Advanced Soil Mechanics
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Lecture – 12
Important Constitutive Relationship

Welcome back in the last lecture, we were discussing about cause effect relationship. Today's discussion is a bit of extension of that particular lecture, where we will discuss about some important constitutive relationship. Again it is not an exhaustive session here, we will introduce certain important relationship between stress and strain and which is relevant in geo mechanics. So, cause effect relationship, it describes how stress and strain is related and we have seen that the most important aspect is constitutive metrics.

Now, depending upon what sort of model you adopt for modelling a given situation, the constitutive matrix keeps changing. So, we will see some of the important constitutive relationship in today's lecture.

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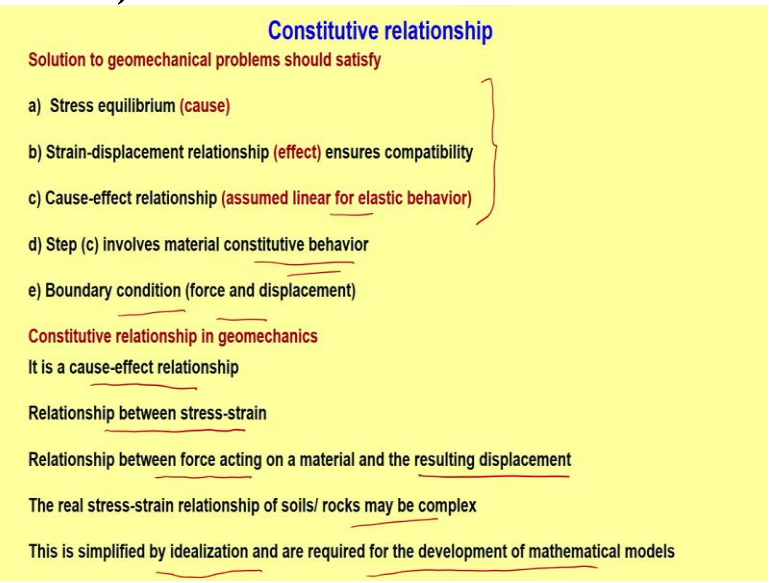

Again, we will stick to the very same issue that is solution to geomechanical problems, it should satisfy stress equilibrium, strain displacement relationship which ensures compatibility the cause-effect relationship which is assumed to be linear for elastic behaviour, this one and the step c involves the constitutive behaviour. So, today's discussion on constitutive relationship will focus around this particular point.

That is the constitutive behaviour and we will skip this we have already discussed set of discussion, why it is essential for one to understand the problems or the solutions in geomechanics involves these 3 steps. The first 3 steps essentially and plus this in some cases, you will see that maybe stress equilibrium is satisfied but the others may not. There are several problems in geomechanics or maybe specific to soil mechanics as well, where we do not achieve the other 2 but stress equilibrium is obtained.

So, how a particular method got evolved, it depends on that. Now constitutive relationship in geomechanics, it is a cause effect relationship. That is what we in the beginning I told is an extension of last lecture session. It is a relationship between stress and strain or you can see it is a relationship between force acting on a material and the resulting displacement, the real stress strain relationship of soils or rocks, it may be complex in nature.

In most of the time, we have seen and we everybody who is an engineer or a non-engineer, they will be able to understand this very fact that soil is pretty complex and very hard to understand and whatever we say, still, there will be some gap left in our understanding, there is so much of uncertainties are there associated with soil behaviour. So, a real stress strain behaviour is beyond question, what we can do is to understand or to develop models which are close to or which closely simulate the real soil behaviour and this is simplified.

So, we understand the fact that it is complex. So, through some idealization, these particular relationships are simplified by idealization and are required for the development of mathematical models.

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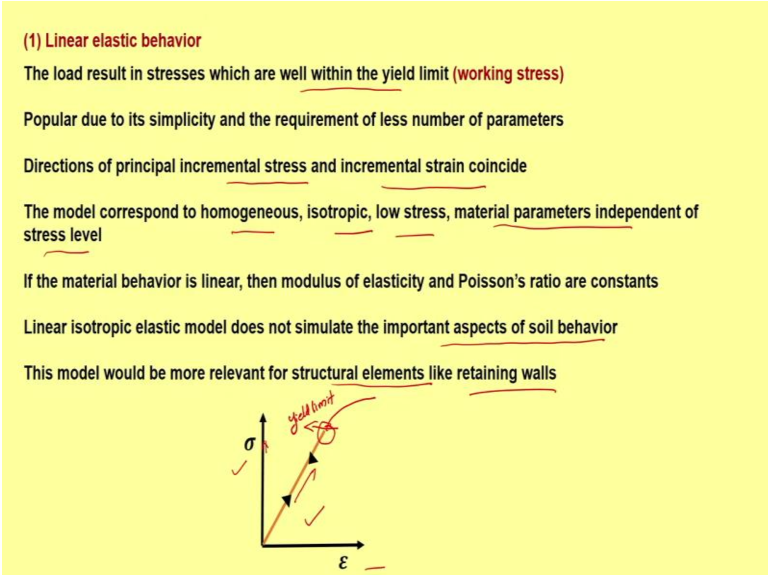

So, first let us begin with a very simple model which is linear elastic behaviour. One may always tell why we should discuss linear elastic model at all in soil mechanics, because most of the cases we understand soil behaves nonlinearly but there are certain aspects just because of its simplicity and simple mathematical formulations and minimal requirements. This model is very popular.

So, linear elastic model, we always start with that and there are very specific problems which consider it to be elastic in nature. Now, how far it is true, that is a question mark but because of its simplicity and to the fact that the results are not really off, under certain circumstances, we can even apply linear elastic model for simulating the behaviour of soils but not always it is far from reality.

