Advanced Soil Mechanics

Prof. Sreedeep Sekharan
Department of Civil Engineering
Indian Institute of Technology – Guwahati

Lecture - 17
Basics of Shear Strength

Welcome back one and all. So, we have been dealing with module 1 on continuum mechanics. Till now from today's lecture onwards, we will see module 2 which is shear strength of soil. Now you may be wondering why I have chosen shear strength of soil in this advanced level course? From the past few years of experience with the students who are coming for M Tech, I have noticed that some of these concepts are not conceived properly, some of these concepts related to shear strength are not conceived properly.

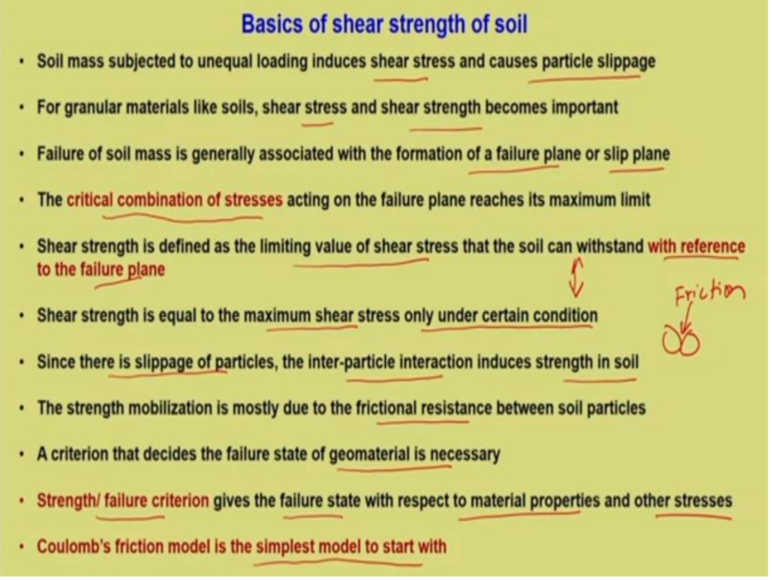
And this is very vital for module 3 and module 4. So, we need to have the concepts clear about shear strength of the soil for getting a hold on module 3 and module 4. So that is the reason why I chose to explain shear strength of soil in detail and some interpretations. So, those who have understood this topic very well during their undergraduate, you may just refresh it. The reason is, there are several ways by which people understand this very concept of shear strength.

If you just read through the textbook once, you may not find it anything complicated, everything is in place, the moment you start reading more, you are there for a lot of surprises and some sort of complexities which induce a lot of confusion in understanding this topic. This is only possible if you start reading more. So, shear strength understanding there are a lot of nuances and these nuances, we need to understand it very carefully.

So that things are clear. And I would also like to make a point here that whatever we understand is based on my viewpoint, I grasp this subject in a particular manner, another person may grasp it in a different manner. And this we have also noted when we talk about this subject to students, as well as our faculty colleagues. So, we have some sort of kind of differences in what we have understood.

So, I do not say or I do not claim that whatever I have understood is the exact one, somebody may dispute it. So, what I will be presenting here is from the majority of the facts which has been reported in some of the best and the popular textbooks. So, it is very important that we understand the shear strength of soil clearly. So, with this, let us start this lecture is about the basics of shear strength and the following lectures will be on interpretations and certain aspects which we need to keep in mind when we deal with module 3 and module 4.

(Refer Slide Time: 04:02)



So, soil mass subjected to unequal loading induces shear stress and causes particle slippage. What is the importance of this sentence unequal stresses? So, if you have noted if the order on stresses are same can you induce shear in the soil? Can it bring about failure? If you look from the perspective of the normal loading, normal loading means not σ_n, the normal range of loading that is subjected to the soil.

Possibly it will not cause any sort of failure when it is isotropic or when the order on stresses are same, it will induce volume change. So, what causes failure in the soil? Soil is a granular material. So, there has to be unequal stresses which results in particle movement and that causes deformation and this is what you call it as shear stress. So, shear stress causes particle slippage and this shear stress is caused due to unequal stresses.

For granular materials like soils, shear stress and shear strength becomes important. Stress the limiting condition is strength. Failure of soil mass is generally associated with the formation of a failure plane or slip plane. So, the whole definition of shear strength theory is based on the assumption that there is a failure plane that gets formed within the soil and I do not have to pictorially show this even though we will be having it in the subsequent slides because these facts you are expected to know by the undergraduate learning.

