Advanced Soil Mechanics
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Lecture – 11
Cause Effect Relationship

Welcome back. So this lecture is about cause-effect relationship. In the last lecture we discussed about strain and the lecture previously we discussed about stress. Now in this lecture we will be combining these two so that is known as cause-effect relationship okay.

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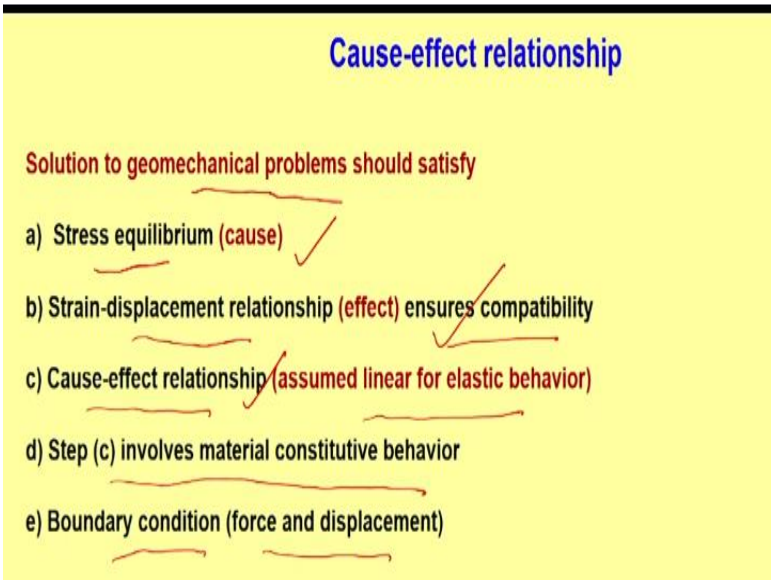

So solution to geomechanical problems it should satisfy the first one is stress equilibrium which is considered as a cause. Now as I discussed before stress is not actually the cause, the cause is the external loading, but the internal mechanism is understood as stress. So do not get confused with that, stress is a response, so how it can become a cause, but this is due to the external load which is acting. The manifestation of external load is conceived as stress.

So stress is the cause that is how it is taken and the effect would be strain. So we need to satisfy stress equilibrium condition which is conceived as the cause. Then we also need to understand the strain displacement relationship which is defined as the effect and to ensure compatibility. Then it is cause-effect relationship and in most of the cases for simplicity it is assumed linear for elastic behavior.

Now why I have specified it here is to make you understand cause-effect relationship really talks about the material behavior. Now cause-effect relationship, why do you need that such a relationship that also will become clear subsequently? We need to study about stress, we need to study about the strain, why you need to connect these two? So that it will be clear in some time. So it is assumed for linear for elastic behavior.

Now it is all about our understanding about a given material. We know that soil and rocks they behave nonlinearly, but there are certain situations and conditions where soil can also exhibit linear behavior. So if you assume the cause-effect relationship essentially depends upon what kind of response the material give for a given set of loading and depending upon the constitutive behavior of that particular material.

Now step c that is what it means, the step c involves material constitutive behavior, whether a material should respond linearly is mostly dictated by the material itself. A very good example would be of that of over consolidated soil. It is the same soil but it has undergone a lot of stress in the past. So the present stress what it is subjected to, is in fact very small for that material and in those conditions it can exhibit a sort of linear behavior.

So what type of response a material should exhibit will depend upon the constitutive nature of that particular material and then we need to also satisfy the boundary condition both in terms of force and displacement. So these are some of the minimal requirements to solve problem in geomechanics. So in that we have discussed about stress, we have discussed about strain, displacement relationship. So in this lecture we will specifically understand the cause-effect relationship.

