River Engineering

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Lecture – 08
Specific Energy, Specific Force and Critical Flow

Welcome all of you for this lectures on river engineering. As we discussed in the last class, hydrodynamic equations like Navier-Stokes equations and Saint-Venanat equations and we have also seen there are the solvers like HEC-RAS models for solving Saint-Venanat equations and continuity equations, but still there are the concepts like specific energy, specific force and the critical flow that what we can use for river flow analysis.

Today, I am going to talk these simple techniques which you may have read it in open channel hydraulics, the same concept we can use for river engineering that is the points what I will highlight today. So we are going to talk about specific energy, specific force and the critical flow. Those are things also we can use for solving this complex river flow problems when we have very, very limited flow geometry data, the hydrodynamics data with certain assumptions.

So today, I will highlight it when you use the concept of critical flow, specific energy and specific force. What are the assumptions we do? Are the assumptions correct for that conditions that is what a river specialist work on that whether that assumptions what you have chosen there in specific energy, specific force and critical flow are valid for that cases, so that is the reasons I am going to teach you specific energy, specific force and the critical flow in the context of river engineering.

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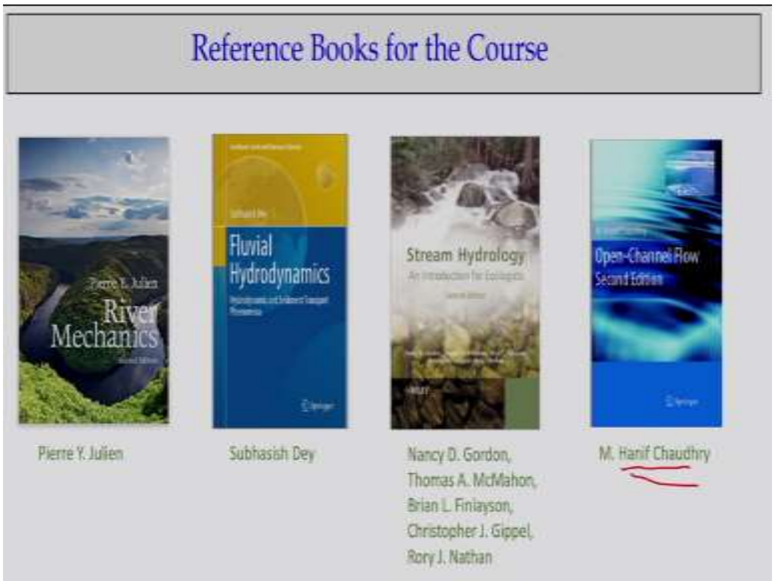

So looking that we can look at the books what we have added is open channel hydraulics with Hanif Chaudhry books along with other books. So we have added new book which we are talking about the critical flow, specific energy and the specific force that the concept we are highlighting through from referring some of the context from open channel flow authored by Hanif Chaudhry.

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So looking that let us come back today what we are going to discuss it with specific energy concept, channel transactions, hydraulic jump, then we can go up to non-rectangular channel looking for the specific energy. So that is what with today's lecture we will cover and next series of lecture also will cover the critical flow concept.

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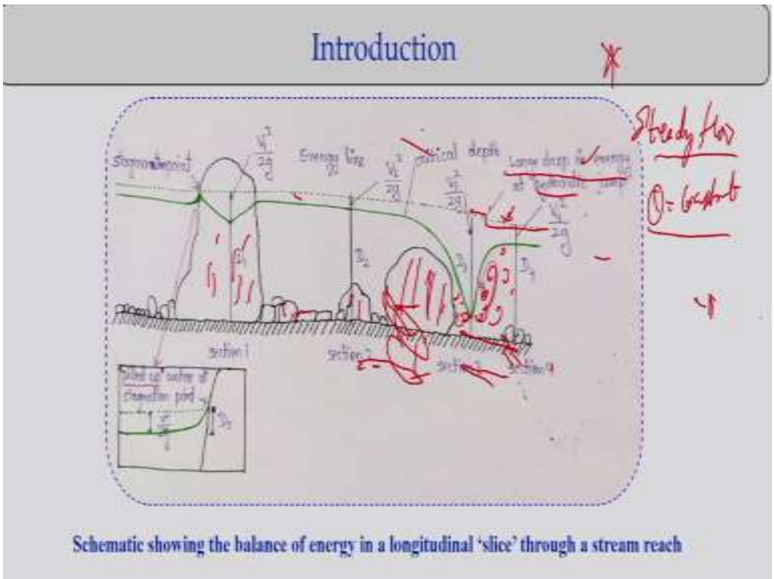

Looking that let me go back to real conditions of the river when you have a river in the hilly regions, you can have series of the boulders. So if you can see this there are the series of boulders and there is sometimes position of the boulders is the higher than the flow depth, it can happen like that. So if you look at that if this is the conditions and if I consider the simple assumption is the steady flow that means my discharge is constant.

So if I am assuming that the flow is a steady flow and considering that the same amount of discharge is happening or we are not bothering about this as variability in a time domain or that is not significant for us, so if that is the conditions if I just try to draw it hypothetically typical behaviors of what could be the velocity variations, what could be the water depth variations?

