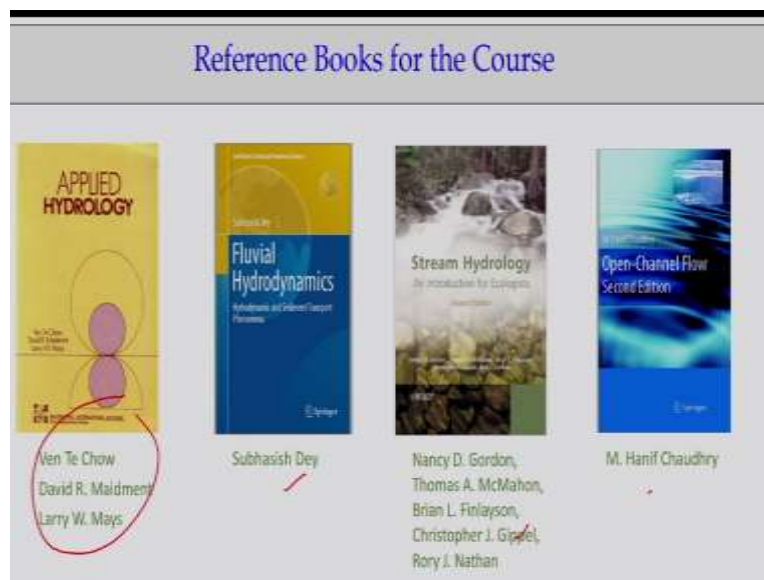


River Engineering
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Lecture – 07
Saint Venant Equation and Solver

Welcome all of you to river engineering course and in the last class we have derived Saint-Venant equations. Today I will derive the next versions of the Saint-Venant equations in terms of discharge and also we will talk about how we can solve these Saint-Venant equations. It is quite interesting lectures today and focusing on Saint-Venant equations and its solver.

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If you look at this book, we have Applied Hydrology by V. T. Chow and D. R. Maidment and Mays book which we are following for the Saint-Venant equations derivation in terms of discharge Q and the flow depth and also we have other books what we have been following it as part of the river engineering course.

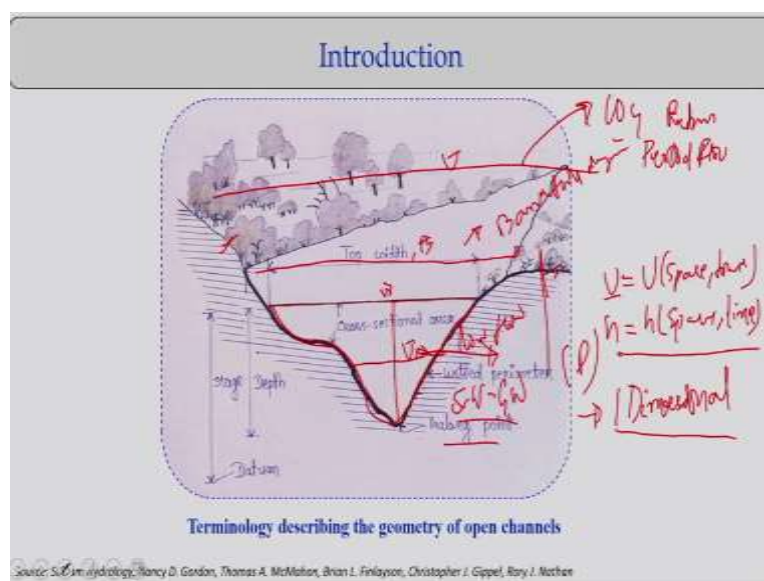
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1. Geometry of Open Channels
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5. Classification of Distributed Routing Models
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7. St. Venant Slover (HECRAS)

Rewrite again Saint-Venant equations and more detail we will discuss in it how we can do a classification for distributed flow routing models. Then, we have a solved examples and we will show demonstrations of HECRAS river models, which is the solver of Saint-Venant equations and the continuity equations.

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Let me go for very basic diagram. If you look at a river, it looks like this, you will have a thalweg line, the deepest depth, where the deepest depth will be there, that is what we call the thalweg line. The regions where we are having the perimeter is wetted that is what the wetted perimeters or symbolically we represent P and you have the flow depth and this is what is showing the top width B of the channel.

If you look at that when you have a river, its flow varies, so you may have the low flow here, this is low flow, you can have the flow at the top width which will indicate is the bank full discharge and sometimes you can have the flow in this level, which includes the river and the floodplain, maybe this is 10 year return period flood. So, we have the flow variabilities. Because the flow varies from low flow, bankfull discharge to 10 years return period discharge when you have the river as well as the floodplain.

So, there will be the left bank, there will be the right bank and when you go for the flood situations if you look at that there will be different roughness will come it because of the floodplain. There is a floodplain, there will be different flow resistance as compared to the main rivers which generally have bed materials like a gravel, sand but when you come to the floodplain regions you can have trees, bushes and floodplain area can have a different land use land cover.

So, if we look at that as the flow varies from the bankfull discharge to 10-year return period flood, when it comes, it also occupies the space in the floodplain as well as in the river and as you go for the low flow the channel is confined within this flow. So, the flow variations are there from low flow to 10-year return period flood or 100-year return period flood. So, you can imagine it that the flow depth variability is there.

