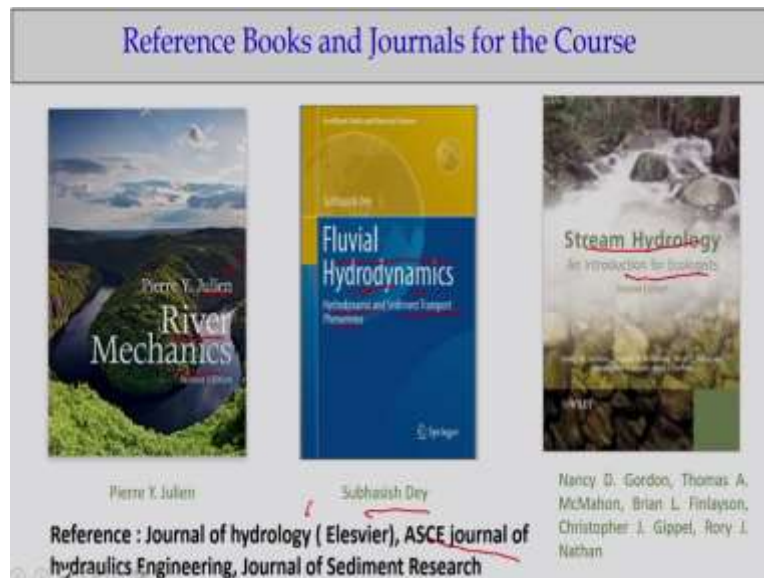


River Engineering
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Lecture – 04
Hydrodynamic Principles

Good morning. Let us come to the next series of lectures. These are the lectures that will be focused on how to develop a mathematical model for the river. So considering the basic concepts that hydrodynamic principles of the basic modeling framework what we do it and the same concept we also can use for analytical solutions of any river flow. So, considering that importance I will go through some examples as well as the mathematical derivations. At the end, I will solve some numerical examples.

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Let us look it again to talk about this, mostly I have been following this book fluvial hydrodynamics which talks about hydrodynamics and the sediment transport phenomena, very detailed and very illustrated way to represent the hydrodynamics and sediment transport phenomena in the river systems. Also, I will talk about having a reference with river mechanics of P. Y. Julien's and also the stream hydrology which is for an introduction to ecologists.

Not only that, we will have a some of the reference from the standard journals like journal of hydrology, American society of journal of hydraulic engineering, journal of sediment research. I do encourage all of you to visit these journal sites and look at advance level of

research what is going on in river engineering.

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Contents of Lecture
1. Brahmaputra River : Scale problem, Turbulence structures and Energy dissipation
2. Approaches for solving river engineering problems
2. Continuity Equation in Three Dimensions
3. Continuity Equation for open channel flow
4. Solving Example problems
5. Summary

Now, let us come back to today's contents. So, let us start with the Brahmaputra river concepts where I will talk about the scale problems, I will talk about the turbulent structures and energy dissipations which are very complex process and that is what I will give at introduction levels before deriving the basic equations, let us learn how complex the river flow in terms of hydrodynamics, in terms of sediment transport, in terms of nutrient transport mechanism in a natural environment like river.

That is what we will discuss in more details how we can solve these river engineering problems. Then there will be derivations for the continuity equations in three dimensions, the continuity equations for open channel flow and then will be a series of the problems what will we solve it to give you confidence that understanding of velocity field, water depth variations, those things you can get the idea while solving some of the example problems for basic mass conservation equations that is what we will talk more details.

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Let us come back to the scale problems. So, we may have awareness about the Brahmaputra river. This is the river which we call the braided river systems because of multiple unstable channels and in between there are series of bars or the islands and intense sediment transport and active channel process what it happens.

And this river if you look at this satellite data set the length of the river is close to 1000 km and the width of the river which varies in a range of 1.2 km to 16 km and the depth varies on average of 12 meters. So if you look at these values, one is 1000 km scale the length, the width is in kilometer scale and the depth is in a meter scale.

So this is what we call the scale problem, that is what the scale problems with a river system because the dimensions of the rivers are not same dimensions in length, width and the depth which is considerably order of difference is there. So that is the reason we need to have a lot of approximations to solve this river problems, it is not a simple way to solve a computational fluid dynamic problems.

But the major issues come is the scale problems because of length, width and the depth. The Brahmaputra River is a classical example of large sand bed river. As I said it if you look at these figures, there are multiple channels and these channels are quite dynamics and there are very, very high flow and sediment variability that is what I discussed earlier and that is the reason we should know it how the flows are happening it and which are very complex process what is happening.

Mostly again I need to emphasize about the scale problems in river flow systems because this length, width, depth the dimensions itself is a scale or a difference that is the biggest issue.