As you can see, the y axis is the stress and the x axis is string and this is a simple linear elastic behaviour, where it is represented by a straight line and we know that it is non hysteretic means it traces back the same path. So, linear elastic model, the load results in stresses which are well within the yield limit. Now, this is the point. Now, if the loading acting on the soil is fairly less, as compared to its strength, we can still see that it is within the elastic limit.

And this is the whole basis of working stress method and the logic of applying factor of safety, we apply a high factor of safety and ensure that the stresses which are acting on that particular body or soil is well within or maybe it is at par or slightly below the elastic limit or to be very specific in geomechanics the yield limit. Now yield limit is the point beyond which the plastic strain sets in or the plastic behaviour starts happening, let us say that we have marked up to here at up to this particular point.

Now, if the same material is further loaded, it may exhibit a nonlinear behaviour. So, this point can be treated as yield limit. So, we will come to that a bit later. So, linear elastic consideration ensures or we make a presumption that the stresses are well within the elastic limit or the yield limit. Yes, I have already told this it is popular due to its simplicity and the requirement of less number of parameters.

Obviously, this we have seen in the last lecture, we have seen that for linear elastic cause effect relationship, you need to know only 2 parameters and what are they? Modulus of elasticity E and the poisons ratio μ. So, with this one can readily model the behaviour of soils or maybe geo materials. Now, there is an inherent assumption involved in most of these models which is directions of principle incremental stress and incremental strain coincide.

Means, the direction in which the incremental stress is it is the same direction for the principle or incremental string. So that is what it means both coincide. Now, when it comes to plastic formulations, it may or may not coincide. So, there are various other models but in linear elastic for that matter, various other elastic models will have this inherent assumption that the incremental stress and incremental strain are in the same direction incremental why incremental. We are talking about a very small increment of stresses that we will come to know a bit later why incremental is important and this model essentially corresponds to homogeneous isotropic low stress that is what I mean, if we want to ensure that it is working stress, then it has to be under a low stress condition and material parameters independent of stress level. Now, if you consider this particular plot, you can see that E the value of E remains constant up to this particular limit yield whatever be the strain level.

We can see that this are different levels of stresses and this various levels of string or whatever be the stress level and the strain level, if your body is considered to be linear elastic in nature, the parameters or the material parameters which is E and μ is considered to be constant or rather it is constant and it is independent of the stress level. So, linear elastic on only this particular requirement is there and it essentially confirms to low stress.

So, if the material behaviour is linear, then the modulus of elasticity and Poisson's ratio are constants which I have already discussed. So, it does not simulate the important aspects of soil behaviour. So, in the absence of other complex models, one can always use linear elastic model but with a proper understanding that it is not going to simulate the real soil behaviour but, it is not fully true the statement.

There are certain situations where soil exhibit some sort of elasticity and one good example is an over consolidated state where the soil is fairly stiff and when it is stiff, it can exhibit elasticity. Again that elasticity is also subject to certain conditions beyond certain point it again subject to yield. So, it is condition specific, it is not like other materials for soils, what type of behaviour it would exhibit, it is also condition specific under certain conditions it can exhibit elastic behaviour.

So, this model would be more relevant for structural elements like retaining walls because retaining walls are very widely used and it is an important aspect of geotechnical engineering. So, for modelling those linear logistic models may be important. So, now, we will go to some other extension of the linear elastic model.

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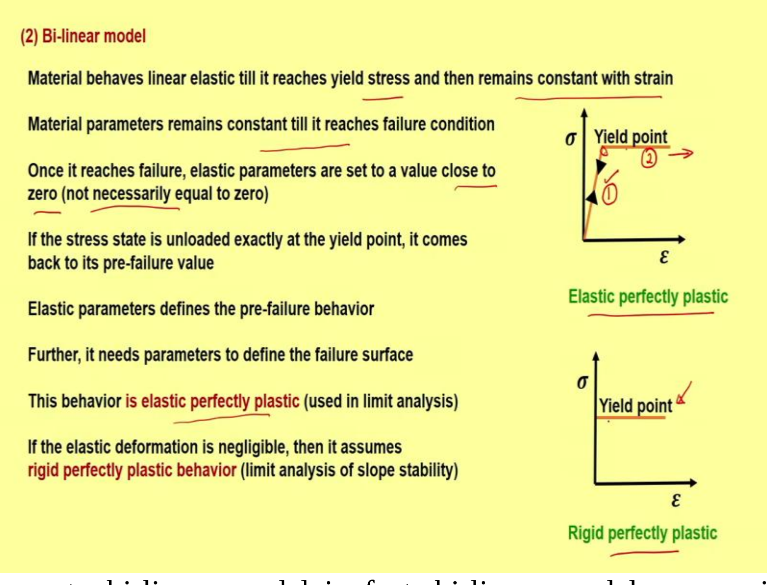

Now, we will come to bi-linear model in fact, bi-linear models are or it represents non linearity but, you are doing some sort of adjustment or you are showing that non linearity using 2 different lines. So that is why it is called bi-linear model material behaves linear elastic till it reaches yield stress, we will be discussing about yielding yield stress in detail in module 4 but here we just need to understand it is the point beyond which the plastic behaviour starts.

So, material behaves linear elastic till it reaches yield stress and then remains constant with strain. So, what it means is that the stress strain behaviour, it is of certain manner till it reaches yield limit or yield stress and beyond which it remains constant. Now, this constant means, depending upon the formulations, there will be some sort of changes as constant, we will come to that in the discussion. Material parameters remains constant till it reaches failure condition.