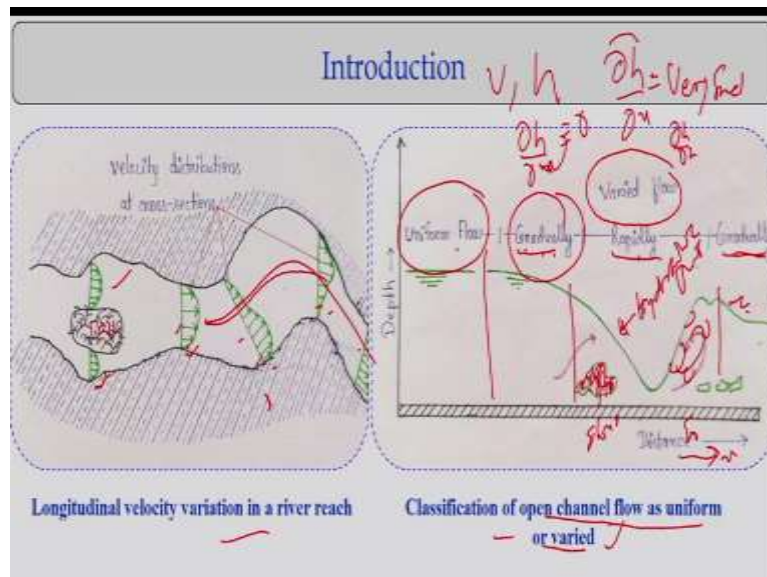
The basic parameters of these in channels like flow resistance, flow depth, the discharge, the velocity varies with time as well as the space. So, that is indicating it, the velocity varies with the space and the time, here I can say that area average velocities and the flow depth can vary with the space and time, but most of the cases we simplified it to the one-dimensional flow. So, we simplified it to one dimensional flow that is what we have done for the Saint-Venant equations.

Please remember that when you have the river flow, not only the floodplain and the river also interact with, there is interaction between the groundwaters and the surface water. There is interaction between surface water and the groundwater. There will be interactions, the water from the river to the ground waters or groundwater to the river, so there are interactions of the lateral flow from the groundwater to the surface water.

So, there are the mechanisms working it with a different flow, different discharge, different flow resistance, mostly we are looking in terms of mass conservation equations, momentum conservation equations, similar way we can look for also energy conservation equation. So, we have 3 basic principal equations what we can derive, but those are all I can say is approximations when you look for real conditions like these figures is indicating for, the real conditions are much more complex.

We simplified it and we tried to write in terms of mass conservation, momentum conservation or the energy conservation equations. So, still we have the assumptions, still we have a lot of simplifications in terms if you look at the complexity of a river systems.

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Now if you look at the next figure which is very interesting figure, it shows that the river is not as straight channels. We are not having the canals which are straight channels with constant slope, in this case slope variability will be there, bed material variability will be there. There are the regions can have the island formations like this, Because of that the velocity distributions changes.

Look at this, these are our major velocity distributions, that is all the velocity distributions along the channel and here the river has the curvatures, the meanders, but in this case there were expansion and contraction sections. There is expansions and the contractions of the river is happening. There are the river bend formations, there is an island, the bar formations are there.

So, if you look at this river is not a simple channel like canal, where you have a constant slope and main bed channels, it has the natural channels, because of that there is expansion, contractions, river curvatures or the river meanders formations will be there. That is a reason if you look at this major velocity distributions, which indicating is a very complex process what is happening in terms if you just look the velocity distribution.

If you look at these figures, which are very interesting figure for us, and we look at to simplify the river flow problems. Like if we look at that I have just a stone hit here and we have a very gentle bed slope S_0 , very gentle bed slopes and assuming that there are some bed perturbations or bed materials like the stone formations are here, what is going to happen?

There will be part where the flow will be uniform flow, means the flow value like velocity and the depth, the variations of the velocity and the depth with respect to the x . That means this is if I define x is longitudinal direction that remains very close to the 0, in that case we can consider the uniform. In other way around the flow depth and the average velocity does not change with the space or does not change with x coordinate directions.

If it is that, then we call uniform flow and this is a very simplified case when you have the uniform flow, but there are the cases because of the stone formations you will have an effect that what will be the gradually and rapidly. Under the varied flow conditions, we will have a gradually variations that means $\frac{\partial h}{\partial x}$ these variations is there, but this quantity is very small.

You can see that slope of water levels variations is there, but variations is not that large, it is very small. If it is that condition, we call gradually varied flow, but if $\frac{\partial h}{\partial x}$ varies considerably high then we call rapidly varied flow. So, these are the reasons we are just here if there could

be a formation of hydraulic jump. There could be a formation of hydraulic jump and there could be formations of Eddies.

So, if you look at this way, if we just conceptualize with simple channel, if you put some obstructions, you can see there are different type of flows are happening or flow reaches we can approximate it for analysis point of view as uniform flow, gradually varied flow or rapidly varied flow. After the rapidly varied flow, there would be a gradually varied flow like free surface change will be there, but that change is not significant.

As the free surface flow depth is a changing it, the similar way the velocity also changes as you know from basic mass conservations equations. So, if you look at that way there is a uniform flow, gradually varied flow, rapidly varied flow. As water resource specialist or a river engineer specialist, we need to look at what type of flow approximations we can do when you try to solve real-life problems.

Otherwise it is not necessary to solve very complex equations for uniform flow or gradually varied flow. So, we need to try to understand the river flow at the reach scales, also at the larger scales like beyond the reach scales what is happening that is what we try to understand it with this longitudinal velocity variations as well as if we look at this the flow classifications in open channel flow.

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Contraction/expansion

Contraction or expansion of the channel causes energy loss through eddy motion. The magnitude of eddy losses is related to the change in velocity head, $\frac{V^2}{2g} = \frac{(Q/A)^2}{2g}$ through length of the channel causing the loss.

Drag force creating these eddy losses is given by

$$F_e = -\rho g A S_e dx$$

Where, S_e is the eddy loss slope

$$S_e = \frac{K_e}{2g} \frac{\partial (Q/A)^2}{\partial x}$$

K_e = non dimensional expansion or contraction coefficient, -ve for expansion of channel
+ve for contraction of channel

Now, we will come back to the basic concept what we have derived in Saint-Venant equation in terms of 3 force components. One is the gravity force, another is the friction force. These two forces are more and that what is equal to mass multiplied by accelerations that is what we have done it, but as we discussed earlier, river is not a constant width channel. There will be expansion and the contractions.

If it is that, let me sketch the simple control volume for you. So, if I look it, so I will have the expansion of the flow, which having Q amount of the flow is coming in that dx distance using the Taylor series concept, I can make it Q variability like this. So, this is my control volume and I can consider small q will be the lateral flow per unit length.