It could be like this because of presence of these big boulders you can have a stagnation point, the point where the pressure is equal to zero. You can have a drop of the velocity head the $\frac{V_1^2}{2g}$, so that is what the velocity head and also you can look at that I am just plotting this water depth, this is the bed and plus the $\frac{V^2}{2g}$is the velocity head is equal to the energy head. So hydraulic gradient line and energy gradient line.

So if you can look it there are the hydraulic gradient line and the energy gradient lines and as these boulders passes through that again the V_1 velocity flow happens, the stagnations point is here, after that the flow again happens with V_1 velocities, again increases and because of the presence of these boulders there are certain energy losses happening that is what is reflected in the energy gradient line.

The energy gradient line, the slopes indicate because of presence of these boulders how much of energy losses happen that is what we can quantify using Manning's equations. So we can have presence of the boulders, because of that there is energy dissipations that is what is highlighted through the energy lines which typically it is indicating that. If you look at another bigger boulders are here the formations of hydraulic jumps, as you know it there is the change of the flow from subcritical to supercritical.

If you look at this energy dissipation what it happens because of the presence of this smaller boulders that is the energy dissipations happens and that is what we can clearly know it from the slope of this energy lines. Slope of this energy line we can always quantify how much energy dissipated because of presence of the boulders. When you come into this location where the flow changes, it reaches to a critical depth.

And there is a change from supercritical to subcritical, the hydraulic jump formations happen. There will be hydraulic jump formations happen. The vortex formations will happen, the energy dissipation occurs. So, because of that there will be the change of the energy losses here. So if you look at a typical sections like this, there are the stagnation point of formations, there are the energy dissipations because of bed roughness.

And there is a positions if more obstruction is there like this boulders it can have a lot of hydraulic jump formations and because of the hydraulic jump formations you can have a large drop in energy due to hydraulic jump formations that is what happens in open channel flow. It also happens in the rivers. As you look at the typical river cross sections, you can see like this, so the energy loss is happening.

Now the point is coming how we have to quantify it, how do know that there is the change where the hydraulic jumps are going to happen? What type of energy dissipations are going to happen? Those are the things we can analyze for conditions of steady flow. That means we are assuming it the flow is a steady flow and this assumption is quite valid.

When you have a high flow condition or design flow conditions what could be the extreme conditions in terms of energy dissipation, in terms of velocity variation, in terms of the flow depth variations. That is what we try to know from that. That is the reason many of the steady flow analysis also enough for us to solve these problems. So that is what we do a steady flow analysis in terms of specific energy, specific force and critical depth analysis that is what in these figures we are very clear cut showing it.

How the hydrodynamics vary in rivers with having a different type of big boulder, smaller boulders. There could be hydraulic jump formation, there will be stagnation point, there could be energy losses because of bed roughness all you can understand if you sketch this figure where hydraulic gradient line, energy gradient line is present and how we can use the concept of specific energy and specific force for this. Let us go forward to the next slide.

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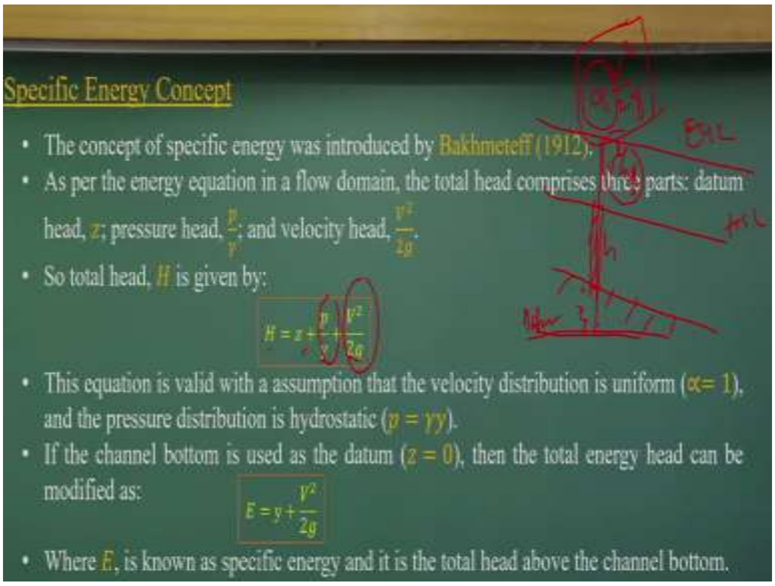

Next slide what I am talking about is we have discussed about 2 basic principles, mass conservations and the linear momentum equations. If you look at these 2 equations beyond that we have energy equations. So we can apply this energy conservation equation in control volumes. So if you can consider a control volume, you can apply the energy equations. As you know it you have the 3 types of energy.

Potential energy, kinetic energy and the energy due to pressure difference that is the pressure force, pressure energy per unit weight. So that is the reasons if you know it very basic fluid mechanics that we use the Bernoulli's equations the same versions we can use for the open channel flow where the total energy per unit weight, we can define as **a** potential energy. This is energy due to the pressure per unit weight, this is the energy per unit weight as a kinetic energy.

So this is the sum of three energy as you know it is a part of Bernoulli's equation, the same total energy available at that particular cross section we can divide into 3 energy components. That means if I have any channels, so I can have this hydraulic gradient line and I can find out the energy gradient line and the h from certain datum and this is z, this is the h and this is $\frac{V^2}{2g}$.

