Advanced Soil Mechanics
Prof. Sreedeep Sekharan
Department of Civil Engineering
Indian Institute of Technology - Guwahati

Lecture – 10
Strain in Soils

Welcome back all of you. Till last lecture, we have discussed about stresses and in today's lecture we will see a bit about strain, strain in soils, and when I say we have discussed about stresses, I would like to make a disclaimer here like whatever we have discussed is just a tip of the iceberg, like this is only for orienting the listener or the participant of this course towards the higher limbs of continuum mechanics.

So, whatever we have learned, it is a very elementary discussion related to stresses because in this course advanced soil mechanics, we cannot go further beyond because we have many other steps closely related to soils which need to be discussed.

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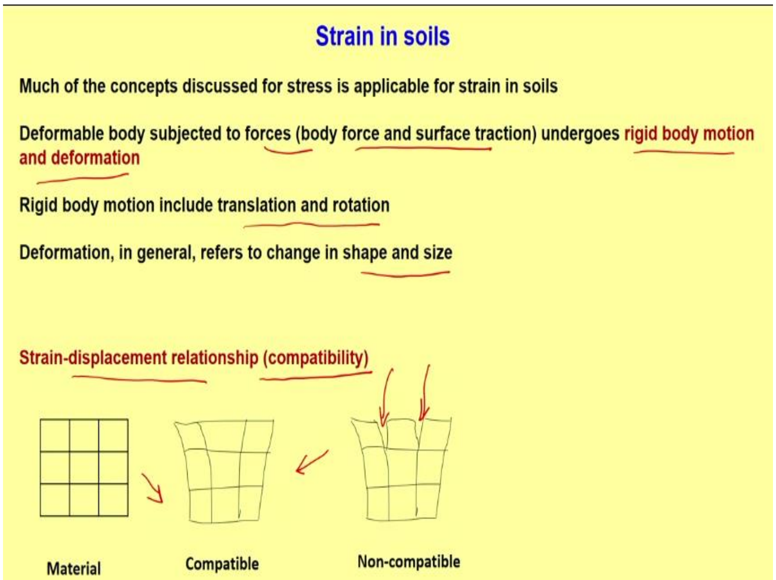

Now in that whatever we have discussed about stresses, it holds true for these strain in soils as well. So, as you can see much of the concepts discussed for stress is applicable for strain in soils as well. Now when we discuss about stress and strain, one would wonder, which is happening first and which is the manifestation of the other. What we know is that we have a body on which a set of forces or a force act.

Now, this application of the force can bring about some changes in the body. Now this changes one can visualize as the strain that is happening within the body. Now, when such changes happen, what is the internal response of the body that we defined in terms of stress. So, both are manifestation of the load acting on the body where you can always visualize strain that is happening in the body and it is more like it is a reality.

Whereas stress is the internal manifestation, you may not be visualizing stress, but it is there in the form of response of the body. So, that is how we need to conceive this whole aspect. Now, deformable body subjected to forces. Now forces it can be body force and surface traction, we have not categorized these forces in our earlier discussion and it was not required also, only when we discussed about equilibrium equations we mentioned about body forces, which becomes very important in soils.

So, geostatic stress or the self-weight stress becomes very important in soil and rocks. So, deformable body subjected to forces, it can be either body force or traction force, it undergoes rigid body motion and deformation. So, the two responses are rigid body motion and deformation. Now, when we say rigid body motion it includes translation and rotation. We will not get into the details of rigid body motion right away.

We will discuss at length about the deformation part of the deformable body. Now deformation in general it refers to change in shape and size. Now, possibly you will understand the implication when I say shape and size because we have already discussed this in the previous lectures. Size basically it relates to volumetric response and shape it basically represents the deformation or the shearing aspect of the material.

So, when I say change in size is a volumetric response and it is decoupled. So, we study an object which undergoes volume change where change in shape is not discussed as a volumetric part of it and the other one is deviatoric part which we discussed earlier. So, deformation in general it refers to both in general, but for soil mechanics or for rocks, the deformation which is associated with change in shape or shearing becomes very important.

Now, strain displacement relationship which ensures the compatibility requirements in the body. Initially we discussed about stress and we have seen the equilibrium stress equation that is one part of the story, the other part is strain displacement relationship, which will help to ensure the compatibility. All these elements are very much important for the study of continuum mechanics.

Now, let us say that this the given material, a compatible material when it is subjected to different forces is in this manner. So, this it is not failing as such but it undergoes deformation, hence for the given situation we can call it it is compatible. In this you can see that certain fractures are being made under the given condition of external forces. Now, this can be termed as noncompatible.

