River Engineering
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Lecture – 05
Hydrodynamic Principles

Good morning all of you. Let us go for the next lecture on river engineering. As I discussed in last class, the mass conservations in 3-dimensional form, we have derived it and also we have solved few example problems to demonstrate how to use 3-dimensional form of mass conservation equations. Today, we will go beyond mass conservation equations that momentum conservation equations in 3-dimensional forms.

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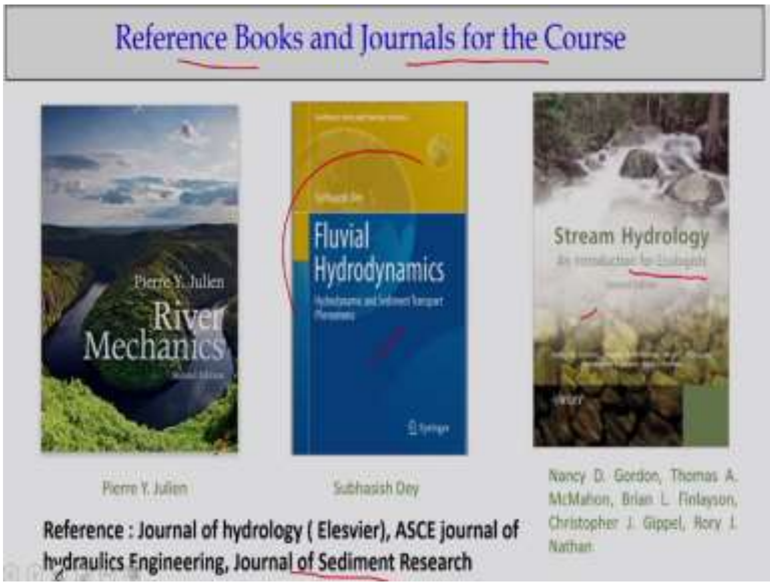

Before going that part, just like the previous class I can see that the same 3 books we are referring. Mostly these books we have been following it now more details to give you the basic fluvial hydrodynamics process what is happening and as soon as we will also follow those other 2 books as we proceed for the next chapters.

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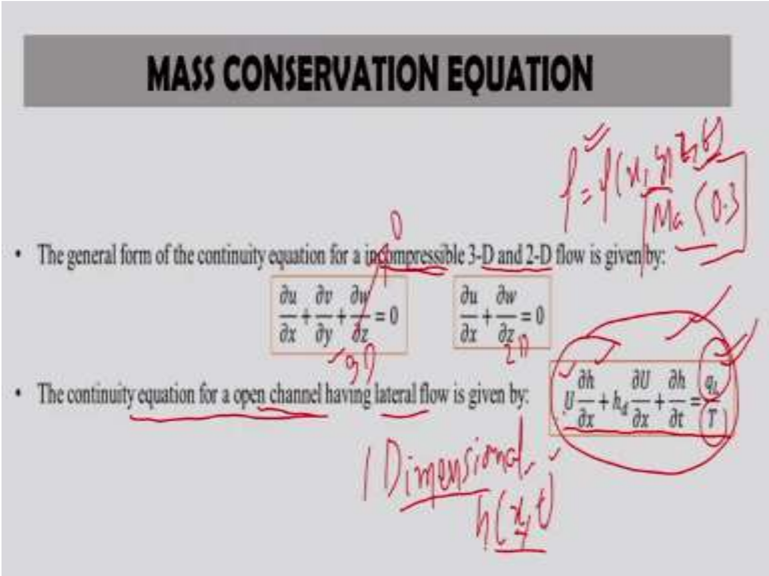

As we go for the next levels that if you look at this mass conservation equation if the flow is incompressible that means density does not change with space and the time. The density remains constant and that is what it happens when the flow mach number is less than 0.3 that is what if you can follow any fluid mechanics book that most of the flow we can consider as an incompressible flow in three dimensional or two dimensional.