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Now if you look at another bigger issue if you take photographs of Brahmaputra river during the flood periods, you can see the turbulent structures like for example this type of vortices formations you can see. From taking the photographs, you can see that large vortex formations are happening. There are the boil formations which is that the vortex is formed from the ground and grow it and then it dissipates.

So this type of process also happens in Brahmaputra river and not only that if you look at a bigger snap of photos you can see the similar type of series of the vortices and boil formations happening and these are called the macroturbulence structures. So these structures are also responsible for the flow behavior change, the velocity, depth and also the change in the sediment transport process.

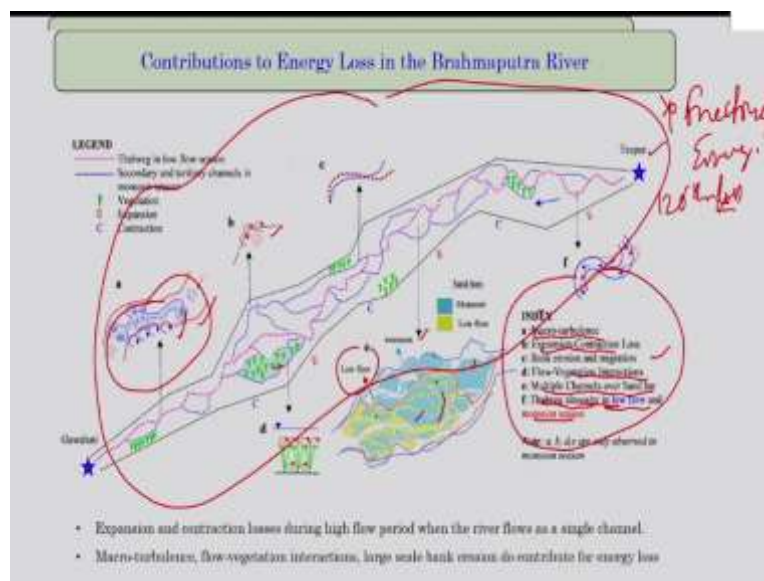
The sediment transport process because of this presence of the boils and vortices is different or macroturbulence presence makes the sediment transport process different in terms of source, in terms of deposition, in terms of transport process. Now if you look at this what could be the dimensions?

The dimensions are as high as it can go to 75 meters and or 240 meters above the sand waves. It works like a sand waves and stay up to 30 seconds. If you look at this, these turbulent structures are visible in Brahmaputra river during the floods, no doubt they are also

responsible for the energy dissipations in the flow systems. So if you look at this microturbulence structures which is visible.

It is quite visible in Brahmaputra river, but any other rivers also you can have smaller magnitudes of the turbulent structures and that is the turbulent structures. Try to understand, these structures changes the flow characteristics, sediment transport mechanisms as well as the energy dissipations. So that is what contribute to the energy losses in the river. So that is the new concept what we are introducing to you.

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Now if you look at, very interesting figures showing to us that how many type of energy dissipations happen in the Brahmaputra river. Like for examples if you take Brahmaputra river is about 120 km in length from Tezpur location to the Guwahati location in Asaam and if you can look it, the river expand and contract at several in between locations. It is not having uniform width throughout. It expands and also contracts. This expansion and contractions also vary from season to seasons.

Like for example, for the low flow, you can see channels which are activated, but in case of the monsoon the high flow you can see which are the channels activated there. The width also expands and contracts from season to season, months to months that is what the expansion and contractions are there, which generally considered as major energy losses as its river expands or contracts as you know it from basic hydraulics properties.

Not only is that because of the presence of the turbulent structures, the macroturbulence

structures, what will be there that a major emphasis here, that also generate the energy dissipation. Energy dissipations also happen because of macroturbulence structures, the expansion-contractions losses like as there is an expansion and contraction are happening as equivalent river width that is what also we have these ones.