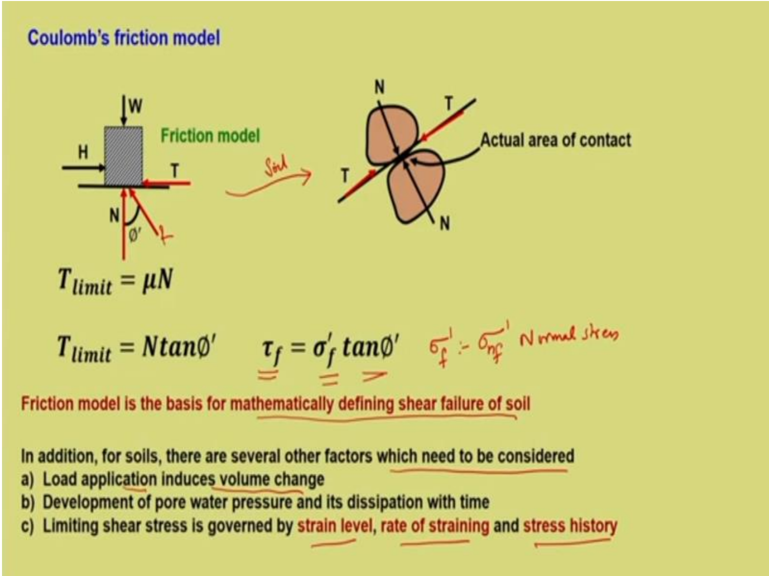
Now a very important point is the critical combination of stresses acting on the failure plane reaches its maximum limit. Now this particular phrase is very important, critical combination of stresses. So, which we will appreciate better as we move further? Shear strength is defined as the limiting value of shear stress that the soil can withstand with reference to the failure plane, see it is always done with reference to the failure plane and it is the limiting value of shear stress.

Shear strength is equal to the maximum shear stress only under certain conditions. Now please read these 2 sentences together, what is that? In the first sentence we say that it is a limiting value of shear stress which is associated with the reference failure plane. So, this failure plane. So, it is not actually equal to the maximum shear stress in the soil but the maximum shear stress becomes the failure shear stress under certain conditions. We will discuss this when we discuss more about undrained shear strength.

Since there is slippage of particles, the inter particle interaction induces strength in the soil. So, any particle which moves over one another, there will be a sort of friction which gets developed. So, strength mobilization is mostly due to frictional resistance between the soil particles. A criterion that decides the failure state of geo material is then necessary. Here geo material is concerned with the soil. So, when will the soil fail, we need to mathematically define this and that definition is what you call it as failure criterion.

So, failure criterion it indicates when the soil fails for a given condition of stresses and material property. So, this is what the strength failure criterion gives the failure state with respect to material properties and other stresses. So, Coulomb's friction model is the simplest model to start with why? Because Coulomb's, Coulomb has talked about the friction. So, this becomes the simplest model to understand what kind of strength the soil imparts.

(Refer Slide Time: 08:50)



So, let us go into the simplistic explanation of Coulomb's friction model. There is a block and it has a horizontal force, T is the resistance, W is the weight, N is the reaction. So, this is R the resultant. So, it acts at φ'. Now if you translate this to soil, we can see that we have 2 particles and there is a fictitious area at the point of contact and this is the N is the force and T is the horizontal force. N is the normal force and T is the horizontal force at a actual area of contact between the 2 particles.

So, we can very well write T limiting condition is equal to μ into N where μ is the coefficient of friction and we can write $T_{limit} = N \tan φ'$, $μ = \tan φ'$. Now we are just translating these 2 soil and in stress form, one can write $τ_f = σ'_f * \tan φ'$. Here $σ'_f$ is the, we can write $σ'_{nf}$ which is normal stress. Now we need to be very specific here, it is the normal stress acting on the failure planes.

So, $τ_f$ is the shear strength or the failure shear stress which is acting on the failure plane. So, friction model is the basis for mathematically defining shear failure of soil. That is what I told there are different ways by which we understand shear strength. Now whatever be we need to define this in a mathematical framework then only as engineers we can use it further. In addition for soils there are several factors which need to be considered.