When the flow density does not vary significantly in the space and the time domains and that is what we can find out when the flow mach number is less than 0.3. That is the conditions we have three-dimensional form of the continuity equation will come like this and the two-dimensional form of continuity equations will come like this, very simple for it is making this part 0, but we also derived the continuity equations for one dimensional flow.

That means we have only the variability in terms of one x direction and the time. Here h stands for the flow depth, U is the velocity. That is what when you apply this continuity equation for open channel flow with qL is lateral flow, T is the top width, we will get this mass conservation equation, the one-dimensional mass conservation equations in terms of flow depth, in terms of the velocity, in terms of lateral flow.

So please refer back to the continuity equation derivations for open channel flow or the river flow with the lateral flow part and that is what we have partial differential equation what you get in terms of flow depth, the velocity and qL as a function of one dimensional taking the x directions and the time part. This is what for the river flow conservation equation.

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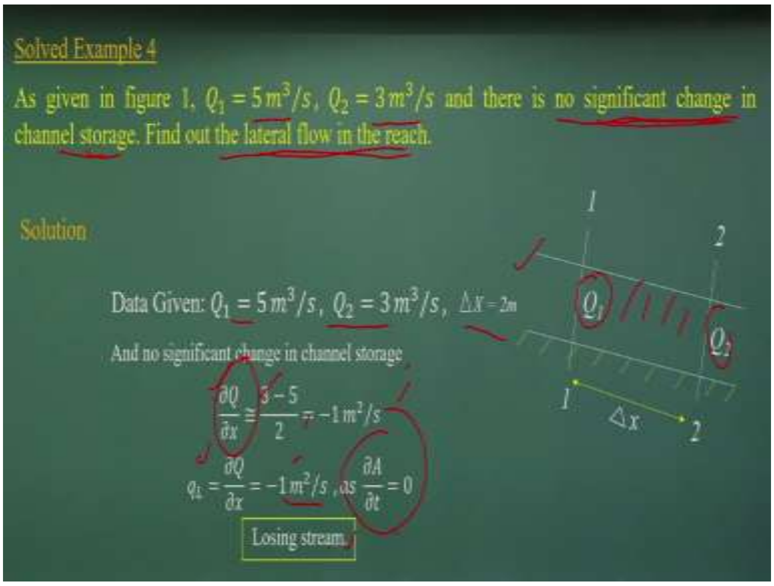

To look at that further to extend that let us have a few examples, the solved numerical examples. Let us say you have a river and the two sections you are measuring Q1 and the Q2 that is what is 5 m^3/s and Q2 is 3 m^3/s assuming that there is no significant change in the channel storage. There is a no significant change in the channel storage, then find out the lateral flow in the reach.

So that is what is happening in real field case. We can measure the velocity at the two different cross section, we can measure the flow area, we can find out the discharge at the two sections. From that if I assume it that there is no significant change in the channel storage or the flood plain storage, we can compute it a simple lateral flow part. Just let us put it in partial differential equations for Q1 is given, Q2 is given.

Δx also given is a 2 m, distance between these two points, if it is that I can find out $\frac{\partial Q}{\partial x}$ the first gradient using this, just Q2 – Q1 by the Δx. I am getting this value and qL will be equal to $\frac{\partial Q}{\partial x}$, -1 m/s that is what it will come as the lateral flow as we are considering this is equal to 0. So, if you look at that, it is a simple balancing equation.

But we have to put it in a partial differential equation with approximations, compute it what will be the lateral flow. As it is negative, it is indicating that it is a losing stream. So, the same way if I have a series of observations of the discharge at the different intervals, we can find out which are the stretches of the river, losing or gaining streams, that we can do it with simple mass conservation equation following it here.

Here we are following the mass conservation equation with the assumption there is no significant change in the channel or the floodplain storage. So, this is a practical problem we generally do it.