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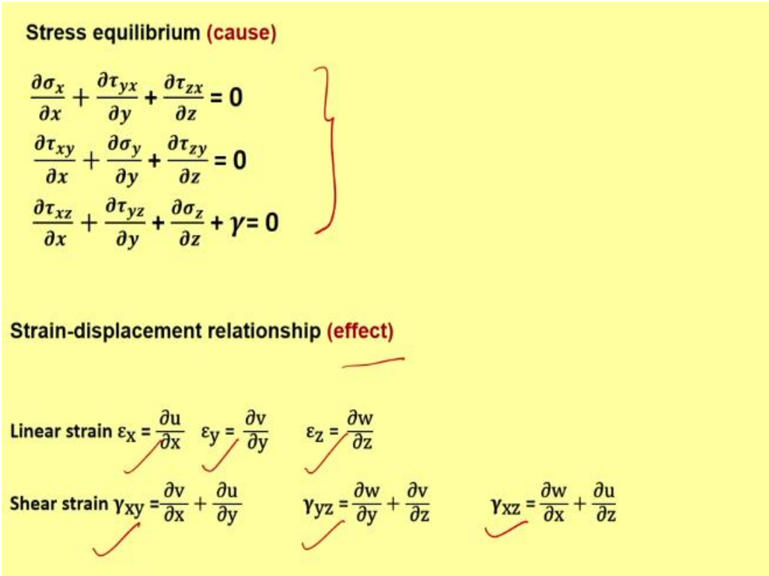

So to summarize this is about the stress equilibrium equation which we have already discussed before, just for completeness I am again repeating it. Again the strain-displacement relationship which is the effect linear strains ε_x, ε_y, and ε_z and shear strain which is the engineering shear strain is given in this manner. I will not spend time here, we have already discussed this.

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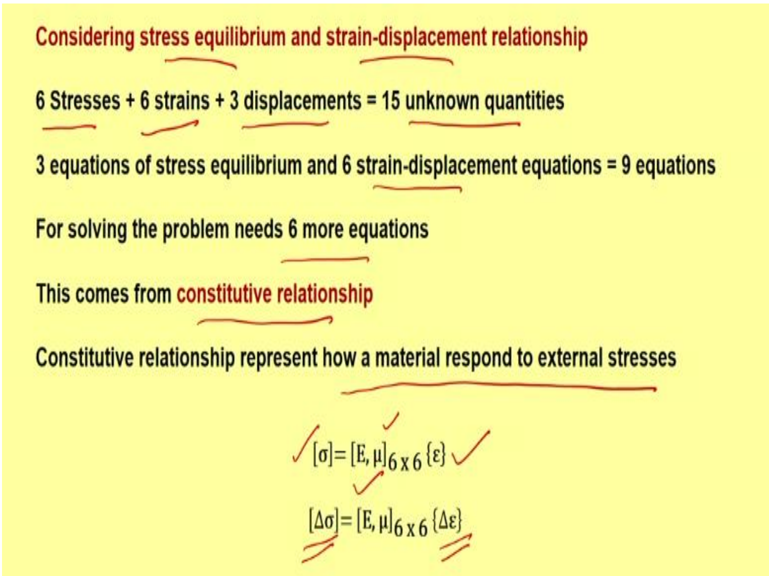

Now considering stress equilibrium and strain-displacement relationship, there are 6 stresses, independent stresses, 6 strains and 3 displacements, altogether there are 15 unknown quantities. Now; 3 equations of stress equilibrium, 6 strain displacement equations amounts to 9 equations, again number of equations are way less than the unknown quantities. So for solving the problem, we need 6 more equations and this comes from the constitutive behavior or constitutive relationship, why?

We need to connect stress and strain or cause and effect because that process will give you additional 6 equations with which one can solve the given geomechanical problem. The constitutive relationship is extremely important for any material and we need to spend a lot of time learning if you want to master geomechanics, this constitutive relationship is extremely important. So constitutive relationship represents how a material respond to external stresses.

For a given problem, rather a given linear problem this is the cause, this is the effect and this is the constitutive behavior or the constitutive relationship is ensured through this particular matrix. For here it is given as 6 by 6, we will come to this a bit later and for materials like soils which are highly nonlinear it is always ideal to talk in terms of incremental stress and incremental strain.

So this is what is denoted here Δσ and Δε, the other thing remains the same, the constitutive matrix that remains the same.