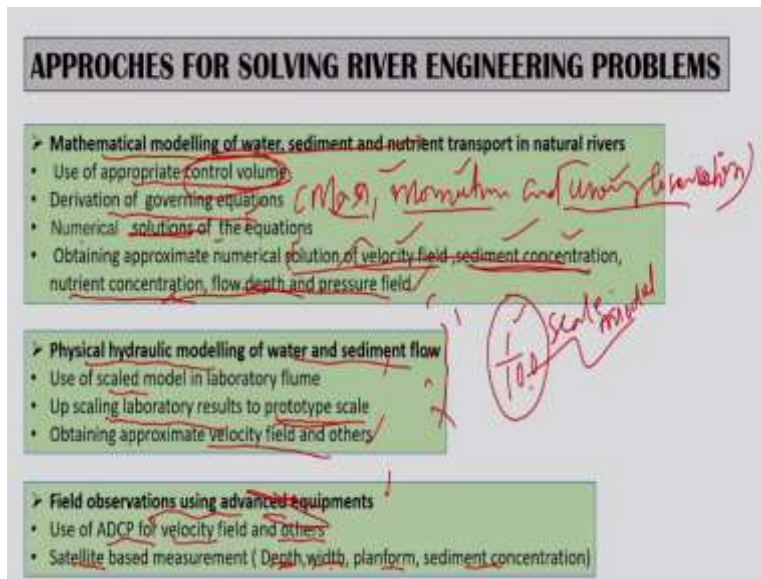
Not only that if you look at this river when you have the river belt is so large, you can also have the presence of the vegetations, certain type of grasses, some type of scrub lands or the forest. So those vegetations also interacts with the hydrodynamics. So you can have energy losses due to the flow vegetation interactions. There are the multiple channels over the sand bars that is also there and there is a thalweg line.

The thalweg line is that line joining the deepest points along the channel, that thalweg line also have the variabilities during the monsoon flow and the low flow. So, if you look that when you talk about the energy dissipations in Brahmaputra river, it is not only this frictional losses okay, frictional energy losses which is well debated in many of river engineering books, but what we are talking about is beyond that.

There are the possibility for energy losses because of expansion-contractions, because of the bank erosion process, because of macroturbulence structures as well as the shifting of the thalweg lines in the low and medium seasons. So that way we try to understand the river in different perspective like how the energy dissipations are happening because that is what it controls how the sediment transportation happens, how the flow variability changes.

How it is making more vulnerable for the bank erosions? how these bed forms of river changes are over bed from mega bed forms like sand bars, how these formations are happening. All this knowledge we should have, try to understand it how a river does energy dissipations, river does sediment transport mechanisms and also rivers how is water flow as a velocity and turbulent structures, the depth and the width.

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Let me go to the next slide that when you have a complex river like Brahmaputra river or any rivers, which is a natural system, we should try to understand this river or try to find out the solutions for that rivers. One approach is that we can follow mathematically, a set of equations we can have and we can solve these equations to find out the basic characteristics of the rivers like the velocity, sediment concentrations, nutrient concentrations, the depth and the pressure field.

What we do it nowadays we have more emphasis, but I can say that each methodology having complementing to each other is not having any superiority that mathematical model does good work or physical model does not work it or all are having the advantage and disadvantage, but as a river specialist what do I look is that all these options and all these 3 options like mathematical modeling, physical hydraulic modeling and the field observations.

All the things we should conduct to understand the rivers in terms of sediment transport, in terms of the flow transport, in terms of nutrient transport, in terms of energy dissipation. What we do in mathematical modeling? First we find out appropriate control volume that is what is you know from the fluid mechanics. You can follow some of the fluid mechanics YouTube lectures developed by me.

We talk about more details about the control volumes, Reynold transport theorem, how to use the control volume concept for deriving mass conservation equations, momentum equations, and energy equation. These are there in any fluid mechanics book as well as you can follow the YouTube lectures on fluid mechanics by me talking about how to choose the control

volume, how to choose an appropriate control volume so that you can solve the fluid mechanics problems.

The control volume can be infinitely small or it can be as large as Brahmaputra river. So, you can decide what type of control volume you have to consider to solve your problems. Control volume can be at the rest or in motions or you can choose it, what type of control volume you have to consider. After considering the control volumes, you can find out the basic physical equations for the mass, momentum and energy conservation equation.

So you look at this mass conservations, momentum and energy conservations that is what we get a set of partial differential equations as a governing equations for us as you know in fluid mechanics. Then you talk about, because these are non-linear equations which always, most of the times will follow start from a numerical solutions of this the governing equations which is 2D if it is possible to have this.

There are a lot of numerical techniques, you can find out what will be the numerical solutions, what we are looking at as a numerical solution for a control volume for a study reach. We try to find out how does the velocity varies, that means we talk about the velocity field. We can find out the acceleration field because if you know the velocity field. Similar way, I want to know how the sediment concentrations variability is there as a suspended load, as a bed load, as a wash load.

How the sediment concentration variability is there that is what if I try to get it as a solution. Similar way if I am talking about nutrient, flow depth and the pressure field, basically we try to get these characteristics of the rivers in terms of hydrodynamics, in terms of the fluvial means with a sediment, nutrient, and energy dissipations. We try to look at all these components from the rivers.

After conducting a numerical solution of these governing equations which are in terms of mass, momentum and energy equations is most often we follow in the fluid mechanics problems, the same percent of the fluid mechanics problems customized for the river flow systems. Many of the times, also we go for a physical model and for water and sediment transport I will show some of the photographs later on.

