesionless soil there is not much influence of drainage and there is no cohesion coming into picture, the understanding shear strength behavior is relatively simple as compared to that of cohesive soil which we will see later.

So, we can conveniently write shear strength of granular soils, $\tau_f = \sigma'_f \tan \varphi'$ where τ_f and σ'_f these are stresses acting on the failure plane which we have already seen in the previous lecture and φ' is the angle of internal friction. Please note that τ_f there is no prime, σ'_f that is for normal stress, it is always in terms of effective stress.

φ' is mostly determined by either direct shear test or drained triaxial test because the permeability of granular soil is fairly high. The initial state of the granular soil whether it is a loose medium or dense state affects the shear strength. Now what is this initial state? The initial state depends upon the initial unit weight or packing or the compaction state and is generally defined by the term relative density which all of you knows.

So, as a guideline, a relative density less than 35% is considered to be loose, 35 to 65% is medium and greater than 65% is considered to be dense. So, we are basically discussing about 3 states of the granular soil which we define in terms of loose, medium and dense. So, this relative density takes care of what is the kind of initial packing of the granular materials.

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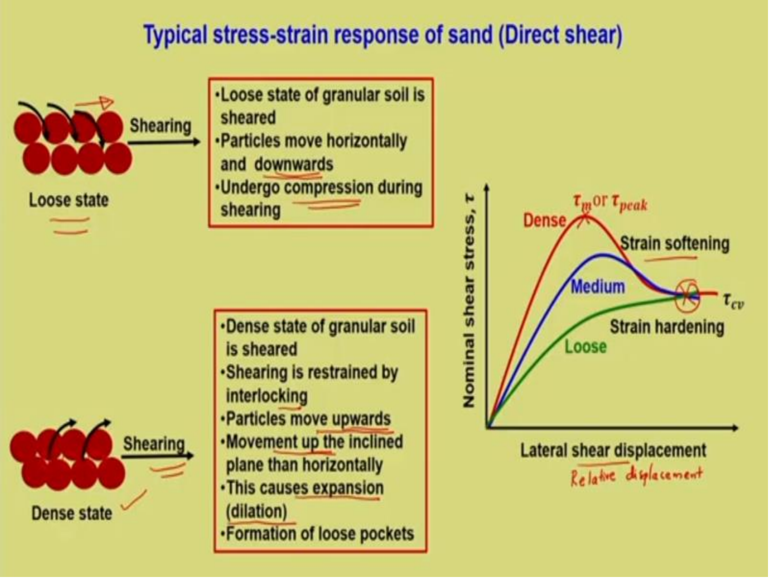

So, now we will see the typical stress strain response of sand which is a granular material from a typical direct shear test wherein the x axis is represented by lateral shear displacement and y axis is given by nominal shear stress, τ. Now we know in direct shear test, it is very difficult to quantify the strain and the true stresses acting on the failure plane or which is acting on a given plane.

The reason is the area of direct shear keeps changing during the test because there is a horizontal movement of the upper box. So, it is very difficult to state the true stresses acting and hence the term nominal shear stress, τ and instead of strain we express this in terms of lateral shear displacement or it is written in terms of relative displacement, intentionally we do not use the term strain there.

So, here there are 3 states we have discussed, we can see that the stress strain response is drastically different for dense, medium and loose, at least dense and loose it is fairly different. What is happening? We can see that for a dense soil or the dense state of the soil initially there is an increase in shear stress reaches a maximum value and further which it starts reducing. Now this post peak reduction is what is known as strain softening.

So, there is a maximum shear stress or peak shear stress in the case of dense packing. Medium it reduces but for a loose state of packing, we can see that there is an increase in stress but there is no reduction in stress. Somewhere at some point there will be yielding of the loose state. So, even after yielding we can see that the stress keeps on increasing and all of these finally reaches to a relatively constant value which is known as critical shear stress, τ_{cv} or constant volume shear stress or critical shear stress.

So, whatever be a dense reaches peak strain softens and a loose keeps on densifying and reaches finally a critical state. So, this is the general understanding of stress strain response of sand or the same type of behavior is applicable for cohesive but in a different framework that we will see when we discuss the shear strength of cohesive soil. Now what is happening is, this is what a loose state represents.

That means soil particles are loose and when this loose state is subjected to shearing that is the horizontal movement what will happen is when the loose state of the granular soil is sheared, particle can move horizontally and when it moves horizontally in this direction, what happens is, the particles moves downwards as you can see the arrow all these particles goes and fill into the voids because it is a loose state and hence, it moves downwards.