So you can easily have these 3 components from this datum, this potential energy per unit weight that is what is the z value, this is because of the pressure difference, the energy per unit mass because of the pressure and this is the kinetic energy per unit mass that is what is $\frac{V^2}{2g}$. Many of the times also we use α as kinetic energy correction factor.

In that time, we consider the flow is not uniform. In the case of the flow is not uniform, we adopt α value which consider the flow velocity variations in terms of computing kinetic energy per unit weight. So this is very simple concept as you see from the fluid mechanics that is what is idea we have brought it that total energy or total energy per unit weight which is called as total head that is what will have a three compositions.

Potential energy, the flow energy or it is called the energy due to the pressures per unit weight, same way you know the kinetic energy per unit weight. So these things we can easily get it for a particular smaller control volumes as a reach we can consider it and we can compute these 3 energy components which represent our total energy. So connecting the total energy point is called energy gradient lines.

So how the energy variability is going in that is what you look at. So what we do is we make it a bit easier for us that with certain assumptions is that we consider it.

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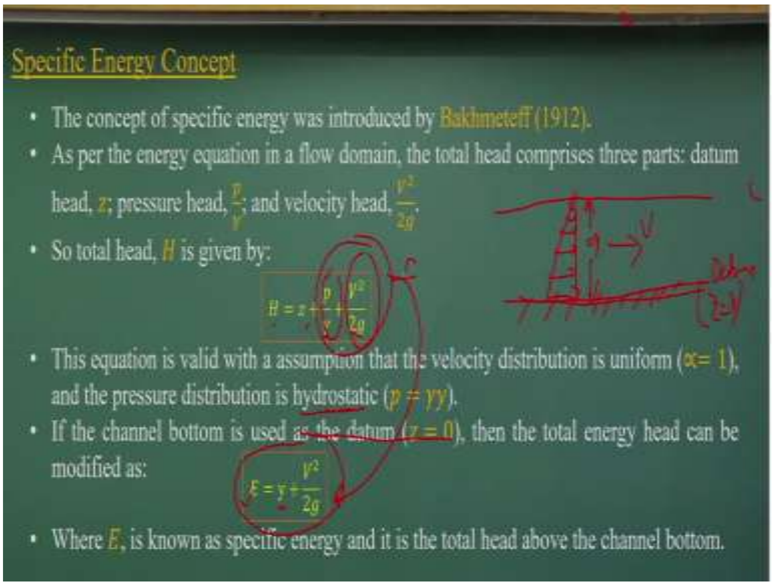

The first simple assumptions where we put it that if I consider is the surface is a horizontal. and the flow is coming. If I consider this horizontal surface as my datum, so z = 0, River bed if I consider as a datum, then z becomes 0. So I have only these two components , the energy components will have 2 components.

Now this flow energy due to the pressure difference and there will be kinetic energy that is what is because of the flow is there. So if you have these two energy concept and if you consider is pressure distribution is hydrostatic, that means you are considering the pressure distribution as flow at the rest, so hydrostatic pressure distributions. The pressure distribution is hydrostatic.

It only varies with the depth, pressure distribution a linear triangle and it varies with the depth, with increase in depth you will have pressure increases, it is a linear function as discussed in any of the fluid mechanics. So if you have a hydrostatic pressure distributions and if I assume it the channel bottom as a datums, this energy equation is very simplified it as this form.

Now the energy that means what I am looking at, we are just getting the energy per unit weight for channels where the pressure distributions is hydrostatic and channel bottom if I consider is as a datum for me, then just that energy we call is specific energy. So it is a simplified version with these assumptions. So we have an energy, specific energy E as a function of flow depth the y, v is the velocity, $\frac{V^2}{2g}$.

So if you look at, the very simple functions what we got it as energy calculations. So many of the times we use the specific energy instead of this equation because these 2 assumptions we can validate for many of the cases. This assumption can be considered for many analysis to know it how the energy dissipations happening. So that is the reasons we compute the specific energy.

That is the assumption, you try to understand it and that is what the specific energy is, total energy per unit weight where the channel bottom is consider is datum and pressure distribution is hydrostatic that is the specific energy, it is a total energy in an open channel flow in a river flow.

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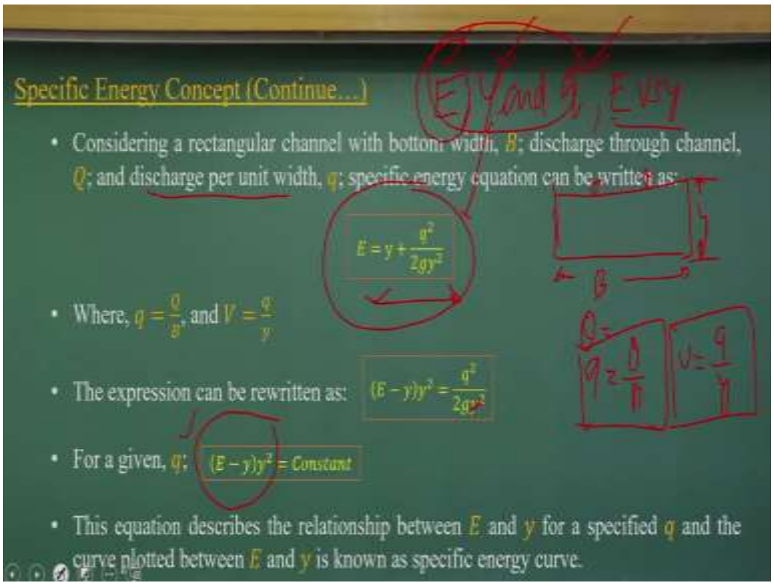

So considering that now if you look at these equations, it is a very simple way, many of the time we can consider the river as a rectangular channel, it has assumptions now. If I consider river is a simple rectangular channel, so having a B width and y is the depth of flow, I can consider channel is a rectangular channel, these are assumptions, it is also quite valid.

