Advanced Soil Mechanics

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Lecture - 18
Stress Representation

Welcome back. So in the last lecture, we have started module 2 and we were discussing about the basics of shear strength, we have discussed about Coulomb's model which is applicable for granular materials like soil then we discussed about integrating Mohr circle with coulomb's model and that is how we got Mohr Coulomb failure envelope. And we discussed about some basic facts about Mohr Coulomb failure envelope and how important it is for materials like soils in defining the failure criterion.

Now, it is a continuation of that particular lecture, we have represented the stress in terms of shear stress and normal stress. Now there are various other ways of representing stresses for different failure criterion. So today's, this particular lecture, we will see what are the different representations of stresses for defining the failure of a material?

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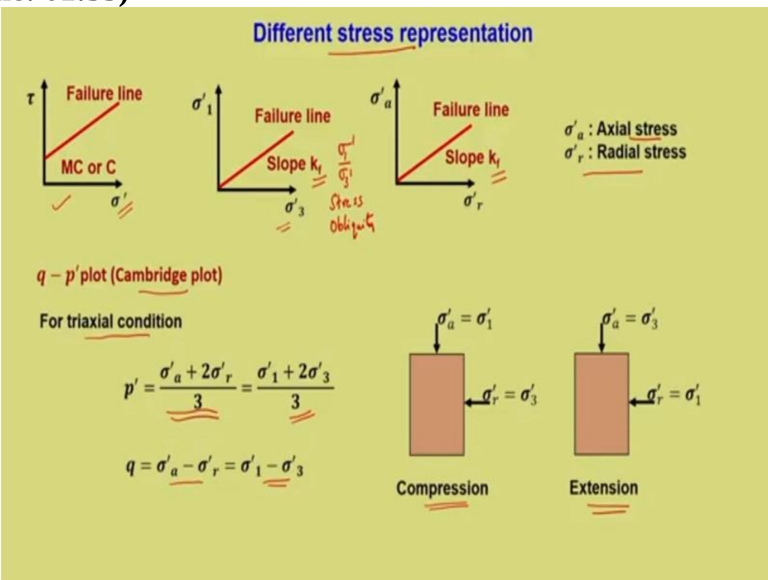

So the first one is about different stress representation, the first one is familiar to us which is τ versus σ' where σ' is the effective normal stress. Now you need to note a specific point here, I have started introducing dash or prime. Now this is very important because now, we have started the actual discussion of strength and we know by effective stress principle strength is completely governed by the effective stress.

So we have τ versus σ' and this is a typical coulomb failure line or Mohr coulomb failure line. We need not discuss about this anymore, we have already seen this. Another way of stress representation is major principle stress versus minor principle stress. And we will get a line here and that is the failure line within which the soil remains. The possible stress states of the soil remains.

We also have discussed and we have also seen $\frac{\sigma'_1}{\sigma'_3}$ is the stress obliquity and we have also discussed about maximum stress obliquity in the last lecture. So this is another way of representing the stress or as a modification of this, this one, we can also represent in terms of a more general term where it is axial stress versus radial stress and all normal stresses need to carry this dash or prime indicating that it is effective.

So σ'_a is axial stress and σ'_r is radial stress. So you can also plot between σ'_a verses σ'_r and again there is a failure line and the slope of the failure line is represented by k_f. So in all these representations the slope of the failure line is k_f. So these are some different ways of stress representation. Now to be very specific in terms of failure criterion, we also have some other representations.

Which is q-p' plot or it is known as Cambridge plot possibly due to the contribution towards this from the research group at Cambridge. So we will see for specifically for triaxial condition. In general we have already seen, what is q? What is p'? What is q? q is deviator stress and p' is mean stress for 3 dimensional state. We have discussed this in general in our previous lectures.

Now here we will specifically confine our discussion to triaxial condition because that is going to be a very prominent condition for soils. All our determinations in the lab confines to mostly triaxial condition or in other cases it will be plane strain condition. Whatever be in today's representation, we will confine to triaxial condition and q and p' for triaxial condition we have already evolved in the previous lecture.