This is very important aspect which we need to understand when a loose state is sheared particle moves horizontally and downwards as it can shown by the arrow and hence it undergo compression during shearing. This is an important point which we need to keep in mind a loose state when it is sheared undergoes compression. This is very important observation. In the case of dense state it is already in these voids.

So, there is a good interlocking of materials. Now let us say that this dense state is sheared. Now for shearing means it has to move horizontally. Now what is the possibility of such a movement? If it has to move horizontally then it has to undergo some movement, so that we will see. So, the dense state of the granular soil is sheared, so it is sheared. Shearing is restrained by interlocking because it is already occupying the spaces it is densely packed, so there is a good interlocking.

Now during shearing, if it has to move horizontally then it has to move in the upward direction, the particles have to move in the upward direction, this is different from the loose soil. So, movements up the inclined plane then horizontally. So, if it has to make any sort of horizontal movement, the movement of this particle is more like moving up the inclined plane. So, it has to do additional work.

Now this causes expansion or what is known as dilation. Please remember this carefully like when a dense state of soil is sheared for executing shear, it has to expand or it is known as dilation and during dilation there are formations of loose pockets because a dense material which is packed when it expands definitely it leads to the formation of loose pockets. So, this is the change in behavior or this is the difference in behavior when a loose and dense state of the material is sheared.

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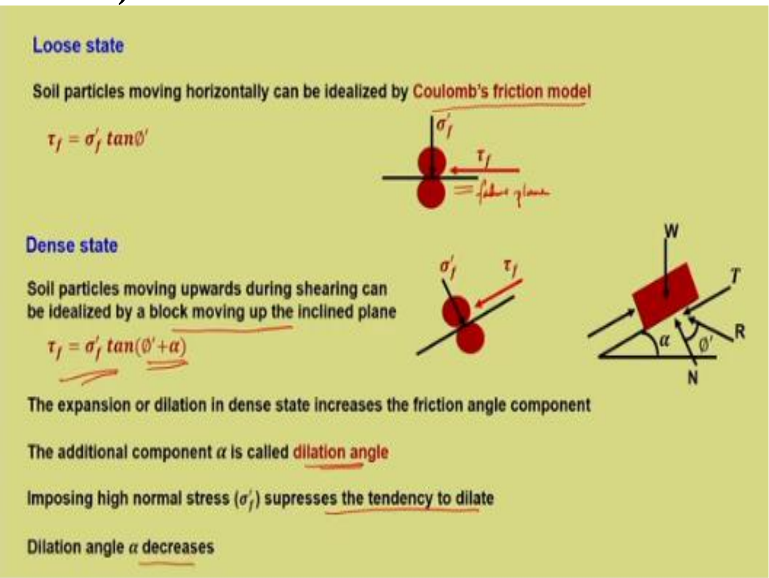

Now let us see loose state in detail. Soil particles moving horizontally can be idealized easily by Coulomb's friction model as what we have learnt. Because here it is loose state there is an intergranular contact where τ_f, σ'_f is acting, this is the predefined failure plane along with it acts. We know the Coulomb's friction model is applicable only when there is a failure plane attached to it, so we can easily write $\tau_f = \sigma'_f \tan \varphi'$, no confusion in that.

So, let us see for a dense state, what is going to happen? Soil particles moving upwards during shearing can be idealized by a block moving up the inclined plane. Now it is already in a interlocked position, so it is more like this. So, the plane now is inclined at the point of contact or tangent plane is inclined. So, same interpreted on an inclined plane upwards. So, what is happening? You can be idealized by the block which is moving up the inclined plane.

So, you can see that there is an, this is a block here, the weight of the block acting downwards R is the resultant, φ' is the angle and here the slope angle is α. Now this is a very prominent friction problem that we have learned and we can conveniently write $\tau_f = \sigma'_f \tan(\varphi' + \alpha)$ because it is moving up the inclined plane. Now if it is down the inclined plane this will be minus, in soils both will be there.

There can be upward and downward movements for particles but generally in the case of dense soil for explaining the shear behavior, we normally refer to dilation or expansion. The expansion or dilation in dense state increases the friction angle component, this is what it is. So, it increases from φ' to φ'+ α. The additional component α is called the dilation angle very important.