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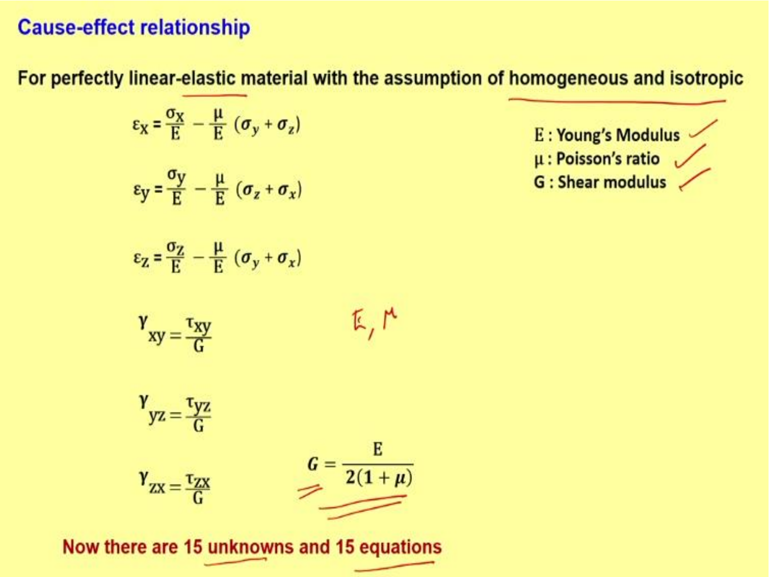

So cause-effect relationship: Now it is always easy to start with most simple part of the constitutive relationship or cause-effect relationship that is why we want to discuss for perfectly linear elastic material with the assumption of homogeneous and isotropic. I hope all these terms are known to you. Homogeneous means spatially it is all same and isotropic it is same in all directions and it is a linear elastic material.

This is a very fundamental relationship between stress and strain where ε_x which is the linear strain in x direction you can write in terms of stresses as

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z)$$

where σ_x is the normal stress in x direction, E is the modulus of elasticity, μ is Poisson's ratio. Similarly you have that is what is written here Young's modulus, Poisson's ratio. There is one more term called G which is the shear modulus.

So similarly you have ε_y and ε_z. In addition, one can always write the shear strain $\gamma_{xy} = \tau_{xy}/G$. Accordingly $\gamma_{yz} = \tau_{yz}/G$ and $\gamma_{zx} = \tau_{zx}/G$. So these are the set of constitutive relationships if you consider a body to be linear elastic homogeneous and isotropic where G is the shear modulus and it takes the relationship E / 2(1+ μ).

One important thing we have to note here is in all these expression E and μ they are the only two parameters which need to be known to define the constitutive behavior. G is a dependent parameter and it depends on the value of E and μ. So we have two independent parameters we should know. Now when you take into account the cause-effect relationship, we are left with now 15 unknowns and 15 equations and the problem can be solved.

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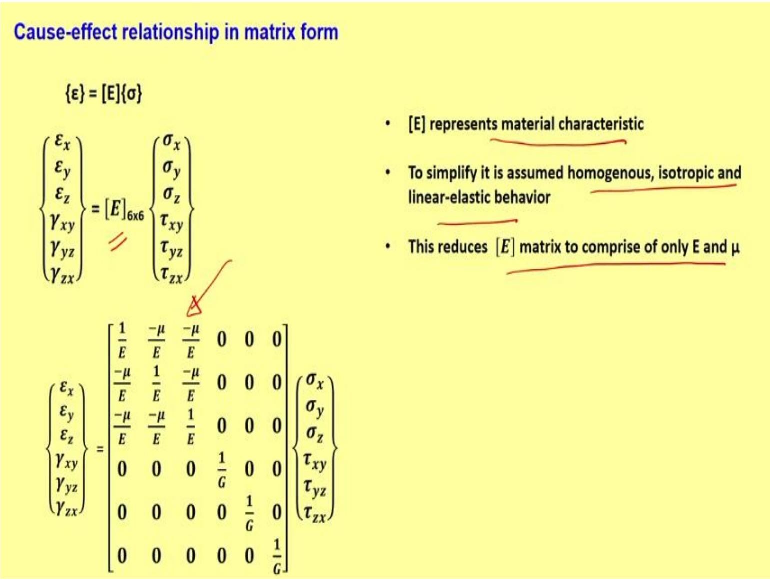

So cause-effect relationship it is written in matrix form where ε is the strain, E is the constitutive matrix and σ is the stress matrix, E represents the material characteristics and here again we will be dealing with homogeneous isotropic linear elastic behavior. This reduces E matrix to comprise of only two unknown parameters which related to material behavior which is E and μ and which can be determined using suitable laboratory tests.

