River Engineering
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Lecture – 09
Specific Energy, Specific Force and Critical Flow

Welcome all of you for this class on specific energy, specific force and critical flow which is a part of steady flow in a river flow system which we try to understand how energy dissipates, how the mass conservation equation we can use and the linear momentum equations for solving the river flow and mostly we are targeting now the steady flow analysis using specific energy concept that is what we discussed in the last class.

And today we are going to discuss about specific force and the critical flow. That is quite interesting to know how the flow happens in a river.

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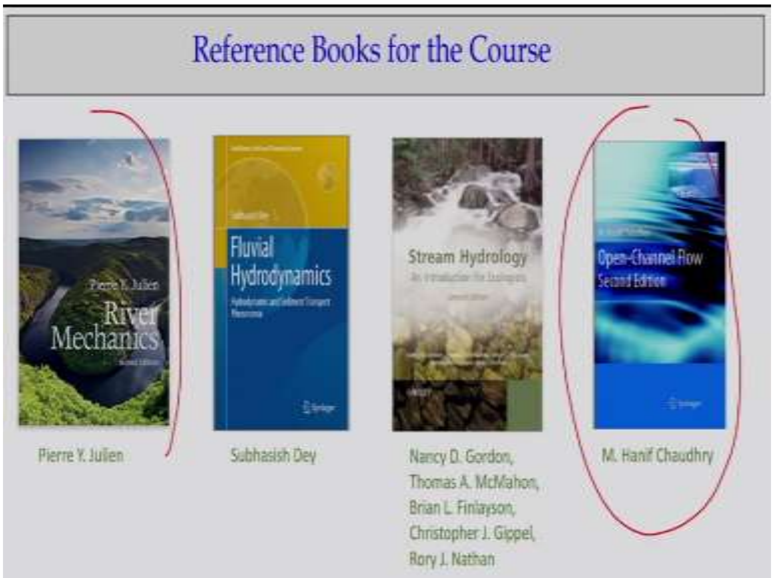

Considering that if you look at the slides what we are following more details of Hanif Chaudhry books okay for the steady flow as well as we are following river mechanics books by P. Y. Julien. These 2 books we have been following for steady flow which is necessary to analyze how the flow behaviors, is it a subcritical, supercritical, critical what the conditions are there and that is things we are going to discuss more as the in the last class what I discussed, beyond that today we will discuss it.

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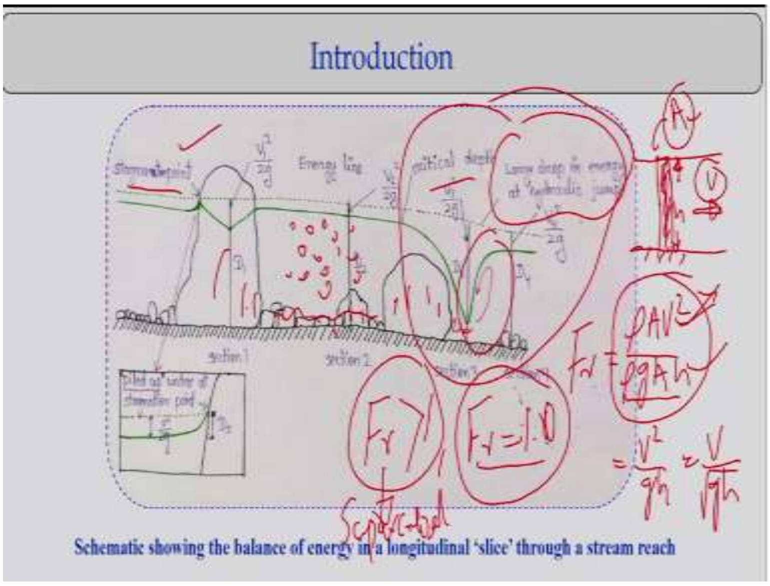

Let us go back to the same slide what I discussed in the last class is that if you look it in river where you have the hilly rivers and you have the stones or the boulders positions like this. So in that case you can as we discussed there will be stagnation points, there will be the critical depth and there will be the hydraulic jump formations and there will be the flow.

Because of this resistance, we can have turbulent structures, the energy dissipations that structures will be there which we can consider how the bed roughness is playing the role for creating the turbulent structure as well as the energy dissipations, but more details we can look at now the critical depth and the hydraulic jumps which is quite related to each other. Before that, you know from the fluid mechanics the flow Froude numbers is the ratio between inertia forces to the gravity force.

If I consider a channel having the flow depth is h and V is the velocity and area of the flow is A, if that is the conditions I have, I can find out the inertia force will be ρ A V^2 is it, ρQ is mass flux multiplied by V is momentum flux which is as equal to inertia forces for a flow when you have h depth is moving with V velocity and area of cross sections A, we can find out the momentum flux will be rho A V square which is the inertia force divided by the gravity force gives Froude Number.

For gravity force, you will have a ρgAh, h is the depth multiplied with area is the volume into the density, then you multiply with acceleration due to the gravity you get the weight that is what is the force due to the gravity. If I have these components if you look at $\frac{V^2}{gh}$ or I can say it $\frac{V}{\sqrt{gh}}$. Please remember it, it is a ratio between two force components, inertia force and the gravity force component which is equivalent.

We are considering the fluid mass is moving with uniform velocity V and it has a flow depth h and you try to find out what is the relationship. So when I talk about the flow Froude number is equal to 1, it indicate the conditions where the gravity force is equal to the inertia force.