Now imposing high normal stress, now if the normal stress is more, let us say, what happens is, this particle will not be able to traverse in the upward direction, you are restricting the movement from the upward there is a confinement. Now this confinement will not allow the particles to move upwards freely which means to say that the amount of dilation that is going to happen is a function of the normal stress. Now in this case normal stress is more like a confining stress from above. So, imposing high normal stress suppress the tendency to dilate because of which the dilation angle decreases.

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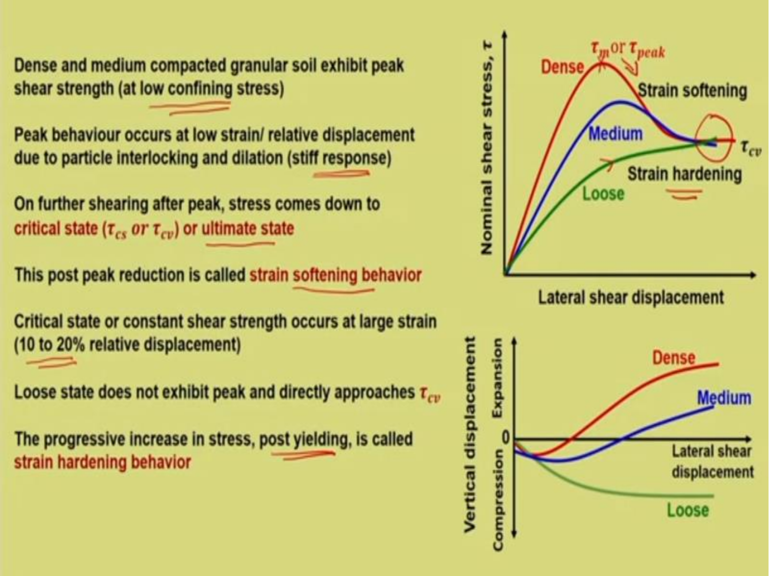

Now we will see this again. There are some facts which we will enlist where dense and medium compacted granular soil exhibit peak shear strength specifically at low confining stress because the dilation has to happen then only the dilated behavior expansion followed by the peak behavior is going to happen. So, this peak which is marked here is a function of this dilation behavior. So, peak behavior occurs at low strain or relative displacement due to particle interlocking and dilation.

And it is mostly a stiff response which means particles are tightly held so, it is a stiff material. Now the peak happens at a low strain. So, it keeps on increasing, reaches a peak but this peak behavior is exhibited at relatively lower strain not at the highest strain or it is expressed in terms of relative displacement whatever and this happens because of the dilation or particle interlocking. On further shearing after the peak, stress comes down to critical state or τ_{cs} or τ_{cv} or it is known as the ultimate state.

We have already told us this particular movement downwards not movement this stress comes down. So, this post peak reduction is called strain softening behavior. Now critical state which is the state here or constant shear strength occurs at large strain, generally it is 10 to 20% relative displacement. Now this is again defined in different codes, it is generally 10 to 20% of strain levels you can expect critical shear strength.

Loose state does not exhibit peak and directly approaches τ_{cv} as can be seen here directly approaches τ_{cv}. The progressive increase in stress after yielding, post yielding is called strain hardening behavior which is denoted here.

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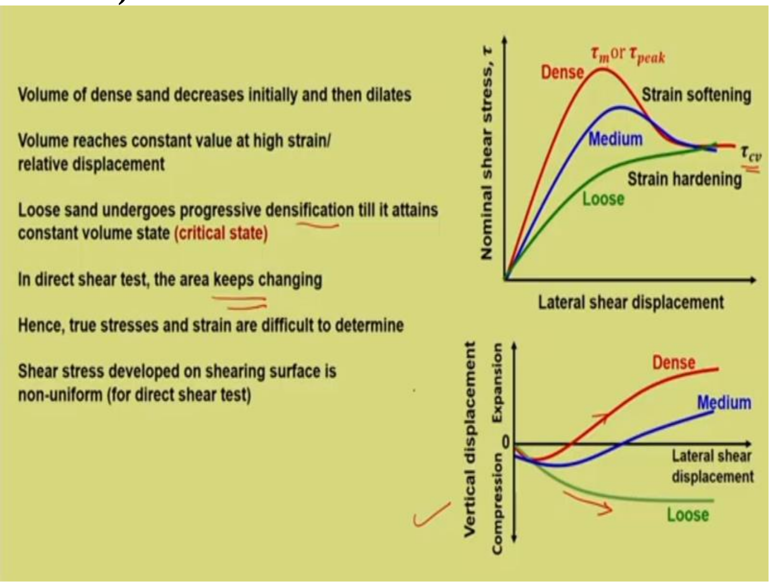

Now that is about the stress strain behavior whatever we have discussed, let us see what happens to the volume? Because we do not generally discuss about undrained or no volume condition for granular soil there will be volume change and how it happens for dense and loose state, we will see that. The volume of dense sand decreases initially and then dilates. So, here on the y axis, it is vertical displacement and on the x axis it is lateral shear displacement or relative displacement.