So that is how it looks like ε_x, ε_y, ε_z, γ_{xy}, γ_{yz}, γ_{zx}. The corresponding stresses and 6 by 6 matrix in of material behavior and for linear elastic material one can always obtain and this formulation of matrix comes from the previous slide. We have the relationship, so that is rearranged in matrix form that is all. So this is a simple relationship wherein we talk about the linear elastic behavior and that is why this particular model is so popular.

So if one can define the behavior of the material using linear elastic, then it considerably simplify the problem and it becomes less computationally intensive.

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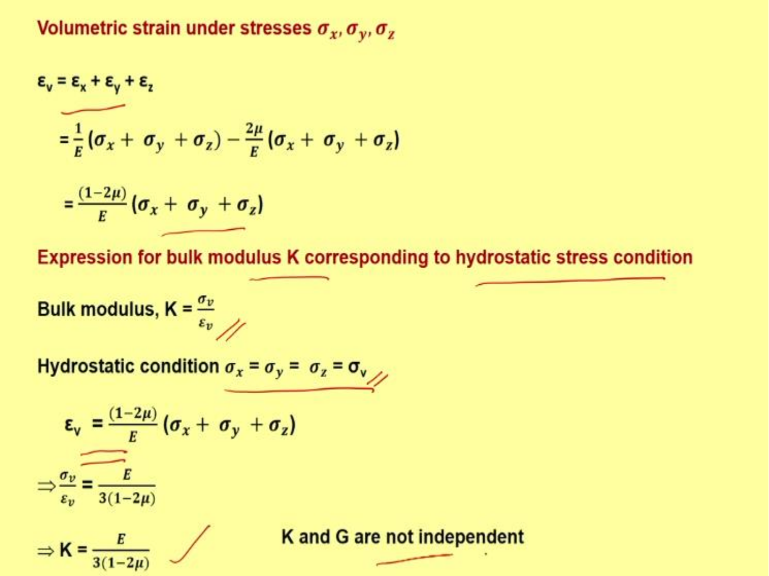

So volumetric strain under stresses σ_x, σ_y, σ_z. This we have already seen and if you expand you can write as

$$\varepsilon_v = \frac{(1-2\mu)}{E}(\sigma_x + \sigma_y + \sigma_z)$$

So expression for bulk modulus now there are two more parameters, sometimes it is expressed in terms of K and G, in some cases it is represented in terms of E and μ. Whatever be all these terminologies parameters should be familiar.

So expression for bulk modulus corresponding to hydrostatic stress condition is just to understand the expression for K which we all already know, but just to have a discussion in terms of let us say whatever we have learnt till now. Now what is meant by hydrostatic stress condition? Stresses are equal in all direction. That means $\sigma_x = \sigma_y = \sigma_z$. So bulk modulus how do you define?

It is volumetric stress upon volumetric strain and this is the condition for hydrostatic condition, this is the expression $\sigma_x = \sigma_y = \sigma_z = \sigma_v$. If you substitute this back into this equation which is the expression for volumetric strain you can get the expression

$$\frac{\sigma_v}{\varepsilon_v} = \frac{E}{3(1-2\mu)}$$

So K is nothing but E / 3(1- 2μ) and you can see that both K and G are dependent on E and μ, so it is not independent.