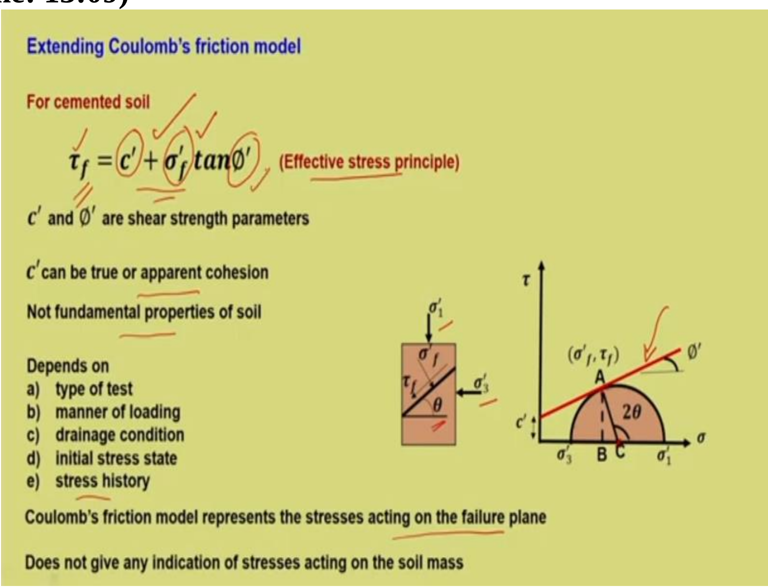
Now for a normal block model several other complications will not come into picture but for a geo material like soils, we also need to consider various other aspects. We need to consider load application which induces volume change. Now if you remember in module 1, we told that we will study the soil in a decoupled manner, we will consider volume change separately and we will consider the deformation separately.

Now for stress strain response of and various other continuum mechanics problem this is fine but when you consider actually the shear strength that is the shear stress which gets induced during loading, you also need to consider what type of volume change response it undergoes under a given load because that is going to change this state, the volume change is going to change the state. It is also complicated because of the voids and the pore water pressure present in it.

Pore water pressure keeps changing. So, there are 2 conditions drained and undrained. So, depending upon that we will come to that a bit later. Depending upon the condition of drainage each one may have volume change or we may be left with pore water pressure. Whatever be throughout the loading process this keeps changing. So, limiting shear stress is governed by also strain level, rate of straining and stress history.

So, what will be the limiting shear stress or the shear stress at failure will be governed by strain level. What percentage of strain is it subjected to? Or rather we can fix the strain level as a reference point; we consider it to be a failure state. Rate of straining how fast it is loaded and stress history, stress history plays a very important role in shear strength. When we discuss further this will be clear.

(Refer Slide Time: 13:09)



Now, extending Coulomb's model that means, we have now consider only the frictional aspect. Now there are certain cases where soil will have cementation for example, certain precipitation of certain minerals can create a sort of cementation, a very good example and some of the state of the art research that is going on these days is microbially induced calcite precipitation which enhances the strength of granular soils. So, what it is done is?

There is a sort of cementation that happens. Now this cementation also enhances the strength of the soil. Now we need to account that till now in the Coulomb's model that we discussed this has not been considered. So, for cemented soil one can always write $\tau_f = c' + \sigma'_f * \tan \varphi'$. Now you need to understand here we have specifically used c prime, σ_f prime and φ prime or φ', we call it in both ways. Now why this is so?

Here it needs to be specified that this model is specific to effective stress condition. And then you may ask why this quantity does not have prime or dash because water does not have shear strength. So, whatever be both effective and total condition will be the same. So, this is dealt mostly in terms of effective condition and here I would also like to remember you effective stress principle.

Now it depends on how well you have been taught this principle during your UG because this is extremely important and it is based on this the whole of the soil mechanics is founded. And we do not talk about effective stress condition in any other material; we talk mostly in granular material and specifically for soils and rocks. Now what is effective stress principle? The first statement defines what is effective stress but the second statement where we state that the strength and compressibility of the soil is entirely governed by effective stress.

Which means to say for, it to be very specific, long term condition or long term strength of the soil is completely governed by effective stress. So that is why this model which specifically deals with long term condition is to be stated in effective stresses. So that is why we have specifically used now dash. Now from here on, we have to clearly distinguish between total stress and effective stress, c' and φ'are the shear strength parameters.