So, here we will discuss 2 bi-linear model. The first one is elastic, perfectly plastic. So, you can see here that we are not talking only about elasticity is a pure elastic model. It is elastic, perfectly plastic, it includes plasticity behaviour as well. So, here it is not a pure elastic model, it is rather elastic and plastic response, both are taken care. But since it is an approximation using linear lines, it has been discussed now, you can see that there are well defined 1 and 2, 2 well defined behaviour one is in this manner. One is in this manner, there are 2 straight lines. So that is why it is called bi-linear model and elastic perfectly plastic. You can see that from this point to this point. You can see it is a linear trend and the material constants E and μ remains constant. So, this part takes care of the elastic behaviour. Now, after reaching the yield point then nothing changes. It has gone to its plastic state or just directly from the elastic limit, it goes to its plastic state or the failure state.

So, here you can see that stress remains constant, whereas, the strain increases to a large deformation but at constant stress condition. So, here it remains constant, whereas, immediately after the yield point, it remains constant, the stress remains constant. So, one more requirement in this particular model is one should be knowing what is the yield point. So, apart from E and μ one should be knowing what is the yield point the Mohr Coulomb failure envelope using c and φ is a good example of this for setting the yield point.

So, this we know that this is the point at which the soil will fail. So, it is assumed up to that particular point it behaves elastically. This is also far from reality. In real condition, soil is not going to reach to its plastic state just at its yield point or at its failure state at its yield point, there are certain other type of behaviour as well in the real soil behaviour. Now, we have discussed about elastic perfectly plastic a similar model or maybe a subset of this, is rigid perfectly plastic.

Where the material is considered to be having no deformation till it reaches the yield point you can see here. There is no elastic portion of the curve here in the earlier case you have elastic portion here there are, there is no elastic portion directly it goes to its failure state at its yield point. So, in this case, only one parameter need to be known and that is the yield stress or the yield point or the failure state whatever, but the response both are same. So, here it jumps to its yield condition directly.

Now, these are different models which are used. So, once it reaches failure, elastic parameters are set to a value close to 0. Now, I would be very cautious to use this term it is close to 0, because, you will see that in certain finite element modelling, if it is set equal to 0 then there will be a problem. So, it is given a very small value close to 0. So that is why I am used this particular term and we have written here not necessarily equal to 0, even though we know according to the model, it is 0 but it is set close to 0.

If this stress rate is unloaded, exactly at the yield point, it comes back to its pre-failure value which means I am not telling any point here and telling at this particular point, if the unloading is done then it gets back to its original. So that is the issue related with this particular model. Once at yield it undergoes some sort of plastic deformation but here even it has reached at its yield point and if you unload, it is bound to come back to its pre failure state. Elastic parameters defines the pre-failure behaviour.

So, pre-failure behaviour is defined by elastic parameters. Further, it needs parameters to define the failure surface that is what I told you need to know the yield point as well. This behaviour is elastic and perfectly plastic and these are used in limit analysis problems that you use in soil mechanics, where in the case of limit analysis, it also satisfies the stress strain relationship but it needs to know what is the limit value according to certain theorem.

So, we will not go into the details of limit analysis but this becomes the foundation block for limit analysis used in geotechnical engineering, if the elastic deformation is negligible, then, it is assumed to be rigid, perfectly plastic, only under the condition that we neglect the elastic deformation it is, so less that it is neglected and straight away it is going to its failure. So, limit analysis of slope stability is a classical example of this.

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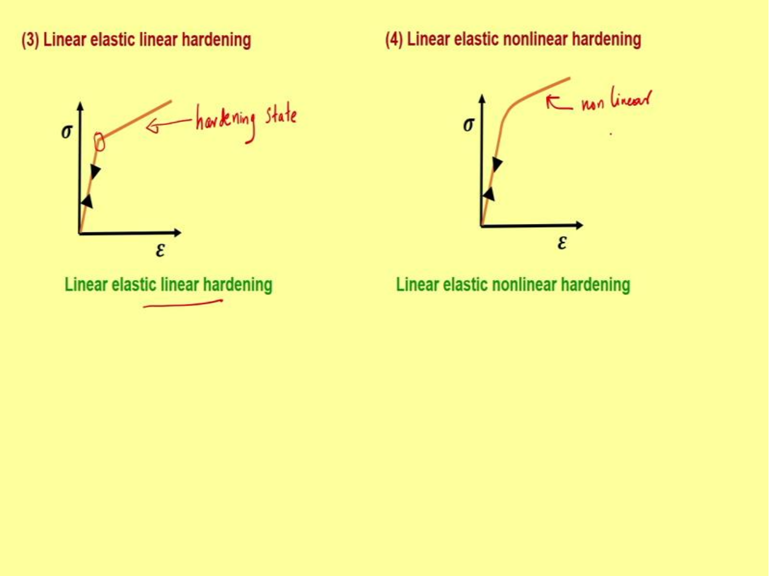

Now, we will quickly discuss about certain other model which are in line with this bi-linear modelling which is linear, elastic, linear hardening, now I am, I am introducing another term which is known as hardening, hardening behaviour, it is a typical behaviour exhibited by plastic response of material after it reaches its yield limit, where the stress increases with strain as you can see here.

So, this particular point is the yield limit, where it shows the transition from elastic state to the plastic state, obviously, the stiffness portion has reduced now, this portion is called the hardening state and this is the linear elastic portion but it also tried to capture the linear hardening. So, this in fact, it is not linear but here it is approximated as linear hardening. So, this is linear elastic linear hardening.