When you have the critical flow or flow Froude numbers is equal to 1, but if you have a flow Froude number is more than 1, the inertia forces is larger than the gravity forces. The interpretations, if you look at this flow Froude number is greater than 1 what it indicates is that the inertia forces is larger than the gravity forces and that is the conditions when you have, you have supercritical flow.

Just reverse when you have the flow Froude number lesser than 1, you can interpret it in terms of gravity forces and the inertia forces. Inertia force is nothing else is a momentum flux. As we consider smaller control volume which is having flow depth h and area of this cross sections A and it is moving with a velocity V, very easily you can derive this the ratio between inertia forces by the gravity forces.

That is what is given as definitions about the flow Froude numbers that is what could be discussed in fluid mechanics book, but let me understand when you have the open channel flow or the river flow you can have a stretch where you can have a subcritical flow and there could be a stretch you can have a supercritical flow, and during this period, the transitions of the supercritical to the subcritical like of these cases, you will have a hydraulic jump formations.

You will have hydraulic jump formations that is the reasons you will have a high dissipation of energy. So use the hydraulic jumps as a design for energy dissipations or creating a turbulent structures for the mixings of any fluid we can do or fluid and sediment mixing we do we try to use a hydraulic jump as energy dissipators or we can use hydraulic jump as a mixing of the flow systems.

That is how we use the hydraulic jump concept, but in case of the rivers we try to understand where the hydraulic jumps occurs and what is the amount of energy dissipations because of the hydraulic jump formations. Let us go to the hydraulic jump part.

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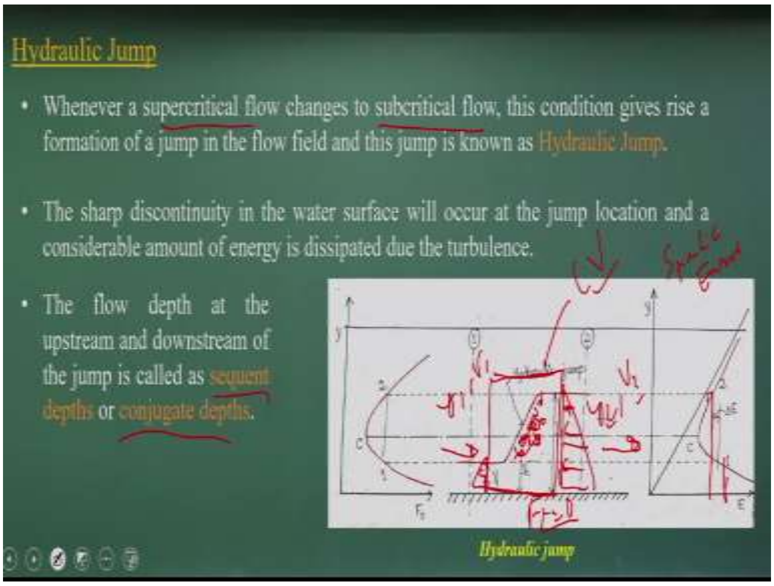

Already we have discussed specific energy, let us come back to the hydraulic jump. The hydraulic jumps it happens when a supercritical flow changes to the subcritical flow. That means very simple way but if you look at in terms of flow Froude numbers the supercritical flow where you have inertia forces is larger than the gravity force. When that changes happen as terms of force that is the mechanism happens from supercritical to subcritical.

That means the supercritical where the velocity is higher the depth is lesser and that is the conditions when it would happen there is a formation of hydraulic jumps. So there is the point what if you look at this, the specific energy curve and you can see that if I consider y1 the energy at this point is this and y2 energy is here.

There are energy losses in this hydraulic jump and there will be noise or the sound production by heat energy and turbulence energy, all they work to dissipate energy in the formations of the hydraulic jump from the supercritical to subcritical. That is what you know it basically we call sequent depth or conjugate depths okay that is the theoretical points. Let me look at this hydraulic jump as a part of specific force concept.

When you consider a specific force concept, I can use a control volume. I can use a control volume like this. So when I use this control volumes, then we try to look at what is the force acting on the control surface. In this case if you look at that in this surface there is no force acting on and at this surface there will be frictional forces and the frictional forces what is acting in the short reaches, we can consider as 0 or it is a negligible, it is not that significant.

So if that is the condition and here if I consider the pressure distribution is hydrostatic pressure distributions and it is quite valid if there is no curvatures of the free surface, we can assume, we can use the hydrostatic pressure distributions for this flow. So I know the pressure distributions, I know the momentum flux what we are coming here y_1V_1, y_2, V_2.

I know this velocity, I know the depth and if I consider unit width I can find out what is the change of the momentum flux happening in these directions that should be equal to the net forces acting along these directions for this control volume. So if you apply that, so what are the forces we have? The momentum flux, the rate of change of momentum flux and the hydrostatic pressure distribution because we have neglected the frictional force component.

And there no significant force will be there in the interface between the water and the air, so that we can neglect. So that way if you look at that if that is the conditions, we have only 2 force components that is the concept is called specific force.

