Advanced Soil Mechanics
Prof. Sreedeep Sekharan
Department of Civil Engineering
Indian Institute of Technology – Guwahati

Lecture – 13
3D to 2D Idealization

Welcome back. So, now we have already completed stress acting at a point, strain, stress strain relationship rather cause effect relationship and we have discussed some important constitutive relationship which is handy for geomechanics. Now, in most of the discussions, we have focused on 3 dimensional state or we have discussed x, y, z 3 dimensional coordinate system. Now, the scale of the problem is in 3 dimension.

There are certain cases or there are certain problems where you can convert 3D to an equivalent 2D in modeling. This will be quite beneficial from computational point of view as well as simplicity point of view. Hence, in this today's lecture, we will see 3D to 2D idealization.

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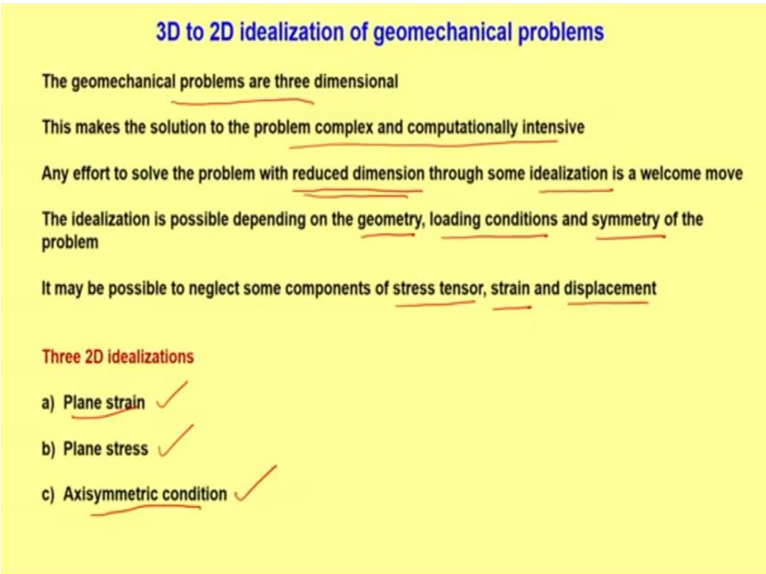

So, 3D to 2D idealization of geo mechanical problems: As we have seen, the geo mechanical problems are essentially 3 dimensional. It is not 2D. This makes the solution to the problem complex and computationally intensive, reality is 3D but when you conceive 3D, it is going to be a bit more complex than 2D and it is computationally intensive. So, any effort to solve the problem with reduced dimension through some idealization, it is a welcome move.

The idealization is possible depending on geometry of the problem, loading conditions and symmetry. So, based on these 3 factors, one can idealize a 3D to a 2D problem, how it is done? It may be possible to neglect some components of stress tensor. It is not stress tensor we are neglecting, some components of stress tensor, strain and or displacement. So, essentially, the 2D idealizations which are considered in geo mechanical problems are plane strain, plane stress and axisymmetric condition.

For completeness, we are discussing all the 3. What is this 3D to 2D idealization based on these 3 formulations? But, in geo mechanical problems or in soil mechanics, what is more important is plane strain and axisymmetric condition. So, these are the 2 important ones when you consider 3D to 2D idealization.

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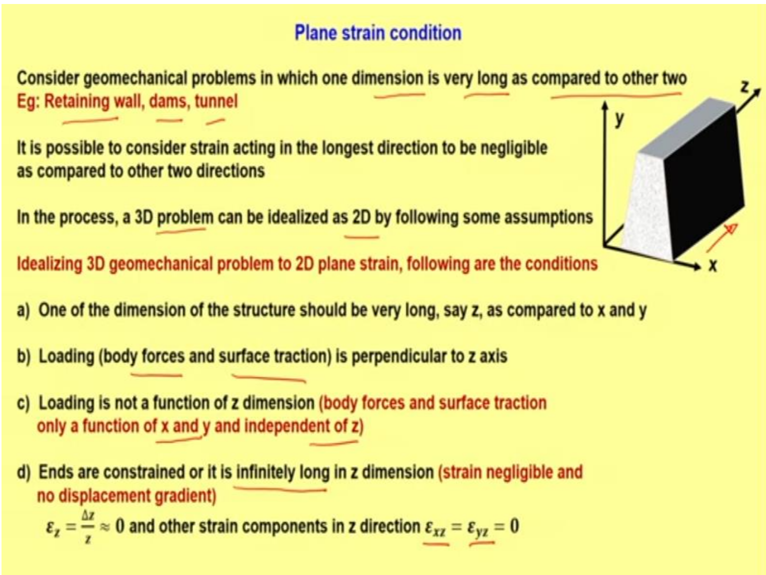