Now it is very easy to state the strength of the soil defined in terms of c' and φ'. And there are several other misconceptions also associated with this which is generally not discussed during UG. So, those are missed that it is better that you refresh that now, c' can be true or apparent cohesion. Now here c', what is c' and from where it comes from c'is either a cementation characteristic.

If it is cementation we call it as a true cohesion. But it can also be due to certain characteristics of the pore water. Pore water can induce a sort of cohesion depending upon the state of saturation. So, you call it apparent cohesion which means, it is not going to be there. Another possibility of this cohesion is for over consolidated state, again I will not get into the details now, we are going to discuss that in length.

But this cohesion is dependent on certain conditions, whether it is going to be there or not is not sure, it is condition specific. That is why it is called apparent cohesion and true cohesion refers to the cementation which is there for sure. So that distinction one has to make sometimes c' becomes a mathematical artifact as well, so that is why in some of the designs, we tend to ignore cohesion part and we consider φ' which is more relevant for granular materials like soils.

Now when I say this that I am sure some of you may not understand or some of you would not agree with me. You would immediately say that a clay will have cohesion and a sand will have friction. So, this is the manner in which we are normally taught but probably subsequent lectures have not discussed what is the extension of that? It is because of that. So, we will see this in detail and this is where we need to clarify our concepts clearly.

And it is to be noted that these are not fundamental properties of soil. And it will depends on the type of test what type of test? What type of shear strength test you are doing? That will also govern what type of values you are getting for c and φ. It will also depend upon the manner of loading, drainage condition and the initial stress state we will not get into the details now, we will discuss in detail and very importantly, the stress history.

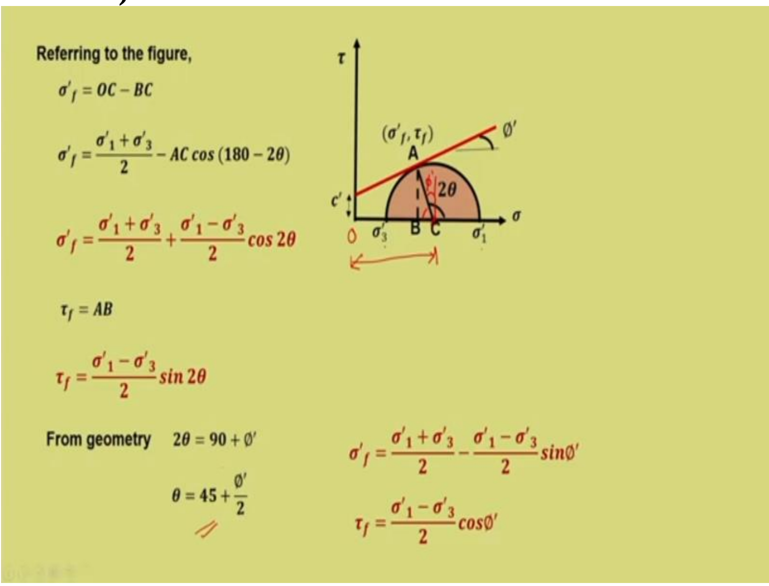
So, Coulomb's friction model represents the stresses acting on the failure plane that we have seen, if you refer to this, it talks about the stress τ_f and σ'_f which is the stress acting on the failure plane when I say what does that mean is a typical soil sample which is acted upon by external stresses, σ'_1 and σ'_3, major and minor principle stresses. So, the loading on the soil is represented in terms of principle stresses.

Now as we said the use of Coulomb's friction model assumes a failure plane within the soil mass. Now this failure plane is at an inclination of θ with major principle axis. Now this is the major principle axis, so it makes an angle of θ. Now the stresses acting at the time of failure is σ'_f and τ_f this understanding also one should have clearly. Now if the same is represented in terms of Mohr circle, this is what it is.

So, here this particular a red line is the Coulomb's model which says $c' + \sigma'_f * \tan \varphi'$. Now if you draw it clearly, so this is the center, this is σ'_1 , σ'_3 of the Mohr circle and please makes a note, like you need to get into the basics of Mohr circle properly and understand its characteristics. Now here this angle will be 2 θ that also we know it is a property of the Mohr circle representation and A C is the radius of the Mohr circle. This is the point A which denotes σ'_f and τ_f.