Accordingly we know p' that is the mean stress is

$$p' = \frac{\sigma'_a + 2\sigma'_r}{3}$$

$$p' = \frac{\sigma'_1 + 2\sigma'_3}{3}$$

Now let me make it a point clear here. It is always better to write in general terms and when I say general terms, I mean in terms of axial and radial stresses, why because whether σ'_a that is axial stress equal to major principle stress will depend upon certain condition, we will see that in the subsequent slides.

So I would always suggest that we write p' and q in terms of axial and radial. For the; time being let us not worry; about the major, minor principle stress. So triaxial condition, what is the triaxial condition? We have already seen I will not discuss that,

$$p' = \frac{\sigma'_a + 2\sigma'_r}{3}$$

because order on stresses. So $\sigma'_2 = \sigma'_3$. So in the order on stress it is same, so 2 σ'_r / 3 q = σ'_a - σ'_r or it is σ'_1 - σ'_3 dash this may change depending upon conditions.

So that is why will confine to axial and radial. So here in compression you will see that σ'_a or the axial stress is equal to σ'_1 and $\sigma'_r = \sigma'_3$. This is a typical triaxial condition that we deal with in the lab. Now if you slap it that is $\sigma'_a = \sigma'_3$, $\sigma'_r = \sigma'_1$, it becomes a typical case of extension, why? It gives more like a squeezing effect. So σ'_1 is more, so it has a tendency to extend so that is a typical case of extension and this is a typical case of compression.

So if you want to specifically define in terms of compression and extension then it is always better that we define these stresses in terms of axial and radial that is the point I want to make. So we will confine to that.

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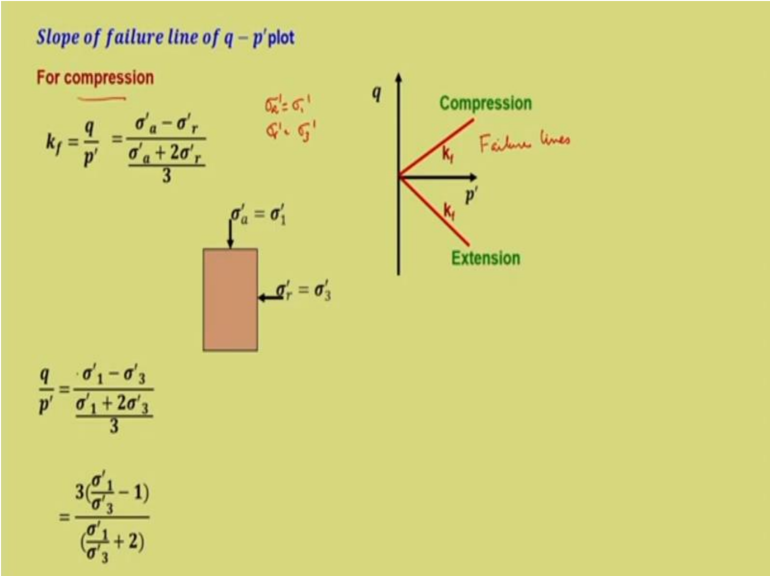

Now, let us see the slope of the failure line in q-p'plot. So what is the slope of τ versus σ plot? It is nothing but φ', so that is the slope. Similarly we want to find out what is the slope of failure of q-p'plot. So this is q-p'plot and I have drawn 2 lines, these are failure lines and one line corresponds to the case of compression, the other line corresponds to the case of extension.

So depending upon whether it is compression or extension, we need to take appropriate stresses and that we have seen in the previous slide. So k_f is nothing but q/p'. Now we are taking the case of compression. This means that $\sigma'_a = \sigma'_1$, $\sigma'_r = \sigma'_3$. Substitute for q and p' that will give $\sigma'_a - \sigma'_r / [(\sigma'_a + 2\sigma'_r) / 3]$ always represent in terms of axial and radial.