If that is the condition, now these equations again we can simplify it that if we have the Q amount of discharge passing through the steady discharge over this, I can compute the q which is the discharge per unit width that means Q divided by B, discharge per unit width q. If I put it that then I have the y as a variables, so I can easily write this velocity v= q/y that is very simple thing, discharge is equal to velocity into area.

Here we have considered this discharge per unit width divided by the depth, this will be the velocity. So that way if I can substitute it, I am getting this equation, again with some assumptions with writing the specific discharge equation in terms of discharge per unit width. If I consider that, my equation comes like this. So in this equation if you look at that I have 3 variables and q.

So I have specific energy, the flow depth, I have the discharge per unit width. So 3 variables I have as established with these 3 equations. Now, I am not going to detail minutely, look at this equation and the derivations which you can follow from Hanif Choudhry books, but let me have a very conceptual understanding that that we have the specific energy y and the q, 3 variables, a relationship of specific energy relationship we got it.

If we got this relationship, now we are looking it, if I make E is a constant, we try to know what could be the relationship between the y and q. Again I am going to repeat it. If I consider E is constant, that means I am considering reach of the river where energy dissipations are not happening that order, so that means energy remains constant at that reach of the river.

If that is the condition, what is the relationship between the y and small q, we can establish it using these equations. Same way, if you try to understand it if q is a constant, that means the river width is not changing, whatever the Q discharge is coming in per unit width, width if it does not change then smaller q, the discharge per unit width does not change.

So, you can have another conditions that one condition is that E constant we look at what could be the relationship between y and the small q, that means this is the cases where energy does not dissipate that much, so we can consider a reach, we can assume it there energy dissipation is not that significant order we can find out a relationship between y and q. Same way if I can assume the Q does not vary.

That means my Q amount of discharge what is coming in is a constant, it is a steady flow and my width of the river is not changing. I am considering a rectangular river and the width of the river does not change. If that is the conditions q also does not change. In that case, I am looking at what could be the relationship between E versus y. So that is the concept and the other derivations and all the things are the basic things, but try to understand it.

There can be two conditions you can have that one is the E constant, energy dissipation is not that significant order, second is that discharge per unit width is a constant and not varying that significant order. If that is the condition, what is the relationship between q and y or in case of q constant what is the relationship between E and y because the flow behaviors happen like that.

So with this relationship, we can understand different type of the flow. So, if you look at that, the first part what we have done is that we assumed this q is constant. So, if you have considered q is a constant, see if you can write it the equations becomes like this, simple way. So we can rewrite this equations when the q is a constant, you can write it like this forms.

So we can get this is a cubic equation and you can get the solutions of the y. So that means if q is a constant, we can get solutions of y, E for this and or we can develop the solution curve for this cubic equation which are very mathematically if you look it is a cubic equation where we can get the behavior of the curves.

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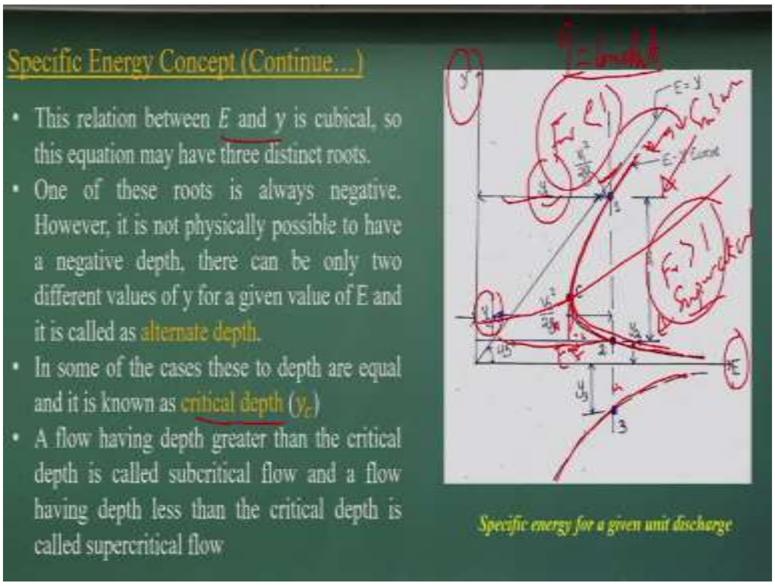

That is what is specific energy curve which discuss more in open channel flow that is what is called specific energy curve that is what give a establishing between now if you try to understand as q is constant, so discharge per unit width does not vary significantly. So if you consider that, then there is a relationship between E and y that is what will happen, it is cubical and nowadays you can solve these equations, it is not big issue.