That is what is the flow coming in this direction, this is my control volumes and the distance is dx . If this is control volume, then if you have Q is the amount of volumetric flow coming into the control volume, we can approximate using the Taylor series the flow variability at the surface will be this way. The same way I can write about $\rho\beta VQ$ is a momentum flux.

So, β is considering the velocity distribution momentum corrections factor. So, ρVQ will be the momentum flux and that way I can also write it, ρ if comes out βVQ plus the variation of ρVQ in dx distance. So, more detail derivations you can look in applied hydrology text book. So, that means you can have the control volumes, you can write the mass flux in terms of Q and the momentum flux in terms of $\rho\beta VQ$ that is what you can consider.

If it is that, there expansions are happening. Because of this expansion if you look at that streamline, it will be expanded, these are the streamlines. These are the streamlines that will be expanded and there are the formations of eddies. The eddy formations will be there and because of eddy formations, there will be energy losses. There will be formations of eddies and that is what will be conducting energy losses there.

So, we are not going into seeks up how the eddies formations are happening and how quantifying of energy, we are just quantifying energy as we are following this hypothesis of pipe flow. In a pipe flow if we look at this eddy formations and all in a pipe expansion joint, we considered as minor losses and we try to establish the energy losses in terms of velocity head.

The exactly same way for these channel expansions, we are following this pipe flow minor losses concept. We are considering the energy losses is related to the change of velocity head that is what we are looking. It is proportional to the velocity head through the length of the course what is causing it. That means what we are talking about that we have considered as equivalent hypothesis that as the river is expanding as equivalent to a pipe expansion.

As equivalent to a pipe expansion, you will have a formation of eddies and these eddies if you know in a pipe flow, it is related to the energy losses related to eddies in the pipe flow to the velocity head. The same concept we are using here to quantify the energy losses due to the formation of eddies as the river expands, as the width increases that what will be related to the velocity head.

And if that is considered as the eddy energy loss slope, S_e stands for energy loss slope, this S_e equivalent to a frictional slope. So, as equivalent concept if you consider the force because of this eddy generated the energy losses on these control volumes can be defined as similar to the friction slope concept. That you just try to understand it. Because of this energy losses, what will be the additional stresses which is going to act on the surface, on the bed as well at the surface?

That as equivalent if you will consider it as we have done for the friction slopes, the same way we can consider it and we can compute what will be the drag force because of the eddy formations, because of river expansions, and S_e is a gradient of this that is the reasons we have a partial derivative of the velocity head and is a proportional constant, K_e is a proportional constants which from experiment we can get it what will be the K_e value.

So, whenever you set up river models, it asks what will be the expansion contraction coefficient that is depending upon your rivers, your physical model information you can include it what will be the K_e value for a particular river system. So, that way if you look it there will be non-dimensional expansion and contraction coefficient, it is negative for the channel expansion, positive for the contractions of the channels.

So, basically, the eddy energy losses because of expansion of the river or the contractions that what we consider with these 2 equations as an energy eddy loss slope and the drag force due to the eddy losses.

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Net momentum outflow

Mass inflow rate to the control volume, $- \rho(Q + q dx)$

Corresponding momentum is $\int_{inlet} \rho V dA = - \rho (\beta V Q + \beta \frac{\partial(VQ)}{\partial x} dx)$

The momentum leaving the control volume is $\int_{outlet} \rho V dA = \rho \left(\beta V Q + \frac{\partial(\beta V Q)}{\partial x} dx \right)$

Net outflow of momentum across the control volume $\int_{outlet} \rho V dA - \int_{inlet} \rho V dA = \rho \left[\beta V Q + \frac{\partial(\beta V Q)}{\partial x} dx \right] - \rho \left[\beta V Q + \beta \frac{\partial(VQ)}{\partial x} dx \right]$

$= - \rho \left[\beta \frac{\partial(VQ)}{\partial x} dx \right]$

Where,
 Q = stream inflow
 $q dx$ = lateral inflow
 β = momentum coefficient or Boussinesq coefficient
 $\beta = \frac{1}{V^2 A} \int V^2 dA$



Now if you look at the next one what I want to just revisit you that when you consider that the same control volumes okay. Same control volumes, I have the Q amount of the flow is coming in and there are lateral flow small q amount of the flow is coming okay and this is the x direction. So, definitely this is making the velocity components, this lateral flow also is not along this q directions, there could be a V_x the velocity component along this x direction.

This is the lateral flow and all the things. So, now if we consider this is the control volume where we have the lateral flow what is coming in and this is the control volume in and out, then we can write it mass influx like this way. You can find out this Q and the $q dx$ this is the lateral flow per unit length. So, mass influx you can write it and corresponding momentum will be the $\rho V Q$.

This $\rho V Q$ multiplied by β which is momentum correction factor, to get momentum flux. Here we have used the momentum flux components because of Q discharge which is coming, it is making a V_x component along the x direction. So, here we are considering this V_x , the lateral the velocity x components into this part into $q dx$. So, this is the momentum flux due to the lateral flow which is not coming along this Q direction, it has the inclination.

Because of that, let us consider the V_x amount of momentum flux velocity is coming as equivalent the momentum flux we can get it. So, you know it and we can find out what will be the moment of flux going out from this control volume that is what will be the $\rho V Q$ and its gradient that is what is the Taylor series. If I try to look at net momentum crosses across the

control volumes, just looking at influx and outflux, we are considering the sign conventions of positive and negative.

For outflux is a positive and influx is negative, To understand this conventions please follow Reynolds transport theorems which has discussed in any fluid mechanics lectures, also I have some lectures on fluid mechanics. So, please go through it, why do we have a positive and negative sign for the moment of fluxes. So net momentum fluxes will come out to this. So, now if we look at that moment flux what earlier we derived, now that will be the different part.