So, strain displacement relationship helps to assess the compatibility requirements of a material when it is subjected to different loading.

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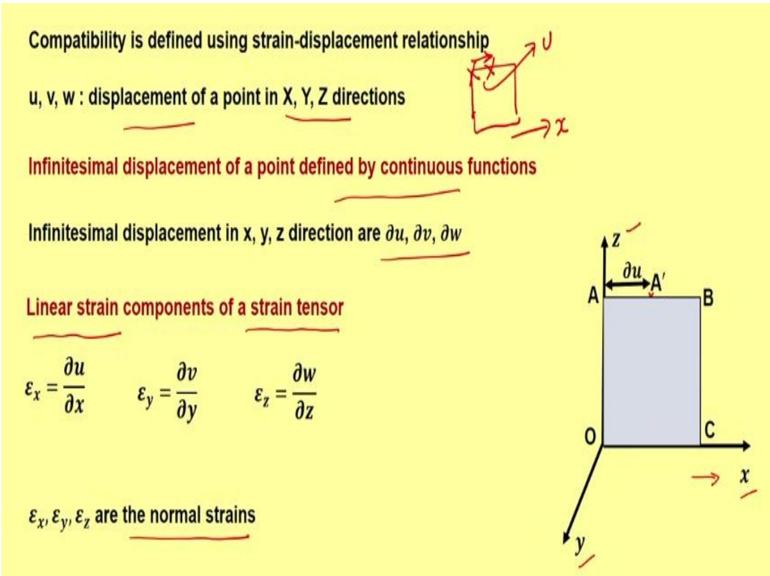

Now, compatibility is defined using strain displacement relationship. Now, we can say that u, v, w these are displacement of a point in x, y, x direction okay. So u, v, w for example if I have a body like this and I am discussing about this point. This point may undergo, it is a microscopic phenomenon, I am just analyzing it and showing you, so this change this displacement if this is an x direction, then this corresponds to u, so that is what it means.

So u, v, w are the displacement of a point in x, y, z directions and infinitesimal displacement of a point. This is the actual displacement and infinitesimal displacement of a point is defined by some continuous functions and infinitesimal displacement in x, y, z direction is given by ∂u, ∂v and ∂w. Now, let us consider this given axis x, y and z and this is a given part of the body. First let us define what is linear strain components.

Now, as I told in the beginning whatever we have discussed four stresses are applicable here. So, I am introducing the term strain tensor and you need to understand it is one of the same or it is very similar to stress tensor. Now, for that in the stress tensor we had normal as well as shear components, same way in the strain tensor also we have normal components that we discuss in terms of linear strain.

So, linear strain components, linear strain components to define that first let us see this is the point A on the body and displaces by around ∂u, ∂u is the infinitesimal displacement in x direction. So, ∂u is represented A is displaced to A’, so that is what it means. So, epsilon x which is the linear stream in x direction is given by ∂u / ∂x. Now, this is the same as that of the definition of strain.

So similarly, we will have $\varepsilon_y = \partial v / \partial y$ and $\varepsilon_z = \partial w / \partial z$. So, these are called normal strains or linear string okay. So, this is ∂u / ∂x, ∂u is the infinitesimal strain, this displacement happening in the x direction and dimension is given by ∂u. So ε_x, ε_y, ε_z. So, these are called normal strains.

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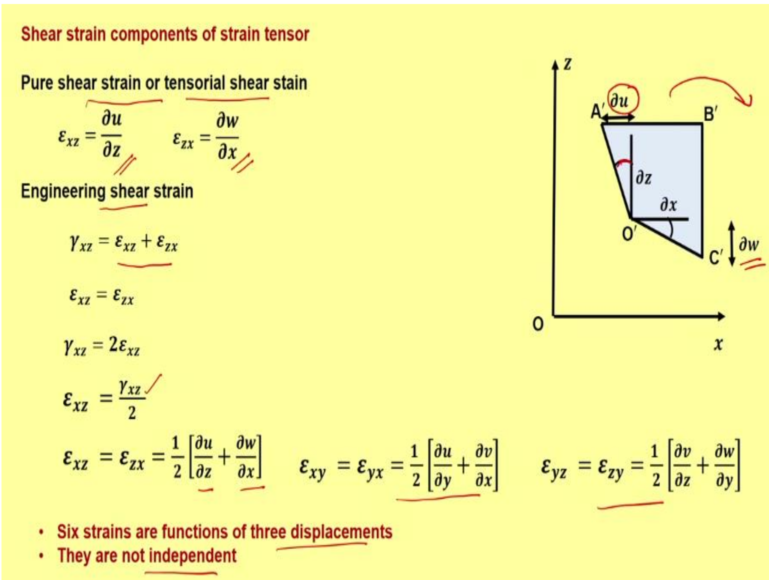

Now, next we will move on to the shear components of the strain tensor. Now, again I refer to x and z axis. Initially there was a body and due to some sort of deformation, due to some response what happens is there is a kind of deformation that is happening for the point C’ which is given ∂u. Similarly, there is a deformation happening in terms of ∂w here and A and C points gets moderately shifted to A' and C'.