So this is what it is, substitute for σ_a and σ_r that will give you this expression. Rearranging you can take σ'_3 outside, so it will become

$$= \frac{3\left(\frac{\sigma'_1}{\sigma'_3} - 1\right)}{\frac{\sigma'_1}{\sigma'_3} + 2}$$

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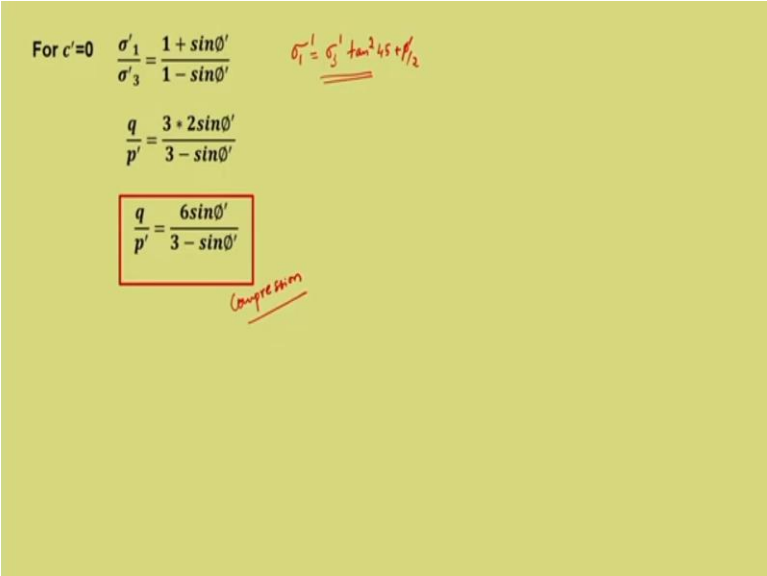

Now, we have $\sigma'_1 = \sigma'_3 \tan^2 (45 + \varphi'/ 2)$ this is as per the Mohr Coulomb failure envelope. Now this is not going to change, this remains same. So we have what is the ratio? Now this is for c' = 0, we are not considering cohesion for the time being, so we will consider the Mohr Coulomb failure envelope that will give $\sigma'_1 / \sigma'_3 = \tan^2 (45 + \varphi'/ 2) = 1 + \sin \varphi'/ 1 - \sin \varphi'$.

You substitute this in this particular expression of q / p' that will give you (3*2 sin φ')/ (3 - sin φ'). So the failure envelope, the slope of the failure envelope will be q / p' = 6 sin φ'/ (3 - sin φ'). Remember this is for the case of compression. Now the same can be obtained for extension as well that we will see later maybe as a part of assignment, for the time being this is only for compression.

So what we have done? For a specific representation of stresses in terms of q and p', we have defined the failure line. Remember this is going to be useful throughout this course, why? We need this in stress path; we also need this in critical state module as well. So please pay more attention to this.

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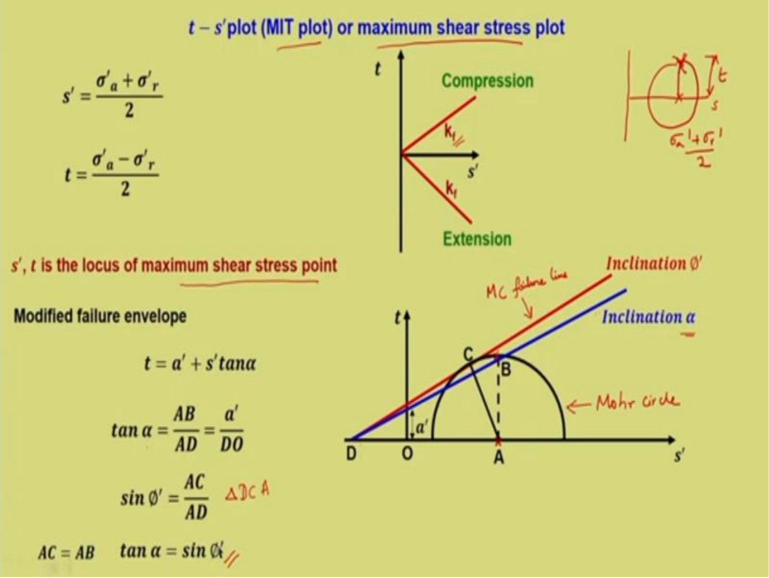