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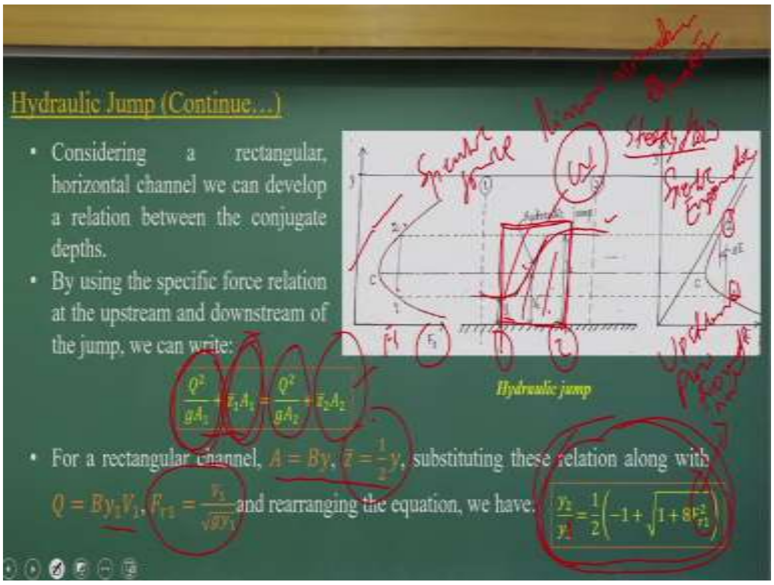

If you look at that way as again I can to scale the control volume and equate the force components. If I look it, this is what momentum flux component, this is the component because of hydrostatic pressure distributions at the section A that what will be equal to the force component per unit weight, force component per unit weight due to the hydrostatic pressure distributions at the two locations.

And that is what the relationship what it gives is nothing else it is a linear momentum equation. When you apply for these control volumes, we will get that and that force component per unit weight if I define it that is called as a specific force. That is we call specific force, that means what we are looking it here, we are looking at for these control volumes we apply.

Where the hydraulic jump happens we apply the control volume, we apply linear momentum equations and as we consider there is no significant frictional force is acting on the short reach, then we can write the force terms and linear momentum equations in terms of two components, one is the linear momentum flux component per unit weight, pressure force component per unit weight.

That is what if I put it that is what is represented as equivalent force component at the E 1 and 2 that is what we are equating, it is nothing else, is a linear momentum equation application for this control volume and per unit weight if you define it that is what will be defined as a specific force. So that is what is indicating it that if you are considering a free surface flow where we have the hydraulic jumps, we can easily find there is no significant other forces.

So, we can use a very simple concept that the specific force at the section 1 and sections 2 should be equal. That is the specific force will be the same, but there is an energy because this hydraulic jump there is energy losses, there is in specific energy diagram 1 and 2 will be in the different locations, but in case of the specific force diagrams it will have in the same specific force what is there at the 1 and that is what equal to 2.

So, in case of the hydraulic jumps, we use the specific force concept or we use the linear momentum equations to derive because we know there are no significant other forces, just to equate it other than the momentum flux and the pressure forces can equate that forces which is a specific force at section 1 and section 2. If you equate it and if you use a simple geometric relation for a rectangular channel like area is equal to By.

And the centroid or the pressure centers if I can find out and if you use these 2 equations and the continuity equations and in terms of the upstream flow Froude numbers, we can find a relationship between y_2 and y_1 which is a function of upstream flow Froude numbers. That is very simple thing, just you use linear momentum equations, mass conservation equations and the definitions of flow Froude numbers, upstream flow Froude numbers at the sections 1.

Before the formations of the hydraulic jumps, using that two equations you can modify it, you can get a relationship between the y_2 and y_1 is upstream, the before hydraulic jump depth and after hydraulic jump depth that is the relationship we will get, which is a functions of upstream flow Froude numbers. These are very simple concepts, only we are using linear momentum equations, mass conservation equations and the definitions of flow Froude numbers to derive these ones.

This is very easy thing and always whenever you solve these problems, any flow problems, you draw for the steady flow problems, we should draw specific force diagram, also specific energy diagram. So you can have these 2 diagrams, one is a specific force diagram, another is specific energy diagram and then you try to interpret it that what happens, pre-hydraulic jump and post-hydraulic jump.

And we use as simple tools of linear momentum equations for this control volume and the mass conservation equations that is what we can derive. It is we have not used any new things except this basic principle of mass conservations, linear momentum equations and energy conservations for the control volume that is what we are doing for a very simple problems like a flow in open channel while the hydraulic jumps occur.

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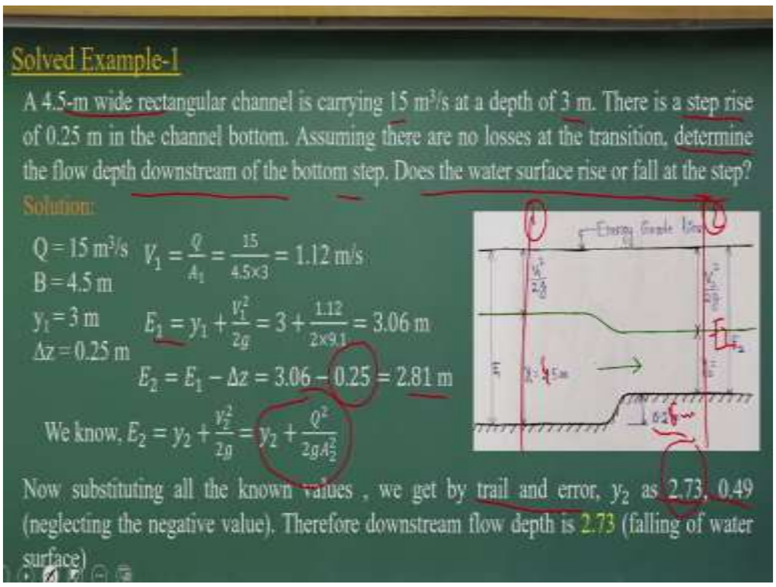

Let us have very simple problems to solve it. A 4.5 meters wide rectangular channel is carrying the discharge is 15 cumec at the depth of 3 m. There is a step rise 0.25 m and if that is the conditions determine the flow depth downstream of the bottom step. Does the water surface rise or fall over the step? So if you look at these problem, it is a very easy problem that there is a rise of the bed.