So, there is a kind of shear happening on this body. So, here ∂u is the kind of displacement that is happening in the x direction and ∂w is the displacement that is happening in the z direction, all those things remain same. Now, here it is all about deformation and hence the rotational aspect of this becomes relevant here. Now, this is in ∂z, this is ∂x. So, we will first discuss about pure shear strain or it is called tensorial shear strain.

So, what are those? Now, the pure shear strain ε_{xz} that is in xz plane, $\partial u / \partial z$. So, what is ε_{xz}? ε_{xz} is $\partial u / \partial z$. So, this is nothing but this particular angle. So, $\partial u / \partial z$ is given as ε_{xz}. So, what will be then ε_{zx}? Which is $\partial w / \partial x$. So, these are known as pure shear strain or tensorial shear strain. Now, there is another definition of shear strain, which is known as engineering shear strain.

Where engineering shear strain $\gamma_{xz} = \varepsilon_{xz} + \varepsilon_{zx}$. So, this is the representation of sum total of the deformation that happens to the body. For example, if I rotate this about this point, one can get something like this, so this becomes γ_{xz}. So, it is a sum total rotation that is happening is represented by engineering shear strain and that is the $\varepsilon_{xz} + \varepsilon_{zx}$.

Now, we can take $\varepsilon_{xz} = \varepsilon_{zx}$. In that case, $\gamma_{xz} = 2\varepsilon_{xz}$. So, one can write the tensorial shear strain is half of engineering shear strain. So, in the tensorial form, it is represented either in terms of ε or in terms of γ whatever be. Hence, we can write now $\varepsilon_{xz} = \varepsilon_{zx} = 1/2\ \gamma_{xz}$. What is γ_{xz}? $\gamma_{xz} = \varepsilon_{xz} + \varepsilon_{zx}$

Now, what is ε_{xz}? That is equal to $\partial u / \partial z$ which is written here and the other one is $\partial w / \partial x$ which is written here. So, that is equal to half, half of γ_{xz}, it is very clear from this expression. Similarly, we have

$$\varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]$$

$$\varepsilon_{yz} = \varepsilon_{zy} = \frac{1}{2}\left[\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right]$$

So, what we have done? We have discussed about the nominal strain as well as shear strain components. So, there are 6 strains, which are functions of 3 displacements. Which are the 6 six strains? Three normal and 3 shear strain components and they are function of 3 displacements and hence they are not independent.

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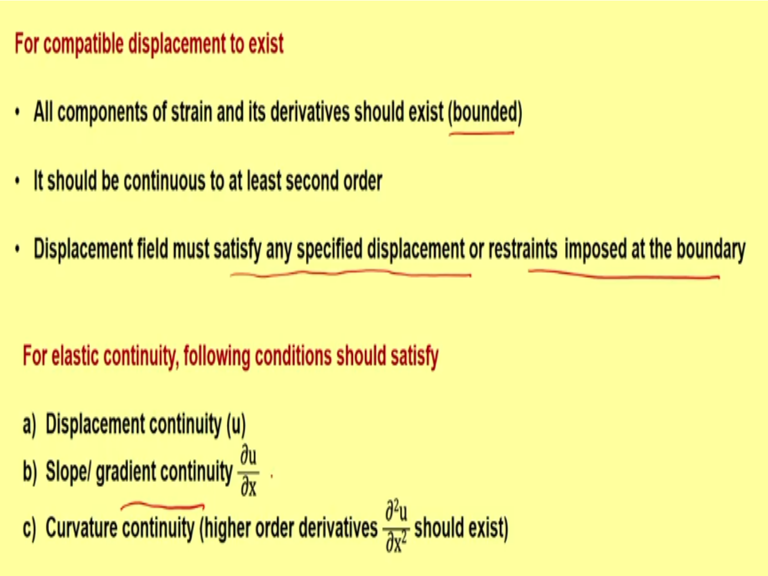

For compatible displacement to exist, now what is the whole idea? When some force act on a body whether it is under stress equilibrium, whether it is compatible, these are some of the questions we need to answer. So for compatible displacement to exist, all components of strain and its derivative should exist which means it should be bounded. It should be continuous to at least second order.