So that is what it is about extension of the Coulomb's friction model to account for the cementation characteristics or apparent cohesion. But Coulomb's friction model if you refer to this, it does not talk about the stress acting on the soil. Stress acting on the soil is in terms of σ'_1 and σ'_3. So, Coulomb's model does not give any indication of what stress the soil is subjected to.

(Refer Slide Time: 21:41)

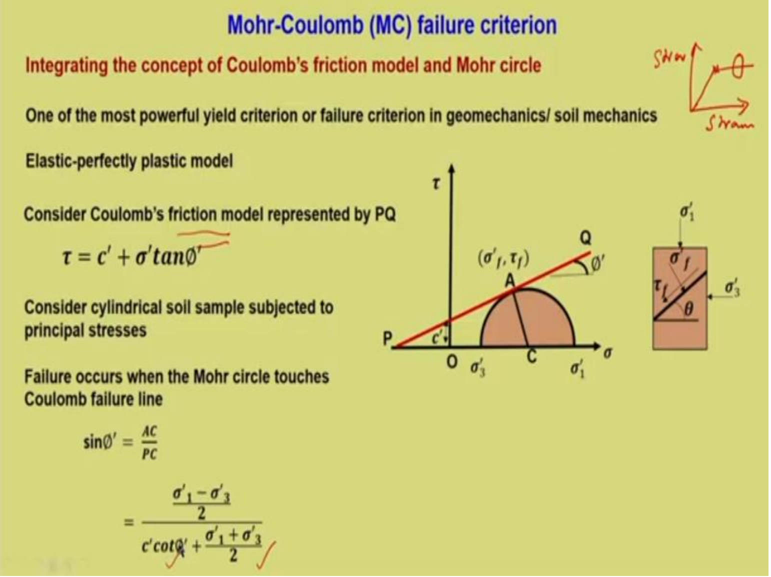


So, referring to the figure again, is the same figure we can write σ'_f = OC, so this point is O. OC - BC. So, here I am talking about σ'_f this is OC - BC. Now you can write σ' what is OC and what is BC? Now OC is from here to the center of the circle, so that we can write it as $\sigma'_1 + \sigma'_3$/ 2 average stress minus what is BC? BC is AC cos (180 - 2 θ), this angle is 180 - 2 θ.

So that will give $\sigma'_f = (\sigma'_1 + \sigma'_3)/2 + (\sigma'_1 - \sigma'_3)/2 * \cos(2\theta)$ and τ_f = AB and that is nothing but $\tau_f = (\sigma'_1 - \sigma'_3)/2 * \sin(2\theta)$. And from the geometry one can always write 2θ is equal to this is 90 + φ because this angle if you draw here, this angle will be φ'.

So, it is 90 + φ' and hence $\theta = 45 + \varphi'/2$ and what is θ? θ is the inclination of failure plane with the major principle axis. Now if you substitute for θ that is 45 + φ / 2 you will also get this equation $\sigma'_f = (\sigma'_1 + \sigma'_3)/2 + (\sigma'_1 - \sigma'_3)/2 * \sin\varphi'$. I have purposely written this why because in certain textbooks it is written in this form and in this form. So, you should not get confused why it is written in this manner. So that is why you just need to substitute it for 45 + φ / 2 you will get this equation.

(Refer Slide Time: 23:53)



So, now, let us come to a more important or prominent failure criterion for soil which is known as Mohr Coulomb failure criterion. Now we have already stated in the previous slide that Coulomb's model does not talk about the stress acting on the soil. So, we just want to make that modification and that is how the concept of Mohr circle has been integrated with the concept of Coulomb's model.