Now, another aspect of it is linear elastic, nonlinear hardening. Now, this particular portion is represented by nonlinear behaviour. So, both captures the hardening portion of the soil behaviour which means, I will again add here hardening it refers to a sort of volume change that is happening during sharing, is the kind of response in linear elastic or maybe the bi-linear model that we discussed, this portion of volume change during sharing is not incorporated.

So, beyond yield point, there will be a volume change now, this volume change results in such behaviour which is hardening or it the another aspect is softening. So, we will discuss those in detail later but here we just need to understand this particular model takes into account the hardening behaviour by certain line and in this case, it is a nonlinear behaviour. We have discussed about linear elastic. Now, the next set which is a bit of refinement over the linear elastic model is nonlinear elastic behaviour.

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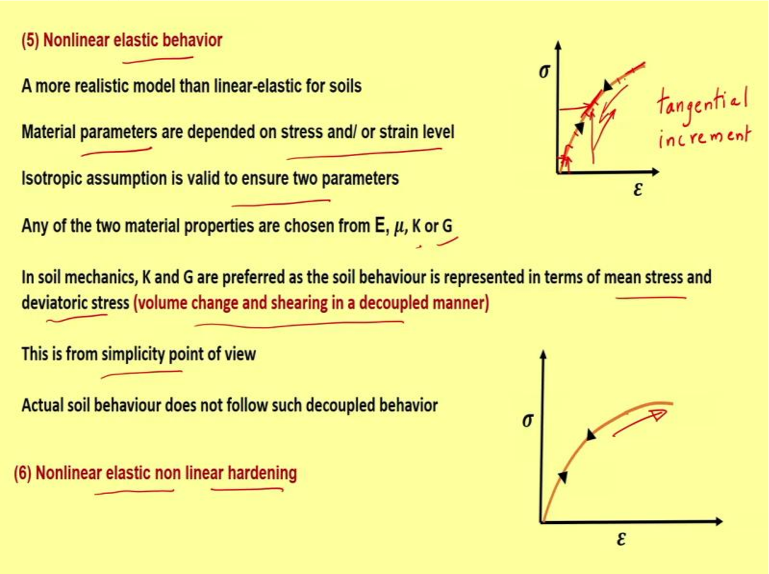

Where the material response is considered to be nonlinear but elastic in nature, how do you ensure this, this is a nonlinear response is a curve response but, if you unload at any point of time, this particular curve trace back. So that is the one very simple definition of elastic behaviour, it does not show hysteretic behaviour, the manner in which you load it traces back So that is what is shown here, you. This is the loading and this is the unloading.

So, non-linearity is in the curvature, it is there but it exhibits the elastic behaviour, it does not result in any sort of permanent deformation and it traces back this is more realistic than the linear elastic behaviour for soils. Material parameters are dependent on stress and or strain level. Now, this we need to understand carefully and that is why the incremental strain becomes important like at every point, if you consider along this curve and we draw a tangent at these points.

For example, if I draw a tangent here, if I draw a tangent here, if I draw a tangent here, we can see that this stress by strain or rather the elastic constant that keeps changing at every point on the curve. So that is why we call it as tangential increment. The increment is represented by tangent at a particular point on the curve and that gives you what is the elastic constant or material parameters.

So that is what it is written here. So, material parameters are dependent on stress and or strain levels. So, if you consider this is a particular stress level or this is a particular strain level similarly, you have this as the stress level and this as the strain level. So, at every stress strain level, the material constants or the material parameters keeps changing. So, isotropic assumption is valid to ensure that it has only 2 parameters.

If it is an isotropic then again the number of parameters keeps changing because in different direction parameters will be different, any of the two material properties are chosen. Now, we know that this is an elastic analysis and hence the very straightforward choice is E and μ. But, we can also represent the model in terms of bulk modulus and shear modulus, why specifically bulk modulus and shear modulus.

Because one represents the volume change behaviour and the other represents the shearing behaviour because it is associated with shear in soil mechanics K and G are preferred as the soil behaviour is represented in terms of mean stress and deviatoric stress. So, when it comes to model that uses invariance it is found that mostly K and G are preferred, why because, one the same reason what I just discussed, it represents the volume change aspect and the shearing aspect.

We have seen in the earlier lectures that we have already told these are decoupled which means to say we study the from the given stress tensor, we decouple the mean stress and the deviatoric stress and we study the volume change behaviour keeping the shape constant and we study the shearing behaviour keeping the volume constant. So, these are taken one at a time. So, volume change and shearing in a decoupled manner and this is purely from simplicity point of view.

The actual soil behaviour, it does not follow a decoupled behaviour. So that is all about nonlinear elastic behaviour, another refinement of this is nonlinear elastic, nonlinear hardening. So, this portion gets added up here, it is not it is up to yielding. So, beyond a yielding, the hardening response is also treated as nonlinear. Another model is K-G model, where K stands for bulk modulus and G stands for shear modulus.