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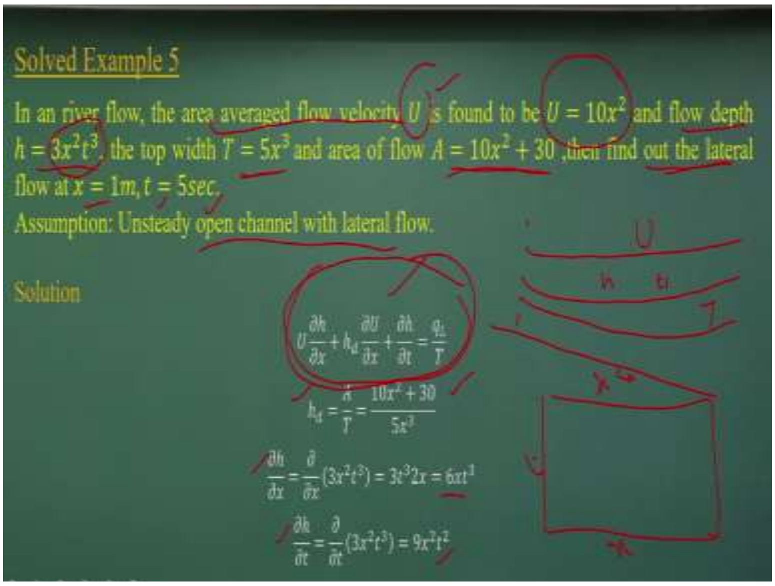

Go for bit unsteady things that example 5 in a river flow, the area average flow velocity is given for us, which is a function of the x i.e. U= $10x^2$. The flow depth is also function of the space x and time t, which is h= $3x^2t^3$. The top width is a function of x i.e. T= $5x^3$. The flow area is a function of x, i.e. A= $10x^2$+30. And then you find out what will be the lateral flow in x and t. Now if we look at that, so that means I have a river and this is the x direction and that is the x direction the U value also varies as $10x^2$.

This is U value, the h also varies with the functions with a different time interval of t1 time interval, this is h value, is varying and that what will vary and the top width T also varies. So, that what I am just representing you that in x and the t space, this average velocity have a function, the flow depth has a function, the top width is having functions and U also have, and flow area is a function of the x.

So, if we look at this problem which looks very difficult, but it is not difficult, it is all these variables, the flow variables of velocity, flow depth, top width, area of the flow we define in terms of x and t functions. Then we try to find out what will be the lateral flow at x = 1 m, t = 5 s. That means this is our governing equation that is what is one dimensional continuity equation of the river having a qL lateral flow and T is the top width.

And if we look at that we can find out the hydraulic depth as A /T that function you can get it. You can have a partial differentiation on $\frac{\partial h}{\partial x}$, you can get it, and here also partial derivative of h with respect to time i.e. $\frac{\partial h}{\partial t}$, you can also get the values. So just to have a partial derivative we are computing it as it is getting h is the function of this. So this is very simple thing, we are just doing a partial derivative with respect to x or the t.

The same way we are looking for the partial derivative of the velocity that is what we have got it. Then, if we substitute all in this one-dimensional continuity equation, then you substitute x value, you can get the qL value at the x equal to this and t equal to this. That means the qL, the lateral exchange of the water it varies with a space as well as the time that is the functional variabilities.

That is what the functional variability of qL in terms of space and the time, at a particular space and time we are getting value, so that is what my point is that. Again, I would draw the space and time and we are getting the q variations at different values. That is the relationship functions of q1, q2, q3 what we are getting as functions of space and time space. That is what we are getting it this value as in this numerical example.

Giving these examples to you give you a confidence that if you have river and you have the flow measurements, if you know this functional relationship with the space domain and the time domain, we can quantify what could be the lateral flow or if you know the lateral flow also we can quantify what is the upstream flow, what is the downstream flow, how the flow variable changes with the top width, the area of the flow and the flow depth and the average velocity.

Those things we can do with just using very basic equations of conservation equations of this, that is the very basic conservation equations, we can apply it, but this is the unsteady flow with a lateral flow. That is what we have derive it and we can also solve these real-life river flow problems adopting these simple mass conservation equations, the continuity equation for us.