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St. Venant Equation

For one-dimensional, unsteady flow in an open channel, the Saint-Venant equation for momentum is given by

$$\frac{\partial Q}{\partial t} + \frac{\partial (\beta Q^2 / A)}{\partial x} + gA \left(\frac{\partial h}{\partial x} + S_f + S_e \right) - \beta q v_x + W_f B = 0$$

Where,

h = water surface elevation = $y + z$

S_f = friction slope

W_f = wind shear factor

S_f, S_e represents rate of energy loss as the flow passes through channel

→ Not in the previous part

So, for more detailed derivations and all you just look at in applied hydrology book, I am just skipping that part of more details, but if we write the momentum equations Saint-Venant equations, you will finally find out like this form. So, if we look at that the additional components we are getting because this expansion contractions point of view, this is because of lateral flow, this is because additional part we call it wind shear factors.

So, many of the times also when you have a heavy wind, big reservoirs are there, we can consider the wind force also acting on that, that part please do the self-readings, which is there in applied hydrology book. So, basically what I am trying to do is that in we have derived the Saint-Venant equations now in terms of Q and h and the A . So, we have derived this equation now the Saint-Venant equation in terms of Q , A and h variables.

And we have included the K_e is the expansion and contractions energy loss gradient, we have considered the moment of flux because of lateral flow, we also consider the force part or the energy what is acting if there will be the wind force acting over the rivers or mostly this is significant when you have big reservoirs. So, there will be certain commentative during the high wind speeds that will be also give a force acting on that.

So we can include this wind shear factor here to estimate that. See if you look at this, again we are getting the Saint-Venant equations in terms of Q , h and A and in terms of partial derivative x and the t , but still it is a nonlinear partial differential equation, still we have that forms. Now, next we are going to discuss that how we can simplify the equations.

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CLASSIFICATION OF DISTRIBUTED ROUTING MODELS

- The terms of the Saint-Venant equation governs the physical processes of the momentum of the flow (local acceleration term, convective acceleration term, pressure force term, gravity force term, friction force term)
- This equation can be simplified to various one dimensional distributed routing models.
- For a subcritical flow condition, the changes in the water level at a point gives rise to the propagation of water back towards upstream.
- This backwater effects can be incorporated into the distributed routing models through the local acceleration, convective acceleration, and pressure terms and for this condition the Lumped routing methods are not suitable.

Now, as I discuss with you that we could look at the Saint-Venant equations is a nonlinear partial differential equation, but solving this nonlinear partial differential equation is not that difficult, but most of the times is difficult to give appropriate river geometry. As I said it in very first that river is quite dynamic, low flow, bankfull discharge, 10-year return period flow systems or 100-year flow return period systems all the floods.

That is the reason we need to give a lot of input data into a river model, even if we solve this Saint-Venant equation. That is the reason and most of the times all elsewhere in the world also getting so detailed data along the rivers, the river depth, the land use land covers, more details it is not possible. So, we try to do a simplification, we try to look the dominance behavior, whether we need to compute as a Saint-Venant equations or we need to do some simplifications on that.

That is what we have to do under this classification of distributed flow routings. So, if you look at this Saint-Venant equations, it has different terms like there are the local acceleration term, convective acceleration term, pressure force terms, gravity force terms, friction force terms and others what we have included expansion-contraction terms, the wind force terms and the momentum because of the lateral flow.

So, if you look at that way which are the significant order, are all these terms are the same magnitudes or some are not significant like maybe some cases the local accelerations may not be significant, more or less the flow is steady, the discharge variability is not there. So we can drop this local acceleration term. Similar way can we drop these convective acceleration term, in which case we can do it?

Can you drop these expansion and contractions part, this is depending on the rivers, if the expansion and contractions is not there, you can drop that part. So the basic idea is to locate the dominance behaviors because each term we have to try to know what is the order of magnitudes, is it a significant? If it is not significant, you can neglect that part. That is the approach we will follow it.

This equation has to be simplified into one dimensional distributed routing as I said that, but many of the causes you will have to try to understand what it actually happens. Like this example what I said it earlier if we have the flow, the flow can change from critical, subcritical or supercritical. More detail about the critical flow we will discuss later on, but you try to understand as you know from basic hydraulics that subcritical and supercritical.

When in subcritical, the flow Froude number is lesser than 1. So in that conditions what it happens is if you do any disturbance, it propagates in both upstream and the downstream. That is what it happens that, that is what is called backwater effect. So that will be the backwater effect, in case of the subcritical flow we will have a backwater effect.

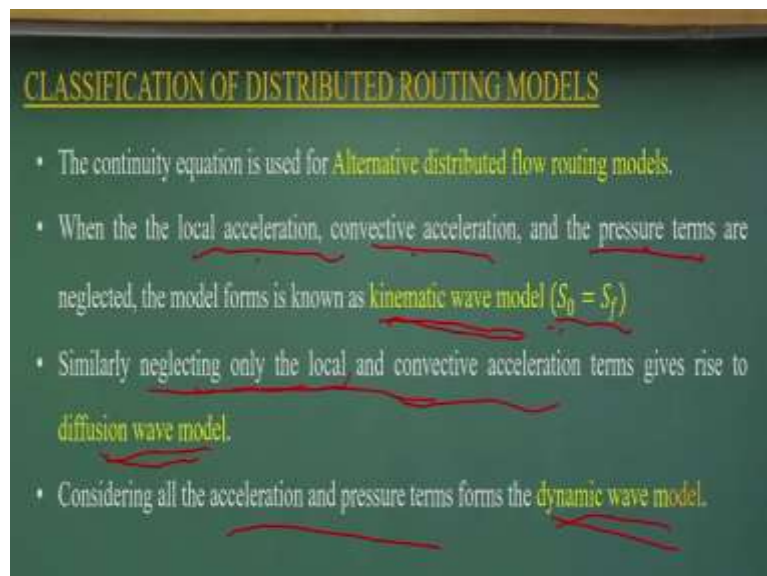
Basically when you look at the Saint-Venant equation, the classifications means we try to look at all these terms in Saint-Venant equations like local acceleration, the convective acceleration, the pressure force terms, gravity force and friction force term which are the dominated component, which are the significant? Like for example, in case of the steady

flow, the local acceleration terms will not be that significant, so we can drop that part, we can consider other 4 terms.