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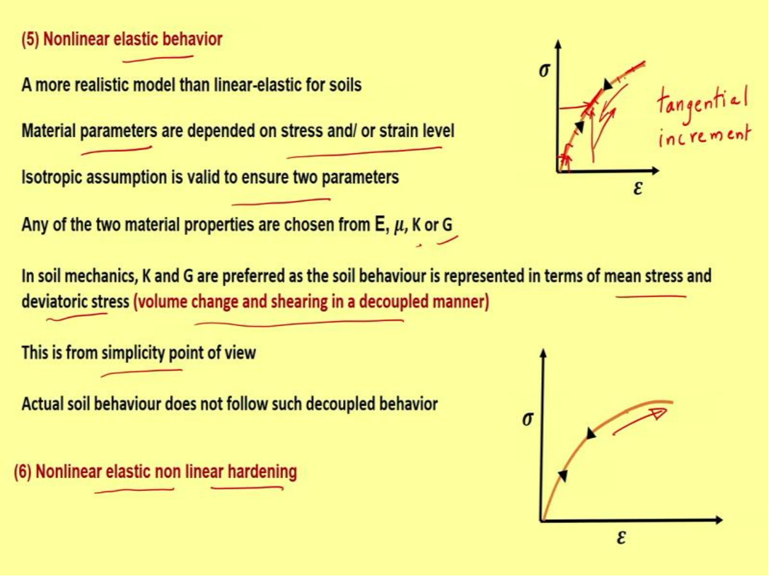

So, the tangential bulk modulus that is incremental at a particular point and shear modulus are defined in terms of stress invariance, how it is defined in terms of P'and J_2. What is J_2? J_2 is the second invariant of deviatoric stress tensor. Now, probably you will understand why we have discussed these invariants before. So, it is just not discussed, why because these invariants are very much used in your mechanics modelling or to express certain models in these invariants are used. So, these invariants are very important.

So, here in the K-G model, the both the bulk modulus and the shear modulus they are expressed in terms of stress invariance that is the mean stress P' and root J_2 where cannot alpha K_0, G_0, α_G and β_G these are all parameters of the model. So, we need to determine these parameters in order to apply K-G model, the stress strain curve for this model is given that is P' versus ε_v.

Let us mean stress upon volumetric strain, it is always associated with volumetric strain and the deviatoric stress that is $\sqrt{J_2}$ is associated with deviatoric strain. So, E_d here represents the deviatoric strain. So, what is deviatoric strain? The expression is given here again it depends upon the various definitions of deviatoric stress or deviatoric strain there is no specific guideline as such different models assumed different forms of deviatoric stress strain expression.

For this in this particular model, the deviatoric strain is given by

$$E_d = \frac{2}{\sqrt{6}}\sqrt{(\mathcal{E}_1 - \mathcal{E}_2)^2 + (\mathcal{E}_2 - \mathcal{E}_3)^2 + (\mathcal{E}_3 - \mathcal{E}_1)^2}$$

So, stress strain curve one important aspect which has we need to note here is in P’ versus ε_v, the slope keeps increasing in this direction which means to say the bulk modulus keeps increasing.

Whereas, in the case of $\sqrt{J_2}$ was deviatoric strain you can see that the bulk modulus keeps reducing as the soil gets more and more stressed or sheared. So, you can see that the shear modulus reduces whereas, the bulk modulus increases. So, this is typical stress strain curve representation which we use in K-G model.

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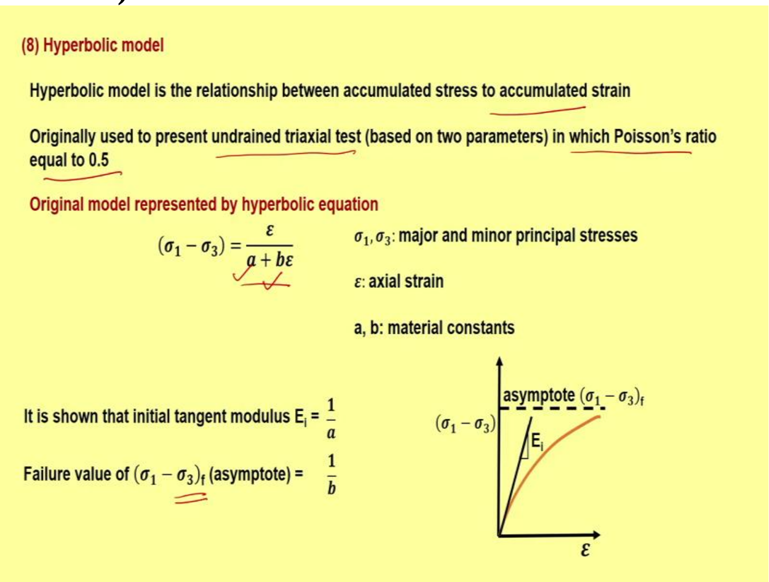

So, next is hyperbolic model; hyperbolic model is a relationship between accumulated stress to accumulated strain. Now, till now, we have been talking mostly about the incremental stress and incremental strain. Here it is the cumulative effect of stress and string it was originally used it was a hyperbolic model was developed to present undrained triaxial test based on 2 parameters in which Poisson’s ratio is equal to 0.5.

I think this point you remember in the previous lectures, we have already discussed this particular point Poisson’s ratio is set to 0.5 for undrained condition why the bulk modulus becomes infinity and then there is no volume change. So, original model is represented by this hyperbolic equation:

$$(\sigma_1 - \sigma_3) = \frac{\varepsilon}{\alpha + b\varepsilon}$$

σ_1 σ_3 major and minor principle stresses, ε is the axial strain and a, b are the material constants. So, it looks like this and the initial tangent model is shown here E_i and the asymptote is shown here $(\sigma_1 - \sigma_3)_f$. Asymptote means, the final value at which the failure happens is represented by this curve becomes asymptotic and that particular value is shown by $(\sigma_1 - \sigma_3)_f$ and the initial tangent modulus E_i is shown here.

Now, it is shown that the initial tangent modulus $E_i = 1 / a$ and the failure value that is $(\sigma_1 - \sigma_3)_f$ that is the asymptote value is equal to 1 / b. So, both these material constants a and b has some meaning in the hyperbolic model that is 1 / a is E_i and 1 / b is $(\sigma_1 - \sigma_3)_f$. So, based on our stress strain data, if we can determine a and b then the constants E_i and the final value is known.