Now for the dense soil, we can see that there is a initial decrement in vertical displacement which is nothing but compression. So, below is compression, no volume change and expansion. For dense soil there is initial compression followed by an increase in volume which we call it as dilation. So, by the time it dilates, it reaches the peak behavior. Volume reaches constant at high strain, relative displacement. So, by the time it reaches the critical state, the volume change also remains constant there is no more volume change.

It reaches a constant volume or a constant value at high strain condition, maybe close to critical state. Loose sand undergoes progressive compression or it is progressive densification till it attains critical, constant volume or critical state volume. That is also at a high strain. Indirect shear test, the area keeps changing. Hence true stresses and strain are difficult to determine which I have already explained. Shear stress developed on shearing surface is non uniform for direct shear test because at the edges and towards the center it will not be same.

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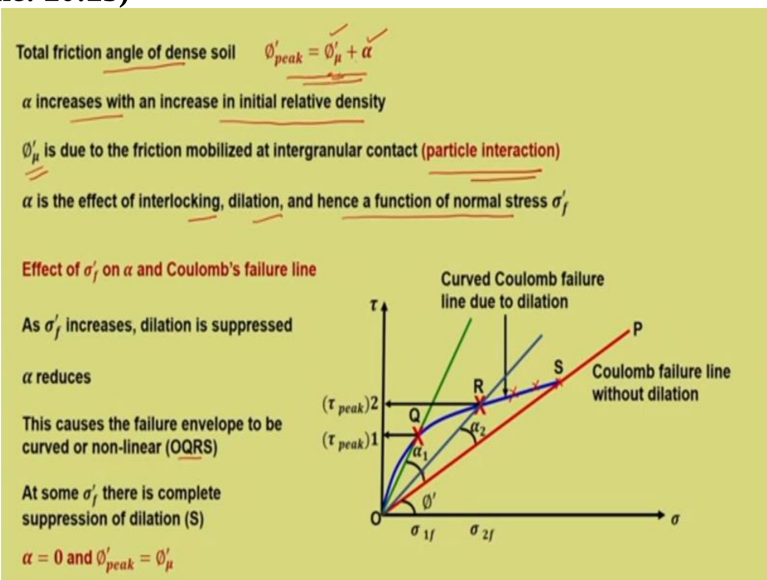

Now, for dense soil, the total friction angle can be written as $\varphi'_{peak} = \varphi'_{\mu} + \alpha$, we have already discussed what is α. Now peculiarity about α is, it increases with an increase in relative density we have seen that. Depending upon what is the relative density the dilation keeps changing, so that will be reflected in α. So, alpha becomes a function of initial state or denseness. If it is more dense α will be more.

Now φ'_μ this component is due to the friction mobilized at intergranular contact there is the particle interaction and this is somewhat a fundamental aspects of a soil, φ'_μ which is mobilized at the particle at contact. α is the effect of interlocking, dilation and hence a function of normal stress σ'_f. So, in this total friction angle of dense soil, φ'_μ is a function of particle interaction, α is a function of dilation.

Now, since α is a function of dilation we have already seen that α is a function of normal stress, if normal stress is more it suppresses the dilation, so we will see the effect of σ'_f on α and Coulomb's failure line, how it changes the Coulomb's failure line? This τ versus σ which we normally represent the shear strength behavior for a cohesion less material, this columns failure line without dilation. So, this is φ', so it is very easy to understand OP is the failure line represented by Coulomb's model.

As σ'_f increases, the dilation is suppressed. So, when corresponding to σ_{1f}, let us say, it is a dense soil, so it will have the dilation angle of α_1 corresponds to a $\tau_{peak\ 1}$. So that is the first state. Now let us say that for a, as the normal stress increases, for example, at σ_{2f}, what is happening? We can see that the dilation angle gets reduced. Now this is the φ' now that is what is denoted by φ'_μ.

Now what is above that? Above φ' is α_1 that is getting added. So, this is α_1, at lower normal stress α_1 is more but as σ_{2f} it increases this α_2 is less than α_1 which is denoted by point R. So, this results in a curved Coulomb's failure line which is due to dilation. If you joins through these points then you will get a curved line. α reduces with σ'_f and hence results in a curved failure line which is represented by OQRS.