The basic idea comes here to know it that how we will be dropping or we will be approximating some of the terms in the Saint-Venant equation so that we can easily solve it with a limited river geometry data set that is the basic idea. When you have a very limited river geometric data, you can do an order analysis of each terms and find out which are the terms that are the significant and which are the terms that are not significant.

Based on that, you can drop the terms. For example, if you have the subcritical flow as you can understand from basic hydraulics books that the subcritical flow it matters for us to know it whenever you do a disturbance, it affects both upstream and downstream and that is the cases you can have this local acceleration component and convective and pressure terms. For these conditions, the Lumped routing methods will not be suitable, when you have the subcritical flow when you have a backwater effect.

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Now, if you will look at basically what we try to do when you talk about these Saint-Venant equation solutions when you go for the hilly area, the floodplain area, the slopes considerably change, the gravity force components considerably change. So we try to locate the force dominance part. Many of the times we neglected the local accelerations, the convective acceleration and the pressure terms.

Then we have a kinematic wave models where it is very simple model, the bed slope is equal to the friction slope, It is very simplified now if you are considering that local accelerations, convective accelerations and the pressure terms are not that significant. So that means you have only 2 terms, the friction slopes and the bed slope, that is what will comes out to be the Saint-Venant equations approximation that is the approximation when do it we talk about kinematic wave models.

That means $S_0 = S_f$, it is a very simple model and we can solve the cases very easily. Even if in Microsoft Excel level we can develop a river model when we have kinematic wave models $S_0 = S_f$, but there are the cases we can neglect only the local and convective accelerations, not the pressure terms. You can consider neglecting only this local and convective accelerations, then you call diffusion wave model.

And if you consider all the terms then we call the dynamic wave models. So, we have now classify the 3 different approximations. The first approximations we consider the local acceleration, the convective acceleration terms and the pressure terms they are not that significant order. So, if that is the conditions like hilly area where the slope is much larger, so we can approximate the kinematic wave models to do the river routings.

But when you come back to the regions where the pressure variations are there, water depth variations are there, but local acceleration and convective acceleration are not that significant, then you can use diffusion wave model. If you consider as a Saint-Venant equation, then we call the dynamic wave models. So, whenever you do the flow routings using the Saint-Venant equations, we can look at the other whether we were doing it kinematic wave models, diffusion wave model or the dynamic wave model.

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Summary of the Saint-Venant equations

Continuity Equation	{	$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$	Conservative Form
		$V \frac{\partial y}{\partial x} + y \frac{\partial V}{\partial x} + \frac{\partial y}{\partial t} = 0$	Non-Conservative Form
Momentum Equation	{	$\frac{1}{A} \frac{\partial Q}{\partial t} + \frac{1}{A} \frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right) + g \frac{\partial y}{\partial x} - g(S_0 - S_f) = 0$	Conservative Form
		$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} - g(S_0 - S_f) = 0$	Non-Conservative Form

Kinematic Wave
 Diffusion Wave
 Dynamic Wave

Now in equations form if you look at that again coming back to this, we define in 2 terms is a conservative and non-conservative. If you look at this first continuity equations which have conservative form because we in terms of mass flux we do it, so that is the reasons you do not have any approximations when you do the conservative form of equations, but in non-conservative form, you write in terms of velocity and the flow depth.

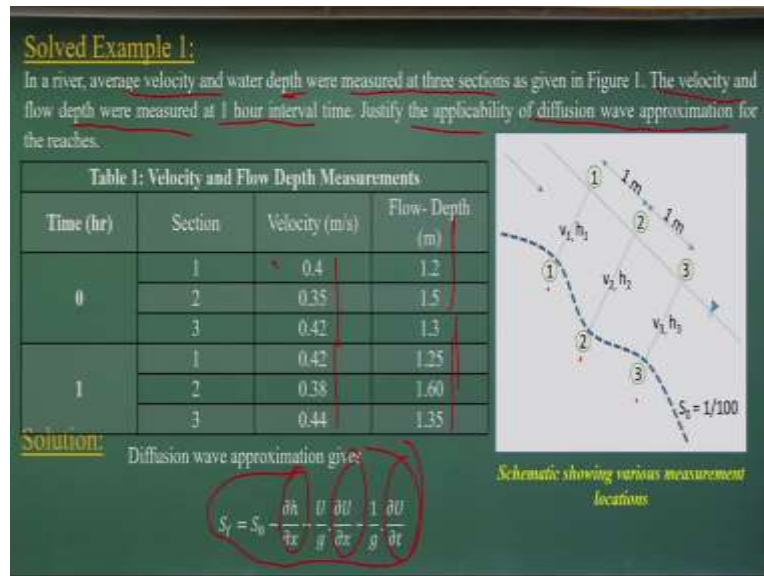
There is not exactly in the mass form there is approximation that the discharge is equal to area into the velocity that approximation which gives a non-conservative form. That is the reasons we put it a non-conservative form because maybe some cases these conservations of mass may not hold good. So, that is the reasons we call it non-conservative forms as we have a conservative equations and non-conservative equations like this.

The same equations only we have put it here and there is no lateral flow. Same way if you look at that, you can have a conservative form which will be in terms of Q and nonconservative form in terms of the V. If we look at the velocity and the flow depth, and there are certain assumptions like if we are considering these two parts. Again just trying to summarize that you will have a kinematic wave approximations.

If you consider these 3 components, then we call the diffusion wave models, if I consider all these components then you will have a dynamic wave. So, if you look at this, these are the equations in Q and A or the Q, V and y, y is the flow depth. All this only 2 equations we can use it to solve the equation, 2 equations for the 2 dependent variables like the discharge and area or the velocity and depth.

If you solve it you can do it, but as I said it, it is not possible to solve these equations mathematically. It is getting the flow geometry data and channel geometry data and the flow resistance all are not easy for natural rivers. So, that is the reason we do certain degree of approximations like kinematic wave models, diffusions wave model or the dynamic wave model.