So, first let us discuss what is plane strain condition and how you can idealize a 3D to 2D problem? Based on conceiving a geo mechanical problem as a plane strain. So, what is meant by plane strain? You can consider geo mechanical problems in which 1 dimension of it is very long as compared to the other 2. For example, retaining wall, dams and tunnels you can see here those x axis, y axis and z axis a typical gravity dance structure is shown.

You can see that this is the cross section, compared to cross section, this direction is quite long. So that is what it means one dimension is very long. So, z direction is pretty long as compared to the its cross section. So, it is possible to consider strain acting in the longest direction to be negligible as compared to other two directions since this length is quite long. So, you can always consider the strain acting in z direction to be negligible.

In the process, a 3D problem can be idealize; to 2D by following some assumptions. So, what are the following assumptions and what are the conditions that are inevitable for considering plane strain condition. one of the dimension of this structure should be very long say z as compared to x and y in this particular example. Loading is perpendicular to z axis that means, there is no effect of loading in the z direction and when I say loading, it includes both body forces as well as surface traction.

Now, these forces are perpendicular to z axis, because of which it does not have any effect in the z direction. Also, loading is not a function of z dimension, body forces and surface traction are only a function of x and y and it is independent of z. So, it is a function of only x and y, it does not vary with z. Now, the fourth condition is ends are constrained or it is infinitely long in z dimension, why? This condition is needed for achieving the most important condition for plane strain.

That is strain in z direction or any 1 direction is 0 or close to 0. So, how it is made, either it is end constraints are there, or the length is infinitely long, so that when you divided by its original length, the strain becomes negligible. So that is how $\varepsilon_z = \partial z / z$ which is approximately equal to 0 and the other strain components in z direction ε_{xz}, ε_{yz} is equal to 0. So, whatever is there, whatever strain components associated with z is neglected.

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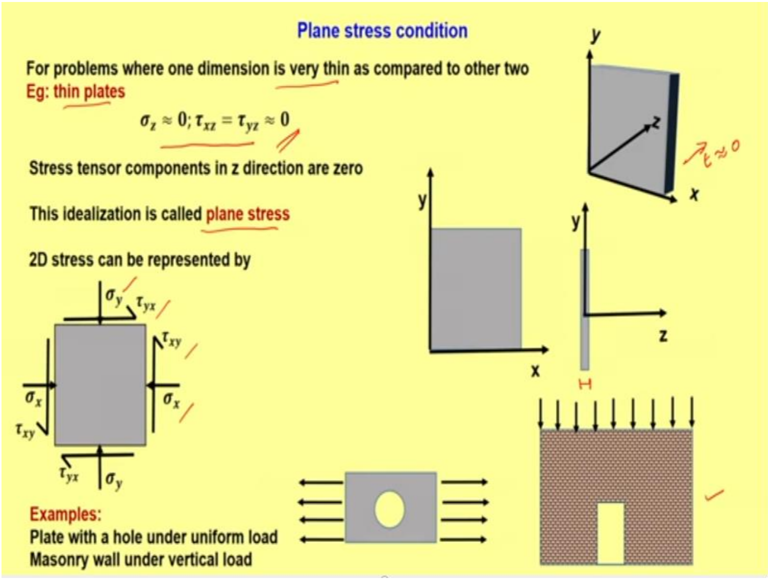

So that is about plane strain condition, we will have some basic formulations done for plane strain and plane stress. But in todays lecture, we will understand how and what is plane strain, how does it exist which problems it is relevant. So, similarly, we will also know plane stress condition, it is not that important in geomechanics but for completeness, we will see what is plane stress condition.