Now another representation of stress is t –s' plot or it is called MIT plot or it is known as Maximum Shear Stress plot. What is maximum shear stress? When you draw a Mohr circle, we know that there is a maximum point that we obtain and we have seen that this maximum point is not the stress which causes failure but a critical combination causes failure. So here we will specifically talk about the maximum shear stress point.

If you draw a Mohr circle, I am talking about this particular point, this is the maximum shear stress point and if you discuss it in terms of σ'_a or σ'_r, this point will be σ'_a this is S, so $\sigma'_a + \sigma'_r / 2$ and t which is this distance is t that will be $\sigma'_a - \sigma'_r / 2$. So again we have represented in terms of axial and radial stress.

So here it is the typical representation of t versus s'. So t is $\sigma'_a - \sigma'_r / 2$ and s' is $\sigma'_a + \sigma'_r / 2$ these 2 represents the maximum shear stress point. So here we are representing in this form and there is compression line and extension line, we need to find out what is the slope of the failure line in s –t' plot. So s' t remember s' we have put prime, t we have not because this is more like a shear stress, so u cancels off.

So you can, t is equal to t', so s' t is the locus of maximum shear stress point, we have already discussed that. Now, let us consider t s' plot, this is the Mohr circle we have represented Mohr Coulomb failure line, we have represented. So that is the point C at which the tangent goes. We also have this particular point through which you can draw a line. So if the Mohr Coulomb failure envelope is given by a red line, from the same point we can also draw a line which passes through the maximum shear stress point that is point B.

So, the; inclination of Mohr Coulomb failure line is φ' and let us say the inclination of the line passing through maximum shear stress point is α, so this inclination is α. What is this point? That represents s' and t point. Now this particular intercept is given by a', the inclination that is the line passing through maximum shear stress point makes an intercept of a' and we know the intercept of Mohr coulomb failure line is c'.

I have not marked it because we already know it, if I mark it; it becomes more cumbersome, so I am not marking it. So this is how it is. We want to find out what is the inclination of this particular line because we are talking about the maximum shear stress point. Accordingly we can write a modified failure envelope which is different from Mohr Coulomb failure envelope that is t, for Mohr Coulomb it is τ.

So here it is $t = a' + s' \tan \alpha$, α is the inclination. So it is more or less very identical to Mohr Coulomb failure envelope. Only thing is the parameters changed. So what is $\tan \alpha$ that is the slope. So we want what is $\tan \alpha$, this one? So that $\tan \alpha$ can be written as AB / AD, now that is also equal to this smaller triangle that is a' / DO. So this triangle and this triangle is what we have considered.

So $\tan \alpha$ is a slope what is needed for us. Now $\sin \varphi'$ if you consider triangle DCA that is triangle DCA, AC / DA is $\sin \varphi'$, so that is what is written here. Now we also know AB and AC are radius, so AB = AC that means, this expression and this expression are same. So, AC = AB, hence we can write $\tan \alpha$ which is the required inclination of the failure line of t- s' plot = $\sin \varphi'$ and φ' we know it comes from the Mohr Coulomb failure envelope.