And at some σ'_f that is along this x axis there is a complete suppression of dilation. So, if you keep on increasing, so there will be another point here at some point here, there is a complete suppression of dilation and α tends to 0 and the failure line it merges with the initial failure line where Coulomb's failure line without dilation is there. So, α becomes equal to 0 at this particular point $\alpha = 0$ and $\varphi'_{peak} = \varphi'_\mu$ according to this equation. So, α is 0, so $\varphi'_{peak} = \varphi'_\mu$.

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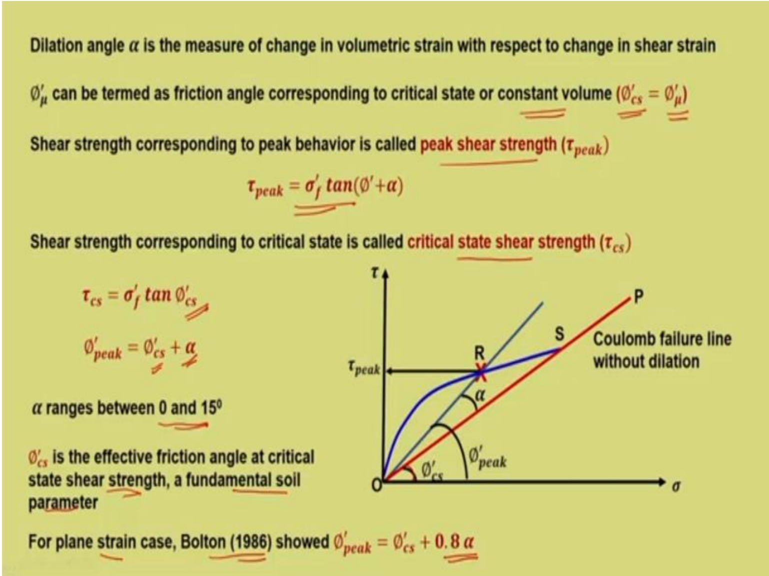

So, dilation angle α is the measure of change in volumetric strain with respect to change in shear strain. And φ'_{μ} can be termed as friction angle corresponding to critical state or constant volume. First we have tried to understand what is φ'_{μ}? Now what is the significance of this φ'_{μ} because this is the fundamental aspect or the fundamental friction behavior of the soil. Whatever be the state, there will be some kind of friction angle which is the critical state friction angle.

We will be able to appreciate this better when we finish the fourth module on critical state for the timing, let us understand that there is always some value which is which remains and which can be mobilized by a soil that is a critical state and φ'_{μ} is termed as the friction angle corresponding to critical state or constant volume. So, $\varphi'_{cs} = \varphi'_{\mu}$. Now, whether dilation happens or not that will determine α or the additional component that gets added to the soil.

So, shear strength corresponding to peak behavior is called peak shear strength that is τ_{peak} and it is expressed as $\tau_{peak} = \sigma'_f \tan(\varphi' + \alpha)$. Strength corresponding to critical state is called critical shear strength τ_{cs} which is given by, $\tau_{cs} = \sigma'_f \tan \varphi'_{cs}$. Now remember this is critical strength, friction angle and $\varphi'_{peak} = \varphi'_{cs} + \alpha$.

Earlier we have written in terms of φ'_{μ} that is for our initial understanding. But where it boils down to? It boils down to φ'_{cs}. So, Coulomb's failure line without dilation and here this friction angle which is the friction angle which is mobilized by soil at any streak that is φ'_{cs}. On top of it we get α which is φ'_{peak}. α generally ranges between 0 and 15 degrees.

φ'_{cs} is the effective friction angle at critical stage shear strength, a fundamental soil parameter. This aspect of fundamental soil behavior can be better appreciated again when we discuss this aspect in module 4. For a plane strain case, Bolton in 1986 showed that $\varphi'_{peak} = \varphi'_{cs} + 0.8\ \alpha$. Here we have taken plus α but there is not much of a difference.

But specifically for plane strain condition, Bolton explained this to find out what is, how the dilation angle contributes to φ'_{peak}. So that is all for now. We will see in the next lecture, the remaining portions of shear strength of granular soil were in 3 aspects we will be discussing, one is the critical void ratio then we will discuss about Taylors model and there is something called intrinsic friction. So, these 3 aspects will be covered in the next lecture. Thank you.