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Now let us come solutions to example problems. In a river the average velocity and the water depth are measured at the three sections okay. The velocities given $V_1 h_1$, $V_2 h_2$, $V_3 h_3$ and that the velocities and flow depth were measured at 1 hour interval. Justify the applicability of the diffusion wave approximations for the reaches. So, looking at this velocity and depth measurements, we can justify it whether the applicability of diffusion wave approximation is okay for us.

So basically, we try to look at all the terms of this Saint-Venant equation and try to look at whether in this case diffusions wave models if you consider is it okay for not considering the velocity and the flow depth data, this is the measured flow depth data. So, if you look at the velocity data and flow depth data, so if you look at these variations and u look at this, so there is a velocity.

So, if you look at this equation, we try to look at the gradient of h along the x direction, gradient of U in the x directions and the gradient of U in the t direction. So, we try to look at

the velocity, the temporal gradient of that velocity, the partial gradient of velocity and $\frac{\partial h}{\partial x}$.

Then we try to look for each term how the significant order of each terms, if they are in not significant order, then you can approximate it as a diffusion wave approximations.

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Solved Example 1:
In a river, average velocity and water depth were measured at three sections as given in Figure 1. The velocity and flow depth were measured at 1 hour interval time. Justify the applicability of diffusion wave approximation for the reaches.

Solution (Continued):
Approximate the values of gradient of U with respect to x and t respectively as given:

At time $t = 0$ hr,

$$\frac{\partial U}{\partial x} \approx \frac{U_3 - U_1}{\Delta x} = \frac{0.42 - 0.40}{2} = \frac{0.02}{2} = 0.01 \text{ (Using Central Difference Scheme)}$$

At time $t = 1$ hr,

$$\frac{\partial U}{\partial x} \approx \frac{U_3 - U_1}{\Delta x} = \frac{0.44 - 0.42}{2} = \frac{0.02}{2} = 0.01$$

Average $\frac{\partial U}{\partial x} = \frac{0.01 + 0.01}{2} = 0.01 = \frac{1}{100}$

At section 1-1, $\frac{\partial U}{\partial t} \approx \frac{U_2 - U_1}{\Delta t} = \frac{0.42 - 0.40}{1 \text{ (hour)}} = \frac{0.02}{60 \text{ (minutes)}} = 3.33 \times 10^{-4}$

At section 2-2, $\frac{\partial U}{\partial t} \approx \frac{U_2 - U_1}{\Delta t} = \frac{0.44 - 0.42}{1 \text{ (hour)}} = \frac{0.02}{60 \text{ (minutes)}} = 3.33 \times 10^{-4}$

Now, if you look at what we have done it for real river cases, we try to find out the gradient okay, $\frac{\partial U}{\partial x}$ is the gradient that is what we follow the Central Difference Scheme. If I follow it we know the U_3 U_2 by Δx , here is 2 meters, I can compute what will be the gradient. Same way at the time t I can know the gradient. So, I can compute what will be the average $\frac{\partial U}{\partial x}$ at Central Difference Scheme, I can compute that part.

Same way, we can find out what is a temporal gradient of the U values. That is what we can put it and we remember we can put it in terms of minutes, 60 minutes and you can get these values.

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Solved Example 1:

In a river, average velocity and water depth were measured at three sections as given in Figure 1. The velocity and flow depth were measured at 1 hour interval time. Justify the applicability of diffusion wave approximation for the reaches.

Solution (Continued):

$$\text{Average } \frac{\partial U}{\partial t} = 3.33 \times 10^{-4}$$

Similarly, at time $t = 0$ hr, the value of gradient of h with respect to x can be approximated as

$$\frac{\partial h}{\partial x} \approx \frac{h_2 - h_1}{\Delta x} = \frac{1.3 - 1.2}{2} = \frac{0.1}{2} = 0.05$$

At time $t = 1$ hr,

$$\frac{\partial h}{\partial x} \approx \frac{h_3 - h_1}{\Delta x} = \frac{1.35 - 1.25}{2} = \frac{0.1}{2} = 0.05$$

$$\text{Average } \frac{\partial h}{\partial x} = 0.05 = \frac{5}{100} = \frac{1}{20}$$

Now, if I look at $\frac{\partial U}{\partial t}$ and I look at $\frac{\partial h}{\partial x}$ values and that is what I can compute for the $t = 0$ as also the same way we have in the Central Difference Scheme to find out the gradient h with respect to the x that can be approximated as ΔU here as well as it is simple central differencing methods to approximate what will be the gradient. We have got the gradient like 1 by 20.

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Solved Example 1:

In a river, average velocity and water depth were measured at three sections as given in Figure 1. The velocity and flow depth were measured at 1 hour interval time. Justify the applicability of diffusion wave approximation for the reaches.

Solution (Continued):

Now, putting all the values together

$$S_f = S_b - \frac{\partial h}{\partial x} \frac{U}{g} \frac{\partial U}{\partial x} - \frac{1}{g} \frac{\partial U}{\partial t} = \frac{1}{100} - \frac{1}{20} \times \frac{1.5}{100} - \frac{1}{100} \times \frac{2}{60}$$

$x_1 \quad x_2 \quad x_3 \quad x_4$

It can be seen that

$$\begin{aligned} x_4 &\ll x_1 \\ x_4 &\ll x_2 \\ x_4 &\ll x_3 \end{aligned}$$

Thus the diffusion approximation is valid for the analysis of this reach.

So, if you look it this way now for each part, we just substitute this value. So, each part we just substitute gradient in terms of the bed slope and all these terms. If you try to look at that you can easily find out this x_4 is much lesser, the x_4 part is much, much lesser than x_1 . So, if you look at these two terms, these two terms are much, much lesser than these two terms. So, we can always tell it this is not significant as compared to x_1, x_2 .

As compared to this x_1 , x_2 these terms are not that significant. If I am considering that part, then I can assume it because this term is very less, the x_4 terms are very less, we can easily tell that we can use diffusions wave approximations for this case. So, these examples are indicating that you can have a velocity, you can measure the depth, and from that we can find out what type of models we can use for Saint-Venant equations.

You need not to know its total dynamic wave models to use it whether you have to diffusion wave models or you can use the kinematic wave models that example is given it here.