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$E = 3K(1-2\mu)$

E and K are positive as they are material constants

$(1-2\mu) > 0$

$\mu < \frac{1}{2}$

In general, $0 < \mu < \frac{1}{2}$

For $\mu = 0.5$, K tends to infinity which implies $\varepsilon_v = 0$

Body undergoes no volume change (incompressible)

Undrained condition is a typical example of no volume change condition

For undrained condition, $\mu = 0.5$

So now we have E = 3K(1 – 2μ), E and K are positive because they are material constants which means to say (1 – 2μ) > 0 ,because E has to be positive. For fulfilling that condition, we can write that Poisson's ratio is always less than half. Now in general Poisson's ratio varies between 0 and 0.5. Now there are certain materials which assume the value of Poisson's ratio close to 0.

For example, cork is a classical example and in general for most of the problems in continuum mechanics related to soils and rocks the variation we will consider around 0.1 to 0.5. We do not talk about μ close to 0 and you will be surprised to see there are certain materials where μ is even negative. These are called auxetic materials, you can refer to literature and basically these are engineered materials mostly and which exhibit negative Poisson's ratio, but that's not relevant for this particular course.

Now for μ = 0.5, in the previous relationship if you substitute μ = 0.5 that is Poisson's ratio equal to 0.5, then the volumetric constant or bulk modulus it tends to infinity which implies that the volumetric strain is equal to 0. There is no volumetric strain, the material is incompressible, body undergoes no volume change it is basically incompressible. This definition has got some meaning in soil mechanics as well.

So undrained condition, we will be discussing about drained, undrained condition in detail, but since this is an undergraduate portion all of you will be knowing what is meant by an undrained condition, loading under undrained condition. So what happens during undrained condition? You are not allowing the water to move out of the soil during loading. So in that case is a typical case of no volume change.

When there is no expulsion of water, then you cannot have volume change, what will happen? Pour water pressure will increase. So there is no volume change, the whole of the soil mass behaves incompressible, so what does it mean? It means that for undrained condition one can take the value of Poisson's ratio = 0.5. So the whole discussion is basically for that.

So in geomechanics we actually refer to the μ value from 0.1, 0.2 to 0.5. For purely undrained case for soils undrained loading, one can always presume μ = 0.5.

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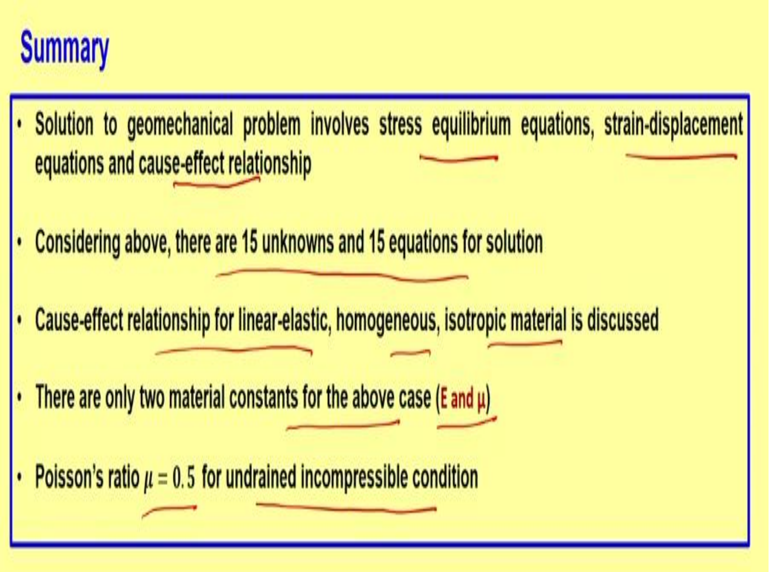

To summarize whatever, we have just told about cause-effect relationship, we have solution to geomechanical problem involves stress equilibrium equations, strain displacement and cause-effect relationship, so 15 unknowns, 15 equations. Considering above there are 15 unknowns and 15 equations for solution. Cause-effect relationship for linear-elastic, homogeneous, isotropic material has been discussed in this particular lecture.

There are only two material constants for the above case and that is the simplicity of this model where we discuss in terms of E and μ and Poisson's ratio μ is = 0.5 for undrained incompressible condition. So, that is all for this lecture on cause-effect relationship. We will briefly discuss about some classical constitutive relationship which you normally see in geomechanics again, it will be a very elementary discussion which we will see in the next lecture. Thank you for now.