We can have a scale models that means you can make a 1:100 scale model and we can solve these problems. So the scale is 1:100. The hundred unit of the prototypes represent one unit of the scale models, so that way you can use a scale model in the laboratory. You conduct the sediment transfer, water flow, then you measure all these the velocities, water depth, sediment transport part.

Once you measure that, then you can upscale it from your scale models to prototype scale, at the river scale you can have some similarity, non-dimensional numbers to upscale this data what you measure in these flumes or the small channels that what you can upscale to the prototype scale. Then we get the velocity field, the depth or the sediment concentrations all you can do it, but here also have the biggest problems with scale effect because you have to scale down the models from 1:100.

The major issue here is also the scale effect as you all also know it have an advantage, or disadvantage. The advantage here is that anybody can, the field engineer can have an insightness what is a flow happening and he can give appropriate suggestions to modify these physical models. So till now there are many places we use the physical models as well as the mathematical models.

They are complementing each other, but how we create this physical model and mathematical model to real conditions, we have to have the field observations. This field observations is very critical nowadays that is what is possible nowadays that we can conduct the field observations with advanced equipment like ADCP i.e. acoustic doppler current profiler to measure the velocity field and sediment concentrations, the water depth.

All we can measure and those measurement of the data can be used for the mathematical model to validate or the physical model to validate that whatever the results we are getting from mathematical models or the physical model are they accurate, are their performance are acceptable for river engineering practice, those we get the confidence based on our comparisons with the field observed data with mathematical models, field observed data with the scale models.

So that is the reasons the field observations always need to do before conducting any mathematical model or the physical models. No doubt nowadays we have another source is

In the next classes we look at how to bring this advanced measurement of the river from the satellite platforms in terms of the depth, width, the planforms and the sediment concentrations. Some of the case studies I will show you in later part of my lectures.

Continuity Equation in Three Dimensions

- For analysis of 2-D and 3-D mode of flow, the differential continuity equation is considered.
- In order to derive of this equation, let us assume a control volume element of a fluid in a flow domain as shown in the figure.
- Considering the Cartesian coordinate system (x, y, z), let the elementary volume be $dx dy dz$ and the mass density be ρ .
- The velocity component are u, v and w in x, y and z direction respectively.

Definition sketch for the derivation of three-dimensional continuity equation of fluid flow in a control volume

If you look at this case as we know from basics definitions is that the mass flux, the mass per unit time

So, I can compute the mass flux passing through a control surface as functions of density, the

velocity scalar component and the area that is the very basic things that we can compute the mass flux. See if this is my control volume which is having the dimension dx, dy and dz, so smaller control volumes and we are considering the u as a continuous function and the velocity vectors

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

If I have that as a velocity scalar component and these scalar components are also the function of space and the time i.e.

$$u=u(x, y, z, t); v=v(x, y, z, t) \text{ and } w=w(x, y, z, t)$$

and similar way the density is also functions of the space and time i.e.

$$\rho=\rho(x, y, z, t)$$

So basically, what I am talking about we go through any fluid mechanics basic things that if I consider this, the density and the velocity are the continuous function in terms of the space and the time.

And if I consider this is my control volumes just to find out how much mass flux is coming into the systems and going out of the system that is what will be indicating me that net change of the storage of the mass within the control volume, which is a basic concept and if you can go through Reynold transport theorem to consider that, but looking it in this case a small control volume here.

I just try to know it what is net mass influx is going in x direction, net mass influx is going in y direction, the net mass influx is going in z directions that summation should be equal to the mass change in the storage of this control volume, that is a very basic things, the inflows and outflows and you can find out the net influx passing through these control volumes and in x direction, y direction the z directions just summing up.

And also we know it what could be the net storage of the mass which is happening within this control volume that is what you equate it as a mass conservation principle that is what we will get the equations. So, let us just look at these derivations that we are looking a very small control volume with having a dimension dx, dy, dz and it have a velocity component, scalar

velocity component of u, v, w which is in x, y, z directions if you can see this control volume.

I am just computing the mass flux. Before computing the mass flux coming through this, these are going out from these, only I have to know it as the mass flux what we can define it is $\rho u dy dz$, where $dy dz$ is a surface area perpendicular to the u directions. This mass flux if you look at, it varies along the x directions if considering the first term of the Taylor series expansions you can get the ρu .

The variations in the x directions will be

$$\left[\rho u + \frac{\partial}{\partial x} \left(\rho u \frac{dx}{2} \right) \right] dy dz$$

So this is a Taylor series expansion to estimate it if a functions varies at the $\frac{dx}{2}$ distance what could be the value. Same way mass influx that is what also we compute it at this surface will be the

$$\left[\rho u - \frac{\partial}{\partial x} \left(\rho u \frac{dx}{2} \right) \right] dy dz$$

So, these are simple Taylor series expressions, you can use it to compute what is the mass flux is coming, coming from this surface and going out from this surface.