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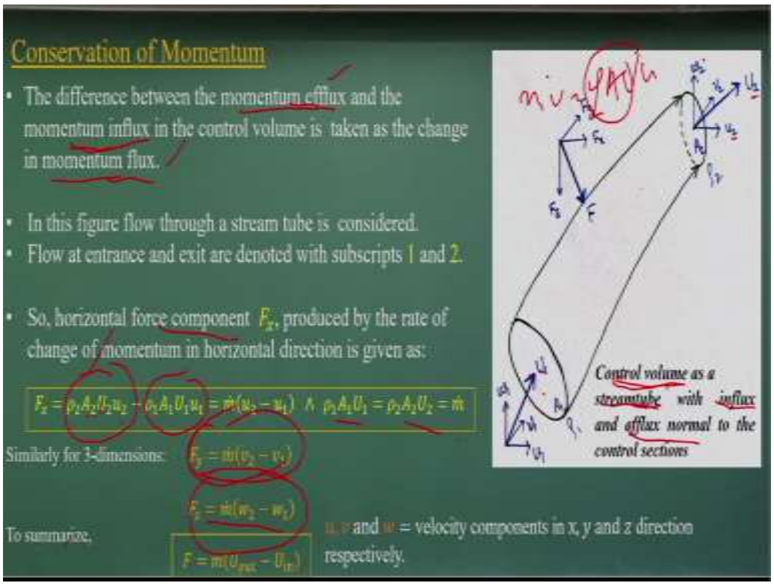

Now, let us go for the next which might already be discussed in fluid mechanics courses. Many of the advanced fluid mechanics courses also are there. To make a completeness of this course, I am just revising this part of the momentum equation. Always I suggest you please refer any advanced fluid mechanics book, F. M. White fluid mechanics or Cengel and Cimbala fluid mechanics textbook, any of advanced fluid mechanics book.

You can understand it how to derive this mass conservation equation and momentum conservation equation, similar way the energy conservation equations. Here, I will introduce you as an introductory level to know about momentum conservation equation at the different platforms. One is the control volume at the stream tube level and other is the control volume at the infinitely small control volume.

Where we try to find out what could be the velocity field, what could be the pressure field that is what the derivations called as Navier-Stokes equations in the fluid mechanics. So, for looking that aspect let me just introduce you this conservation of momentum which generally we use in the river flow with very simplified forms or also in advanced forms like considering it the velocity field, pressure field, acceleration field.

We try to solve these Navier-Stokes equations that is what we try to do it. As introductory as a briefing of these concept, I will introduce to you more detail, again I can suggest you please refer to advanced fluid mechanics book. Let us start about momentum conservation equation. As you know from basic thing that the force vector is equal to mass multiplied by accelerations vector, which is Newton's second law.

Talking about the relationship between any flow continuity systems that force will be the mass multiplied by acceleration or the force will be the rate of change of momentum part. That is the basic Newton's second law, where here we are going to apply the same laws for a control volume like the stream tube. So, this is what my stream tube, this is my control volume where I will apply the same equation, the equation Newton's second law for these control volumes.

As I am considering the stream tube, there is a no flow across from this, there is no flow is coming. So there is no flow passing through this, only flow comes from the surface, goes out from this. That is the reason we consider the stream tube as a control volume for applying linear conservation of momentum. So if I consider a stream tube, then I have a just influx and outflux.

The influx and efflux can have the resultant velocity vector and, U1 and U2 can have a velocity scalar components like u1, v1, and w1 and u2, v2, w2. So you have the velocity vectors here coming into this and there is no variability of the velocity here in a space domain, also no variability of velocity from this. If we consider that part, the net force acting on these control volumes can also result in the three scalar coordinate directs Fx, Fy and Fz.

And this is a space time, x and y, z coordinate systems or Cartesian coordinate systems. Basically, we are talking about the change in the momentum flux, which is the difference between momentum efflux, means momentum going out from this control volume, momentum influx into this control volume. That is what is coming into through the surface and going out from this that difference will be the change in the moment of flux.