There is a step rise, rise of the bed because of maybe the sedimentations, may be manmade structures we can have the weir. If that is the condition, we can use the specific energy concept now. We can assume that energy dissipations are not that significant. So I can use the specific energy at the two sections, section 1 and section 2, I can compute this specific energy at the section 1.

The section 2 the specific energy will be lesser than E1 because of this rise height which is 0.25 m and that is the reason we will get this E2 value. So, as I know this specific energy at the E_2 values and from that substitutions, I will get a relationship between y_2 and A_2. So that is the reason there are 2 unknowns and A_2 is a now function of the y_2 because B is known to us.

So if that is the condition, we can use a hit and trial methods because this is a cubic equation level and we can compute it at the hit and trial methods. It will have the 3 values, one of the things will be negative which we do not consider and another is 2.73. We can ask why do you consider 2.73 not the 0.49 that is the same concept what I have used earlier in explanations in the specific energy.

We have discussed is that we should try to know it whether the flow is subcritical and supercritical. If this flow is a supercritical, then the water level rise will be there, in case of the subcritical decrease will be there and in this case as you know the flow of upstream flow Froude numbers you can compute it and you can find out flow is subcritical, so that is the reason the depth will be the falling.

That is what we will have a falling of waters that the conditions what you will get it, just try to solve numerically it is quite easy.

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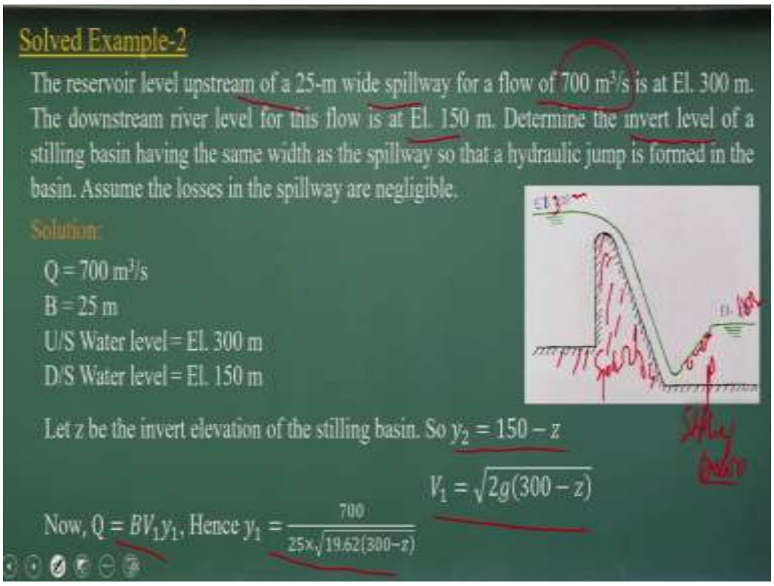

Let us talk about another simple example problem where we are telling it there is a reservoir which is 25 m wide spillway, just look the figures this is spillway, in generally whenever you look at the river, dam projects you can have the spillway. It is having a discharge of 700 cumecs. The elevations is 300 m. At downstream river level the flow at the elevation is 150 m.

Now the question comes what could be the invert level of the stilling basin, this is the stilling basin, basically this structure we created just downstream of a dam structures, the spillway structures so that energy should dissipate it. There should be a hydraulic jump formation for the energy dissipations. So, this is what the stilling basin which convert the flow from supercritical to subcritical and does the energy dissipations in quite significant order in terms of turbulent structures and all.

So if you look at this curve, here it is saying that this z level, the level at this location is unknown to us. Stilling bed level elevation, the invert levels of z is unknown to us. So, if you have 150 m minus z that is the y_2. So you can compute V_1 values because you know the discharge.

Substituting that you have a mass conservation equation and you can compute what will be the y_1 value. So because you know this V_1 and you can compute what will be the y_1 in terms of z.

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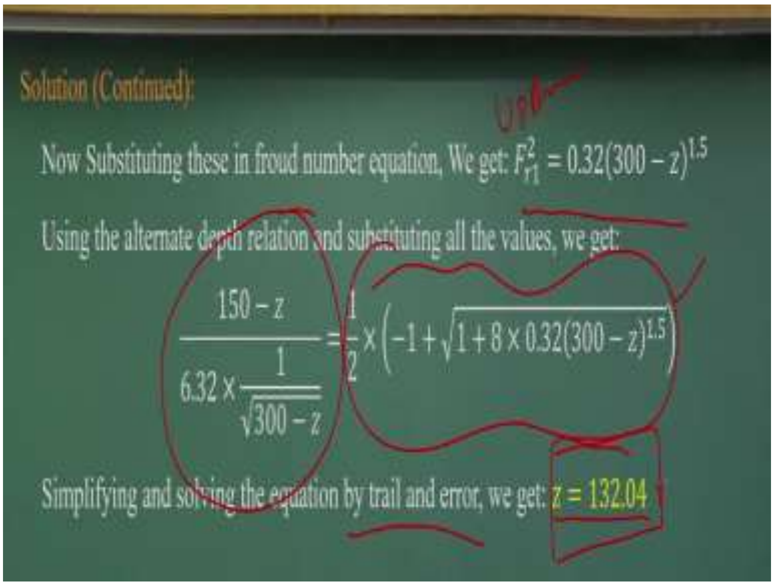