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Continuity Equation in Three Dimensions (Continued...)

- The net mass influx through the CV in the x-direction: $\left[\rho u - \frac{\partial}{\partial x} \left(\rho u \frac{dx}{2} \right) \right] dy dz$
- Here, the first term $\rho u dy dz$ represents the mass influx through the central plane normal to the x-axis, and the second term $\left[\frac{\partial}{\partial x} \left(\rho u \right) \frac{dx}{2} \right] dy dz$ represents the change in mass flux with respect to distance in x-direction multiplied by the distance $\frac{dx}{2}$ to the back face.
- Similarly, the mass efflux through the front face of the control volume in x-direction: $\left[\rho u + \frac{\partial}{\partial x} \left(\rho u \right) \frac{dx}{2} \right] dy dz$
- Therefore, the net mass flux out in x-direction through these two faces is obtained as $\frac{\partial}{\partial x} (\rho u) dx dy dz$

Then we can compute net mass flux, it is very simple that is what is in net mass influx passing through this x directions I explained it, and this is what you will have here the net

influx of mass in the x directions will have just subtracting of these two we will get this part okay. These two are cancelled out each other and $\frac{dx}{2}$ will be combine, you will get it net mass flux out in the x direction through the two faces will be this part. Same way you can do for y directions, also we can do for z direction.

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Continuity Equation in Three Dimensions (Continued...)

- Similarly this yields for other two directions also and hence, the net mass flux out of the control volume is:

$$\left(\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) \right) dx dy dz$$
- As per the concept of conservation of mass, the net mass flux out of the control volume plus the rate of change of mass in the control volume, given by $\left(\frac{\partial \rho}{\partial t} \right) dx dy dz$, equals the rate of production of mass in the control volume i.e Zero.
- Thus, the three-dimensional continuity equation of fluid flow is:

$$\left(\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) \right) + \frac{\partial \rho}{\partial t} = 0$$

If I combine that, so all the other 2 directions I can do it and so net mass out flux will be, this is the x direction mass, this is the y direction and this is the z direction multiplied with the dx dy dz is indicating for us the net mass influx in x direction, y direction and the z direction. So you know from this control volume what is the net mass flux is going in the x direction, y direction, z direction in terms of a function of ρu .

ρ is the density, u is the velocity scalar components in the x direction. Similarly, v is a scalar component in the y direction and w is the z direction component. So, we can get the net mass flux out of the control volume is coming up to that. Based on the mass conservation equations, this net mass outflux from the control volume that become will be 0, the net mass changing with the control volume will be $\frac{\partial \rho}{\partial t} dx dy dz$.

It will give us the rate of change of mass with respect to time that is what will give you $\frac{\partial \rho}{\partial t}$ multiplied by volume that is what if you look it and change with respect to time because $\frac{\partial \rho}{\partial t}$ that is what it will gives us the rate of change of the mass in the control volume. That is if you two equate, I will get this equation. If you look at this equations part, this is the net mass

influx in the x direction, next mass flux in the y direction per unit volume.

Net mass influx in the z direction per unit volume, this is the net storage mass, rate of change of the mass storage within the control volume that is what we represented like this.

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Continuity Equation in Three Dimensions (Continued...)

- For incompressible fluid flow ($\rho = \text{constant}$), it can be simplified as: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$
- The kinematic relation in terms of the components of normal strain rate can be calculated from the following expression:
$$\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = 0$$
- For a 2-D incompressible flow in xz plane, the continuity equation becomes:
$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

So if you look at the next part that many of the times we try to look at fluid as incompressible, density does not vary it, but that may not be conditions for river flow. So that is what again I highlight that in fluid mechanics many of the time we simplified it density does not vary it, but as we discussed earlier that we can find out the sediment water mixtures density which may vary depending upon the concentrations.

So, we cannot consider it as a constant parameter, but if you consider a constant parameter as water flow as there is not significant sediment transport is happening, we can simplify that equations which is there in any fluid mechanics books. That is what will come in to be the steady flow. It also has assumption of steady flow, the flow does not change with the time, so you can get these components.

If we consider ρ is a constant and ρ does not vary with the time and that is the steady flow and incompressible. If you consider it that is what is coming. Again I am highlighting it, there are lot of assumptions that are involved here and when you solve the river engineering problems, please look at whether these assumptions are valid for us. If you have a 2-dimensional flow which is in xz plane you can drop other components and you can have the flow fields in only the x and z directions.