For problems where one dimension is very thin. So, in the plane strain, it is very long, here it is very thin as compared to other two. For example, thin plates, you can see here this is a thin plate and dimension along z direction that is thickness is very less as compared to x and y. So that is what it means, here less means it is relative. So, as compared to xy plane, the thickness is quite less, so, it is becoming like a thin plate.

So that is what it is you can see here the thickness is very small as compared to the area of cross section in the xy plane. So, because of this one can always approximate the stress acting in the z direction σ_z as well as shear stress components τ_{xz} and τ_{yz} this can be approximated to 0. Stress tensor components in z direction are 0, this idealization is called plane stress. So, stress in one direction is 0 that is plane stress, strain in one direction 0 is plane strain.

So, 2D stress can be represented by in this manner. So, we are left with σ_x, σ_y and the shear stress component acting in the xy that is τ_{yx} and τ_{xy}. So, what are the example is plate with a hole under uniform load, as you can see here, masonry wall under vertical load as you can see here. So, these are some of the examples where you can approximate the given problem to 2D plane stress.

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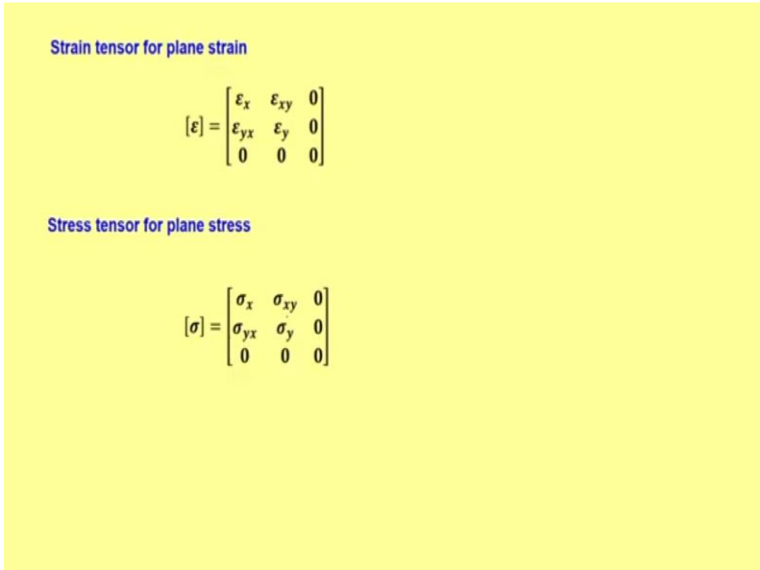

Strain tensor for plane strain can be written in this manner. Now, we have strain in one direction to 0. So, all these components have become 0. So, you have ε_x, ε_{xy}, ε_{yx} and ε_y. Similarly, you have stress tensor for plane stress now, here the other components are all 0; you are left with σ_x, σ_y σ_{xy}, σ_{yx}.

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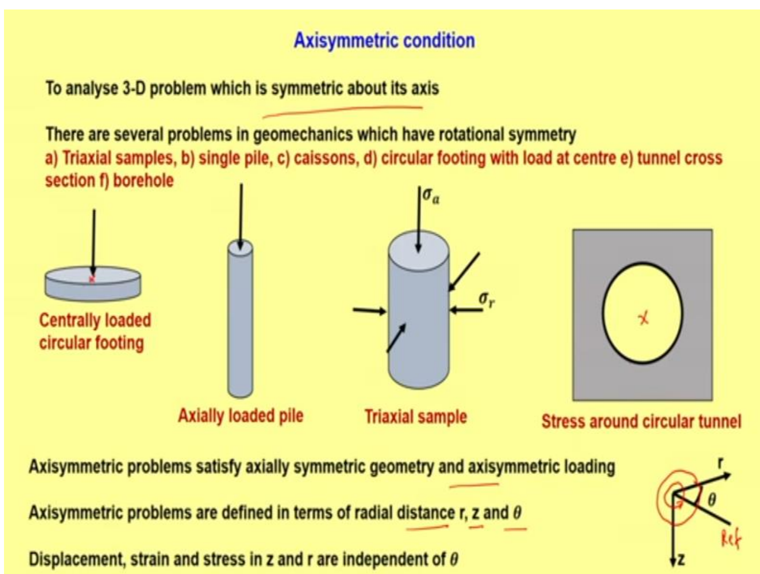