But let us see what is the behavior of these equations, the solutions behaviors that if I plot E versus y, E is the specific energy, y is the depth, so that is what is called specific energy curve. The equation of E-y will follow this curve, so follow this curve and third one will be negative values which will come here. So there are 3 solutions, y_1, y_2, and y_3 for this q equal to constant one will be negative which we should not consider.

It is not physically possible to have a river flow depth is negative, so we cannot consider that part. So there are the two depths y_1 and y_2 which is called alternative depth. That means what we try to understand it from the specific energy curve. For a constant q, you will have the specific energy variations with respect to y, it vary like this. So in these points if you look at from this specific energy curve, very interestingly there will be a point where E is minimum, beyond that the flow is not possible.

That means for a particular q value, we will have the E minimum, the minimum specific energy, beyond that flow is not there. That minimum specific energy if a flow is happening and that corresponding depth is called critical depth, that corresponding flow depth is called critical depth. That means a critical depth where at that particular discharge per unit width the critical depth occurs the point where specific energy is minimum that is the critical, beyond that this flow is not possible.

The flow cannot be possible for this q value beyond this E minimum value, so we cannot have that, the flow cannot be possible, it can go to the E minimum value of this. This is what we call the critical flow. So we divide the flow into two regions, subcritical and supercritical. So the supercritical is this part and this part will be the subcritical. If you look at this behavior, what it actually happened between the subcritical and supercritical?

No doubt you know it that we can compute the flow Froude numbers in fluid mechanics or open channel flow to know it whether the flow is subcritical or supercritical based on the flow Froude numbers, as flow Froude numbers lesser than 1 that is what will be the subcritical flow and flow Froude numbers greater than 1 will give a supercritical flow that is by definitions you know it, let us understand physically what it actually happens.

In subcritical flow if you look at the flow depth is more, velocity head is less. In supercritical flow, flow depth or the pressure energy is less but kinetic energy is more. Just try to understand it, I know it you all know about flow Froude numbers and you know it by definition what will be subcritical, supercritical, but try to understand it that if the flow is happening, it has two energies now as we have discussed earlier that specific energy representing the two energy.

One is the pressure energy per unit weight another one is the kinetic energy per unit weight. So if you look at these two energy, these two energy in a subcritical and supercritical what is the difference? The difference is that in case of subcritical the pressure energy is quite significant as compared to your kinetic energy, but when you go for supercritical flow the kinetic energy is much higher as compared to the pressure energy.

That is the reason $\frac{V^2}{2g}$ is a higher value. If you look at this graphically, it is a higher value. Here the pressure energy, the kinetic energy lesser. That is the way try to understand it, it is not like flow Froude number is lesser than and greater than one to define in the subcritical, supercritical flow, but try to analyze in terms of energy available to the flow, pressure energy and kinetic energy.

So the pressure energy what we found is quite significant high in case of subcritical flow as compared to the supercritical flow. In supercritical flow the kinetic energy is more dominant as compared to the pressure energy that is what the point we should try to understand instead of just remember it the flow Froude number is lesser than 1 or greater than 1. So that is the point what I want to highlight you.

So whenever you get a problem try to understand it, what are the energy components where using it and how these energies varies with respect to the flow depth is a specific energy curve, very interestingly indicate to us there is minimum points which is the minimum specific energy happens, beyond that flow is not possible that is the point is called the critical flow and that is the flow depth represents the critical depth.

Beyond that we have two divisions we have divided, subcritical and supercritical regions and if you look it in terms of flow Froude numbers we can also find out flow Froude numbers greater than 1 is supercritical, lesser than 1 subcritical and flow Froude number is equal to 1 at the point of the critical depth. So that is the concept we should understand it.

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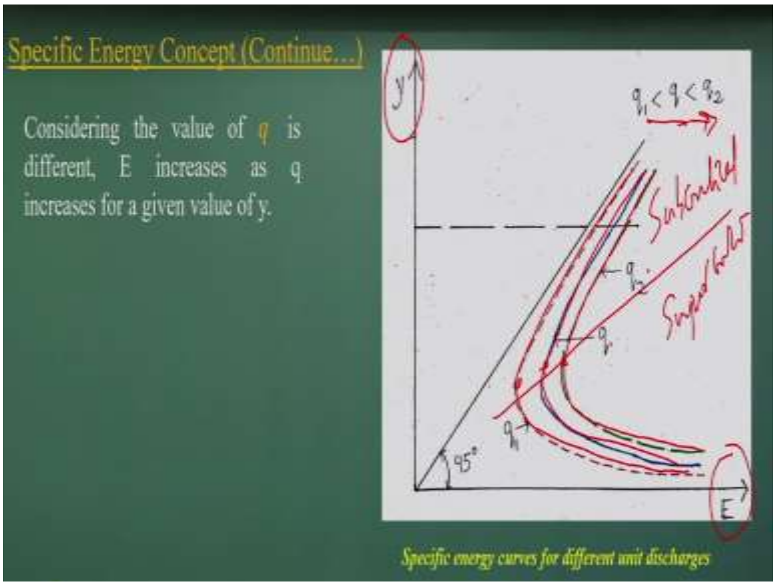

Let us understand the specific energy concept in a river point of view. Let me consider the river width variation are happening. The same discharge of q is coming it with variations there. Because of that as the width increases, the q will decrease, smaller q, the discharge per unit width will be decreasing. So if that is the conditions and that means if I have the conditions the discharge width is varying it from q_1 to q_2.