That is what we equal to the force. If I am looking for horizontal force component, I am looking at the momentum flux changes happen in x direction. That is what if I substitute it ρA U y1. That means if I look at this part, if I look at this momentum flux, the momentum flux will be the m.V, that is what ρ A V and the velocity so that is the components here ρ multiplied by A2U2 that is what the momentum flux going out from this control volume coming into these control volumes.

That is what will be the force acting in the x direction, where mass influx is given it as we are equating for the mass conservation equation, we are applying for this control volume. Same way, we can write for Fy and Fz. So, we can find out what will be the forces acting in the y directions and the z direction for this control volumes as usual just writing this mass flux.

And change of the velocity the scalar velocity component v2 – v1, w2 – w1 indicating for us what will be the force acting in the x directions, y directions and the z directions. This is a very simple way to apply conservation of momentum equation for the control volume as a stream tube with the influx outflux which is normal to the control surface.

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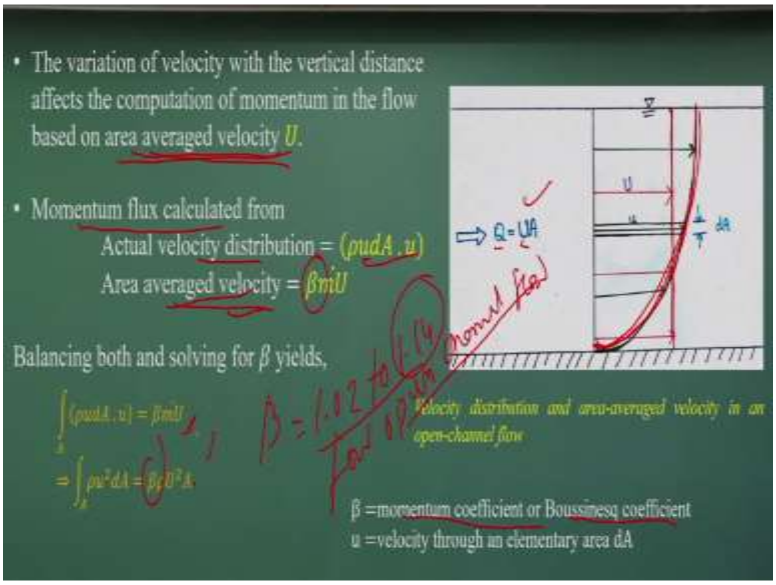

So, that is a very easy thing, but when you come to the river flow, please remember that you will not have uniform velocities when you have a small channel or big rivers like Brahmaputra or smaller river like always you have with the velocity variations, but most of the often we measure the average velocity, so we find out area average velocity. We can measure the area average velocities.

If we are measuring the area average velocity now like how can I compute it, how can we can estimate it, what will be the momentum flux if I take it the variability, the variations of the velocities in an open channel flow from knowing this area average velocity part. So that is what we do that if I have been considering the velocity distributions, which follows maybe a logarithmic velocity distribution and where assuming it is the average velocity U.

Area average velocity U multiplied by area A will give the discharge. If I am computing the momentum flux based on this capital U and the momentum flux based on the small u the velocity variations if I compute it, this is actual velocity distributions, momentum flux, this is area average momentum. So, we have a correction factor. This is what we call momentum coefficient or Boussinesq coefficient.

That is what is the correction factor we adopted for average velocity if I compute the momentum flux, what will be the correction factor instead of taking actual velocity distributions that is what we are equating to here to compute the β that is the momentum coefficient and these beta values varies from 1.02 to 1.14 for open channel flow by doing like this.

That means if we are computing the area average velocities and using $\rho U^2 A$ you are computing the momentum flux, that momentum flux will not be actual momentum flux in open channel flow. You need a correction factor which is more than one value. That means we are underpredicting the momentum flux when we consider capital U, area average velocity in momentum flux correction.

That value increases 1.02, goes to the 1, that means some of the river flow it can go to the 1, some of the river it can go to excess of 15%. So, that is quite large number of momentum flux just to have considering the velocity variations, if you consider it the momentum flux corrections can vary from 1.0 to 1.14 for the open channel flow that is what indicates it.