And then we are just substituting the relations. Similar way the upstream flow Froude numbers we can write in terms of the V_1 velocities and we can have a relationship between y_2 by y_1 and that is what will give us a relationship with a flow Froude numbers that is what we are giving it and these equations because these are functions of z, but you can see it is not a linear equation, so you have to do hit and trials.

There are lot of tools nowadays available, you can do a hit and trial methods to find out what could be the z value. So in this case z comes out to be 132.04 meters. So this type of typical examples we can do it for energy dissipations of downstream of a dam or energy dissipations in a river if you know the flow characteristics assuming it is a steady flow. So, that is the discussion we are doing it for.

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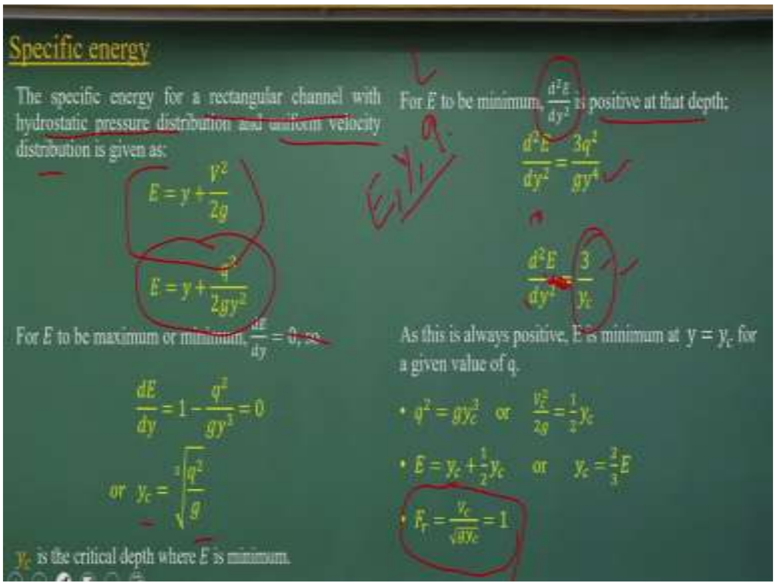

Now if you look it this part as a concept wise I will tell it because this is supposed to be discussed in open channel flow and since we are talking about the river flow it is just a rectangular channel, we just having discussions levels which already we have that. If you are considering a rectangular channel and that is the case and you have a hydrostatic pressure distribution and you consider is a uniform velocity distribution.

So, we can define the specific energy, this is just repeating on that. Now we are just using these functions as I said it earlier in a flow we have 3 cases, one is specific energy, second is the flow depth, q is the flow discharge per unit width. So basically what we are looking it at which locations we will have specific energy its minimum that is very simple thing. We know the E is a function of y and q.

If I consider q is a constant, at what y depth we should have E is minimum. So if you look at this, it is a very easy because it is a quadratic equations, E is a relationship with a quadratic equation in terms of y and q, and q is considered as a constant here, so you can have a basic calculations knowledge is that you can find it $\frac{dE}{dy} = 0$, either function will be maximum or minimum at that point that is what you compute it and find out that the y_c.

Same way if it is to be minimum, then second derivative of E with respect to y should be equal to positive at that, that is what you have equated and find out that always will be positive, and if you substitute all the following you will find that it happens that E is equal to minimum, it happens only when you have flow is critical that is what we did it. In graphically we presented in a specific energy curve, we discussed it.

This is just a mathematical form, to talk that when you draw a specific energy curve with respect to y, you will find a point where you will have a minimum energy because E and y have a cubical function relationship with for a constant discharge. So because of that you can just have arrange it and just do the differences first derivative and second derivative and equate it you will get at the critical flow depth you will have E =E minimum.

And the flow Froude numbers is equal to 1 that is the same concept what we are deriving specific energy curve, the same things we have given it in terms of mathematical justification.

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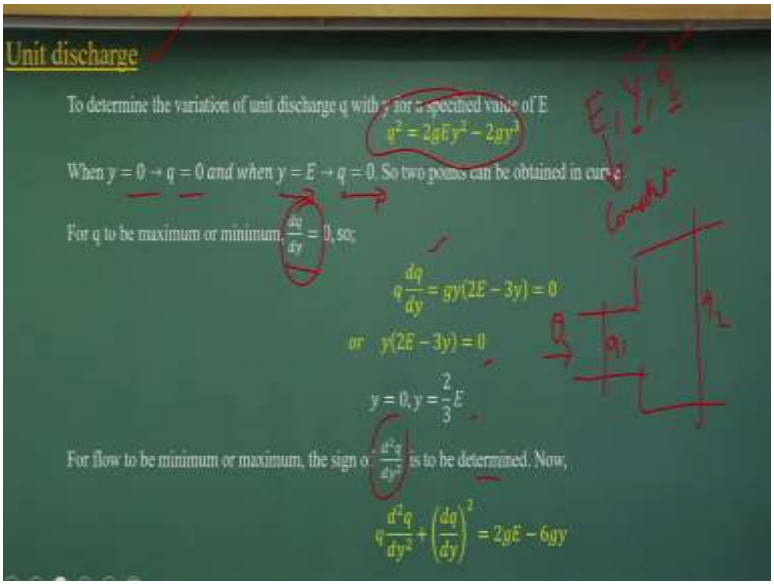