So that means q_2 is higher than q and q is there, so this is the order of increasing. If that is there and if you draw the specific energy curve that is the same equation if you consider the different q value, the q_2 will follow like this curve, q will follow this curve and q_1 will follow this curve, the relationship in E and y space. In any y space, we can find out how the solutions of this value.

The possibility of the flow can happen different E and y for this constant q values that is what we have drawn it q_2, q, and q_1. Now let me analyze this curve. Very interesting fact they are showing in this curve as I say that these points are reaching point of this critical flow. The point where we have specific energy is minimum that point is shifting. So that means as the q increases that we need to have a higher minimum specific energy minimum to maintain that flow.

But if the q is lesser you can have a minimum or specific energy minimum is a lesser as compared to these ones. So that way, we can understand it that as the river width increases or decreases, how this minimum energy concept also changes the critical flow depth also changes it. The critical flow depth also changes it and the minimum energy concept is changing.

So it is a very simple way to trying you the two energy partitioning between E and y with a different value of the q and how the regions of subcritical and supercritical, that is the subcritical and this is what the supercritical regions with a different q value. We can plot it and we can analyze it how the flow can happen if I have the variations of the q as the q variations shown in these graphs. So try to understand these things for river analysis.

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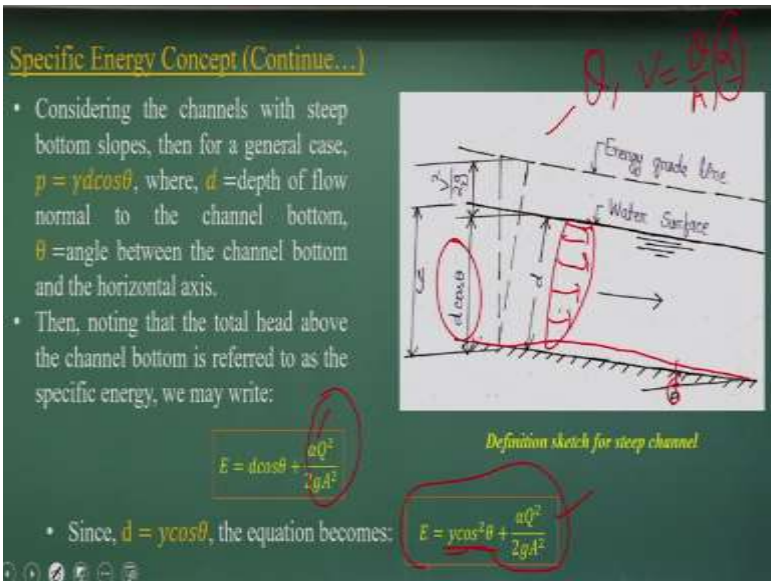

Now if you come into that river having bed slopes, it is not the flat. So in that case that means if the river is having this Ө angle of the slope that is the real condition the river can have a very gentle slope or the mild slope like the Brahmaputra river the slope is in Assam valleys is about 1 by 10,000 or 1 by 8000 slope that is very gentle slope, but if you go to the Himalayans the slope could be an order of 1 by 20.

The slope variability will be there. So the slope has decreasing trend as you go from hilly to the flood plain areas. So looking that things if I consider it there is a slope in the channels in the river and apply the same specific energy concept for q. Now we are writing in terms of Q, so because of that the v = Q by A and also we are considering the α which is the kinetic energy correction factor.

So we have now used the concept of the velocity distributions which is as we discussed it will be logarithmic velocity distributions in an open channel flow. For that we can compute what could be the kinetic energy correction factor. So that is the reason we have introduced α value here. Considering the velocity distributions, we can find out what is the kinetic energy correction factors, the kinetic energy per unit width.

But being a slope here we cannot result the flow depth here and the vertical distance in terms of d cosΘ and in terms of y we can get it $y \cos^2\Theta$ into this part. This is a very simple way, just we have used the combo, an inclined channel or inclined river having a Θ angle okay that is what we can assume. The slope of the river does not change that significantly.

So we can assume it that it is having angle Θ and as it is having angle theta what could be the specific energy equations if I consider the velocity variations in terms of α and Q by A, Q is the discharge, area is the flow area and we can write the specific energy more detailed form of a real for a river conditions where it is having angle Θ, velocity distributions are there as well as we consider it the y in terms of the normal depth, the projections part.

So simple geometry if you look at that, we will get it the specific energy $y \cos^2\Theta$ $\alpha\, Q^2/2gA^2$. Nothing else, it is an equation energy, specific energy equations for a channel with having angle Θ and also considering velocity distributions in terms of α. The α is kinetic energy correction factor. So that is what we can write to get this formula.