We should know accurately the velocity distributions to know what is the momentum flux coming into the control volume or going out of the control volume. If we are following average concept, we can get the value, which is under predicted momentum flux as compared to the actual variations of the velocity in an open channel flow.

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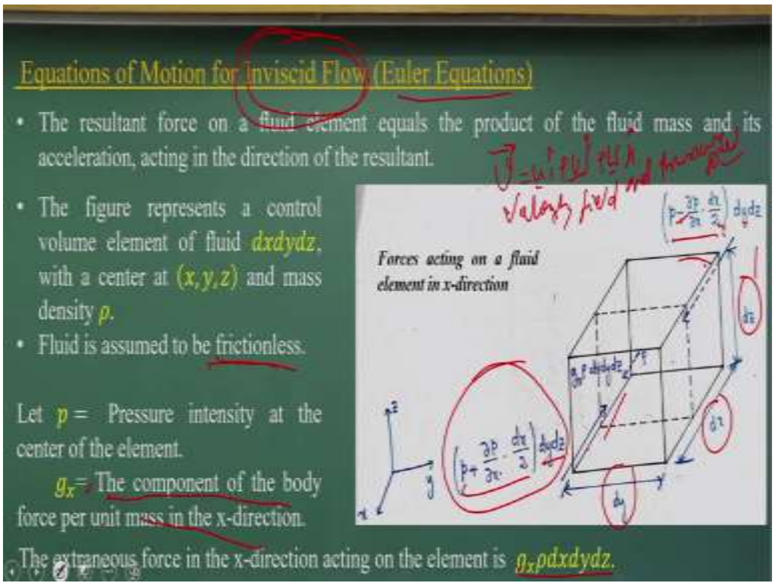

Now, let us come write the Euler equation which is there in any fluid mechanics books, but I have just to summarize you with that. Now, we are considering a infinitely small control volumes and also we are considering that the velocity factor which is have a scalar component and these all are having u, v, w the scalar component parts having the velocity field. They are having variability in x, y and z, the space domain as well the time domain.

If I am considering the velocity field and the pressure field, how I can apply this mass conservation equation and momentum conservation equation? So, earlier we have used the velocity field to derive this mass conservation equation, for momentum conservation equation we have to apply velocity field as well as the pressure field. That is what we will discuss here.

First is very simple way that we are writing this momentum conservation equation, linear momentum conservation equations for inviscid flow, there is not significant viscous effects were the regions where the frictional resistance that is not that significant. The regions where the viscous is not dominated those are the regions we can apply this Euler equation. Where the viscous does not dominate that much we can have the basic equations which is called is Euler equation.

Now, let us come back to how I am deriving that. Again, I am considering a small control volume as you look it that is having a dimension of dx, dz and dy in x, y, z coordinates systems. If I consider p is the pressure at this plane which is just a centroid plane and how the pressure is varying at the dx/2 distance forward or the backward, from the Taylor series I can find out the presser variations like this, multiplied with the area I get the force due to this pressure.

So, I know this pressure, I know this area of this thermal surface is dy by dz that means I know the force. Same way what is the force is acting on this, same way what is the force is acting on this, here the force directions are given. So, we can find out the net force is acting on this control volume in the x direction. So, we can have the force component due to the gravity that is what will be the g_x component of the body forces that is the gravity forces per unit mass in the x direction.

So this control volume having the gravity forces but there is no friction, this is frictionless, there is no shear stress component here, only the gravity force components are there and the pressure force components that is what is the gravity force, which is acting in the x directions in this control volume.