Now, these are certain mathematical requirements to understand whether a given system is compatible or not. The displacement field must satisfy any specified displacement or restraints imposed at the boundary that is satisfying the boundary condition. So, all components of strain and its derivatives should exist, again the mathematical requirement for continuity and it should be continuous to at least second order and displacement field it should satisfy some specified boundary condition.

For elastic continuity following conditions should satisfy, that is displacement continuity, the way in which we define u. Slope gradient continuity, the slope gradient continuity which is given as $\partial u / \partial x$ and the curvature continuity which is the higher order derivatives $\partial^2 u / \partial x^2$ should exist. Similarly, the other partial derivatives also should exist.

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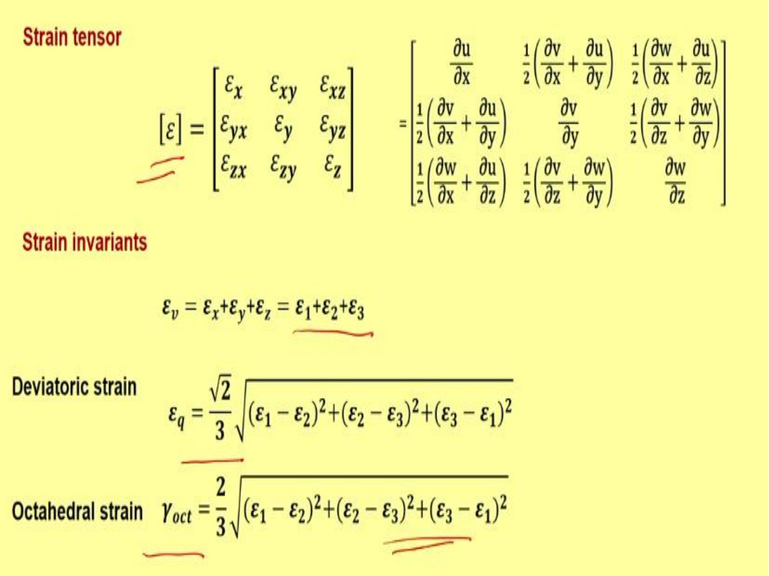

Now having done that, it is quite easy now for you to visualize. Since you have done that strain tensor it is very easy to visualize strain stress as well, we do not have to discuss more. So, this is the representation of strain tensor ε_x, ε_y, ε_z, these are linear strains or nominal strain components and other components ε_{xy}, ε_{xz} these are shear strain components of strain tensor. And to expand this, you can see that it is expressed in terms of pure shear strain or tensorial shear strain.

And hence, we have half of engineering shear strain, which is expressed here. I will not go into the details, it is self-explanatory, just like we had stress invariants it is true for strain also it has strain invariants, but again I will not go into the details of it and a very prominent strain invariant is the volumetric strain ε_v, which is the sum of the linear strains or $\varepsilon_x + \varepsilon_y + \varepsilon_z$ and in principal strain is $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$.

Again, deviatoric strain as we have seen previously there are different ways by which deviatoric strain or stresses are discussed or it is defined. The same way deviatoric strain is also defined and one such definition is

$$\varepsilon_q = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_1-\varepsilon_2)^2 + (\varepsilon_2-\varepsilon_3)^2 + (\varepsilon_3-\varepsilon_1)^2}$$

This is similar to whatever deviatoric stress that we have discussed before. Now, deviatoric means we understand that it is going to create some sort of sharing component and deformation the associated deformation. So, again octahedral stress, we have defined here octahedral strain

$$\gamma_{oct} = \frac{2}{3}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}$$

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So to summarize the strain acting in the soil, similar to stresses there are 3 normal strains and 6 shear strains that form a strain tensor. It can be expressed as linear strain or it has a tensile strain engineering shear strain. So, one is normal component and the other one is shear component of strain which is tensorial shear strain or it is known as engineering shear strain. The rule of transformation of coordinate axes we have discussed for stresses.

Strain invariants, principal strains, volumetric strain, deviatoric strain; all are valid for strain tensor as well. So, this is all about strain acting in the soil with relevant to this particular course and there are a lot more which you will study when you do a course in continuum mechanics or in geomechanics okay. So, this is all for now in this particular course related to strain in soils.

Now, why we have discussed about stress? We have discussed about stress; we have discussed about strain. Now, where are we leading to? We are leading to what is known as cause and effect relationship, which we will see in the next lecture. Thank you.