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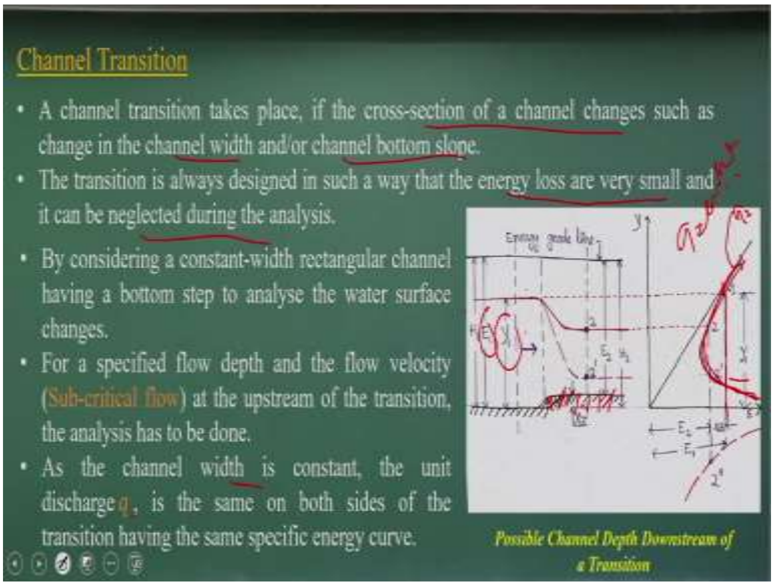

Now let us coming into to a real problem is channel transition. Many of the time as I say that width of the river can increase and decrease. Same way in the river, we can have a rise and the fall because of presence of the boulders you can have increase of bed levels or there could be a natural fall. So there could be a rise and fall, expansion and contraction. The river width can increase and decrease.

Because of that let us use that is what is called channel transitions that means the cross section of the channel changes such as channel width, channel bottom slope. You can have width of the river can change it or channel bottom slope can change it and there could be a rise or there could be fall and that is what is transitions and most of the times this energy losses due to the frictions at these transition locations can be considered too negligible.

It is not that significant order. Also many experiments are conducted that it is not that significant order because if have a gradually transitions happens that is what we can consider. If you have that is the consideration let me try to understand it there is a rise. This rise, the increasing the bed level maybe you consider the boulder presence or be maybe a small weir construction by engineers.

So if you have that conditions, then what will be the flow behaviors that let us understand it. So if that is conditions, there is a flow coming in which is having the flow depth y_1, specific energy as we can consider the channel is horizontal you can find out the specific energy and the total energy is the same $H_1 = E_1$ and you have y as a flow depth and Δz as the rise depth, energy equivalent rise if I consider it.

Now the point is coming if I draw this, that means what will be the specific energy at the section 1 and section 2. If i compare it and if channel width is a constant, discharge is q which is the same for us. That means q is a constant for us and we are drawing the specific energy curve for that. We are considering specific energy curve if you look at that where the q is a constant, we can draw specific energy curve.

That means if section 1 is here, then what it actually we are computing if you look at very simple things.

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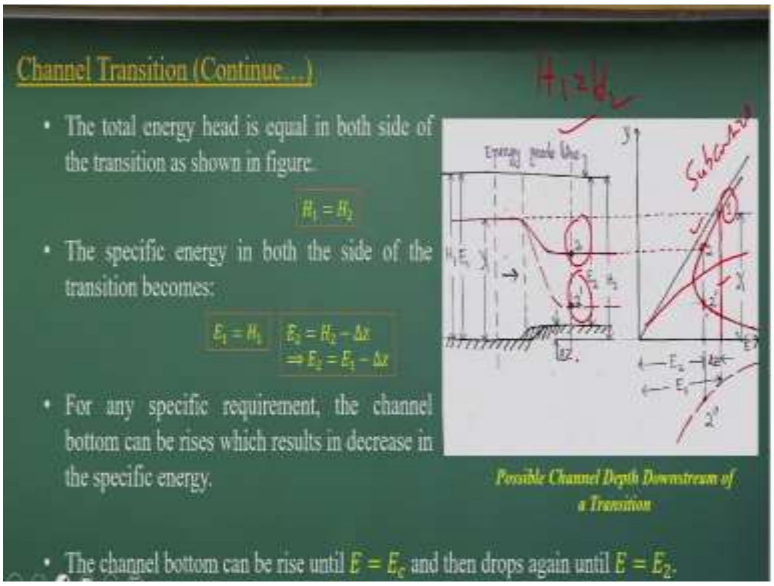

If you look at that as we have considered there is no energy losses, so energy gradient lines can be like this. So $H_1 = H_2$, but the specific energy at this E_2 because energy gradient this flat is here so specific energy will be at the two locations will be E1 - Δz. So if the Δz is a rise, the specific energy drop will be there that is what will be the E2 = E - Δz, very simple thing because of increasing the level.

So the specific energy is going to reduce here as E1 - Δz. If that is the conditions that means in a specific energy diagrams one point lies here, the two solutions will lie either two point here or two dash here, is it correct? So that means the E_1 is energy at the upstream point, E_2 is energy at the rise point and that could have two solutions, one is two is here, two dash is here.

So if you look it that way, whether the solution is two or two dash that is what we have to try to understand here that as the flow is subcritical, so one lies here, the flow is subcritical. The two is a possible solution for us because the flow will try to remain on the same regime, the subcritical regimes. As we discussed earlier that when the flow changes from supercritical to subcritical, there will be some sort of energy dissipations will happen or it will go through the critical flow depth.

It cannot happen suddenly at points with flow will be transit from the flow will change from the subcritical to supercritical. So if you look at this curve if the flow is at the one point, the flow is subcritical, then the solution is two. If the flow is supercritical, the solution could be two dash that is the understanding what we can do it if there is a rise in the river bed that is what we can compute the specific energy and draw the specific energy curve and try to understand where it lies.