Now let us discuss about as I said it we look at the specific energy y and q. So now if I consider E is a constant. That means I am considering a channel where the energy dissipations are not happening, the energy dissipations is not that significant, there the flow will be varied with variations of y and q, q is discharge per unit width. That means if same amount of capital Q is coming in and the channel is expanding, so your q value at this point will change.

And if we are assuming there is no energy losses and we adopt the basic concept that if the q is varying, what will be the relationship with y, whether we can get a maximum discharge q value, the maximum discharge value for a particular specific energy. That is the concept what we are looking at. The same way, now we are looking the same specific energy equations we have written in terms of the q, the discharge per unit width that is we also defined as unit discharge.

Again we have to say that when you have a depth equal to 0, q = 0, no doubt about that and when you have y = E, the q = 0. So when you all the energy is now the flow depth there is no energy is available for the momentum. That means there is no velocity, so there is no flow okay that is the condition we can say it. These two points can be obtained in the curve, so we know these values, we just looking it where it will be the maximum or minimum.

So we try to look at the first derivative q and the y and the second derivative and if you compute the second derivative just same equations we are just rearranging it and resubstituting it, and if you look at that part if you look at the second derivative part to be determining and to for flow rate is maximum or minimum.

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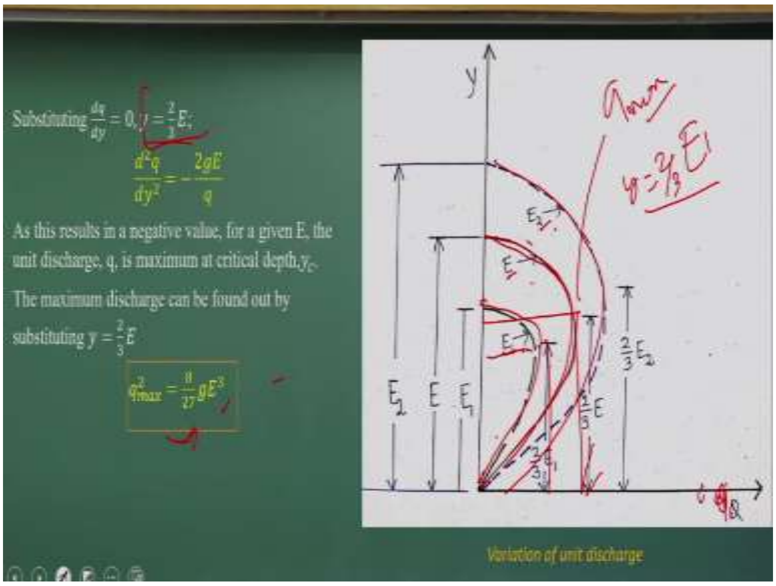

What we interestingly found it that if I draw the q values and the y values we will get typical curve like this for a constant specific energy. For constant specific energy, you can have it like this and that is what we have discussed is that specific energy will be the E1 when you have the q = 0. So there are the two points you will have, we are just looking it from these derivations we can find out q will be the maximum when the y is equal to two-third of E1.

That is from simple derivations you can find out that is what is here. So you can find out, so in this curve in the two-third locations you can have this values and this is what the q maximum, beyond that it is not possible. So that means what is indicating it that if you are taking a channel and you have the specific energy is a constant or the energy dissipations are not happening.

In that case what will happen there will be a critical point or a critical flow depth where you have the q will be the maximum and y will have a relationship like this two-third of specific energy. So simple derivations, if you do it same way you can just substitute it, you will get **it** the q_{max} will be a function of the specific energy that is what, just substitute the values, we can get this relationship which is very easy to do it.

So we can draw this curve and remember it for a constant E how do y and the q the unit discharge variations are there that is the relationship what we have got here.

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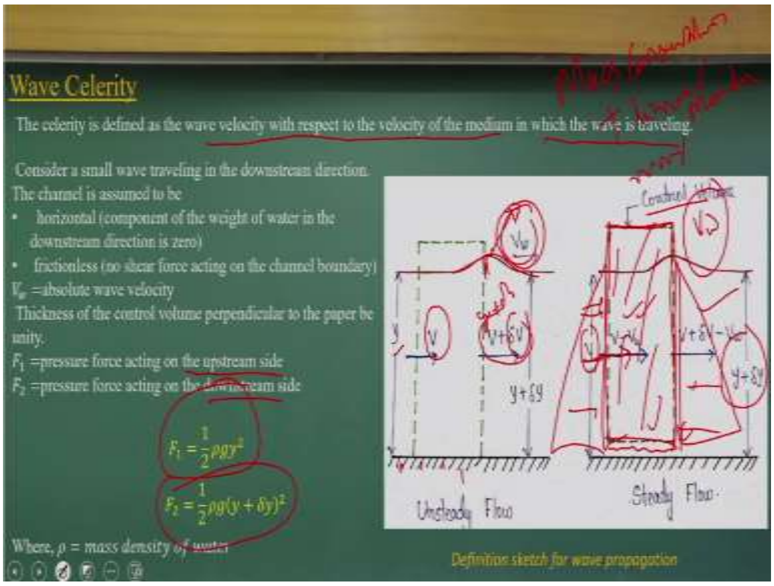

Now let us discuss very interesting concepts is that we know it very basically that critical flow occurs when you have the gravity force is equal to the inertia forces that the concept we know it, but how can you know it that there will be so critical and supercritical flow in a river flow systems. The most often the river specialist say is that you go to the bank of rivers and through stones and create a disturbance over the river water surface.