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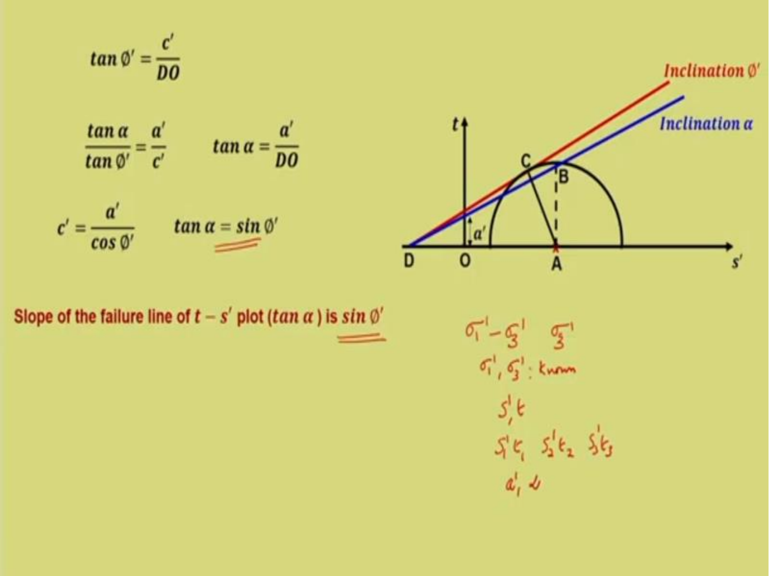

Now we also note tan φ'= c' / DO that is tan φ', considering the red line, the intercept is c' divided by DO. If I write tan α / tan φ' where tan α = a'/ DO, we can write a' / c'. So tan α / tan φ'= a' / c' from which we can write, how will we get this? So tan φ', we have already seen tan φ'= sin φ', we have already proved. Substituting that here in this expression tan α = sin φ'.

Here it will be sin φ' that will cancel though and cos φ' goes up. So it will be c' = a' / cos φ'. So slope of the failure line of t- s' plot, so this plot is tan α and that is equal to sin φ'. So what we have done? We have defined the k_f or slope of the failure line k_f of t-s' plot. Remember when you do a triaxial testing, those who have not gone through this during the undergraduate, I am just adding this detail here.

When you do the triaxial testing, what you will get is, you will get the deviator stress at failure that is σ'_1 - σ'_3 at failure and you know the confining stresses σ'_3. So you know what is σ'_1 and σ'_3? σ'_1 , σ'_3 known. Now what will you do? You need to plot the Mohr circle and get the shear strength parameters c' and φ' as applicable.

Now if you know this s'- t plot you will need not plot the Mohr circle. From the known values of σ'_1 and σ'_3 one can always find out different points that is s'_1 t_1, s'_2 t_2, s'_3 t_3 and plot this blue line, we will get maybe 3 points, corresponding to that we will get 3 points here. In t s' plot, one can always plot this blue line, once the blue line is plotted we can get a' and α.

And once you know a’ and α, we also have the expression tan α = sin φ’. So α is known, tanα is known, we can always determine φ’. Similarly we have this expression c’ = a’/ cos φ’, φ’is known, a’ is known, c’ can be determined. So this, Mohr Coulomb failure parameters can be obtained without plotting the Mohr circle once you know the concept of t s’ plot.

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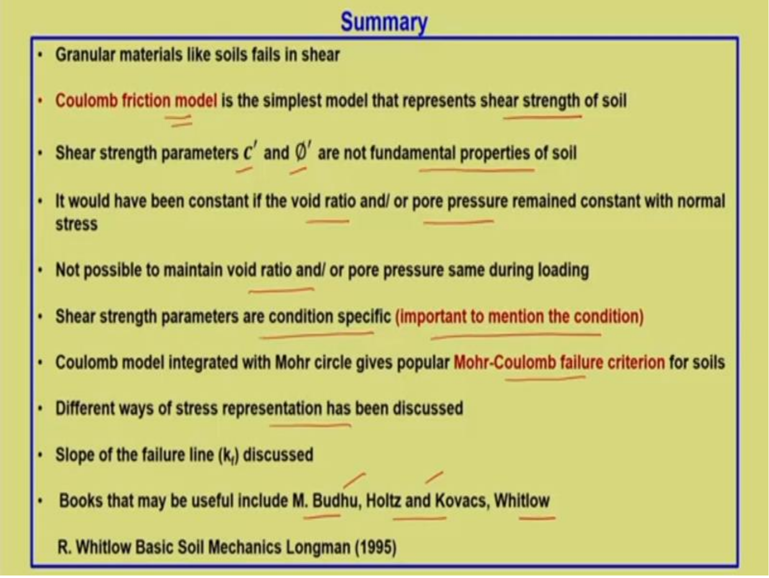