From that we can find out what will be the energy at the E_2, whether subcritical or supercritical. In case of the subcritical the solution will be the two part, in case of supercritical the solution will be the 2 dash, so that is the point, the channel bottom now if you try to understand it. If I try to increase the channel bottom, the Δz I am going to increase it.

Because of any obstruction or anything that if I try to increase the channel bottom, I have control over that increasing the channel bottom. The flow is really possible still it reaches the critical depth, that means the Δz if the flow at this point, the Δz I can increase and that what I can increasing up to the point where the critical depth will occur, the critical flow rate, beyond that flow will not be possible. So that is what it happens.

The bottom can rise until E equal to critical E value and that will be the orbits and if that is not conditions then what it happens, we will discuss in the next class. So basically if you look it, try to understand it that if I have a rise we can draw the specific energy curve, you can compute E_1 and E_2 and you try to find out whether the flow what is coming from upstream is it subcritical or the supercritical.

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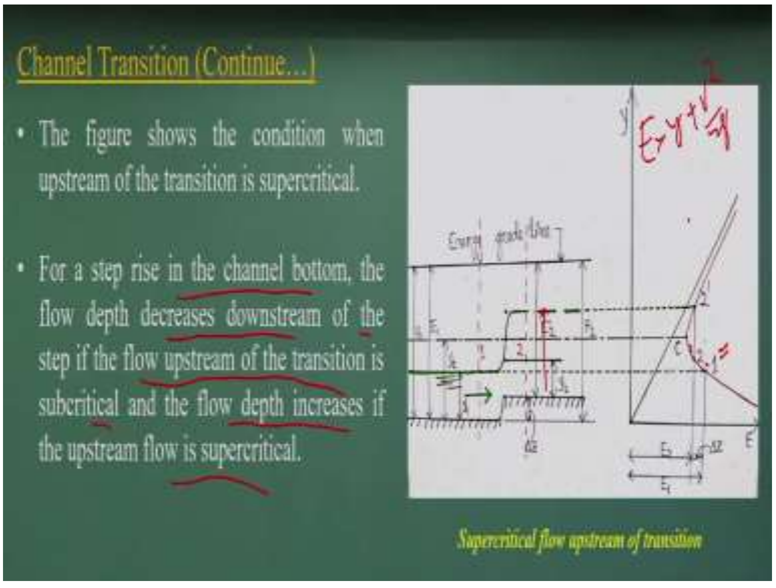

So looking that we can always make it like very interestingly these figures is indicating for us. So if I have this subcritical, the flow conditions will be, look these conditions that when the flow is supercritical at the upstream levels that is where the point one is here, in that case the solutions will be the 2A that is what two points the flow depth is going to increase okay. Flow depth is going to increase and you can find out.

That is what is this curve you can see that the flow depth increasing is there and the velocity decreasing pattern is there as the rise is happening as the flow is supercritical. So basically, let me summarize that if the step rise in the channel bottom okay and the channel flow depth decreases the downstream of this step if the flow upstream transition is subcritical. So if the flow upstream is subcritical reasons, then the flow depth decreases in the downstream.

The reverse happens if the flow is supercritical the flow depth increases, other way around you can talk about also the velocity increase or decrease patterns. As you know it from these diagrams it has a flow depth also the velocity. So specific energy is having the flow depth and the velocity head. So if you try to understand it because of the flow changes from subcritical to supercritical.

In case of the subcritical flow the rise of bed it has effect different as compared to the supercritical flow where the same rise can have a different effect in terms of flow depth increase or the velocity increase that all the interpretations you can do it with this specific energy curve analysis.

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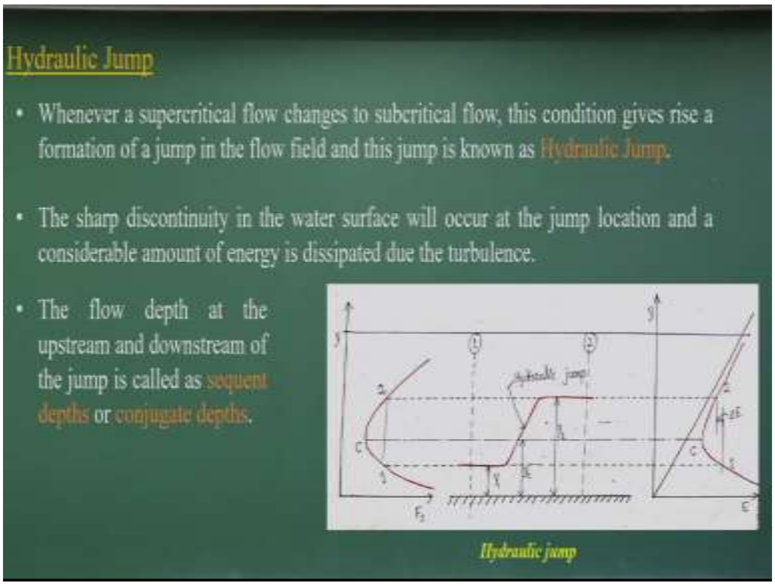

That similar way we can do it if there is a drop that is what we can conduct. With this let us conclude this lecture.