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Solved Example 2:

A series of velocity and flow depth measurements were carried out for a river. From the field measurement data, the relations of flow depth and velocity with longitudinal distance (x) and time period (t) are given below:

$$U = 5x^3 + 3t^2 + 5 \text{ m/s}$$

$$h = 3x^3 t^2 \text{ m}$$

The bed slope (S_0) is found to vary as $S_0 = 5x^2 + 6$. Estimate the friction slope of the river at $t = 1$ hour and $x = 100$ m.

Solution:

$$\frac{\partial U}{\partial x} = 15x^2$$

$$\frac{\partial U}{\partial t} = 6t$$

$$\frac{\partial h}{\partial x} = 9x^2 t^2$$

$$S_0 = 5x^2 + 6$$

If you look at the next one is example where we are talking about a series of velocity and flow depth are measured along a river and the flow the field measurement data show the relations of flow depth and velocity with longitudinal directions and time and as given below the bed slope also has a variability along the longitudinal directions. Estimate the friction slope of the river at time equal to 1 hour and x equals to 100 meters.

This is very theoretical questions, but if you have that velocity variations as a function of x and t and you have the flow variations in x and t and their slope variation in x and t , then we need to compute the friction slope. So here we need to use the Saint-Venant equations. Because the functions are given, we need to compute the partial derivative of velocity and h in space and time domain, in this case only x domain and the time domain.

Once you know it, substituting this value we can solve this problem. So, we compute this $\frac{\partial U}{\partial x}$, $\frac{\partial U}{\partial t}$ and we have the S_0 value. So you just compute $\frac{\partial U}{\partial t}$, $\frac{\partial h}{\partial x}$ you know all these functions, substitute these values.

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Solved Example 2:

A series of velocity and flow depth measurements were carried out for a river. From the field measurement data, the relations of flow depth and velocity with longitudinal distance (x) and time period (t) are given below:

$$U = 5x^2 + 3t^2 + 5 \text{ m/s}$$

$$h = 3x^2 t^2 \text{ m}$$

The bed slope (S_0) is found to vary as $S_0 = 5x^2 + 6$. Estimate the friction slope of the river at $t = 1$ hour and $x = 100$ m.

Solution (Continued):

Now, putting all the values together

$$S_f = S_0 - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} - \frac{1}{g} \frac{\partial U}{\partial t}$$

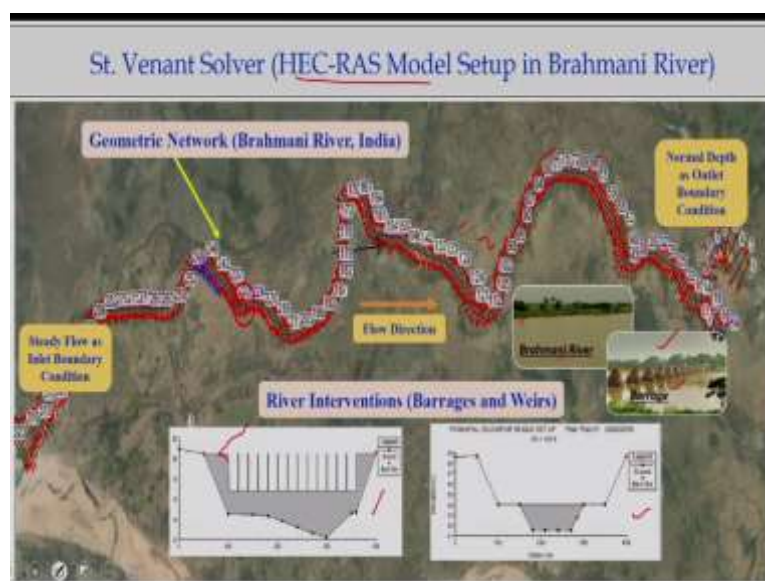
$$= (5x^2 + 6) - (9x^2 t^2) - \left(\frac{5x^2 + 3t^2 + 5}{10} \right) \times 15x^2 - \frac{1}{10} \times (6t)$$

At $x = 100$ m and $t = 1$ hour = 3600 seconds,

$$S_f = -1.824 \times 10^{12}$$

Like this case, substitute these values and as you would be substituting these values into the S_f all these equations and finally you substitute the time, you will get the S_f value. So this is the examples where you may have thousands data and if you can develop the relationship functions between the flow depth, velocity and the bed slope you can find out what could be the friction slope factors. That is the demonstrations we have given.

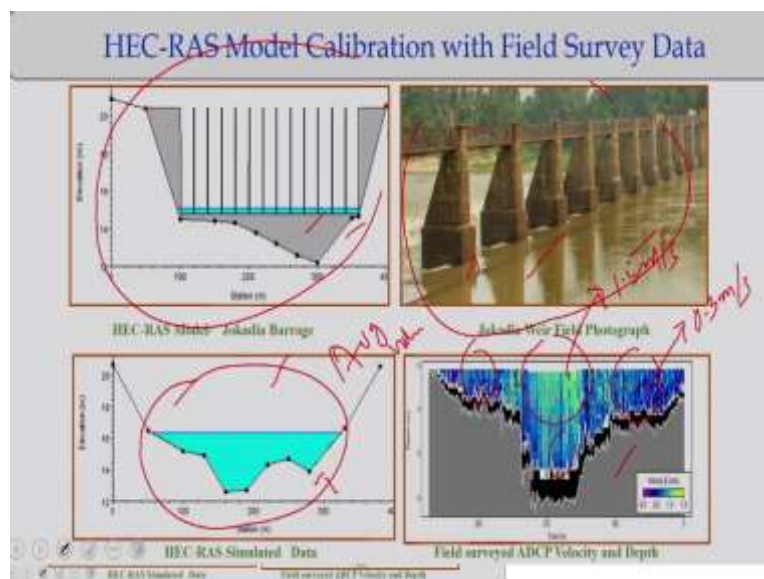
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Before concluding this lecture, I want to just show the few minutes on the models what we have set up is called the HEC-RAS river model which is the more or less the Saint-Venant equation solvers and that is what we have set up for these river stretch. If you look at these all are the cross-section number. So if you look at, very detailed cross section are there and we got all these river cross section, there are the barrages.