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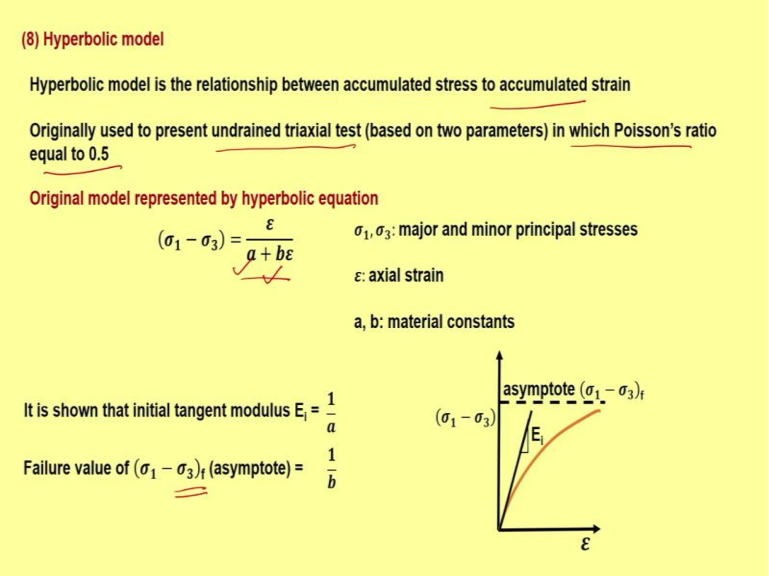

So, for that, if we rearrange the hyperbolic model in this particular form that is:

$$\alpha + b\mathcal{E} = \frac{\varepsilon}{(\sigma_1 - \sigma_3)}$$

Now, this is a linear response $\alpha + b\mathcal{E}$. So, plotting laboratory data to this re-arranged form a and b can be determined how if you plot $\frac{\varepsilon}{(\sigma_1 - \sigma_3)}$ this portion as a function of ε this then we are likely to get a straight line and the intercept is a and the slope is b. So, once you get a and b, E_i and $(\sigma_1 - \sigma_3)$ at failure or the asymptotic value can be determined.

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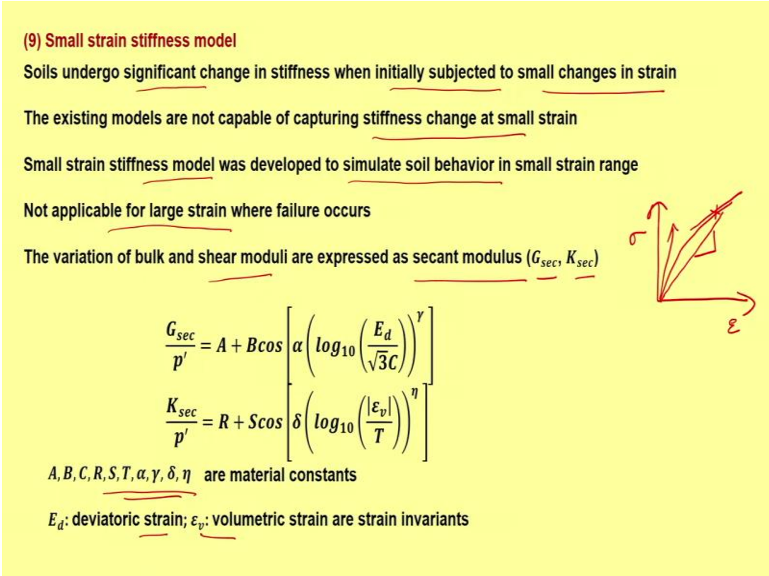

So, another model is small strain stiffness model which is based on the understanding this is a relatively newer model as compared to the earlier discussed models which were very much there in the literature. Now, this particular models small strain stiffness model is based on the understanding that the soil undergoes significant changes in stiffness when initially subjected to a small change in string.

That means, if it is subjected to for the first time or initially a small changes in strain, it can bring about a significant change in stiffness. Now, none of the above models what we have discussed can capture this particular behaviour. So, this particular model has been developed the existing models they are not capable for capturing the stiffness change at small strain. So, small strain stiffness model it was developed to simulate the soil behaviour in small strain range not applicable for large strain where failure occurs.

So, frankly we are not discussing about the failure in this model. We want to capture this small strain behaviour. The variation of bulk can share moduli are expressed as secant modulus. Now, what is secant modulus? It is expressed as G_{sec} and K_{sec}. So, what is the secant modulus? So, if this is σ versus ε and it represents a nonlinear curve, so this is the initial tangent modulus and this is tangent modulus.

So, if I want to find out the modulus here, the secant modulus is this, so this slope is the secant modulus. So, here the variation of bulk and shear moduli are expressed in terms of secant modulus and the expression is given as

$$\frac{G_{sec}}{p'} = A + BCos\left[\alpha(log_{10}\left(\frac{E_d}{\sqrt{3}\ C}\right))^{\gamma}\right]$$

$$\frac{K_{sec}}{p'} = R + SCos\left[\delta(log_{10}\left(\frac{|\varepsilon_v|}{T}\right))^{\eta}\right]$$

So, there are lots of parameters along with this small strain stiffness model. So, practically it becomes difficult in certain cases where the numbers of parameters are too large. So, you need to determine all of those. So that becomes sometimes a deterrent for choosing certain models and why models like linear elastic models are so popular. So, all these are material constants whatever has been discussed here, they are material constants and hence, those issues make these model less popular but for taking care of specific behaviour one need to go for these models.