So, the third idealization is axisymmetric condition to analyze 3D problem which is symmetric about its axis. Now axisymmetric condition is not new to geotechnical engineering is most of the problems this particular aspect comes into picture knowingly or unknowingly we have already dealt in detail, the axisymmetric conditions or axisymmetric problems, this is also nothing but you are idealizing a given 3D problem to a 2D problem, how? It is based on axis of symmetry.

Let us see how there are several problems in geomechanics which have rotational symmetry. For example, a very good example is triaxial samples, everyone has worked with triaxial samples, you can see that there is a deviatoric stress acting actually and all round stress acting now, because of this all round stress, there is a sort of axis of symmetry, with respect to geometry as well as with respect to loading.

So, triaxial sample is a very good example of axisymmetric condition, where you change the 3D to a 2D state. Then we have single piles actually loaded single piles, caissons, circular footing with load acting at the center, tunnel cross section and borehole that is what it is. So, along this direction, if you consider this is acting at the center. So, along this axis, it is axis of symmetry, same is the case with actual loaded pile or triaxial sample which we have already told stress around a circular tunnel.

So, this is an overly simplified representation of tunnel where this is the rock medium and this is the tunnel, you can see that with respect to the center, if you take there is an axis of symmetry along this longitudinal direction, if you take the longitudinal axis, you have an axis of symmetry. So, axisymmetric problems these are mostly defined in terms of polar coordinates in terms of radial distance r, vertical distance z and the angular plane which is given by θ, so, it is basically a polar coordinate approach.

So, axisymmetric problems, it also satisfies actually symmetric geometry and axisymmetric loading. So, the symmetry can be both, because you need to have symmetry with respect to geometry that is the first condition and it should also be symmetric with respect to load, because if I place another load somewhere here, then there will be sort of eccentricity acting, so then you cannot consider it exactly to be an axisymmetric problem. So, both geometry and loading are important to define the symmetry.

So, here is the representation, let us say this is the reference axis and this is the radial distance r and this is the vertical distance z. So, if you consider a cylinder like this and this is the reference, so you have θ beginning from here and then all around to 360 degrees. So, one need to have the properties to be seen, if; you go from here to 360 degrees. So, if this is considered as a plane, if you rotate this plane from 0 to 360 degree, nothing should change; neither the loading nor the geometry should change.

So, then we say that the axisymmetric condition is satisfied. So, displacement, strain and stress in z and r are independent of θ. That is what I just explained. What it means, if you consider stress displacement or strain in r direction and z direction, then if you rotate θ, this is not going to change rather, whatever is there in r and z direction, it is independent of θ, the value of θ = 0, θ = 90 things are seen. So that is how we say that the axisymmetric condition is met.

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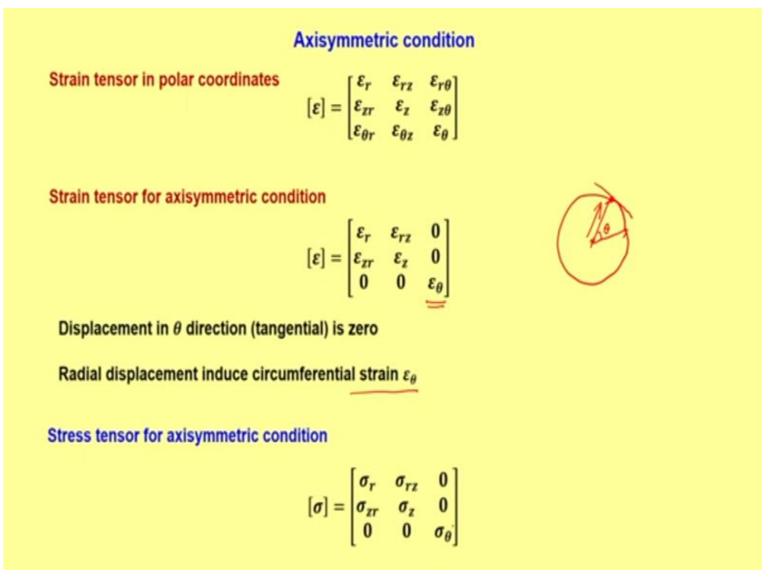