So, your 2-dimensional flow and xz plane, the continuity equations become this which is in the fluid mechanics. In terms of shear strain rate, we can write it in this form, which is exactly this part, just to look at some fluid mechanics books to try to understand that part.

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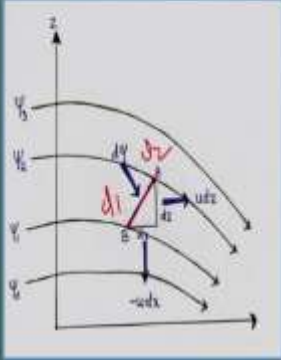
Continuity Equation in Three Dimensions (Continued...)

- In this case we can introduce a well known function called *stream function* ψ , such that:

$$u = \frac{\partial \psi}{\partial z}, w = -\frac{\partial \psi}{\partial x}$$
- Assuming that the ψ is continuous to the second-order derivatives. The stream function has a useful physical significance:

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial z} dz = -w dx + u dz \Rightarrow \psi = - \int w dx + \int u dz$$
- The difference between the values of two neighbouring streamlines is numerically equal to flow rate per unit width:

$$\psi_2 - \psi_1 = \Delta\psi$$



The stream function

But often we use a stream functions when you look at the river from the top, you can see the planforms of the rivers and if you can draw the stream lines you can solve these problems, so that is reasons I am just introducing you with the stream lines which are very basic in the fluid mechanics, as the definition of the stream line the relationship with stream functions and u and w is given like this, then you can find out the change in the stream is a continuous to the second-order derivative.

The stream functions has this, if you put it all these values you can find out the stream function values and you can find out the difference of the stream functions like you have a stream lines and the difference of the stream functions like you have Ψ_1 and Ψ_2 , difference of that will give a net flow what is going through these two stream lines. So that is a very basic ways I am just revising you for the fluid mechanics thing stream functions also we can use for the river flow.

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Continuity Equation for Open-Channel Flow

- The difference of mass influx in and mass efflux out of the control volume must be equal to the rate of increase in fluid mass within the control volume.
- The figure shows a unsteady open channel flow condition, solid lines shows the initial free surface, while the final free surface after a small interval of time dt is shown by the dotted lines.
- The flow in the channel is fed laterally with a uniform flow rate q_L .

Continuity of an unsteady flow in an open channel

Let us come for the continuity equation for open-channel flow. So we are coming from fluid mechanics to the open-channel flow when you are considering a river. So in case of the rivers, what we will have? There will be lateral inflow to the river if you look at this control volume, there will be lateral inflow to the river. That lateral inflow or the outflow depending upon from the surface as a rainfall or evaporations going out or there will be an exchange between surface and the ground water.

Sometimes the river water goes to the ground water or ground water comes to the river waters. So you will have a lateral inflow to this control volume which is quite different than other fluid mechanics problems. When you take a river, there is exchange of the waters from river to the ground water, groundwater to the river and also the exchange of the waters to the atmosphere, from the river to atmosphere or atmosphere to river as raining to happen it.

So that is the reason we can say lateral flow happens to the control volume if I consider it. Say there is a q_L amount of the lateral flow happens, fed laterally with a uniform weight, this is a quite simplified assumptions that q_L does not varies with time but we can do it, but let us consider within this control volume the q is more or less uniform. If you are considering that now if you see these figures, this is the control volume, this is the river.

You have a river shape and that is what it says that unsteady open-channel flow. Solid line shows the initial condition whatever the solid line is indicating is initial level $t = 0$ that is the condition. At the $t = dt$, your surface becomes a dotted part. There is change of flow depth, the change of the storage, there is a change of the velocity field from the $t = 0$ initial time step

to the dt times.

See if this is my control volume, with this lateral flow I want to derive this mass conservation equation that is the basic idea, to derive mass conservation equation. We will talk about how the storage changes in it, how this mass influx changes these terms, we will discuss it more detail in the next slide.

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Continuity Equation for Open-Channel Flow (Continued...)

- The mass influx in time dt into the control volume is: $\rho U A dt + \rho q_L dx dt$
- where U is the area-averaged flow velocity through left section and A is the flow area of the left section.
- The mass efflux in time dt out of the control volume is: $\rho (U + dU)(A + dA) dt = \rho \left(U + \frac{\partial U}{\partial x} dx \right) \left(A + \frac{\partial A}{\partial x} dx \right) dt$
- Where $U + dU$ is the area-averaged flow velocity, $A + dA$ is the flow area and here, $dU = \left(\frac{\partial U}{\partial x} \right) dx$ and $dA = \left(\frac{\partial A}{\partial x} \right) dx$
- The rate of increase in fluid mass in time dt within the control volume is: $\rho T \frac{\partial h}{\partial t} dx$

(Where T is the top width of the flow and h is the initial flow depth.)