So, where E_d in this expression is deviatoric strain we have already explained ε_v is the volumetric strain and these are strain invariance.

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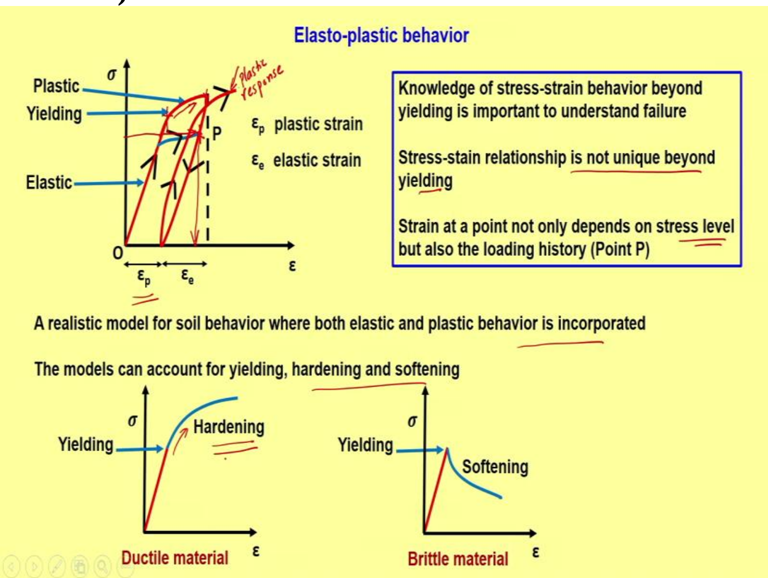

Before ending this lecture, we need to actually discuss what is the real behaviour and how it can be captured for problems in geomechanics and that is elasto-plastic behaviour, a more realistic model, different version of elasto-plastic behaviour we have discussed in bi-linear section where we discussed about elastic perfectly plastic and rigid perfectly plastic, this is yet another and more refined and considerable improvement of those where we discussed about the elastic as you can see here.

This is the elastic portion here the yielding happens, the plastic behaviour sets in so elastic yielding and plastic. Now, if we try to unload it from this particular point, it is not going to trace back. Now that is the difference between nonlinear elastic and elasto-plastic behaviour. In this case it is not tracing back rather it takes another route and this is more or less parallel to this linear portion.

So, it comes down and if it is again loaded, it traces more or less the same parallel as that of elastic behaviour, why possibly, it is because it has been loaded up to this. Now, whatever loading you are making here, it is always less than the load which it has been subjected. So that is why here this is the elastic response, this is the plastic response, here it is unloading and reloading and then beyond a certain point again the plastic response sets in. So, here another important aspect to be looked at as this is the string.

Now, from here we have unloaded. So, whatever string it has regained that is elastic portion of the strain and whatever it has undergone the permanent setting or permanent deformation that portion is given by the plastic strain. So, the total strain in elasto-plastic response is the summation of elastic strain and plastic strain. So that is what is shown here. So, knowledge of stress strain behaviour beyond yielding is important to understand failure this we have already seen, unless we know where it is going to fail, it is difficult.

So, now, here it there is a sort of behaviour which the soil exhibit beyond yielding. So that is captured in elasto-plastic models stress strain relationship, it is not unique beyond yielding which means to save once it has yielded, then the stress strain response is not unique the way we have it for elastic response and strain at a point not only depends on the stress level but it also depends on the loading history.

That is the point P, what it means to say is, let us say it has taken this is the elastic portion, then there is plastic deformation that is happening at this point it is unloaded. Now, let us say that it reaches this particular point P. So, we have reached this particular point P. So, what it means strain at a point that is strain at point P, this one it not only depends on the stress level. Now, here at this particular point, you can say that, this is the stress level.

Now, from here it is loaded, it can take this particular route to P or it can go it can yield and it can come back here and here there is a point P. Now, the strain at a point here it has crossed the strain and then it is unloaded. So, strain at a point, it not only depends on the stress level but also the loading history at the point P. So, it is not only the stress level, if I know a particular stress level, this is the strain it is not like that.

That strain would be a result of the stress history as well or the loading history as well and in soils this is extremely important, what is the loading history and that is how the concept of normally consolidated and over consolidated or dense or loose state comes into picture. So, what is the stress history or loading history also play a major role in defining what is the strain which the body has undergone in this case, it is purely elastic strain, whereas, in this case, it has undergone some plastic deformation and certain regaining has happened.

So, totally these are different path which it has followed. A realistic model for soil behaviour, where both elastic and plastic behaviour is incorporated the models can account for yielding hardening and softening we will come to hardening and softening later. So, here it is yielding and it is hardening now, this is a typical behaviour of ductile material where with stress the strain is increasing.

So that is a typical behaviour of hardening and in the case of softening after yielding the stress reduces with strain. Now, why this behaviour we will see it later in the subsequent modules. So, this is a typical behaviour of brittle materials. So, yielding with hardening yielding with softening. So, these 2 aspects can be captured by different elasto-plastic models.