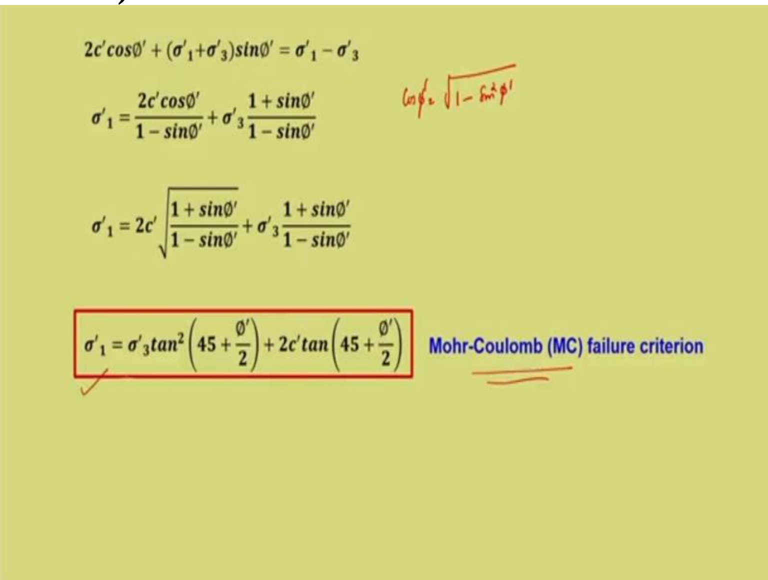
So, one of the most powerful yield criterion or failure criterion in geo mechanics or soil mechanics, this is elastic, perfectly plastic model. Hope you remember whatever we have discussed about elastic perfectly plastic in module 1. So, for up to certain limit, for example, this is stress and this is strain, so it means you have a response like this. So, up to a certain limit you have elastic characteristics and once it reaches this elastic characteristic the soil yield.

So that is why it is called elastic perfectly plastic and what is this value? This value is the limiting condition of stress. It consider Coulomb's friction model represented by PQ. So that is what it is, the same figure again τ versus σ, plot, PQ is the required Coulomb's friction model which is stated and that is $\tau = c' + \sigma' \tan \varphi'$. Now consider a cylindrical soil sample subjected to principle stresses again it is the same figure.

So, failure occurs when more circle touches Coulomb's failure line. Now when will the failure occur? The failure occurs when the Mohr circle touches the failure line and Mohr circle is, it represents the stress state of the soil. If that is a condition, if this is the condition this is met, like at this particular point, the soil is bound to fail. So, sin φ'. this is the inclination as φ', sin φ' one can write AC / PC.

And this probably you would have derived during your UG but I am just revising it sin φ' is AC is nothing but the radius, PC is PO + OC, OC is $(\sigma'_1 + \sigma'_3)/2$ and PO is c' cot φ'.

(Refer Slide Time: 26:33)



So, if you rearrange it you can get in this manner and $\sigma'_1 = [2c' \cos \varphi'/1 - \sin \varphi'] + [\sigma'_3 (1 + \sin \varphi'/1 - \sin \varphi')]$and if you write

Cos $\varphi' = \sqrt{1 - \sin 2\,\varphi'}$

So, if you substitute this for cos phi dash then one can always get this expression,

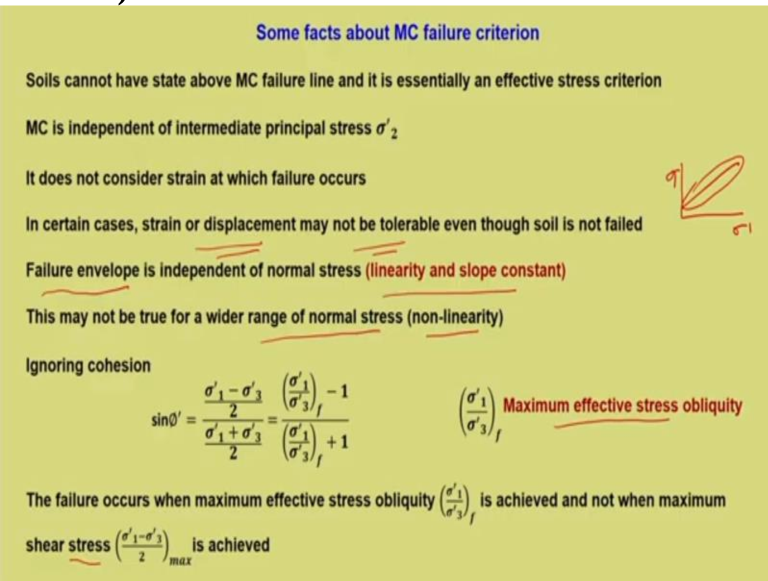
$$\sigma'_1 = 2c'\sqrt{\frac{1 + sin\varphi'}{1 - sin\varphi'}} + \sigma'_3\frac{1 + sin\varphi'}{1 - sin\varphi'}$$

Now this gives the popular Mohr Coulomb model which is

$$\sigma'_1 = \sigma'_3 tan^2\left(45 + \frac{\varphi'}{2}\right) + 2c'tan\left(45 + \frac{\varphi'}{2}\right)$$

This is the popular Mohr Coulomb failure criterion. Now if you see this expression you are representing this in terms of the stresses acting on the soil.