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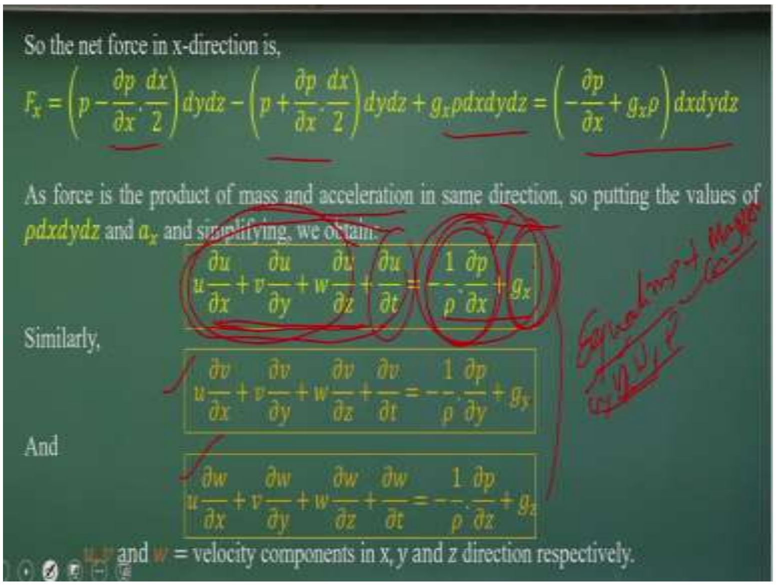

If I just equate it the sum of the forces acting in Fx direction, this is the force acting as you can see that force directions and the g_x component is gravity force. So net force is I am having this that net force will be equal to mass multiplied by acceleration in the same coordinate directions. So, if you have that part that is what you can apply, this is the acceleration part and this is the force per unit mass okay unit width.

And this acceleration part is having 2 components, local accelerations and the convective accelerations in the u directions is equal to 2 components, one is the net force because of the pressure variations in x direction and this is the force due to the gravity due to the weight of this fluid element, weight of this fluid control volume in the x direction. So, this is acceleration component, this is the force per unit mass, that is also the accelerations component.

This is because of pressure difference what you have in the x direction, because of that what is the net force is acting it this is the gravity force component. This is the acceleration component as is u and all these components we consider as scalar fields that is what we define it as a local accelerations and the convective acceleration. As I derived for the x direction, similar way I can derive for the y direction and z direction.

The same way we can have these 3 equations, which is called the Euler equations in a partial differential form with the scalar components of the velocity u, v, w which is a function of the space and the time and the pressure and the accelerations component of g_x, g_y, g_z. That is the variabilities what we have that which in these equations we have u, v, w that is the scalar velocity components are unknown to us, which is a function of the space and time and also the pressure.

Four unknowns are there, g_x, g_y, g_z is known to us because we can see what will be the gravity force components in the x direction, y direction and z direction. So, these 4 unknowns we can get the solution of this partial differential equation with including mass conservation equation. So, 3 momentum equations and 1 mass conservation equation, the 4 equations, partial differential equations.

If we solve it, we will get the four scalar fields like u, v, w, and the pressure. So, if you look at that, nowadays a lot of tools are there, computational fluid dynamics tools are there, we can solve these ones in any complex fluid problems to get it what is the u variations, v variations, w variations and the p variations. Only that we have to solve this nonlinear partial differential equation to find out what will be the u, v, w and the p value and this case we have an inviscid flow.

Here the frictional part, the energy loss is due to the frictions or the momentum, linear momentum equations because of pulls and storms we are not including it. That is the reason we are talking about Euler equations formats.