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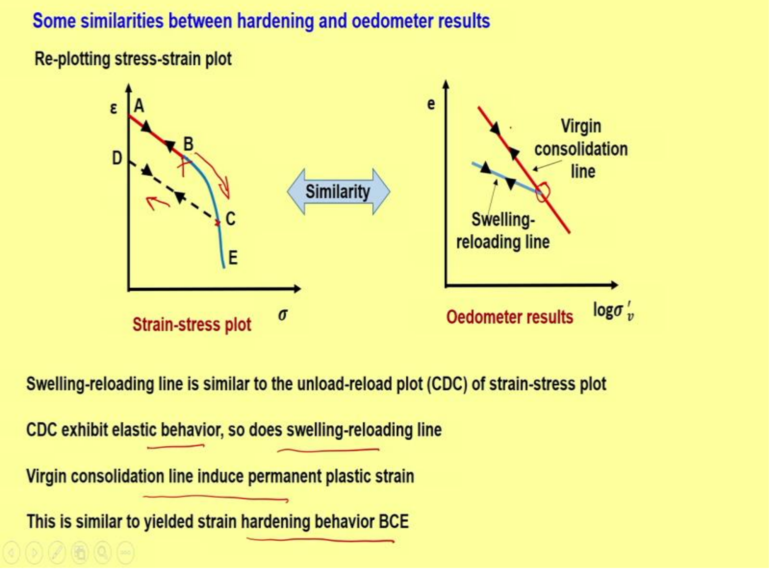

So, some similarities between hardening and oedometer results. So, if you re-plot the stress strain plot like this, so what I have changed, I have interchange the x and y axis now, the strain is on y axis and stress is on x axis, you can see the same plot as before. So, A to B and then yielding and then the plastic behaviour happens from here it is unloaded. So, this is unloading reloading path CD which is more or less parallel to AB.

Now, if you compare this with a given oedometer results, you can find that there is a sort of similarity here it is E versus log σ'_v. This is a very popular representation of odometer results where you talk about the virgin consolidation line and swelling-reloading line you can see that this particular response of this is more or less similar to this behaviour this particular behaviour. Whereas swelling-reloading line it is more or less mimicking this particular behaviour.

So, swelling-reloading line is similar to the unload reload plot of this particular stress strain behaviour CDC. So, you can consider this as the yielding point which is somewhat similar to here or maybe similar to here. Swelling-reloading line is similar to the unload-reload plot that is CDC of strain stress plot because it is plotted on y axis here, this particular line and this particular line it closely resembles and the way in which it responds, it is also similar.

CDC exhibit elastic behaviour, so does the swelling-reloading line and it has something to do with the stress history. Virgin consolidation line induces permanent plastic strain, so beyond BC, so this is the yield point. So, beyond yield point it induces plastic deformation and same is the case with the virgin consolidation line where there is permanent setting which is happening and this is similar to the yielded strain hardening behaviour of BCE. So, this is yield point and this is the hardening behaviour BCE. So, this and the virgin consolidation line behaviour these are same.

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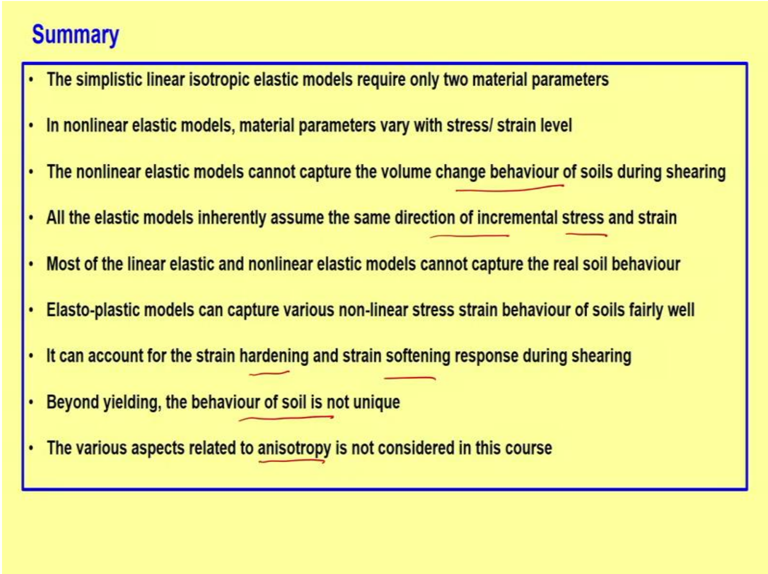

So, to summarize today's lecture, the simplistic linear isotropic elastic models require only 2 material parameters and that is why it is simple. In nonlinear elastic models, material parameters vary with stress strain level, whereas, in linear elastic model it is constant. The nonlinear elastic models it cannot capture the volume change behaviour of soils during sharing, we are talking about the hardening and softening behaviour. All the elastic models inherently assume the same direction of incremental stress and strain.

Most of the linear elastic and nonlinear elastic models cannot capture the real soil behaviour, we have already explained that. Elasto-plastic models they have been developed to capture the nonlinear stress strain behaviour of soils fairly well, it can account for the strain hardening and strain softening response during shearing and beyond yielding the behaviour of soil is not unique.

But, having said that, we also need to keep in mind that as we proceed from linear elastic to elasto-plastic models, the complexity of mathematical solution also increases and the computational effort also increases. So that we have to keep in mind. So, the various aspects related to anisotropy is not considered in this course. So, mostly we will confine all our discussion to isotropic soil behaviour.

So that is all for this particular lecture, we have just discussed some important aspects of the cause-effect relationship in terms of constitutive behaviour or constitutive model behaviour. So, we have discussed about different models which relates the stress and the strain. But, let me again make a point here that this is not an exhaustive discussion, we there are several aspects to what we have discussed now.

This is only for you to get oriented towards such a constitutive behaviour. So, with this we will end this particular lecture and till now, we have been focusing mostly on 3 dimensional aspects. Now, for actual problem solving, we need to transform this 3D to 2D behaviour for simplicity. So, for certain problems, one can always idealize a given 3D problem to 2D. So that will be seen in the next lecture.