If you look it, come back to the initial dt time in mass flux, we will have the mass influx is coming as $\rho U A dt + \rho q_L dx dt$.

Where U here stands for area average flow velocity.

A stands for the flow area in the left section and

If you look at the mass influx in time dt , in this case you have U and A is a variable. So after dt time, you can consider U vary as $U + dU$, A varies as $A + dA$. That is the reason the mass efflux or going out from this control volumes will be the $\rho A U$, the change of the velocity and change of the area into the dt that is what is the mass efflux or out from this control volume.

And mathematically you can derive it as U and A is a function in the x directions that is what we can define as U and A in this form and if you look it that is what I said here U as the area average flow velocity, A stands as a flow area and that is what in the different time. Now if you look at the net increasing of the fluid mass in the control volume that is what will be net increasing will be ρ multiplied by volume by dt .

That is what is rate of change of the mass with respect to time, $\rho \times \text{Volume}$. In this case, we have this volume we write in some T is the top width, h and dx as we have included h is varying with respect to time, so δ is equal to δt and δt is component is there that is what is the mass. Rate of the change of the fluid mass in dt time comes out to be this. So, I know influx, I know outflux, I know the change of things, just have to equate it to get the basic equations.

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Continuity Equation for Open-Channel Flow (Continued...)

- The continuity of flow in an open channel is therefore given by

$$\rho U \delta x dt + \rho q_L \delta x dt - \rho \left(U + \frac{\partial U}{\partial x} \delta x \right) \left(A + \frac{\partial A}{\partial x} \delta x \right) dt = \rho T \frac{\partial h}{\partial t} \delta x dt$$
- Further simplifying: $U \frac{\partial A}{\partial x} + A \frac{\partial U}{\partial x} + \frac{\partial U}{\partial x} \frac{\partial A}{\partial x} \delta x = q_L$
- Using, $Q = UA$ and $\partial A = T \partial h$ at a given section, the equation becomes: $\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_L$
- Further, using hydraulic depth $h_d = \frac{A}{T}$ and $\partial A = T \partial h$: $U \frac{\partial h}{\partial x} + h_d \frac{\partial U}{\partial x} + \frac{\partial h}{\partial t} = \frac{q_L}{T}$
- For a rectangular channel with no lateral flow, the equation becomes: $\frac{\partial q}{\partial x} + \frac{\partial h}{\partial t} = 0$
- Where q is the discharge per unit width.

This equation was first introduced by de Saint-Venant (1871).

So, the influx part, the outflux part that's what the net change of the mass within this control volume which is the open channel control volume with lateral flow of q_L per unit length. So that way you can look at this equation which is just a mass conservation equation of influx, outflux and changing the mass within the control volume that is what if you simplified it, it comes like this.

So if this equation you can just simplify it, it will come that and if you write $Q = UA$, area is top width multiplied by dh value, then this equation again simplified it as just a transport equations as

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_L$$

Be remember it is a mass conservation equations, but when you write down in a partial differential equations form that is what comes in terms of q variability in x direction.

That is the gradient of the q in the x direction and the A variable is equal to the q_L . You can

substitute the unit of all these terms and verify are they accurate. Similar way if I define Hydraulic Depth = Area/Top width

i.e. $h_d = A/T$ and $\partial A = T \partial h$

you can get it, expanding that not in terms of q , in terms of capital U which is the area average velocity and h , I will get this equation form and without lateral flow. This is the equations long back given by de Saint-Venant in 1871.

So, if you look at these equations, if you can remember this, you can simplify this and this only the things are there certain assumptions we have, like for example there is no lateral flow the equation becomes this, this is very simple thing with discharge per unit width (q). If you have in terms of a hydraulic depth, in terms of velocity if you try the equations it will be like this, and if you want to have in terms of q and the area, the equations will be like this.

But all these are derived equations from these basic mass conservations equations, which again I am to show you the basic control volume to understand always, you try to sketch the control volume. If you try to sketch the control volume and find out how mass influx and outflux rate and just write a mass conservation equation to solve these problems and just try to know what are the dependent variables you are making it or what is the flow variability you are making it for these control volumes.

Like for example for this case, we are considering as the river is open channel flow, there is lateral flow, we have given emphasize, there will be lot of exchange of the flow of waters to the river or out of the river that is what is defined is a lateral flow and here the mass influx we defined in terms of ρU , U is the area average the velocity component that is what we are looking in only this in x directions.

So we are not having a three dimensional, we are just looking it in the space only x directions and the time, so we have a ρU that influx and outflux will compute it and you compute the change in the storage part that is what you equate to find out basic mass conservation equation. So basic things what I have to tell you that, first you choose the appropriate control volume, find out the influx and outflux and the change in the storage of the mass within the control volume, equate it.