And try to look at the propagations of that disturbance whether it moves to upstream or the downstream, whether it propagate in both the directions. Based on that propagations, we can identify whether the flow is subcritical or supercritical. We should discuss now the critical flow concept with respect to the disturbance propagations or the river flow is happening immediately there is a wave.

The huge runoff is coming and it creates wave as a disturbance, how does that propagate? Based on that propagations, what is the directions of the propagations we can find out that whether the flow is subcritical or the supercritical or the critical flow that the concept we will discuss. As I said it just throwing a stone in a river we can find out whether flow is a subcritical and supercritical, how it happens.

Let us go for a simple derivation which is more detail you can go through this open channel flow by Hanif Choudhury book, but let us have the concept is that we can define is a celerity that is what called the wave celerity. It is defined as the wave velocity with respect to velocity of medium in which the wave is traveling. Again I am just repeating that we talk about celerity.

Celerity means that wave velocity with respect to the velocity of medium that we create a wave and we try to find out the wave velocity. The velocity of the medium is a flow where the wave is travelling that the velocity we call the celerity. Now if you look at these cases that I have the channel flow and we have created a small disturbance here, small wave. We try to look at what does it happen in this small disturbance that we have created., what it actually happens to that disturbance?

If there is a y depth, suddenly there is an increasing of the Δy depth and you have the cross section like this, so that is what the $y + \Delta y$ depth. If I have a Δy of depth and you have the y depth here and upstream and this and you create a disturbance like this and we talking about the wave celerity which is the V_w. What is the wave velocity with respect to the mediums, the medium is moving with velocity V.

And because of this change in the wave, we have a change of the velocity to $V + \Delta V$ that is an increase in the velocity. Similar way, there is an increase of flow depth from y to $y + \Delta y$. So if this is the conditions, this is unsteady flow conditions. What we can do is we can consider a moving control volume with V_w, the problems become a steady problem.

It is quite easy now, the problem becomes the steady problem neglecting the friction forces we can use these control volumes for mass conservations and the linear momentum equation to get it what would be the V_w value that is the basic idea. Again I have to repeat it that we consider a disturbance or the wave, in a flow where you have a velocity V and because of the disturbance the flow depth has changes from y to $y + \Delta y$, V changes from V to $V + \Delta V$.

And let me consider the V_w is the wave velocity which is moving on this. To consider this unsteady problem, we consider moving control volumes with a V_w velocity and that is what again we change the relative velocity components and we have the y depth, $y + \Delta y$ depth and for this control volume we apply mass conservation and linear momentum equations. To know it what is the relationship between V_w with V and y.

The basic idea is that whether you can establish the relationship with the V_w and y, here again we have considered that the hydrostatic distributions pressure distributions. The pressure distribution is hydrostatic pressure distribution that is what if you look at the force acting on the upstream side and the downstream sides that is what you can just hydrostatic pressure distributions for y depth and y + Δy depth that is what we can compute what is the amount of force is acting as it coming as a pressure force onto this term.

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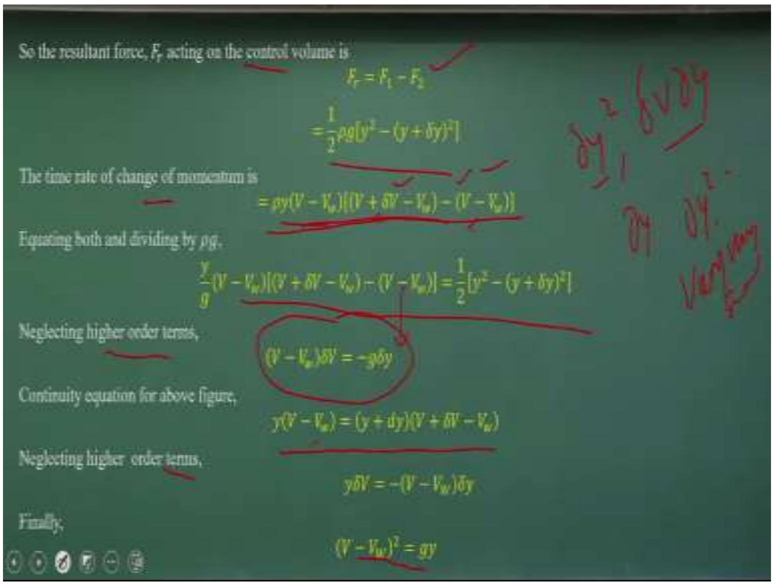

So net force acting on these directions neglecting the friction forces you will get this part that is what will be equal to same concept is the time rate change of the momentum flux. So, we find out the relative momentum flux and the mass flux. If you look at that this is the mass flux and finding out the change in the velocity at upstream and the downstream and that the faces is the momentum flux we are getting.