(Refer Slide Time: 27:39)



So, we will discuss about some facts about the Mohr Coulomb failure criterion. Soils cannot have a state above Mohr Coulomb failure line and it is essentially an effective stress criteria. Mohr Coulomb is independent of intermediate principle stress σ'_2, we do not discuss about σ'_2. It does not consider strain at which failure occurs, there is no mention about the strain at which the failure occurs according to Mohr Coulomb failure criteria.

But in certain cases, strain or displacement may become a governing factor which means to say, it may not be tolerable even though the soil is not failed. So, sometimes the strain based criteria becomes important. Failure envelope is independent of normal stress that means, linearity and slope is constant. So, this we need to understand a bit carefully. Failure envelope is independent of normal stress which means to say τ and σ', this line has a constant slope. So that is what it means.

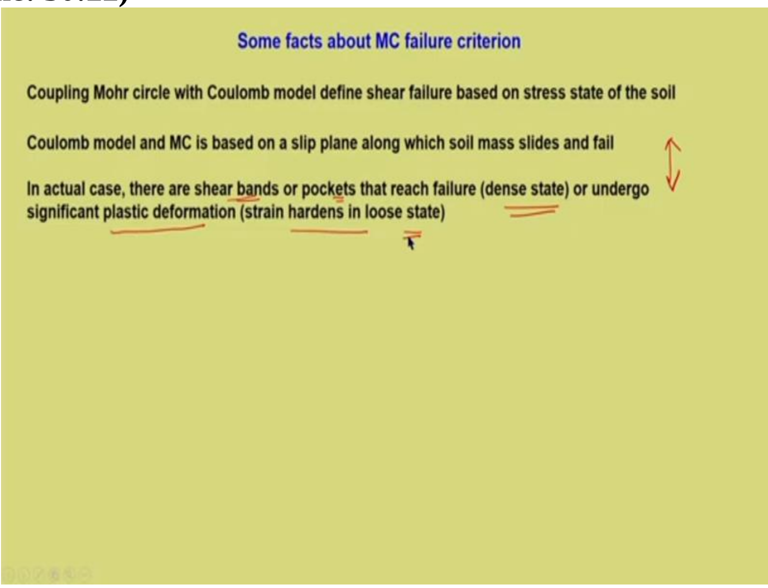
So, this entire failure envelope is not dependent on normal stress rather it is a constant you should not confuse it with failure stress because failure stress is dependent on the normal stress but failure envelope is not because it is a constant. This may be true, this may not be true for a wide range of normal stress. May be for a wide range, there is a possibility of non linearity. For a moderate range, we can still approximate it to be a straight line.

Now if we ignore cohesion from the Mohr Coulomb failure envelope, one can rearrange it in terms of this and if you take σ'_3 outside it will give you

$$\sin\varphi' = \frac{\left(\frac{\sigma'_1}{\sigma'_3}\right)_f - 1}{\left(\frac{\sigma'_1}{\sigma'_3}\right)_f + 1}$$

Now this term $\left(\frac{\sigma'_1}{\sigma'_3}\right)_f$ is called maximum effective stress obliquity. Now you can see that according Mohr Coulomb failure envelope the failure occurs when maximum effective stress obliquity is achieved and not when the maximum shear stress is achieved. You can see the maximum shear stress is $(\sigma'_1 - \sigma'_3)/2$. Now what we are concerned about is maximum effective stress obliquity.

(Refer Slide Time: 30:22)



Coupling Mohr circle with a Coulomb model define shear failure based on stress state of the soil. Coulomb model and Mohr Coulomb is based on a slip plane along which the soil mass slides and fail. In actual case, there are shear bands or pockets that reach failure which is specifically pertain to dense state or it undergoes significant plastic deformation that is strain hardens in the case of loose state. Now we need to read these 2 sentences together. What it means?

The Coulomb model and more column is based on a slip plane along which the mass slides and fail this is the assumption but in reality, you may see that there are formation of some shear bands or pockets which reached the failure faster than the other part of the soil. And this is a specific, this is mostly specific for dense state of the soil or it may undergo ductile behavior or a plastic deformation for loose state of the soil. So that is all. We will see the remaining part in the next lecture.