So, strain tensor input polar coordinates in general, you can represent it by in this particular form,

$$\varepsilon = \begin{vmatrix} \varepsilon_r & \varepsilon_{rz} & \varepsilon_{r\theta} \\ \varepsilon_{zr} & \varepsilon_z & \varepsilon_{x\theta} \\ \varepsilon_{\theta r} & \varepsilon_{\theta z} & \varepsilon_\theta \end{vmatrix}$$

So, it takes the form. So, basically, you have epsilon r, epsilon z, epsilon θ and the shear stress components. Strain tensor for axisymmetric conditions specifically, what is there what will go and what will not be there?

So, you can note that the shear stress component, $\varepsilon_{r\theta}$, $\varepsilon_{z\theta}$ becomes 0 and since it is symmetric, so, this is also 0. But another important aspect is epsilon theta is not equal to 0 in plane stress plane strain, in fact that was also 0 in axisymmetric condition, ε_θ is not 0. Why? Displacement acting in θ direction, it is 0. So, if I represent it, so, this is the reference, this is θ. Now, θ direction means this is the θ direction and this is the tangential location.

So, if you draw a tangent here, we define theta based on the tangent. Now, what it means is that displacement in this direction is 0, if displacement is 0, then why? Epsilon theta is non-zero, because it has got component from some other axis for example, radial displacement induce circumferential strain ε_θ, epsilon theta is the circumferential strain. Vertical component it will not induce any sort of circumferential strain.

But radial displacement that is displacement in this direction, this can induce ε_θ or circumferential strain. So that is why epsilon theta is present. So, stress tensor for axisymmetric condition is σ is equal to σ_r, σ_{rz}, σ_{zr}, σ_z and σ_θ more like the strain tensor.

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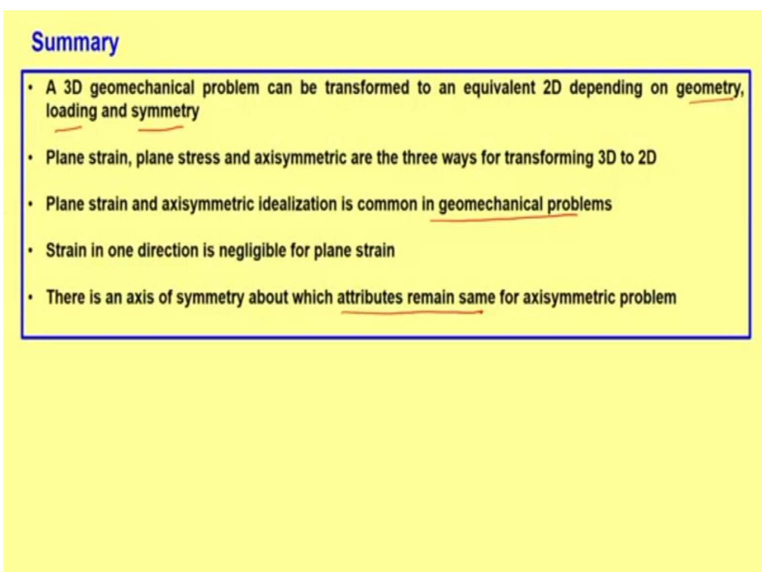

So, now, we have just introduced how and what are the possibilities of 3D to 2D idealization. So, we will summarize a 3D geo mechanical problem can be transformed to an equivalent 2D depending on geometry loading and symmetry. Plane strain, plane stress and axisymmetric are the 3 ways for transforming 3D to 2D. Plane strain and axisymmetric idolization is common in geo mechanical problems. Strain in one direction is negligible for plain strain.

There is an axis of symmetry about which attributes remain same for axisymmetric problem. So, this is all about 3D to 2D idealization. In the next lecture, we will see some basic mathematical formulations relevant to plane strain, plane stress and axisymmetric cases. Thank you.