So that is all about that. So we will summarize this first lecture that is granular materials like soils fails in shear. We have already told this we need to keep this in mind. Coulomb friction model is the simplest model that represents shear strength of soil but remember Coulomb friction model, it depends upon the development of a failure plane within the soil mass. Shear strength parameters c’ and φ’, these are not fundamental properties of soil.

Now when I say this, there will be some for whom this is difficult to understand, why? It is not a fundamental property because it is governed by the condition under which c’ and φ’ has been obtained. You are not likely to get the same value all the time. But having said that, there are cases where or there is a specific condition where it can become, it can become means, mostly φ’ can become a fundamental parameter that we will see in the fourth module in detail.

So for the time being, let us understand that c’ and φ’ are not fundamental properties of soil. It could have been a fundamental property if the void ratio and or pore pressure remained constant with normal stress. Now this is a difficult proposition to make in soils either the void ratio because in drained condition there will be volume change and when there is volume change, there is change in void ratio.

If it is undrained condition, there is no volume change only pore water pressure changes that is why it is written void ratio and, or pore pressure. If these would have remained constant with normal stress then c’ and φ’ could have been a fundamental property. You remember when we discussed in module 1, we said that the volumetric component and the deviatory component we decoupled.

Even though we said that in continuum framework during that lectures, it is very hard to conceive soil without volume change during shearing which means to say deviatory component also involves change in volume. So it is very difficult to decouple. So now, subsequent lectures and all our further understanding, volume changed component is still there during shearing. We are not actually able to decouple it, even though in the tonsorial framework, it is decoupled.

This understanding one should have. So it should not lead to confusion. It has been said there it is decoupled, why it is not decoupled? It is decoupled in that framework, mathematical framework but then it is difficult to decouple to understand the behavior of soil because there will be volume change. Now if the sharing would have happened at constant volume, probably c’, φ’ components can be a fundamental parameter which we are not able to achieve.

Either the volume change will happen or the co order pressure would change. So this prevents the phi dash component to be slightly tricky and not a fundamental parameter. So this is what I told, it is difficult to maintain this to be constant during loading. So shear strength parameters are condition specific and it is very important to mention the condition. Now later we will see that in under certain circumstances it becomes fundamental rather it will not depend upon the condition.

So you underline this particular sentence here for the time being with our current understanding, we say that it is not a fundamental parameter but under specific conditions it can be treated as a fundamental parameter. So Coulomb model integrated with Mohr circle gives the popular Mohr coulomb failure criterion for soils. There are different ways of stress representation for failure criterion which has been discussed.

And the slope of the failure line, how to determine? That has been discussed for the important ones. And books that may be useful for this particular module is Muniram Budhus book, Holtz and Kovacs and Whitlow. These 2, I have already specified it in the reference but Whitlows book is not there. It is R Whitlow, Basic Soil Mechanics, Longman in 1995. This is also a good book to follow this. I am basically following these books for generating these lectures.

So that is all for now, related to basics of shear strength we have splitted into 2 lectures, both confines to basics of shear strength. In the next lecture, we will see the shear strength of granular soil. Now what we have done is, we have splitted it into granular, granular means basically cohesion less soil and then cohesive soil. So in the next lecture we will see, what is the sheer strength of cohesion less soil or granular soil? Thank you.