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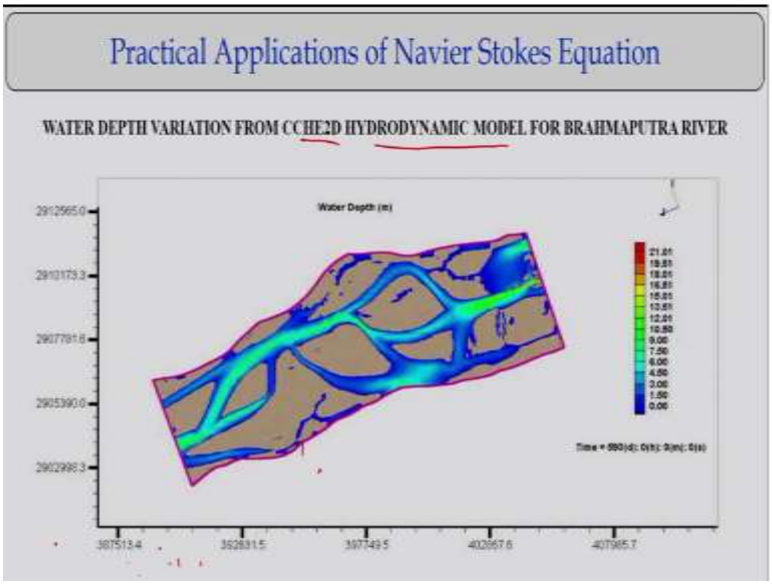

Let us have a few examples what we have done it for the Brahmaputra river using hydrodynamic model, which is the CCHE2D models to simulate the water depth variations at the seasonal scale, the monthly scale and the daily scale, how the water depth variability is there. So, using these equations when we solve it, if you look at this is the example file what we are demonstrating to you water depth variations at the daily scales, the seasonal case and the monthly scale which is the solutions of the Navier-Stokes equations.

If you look at that we can measure the depth variations as if you look the scale that how the flow depth is increasing or decreasing, how the channels are forming, how the flood plains are getting more waters. All these things we can do it in a mathematical model by solving the Navier-Stokes equation, that is what is today's possibilities, that is what we are set up these models to find out how these water depth variations are there.

As you look at some of the red colors are there, the water depth as go as high of 25 meters and there is the water depth lesser than 3 meters, so much variability of water depth, which varies with the space, which varies with the time. That is what we are plotting it as a 2-dimensional flow equation. We have solved it using these models to find out.

So, what I have to say is that the understanding of this basic equations, developing in the models, implement for the real river case that why it is quite challenging to know about the basic of governing equations for the river flow.

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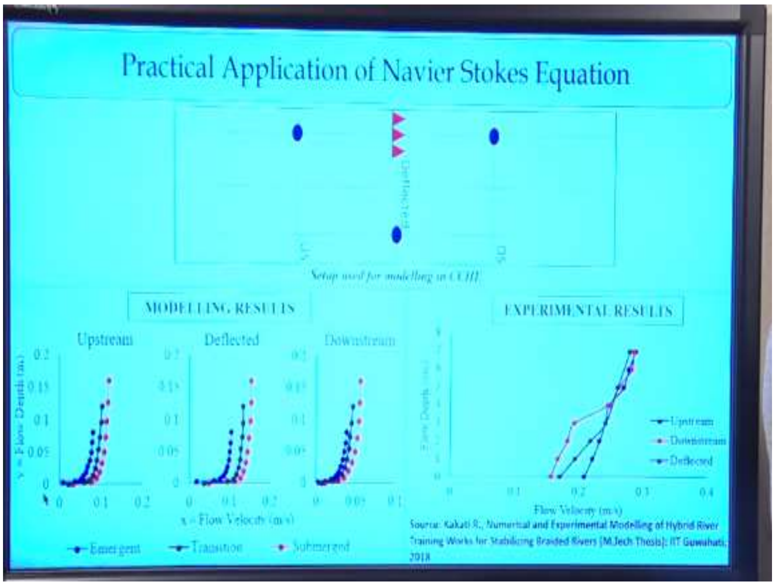

Considering that we have been discussing about this, the next class we will discuss more detail how we will make it more interesting of this in terms of Navier-Stokes equation solvers.

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With this, let me conclude today's classes with citing the students who are contributing for preparing these presentations and solving the problems, acknowledging their efforts. Let us have complete this week. The quote from Mohandas K. Gandhi is that earth provides enough to satisfy every man's needs, but not every man's greed. So that is what is the earth, that is what is the river process and mechanisms water wealth.

We should not be too greedy for the water wealth, but enough what is there for satisfying these human requirements or the nature requirement. With this, let us conclude this class. Thank you.