There is a river photograph showing if you look at very detailed cross section data are there and each cross section we have showing the flow geometry. The cross section like the depth, the floodplain, the width, the geometry all we have set up for the HEC-RAS models and there are the river intervention structures like this, barrage are there and the channel part is there. So, we could set up in HEC-RAS model which is nothing else, is Saint-Venant equation solvers.

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If you set up this models and see this some field photographs, this is the field re photograph of weir systems if you look it that in model setup it looks like this which is having this structure and you have this bridge weir that is what you can see and you have abutment, all these can show like this. This is the model framework, this is the real condition and this is the model simulations, the flow depth.

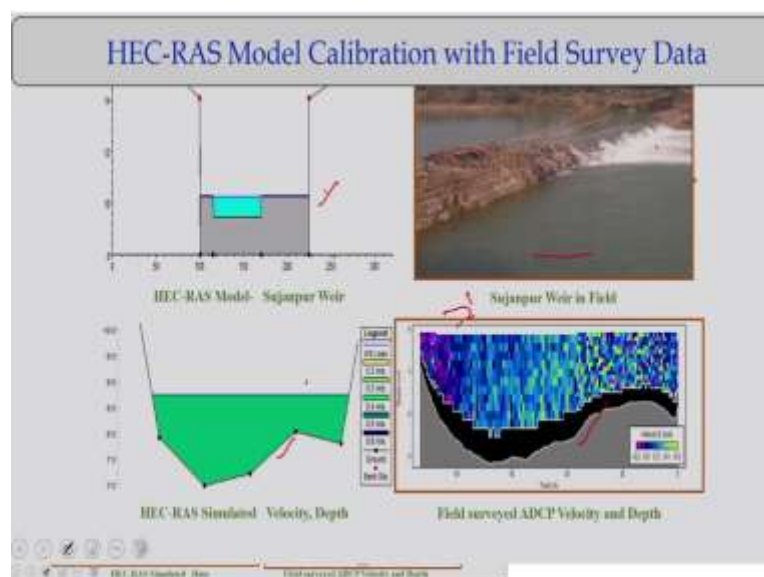
This is what we have done with ADCP survey to show this velocity variations, if you look at this main channel and velocity variations and this is what because in models this is one dimensional model, we get an average velocity and the flow depth. So that is the reasons in

the models we get its average velocity, but in case of ADCP survey we get very detailed velocity distributions. From that, we try to find out what will be the average velocity.

Here if you look at in some reaches velocity goes as high as 1.5 m/s and some cases it can be as low as 0.3 m/s, so some cases it will be 1.5 m/s, but HEC-RAS gives an average velocity condition that is what you try to understand. This is the field data, this is the average velocity condition and this is the field photograph how it looks, but in a modeling framework how it looks.

So, more details about HEC-RAS and all as well we will talk with more details with our case studies, but you try to know it as HEC-RAS as a Saint-Venant solver, we can set up the models.

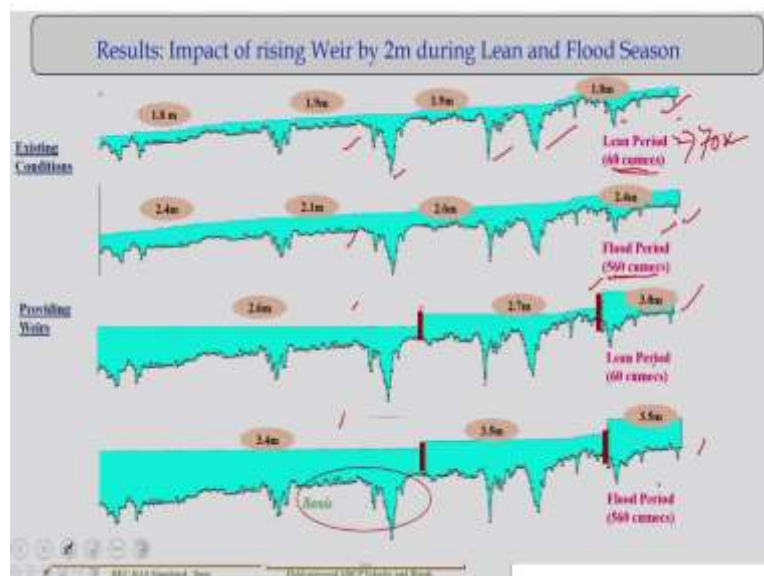
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And we can have case studies like this. Also, we can look at that this is the real conditions where you have the bridging of weir with old weir is the bridging part. The same things are there in HEC-RAS model setups. How we have put it and how this velocity we have measured it and how the velocity has simulated that. So that is the reason, one is a field condition, another is HEC-RAS modeling setup.

Those variations you try to reflect it how does it look in field conditions. When you observe this is the field level photographs and the velocity measurement and this is the one-dimensional HEC-RAS model output which shows that how the things are different when you do the modeling of Saint-Venant equations and we are going to give confidences on that.

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So before leaving it that just time to put it like this. If I look at this river length is considerably around more than 70 kilometers. So if you look at that in the river there will be the longitudinal profiles like that. There are deeper zones and the smaller zones and if have a 60 cumecs of discharge the depth variations will be there, 560 cumecs discharge we will have depth variations like that.

So, the model can simulate more detail of the longitudinal profile, how the velocity varies, how the flow depth varies as we have a 60 cumecs or 560 cumecs discharge. If there will be a barrage in case of the 60 and 560 cumecs, there are 2 two barrages are there, how the flows are happening. So if you look at that, you just look at this figures the HEC-RAS models how it help us to know if I have a multiple reservoir structures.

How the depth is increasing and where the depth is more and less that all details we can understand in mathematical frameworks when you setup HEC-RAS models. Like what we have done it for this is the present case without the weir structures, these are 2 cases with the weir structures, how much flow depth value availability will be there, how much the velocity variations are there, all we can compute it in HEC-RAS models. With this, let me conclude this lecture. Thank you.