And if you are just using this force is equal to momentum flux you will get this equation, this simple derivation is here and if I neglecting very higher order terms like I am not interested to know the multiplications of Δy^2 or ΔVΔy because these are higher order terms because Δy is small and Δy^2 will be very, very small okay. So ΔVΔy will be the very, very small.

So similar way if you use neglecting the higher order terms, these equations comes out to be only this with very simple things. It looks very complicated here, but if you are neglecting these higher order terms, we can come back to these ones. Now we just use continuity equation the mass flux in mass flux out for these moving control volumes you can get it.

Again, neglecting the higher order terms and combining these finally you are getting this relationship between the V the velocity y and V_w.

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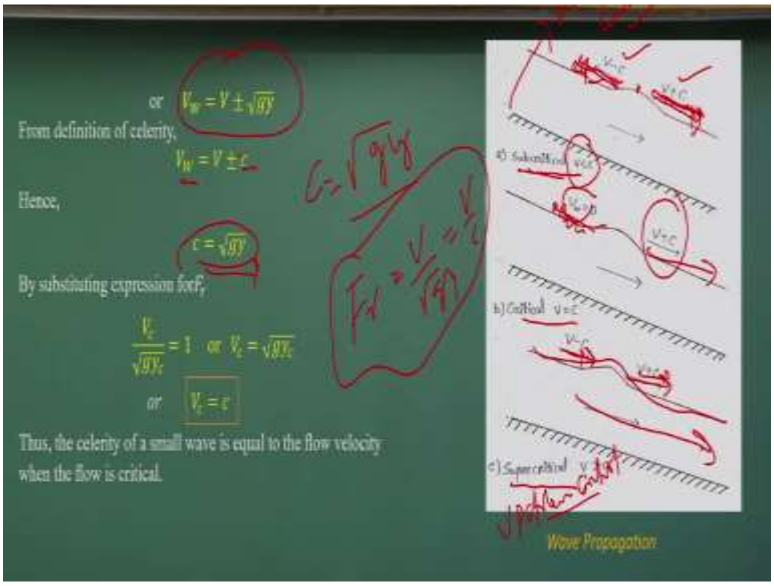

If I rearrange it, I will get $V_w = V \pm \sqrt{gy}$ and c stands for the celerity that is what we are looking with the V value. So, the celerity comes out to, $c = \sqrt{gy}$. So, if you try to understand it that is what we are talking about. So when you have flow Froude numbers $\frac{V}{\sqrt{gy}}$ or $\frac{V}{c}$ you can understand it what it happens to the Froude numbers. It is a very simple thing what I try to show you.

So, if you look in that way the flow Froude numbers is equal to 1 when you have the wave celerity, the speed of the wave is equal to the flow velocity, then we have the conditions when we call this critical flow that is the conditions happen the critical flow in definitions of the flow Froude number wise. Earlier we discussed that flow critical numbers is a ratio between inertia forces by gravity forces.

Here we are talking about the same concept is that flow critical happens it when you create a disturbance and that disturbance speed and the V the flow velocity they are equal each other, then you will have a flow Froude number 1. So, in terms of wave celerity we are defining the flow Froude numbers. Now very interestingly if you look at there will be two parts. So the V_w can have positive values and the negative value depending upon the V and c value.

The cases when you have a flow in with the subcritical V is lesser than the c, you will have both the components positive and V - c will be the negative component. So that means if you do any disturbance, it will propagate both upstream as well as the downstream, just try to look at how the wave propagations. In case of the subcritical flow if you look at this V_w the as the V is lesser than the c, so you will have a negative component, you will have a positive component.

You will have the upstream directions and also downstream, so that is what the subcritical flow happens. That is the reason when you throw a stone you can see the propagations of the wave in both the directions, then it is a subcritical flow, but in a critical flow there will no propagation of upstream directions, so there is no flowing of wave propagations of upstream directions, all what it happens it will propagate the downstream that is the subcritical flow.

The wave will be just propagated the downstream on this. So that is the critical flow conditions will happen when you have this case, but if you look at the supercritical flow both the case will have the positive because as V is greater than c, these two roots V + c or V - c they both will be the positives. That means in case of supercritical flow we will have the wave propagations both are directing to downstream directions.

That means whatever you do the any disturbance, it propagates downstream, but the flow point of view what we call it that when you have the supercritical flow that means it is upstream controls because if you do anything from the upstream is propagated to the downstream. So the supercritical flow we call upstream control system.

But in case of the subcritical flow what it happens if you create any disturbance it moves both upstream as well the downstream. That means it can start from the downstream to the upstream, so that is the reason it is called downstream control because these are the information necessary when you do the river modeling, whether flow is subcritical or the supercritical.

The critical flow happens in very rare conditions, many of the times what the river flow you consider it subcritical turbulent structures. Please try to understand that most of the river flows are classified as subcritical and turbulent flow, so there is no laminar flow, and some of the cases like if there is sudden hydraulic drops are there, the relief is there, guards is there, there could be the formation of hydraulic jump and the supercritical flow.

Those are the conditions we can see that it will be the upstream controls. The disturbance what will propagates only in the downstream directions. So, anything you control in the upstream that is what really affect the flow things that is the way we define it, that is in terms of flow behaviors we can understand it is it a subcritical, supercritical or the critical flow with respect to wave celerity, so we defined as wave celerity. With this, I wish to conclude this lecture today.