The equation may be a partial differential equation, but the basic concept is the mass conservations applied to control volumes like for a river like problems where we consider only longitudinal directions and dominated directions as well as the lateral flow that also we have considered here.

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Solved Example 1

In a fluid flow, velocity field and density variations are respectively defined, $\vec{V} = 5x^2\vec{i} + 6y^3\vec{j}$ and $\rho = 5x^3$. Does they satisfy the flow continuity equation? at $x=1$ and $y=1$?

Solution Data given: Two dimensional and steady flow.

Hence, the continuity equation becomes,

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

$$\Rightarrow \frac{\partial}{\partial x}(5x^2 \times 5x^3) + \frac{\partial}{\partial y}(5x^3 \times 6y^3) = 0$$

$$\Rightarrow 125x^4 + 90x^3y^2 = 0$$

Does not satisfy.

Looking that before concluding I can have a few examples to just to give you a confidence that we try to find out the velocity field when you solve these problems. Like for examples if you have this example 1 in the fluid flow you have a velocity field and density fields which is given is a velocity vectors and the density as a function of the space. Does it satisfy this flow continuity equation at $x = 1$ and $y = 1$. So this is simple.

Since only x and y is there, time is not there, so we can find out it is only the 2-dimensional flow equations mass conservation equations with ρ and U and if you just substitute. This is the 2-dimensional steady flow equation, just substituting all these equations finally you can find this equation and substitute x and y , this is not coming to 0. So that means it does not satisfy. So just to give you that many of the times you can have analytical functions to find out do they satisfy the continuity equations or not.

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Solved Example 2

In an open channel flow, velocity field is defined as $\vec{V} = 2x^2\vec{i} + 3y^2\vec{j} + 2zx^2\vec{k}$ and density function $\rho = 3x^2t^2$. Verify if this satisfies the continuity equation. at $x=1, y=1, z=1$ and $t=1$?

Solution:

Data Given: Three dimensional and steady flow.

Then the continuity equation becomes,

$$\begin{aligned}\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) &= \frac{\partial \rho}{\partial t} \\ \Rightarrow \frac{\partial}{\partial x}(3x^2t^2 \times 2x^2) + \frac{\partial}{\partial y}(3x^2t^2 \times 3y^2) + \frac{\partial}{\partial z}(3x^2t^2 \times 2zx^2) &= \frac{\partial}{\partial t}(3x^2t^2) \\ \Rightarrow 6 \times 4x^3t^2 + 9 \times 2x^2yt^2 + 6x^4t^2 &= 6x^2t \\ \Rightarrow 24x^3t^2 + 18x^2yt^2 + 6x^4t^2 &= 6x^2t \\ \text{Does not satisfy.}\end{aligned}$$

Same way next example, the solved example in open channel flow. The velocity field is defined as

$$\vec{V} = 2x^2\vec{i} + 3y^2\vec{j} + 2zx^2\vec{k}$$

And

$$\rho = 3x^2t$$

Does this satisfy the continuity equations $x, y, z = 1$ and $t = 1$. So same. We can use the continuity equations, substitute all these values, you can have this partial differentiation of this equations part.

Then you substitute. Finally you will see that and when I substitute x, y, z, t on this equation this is what we find out. It does not satisfy the basic equations for $x = 1, y = z = 1$. So this does not satisfy it. So this is what is very simple example is given you to get the idea that we can look at whether you can solve this problem.

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Solved Example 3

In an one dimensional open channel flow, the discharge (Q) is defined as $2x^2 \text{ m}^3/\text{s}$ and area of the flow $A = 3t^2 x \text{ m}^2$. Find out the lateral flow at $x = 30\text{m}$ and $t = 60 \text{ sec}$.

Solution

Data Given: One dimensional open channel flow.

$$\frac{\partial Q}{\partial x} = 4x$$

$$\frac{\partial A}{\partial t} = 3 \times 2t = 6xt$$

$$4x + 6xt = q_L$$

at $x = 30\text{m}, t = 60 \text{ sec}$

$$q_L = 30 \times 4 + 6 \times 60 \times 30 = 120 + 10800 = 10920 \text{ unit}$$

This is a very simple problem again. In a one-dimensional open channel flow that is when we are talking about only x directions, discharge $Q = 2x^2 \text{ m}^3/\text{s}$. So area is given as a function of t and x. Find out the lateral flow at $x = 0$ and $t = 0$. So basically, you want to find out the $\frac{\partial Q}{\partial x}$ and $\frac{\partial A}{\partial t}$ then you substitute to find out what will be the q_L .

So you can do partial derivative of this part and then you put this and $x = 30 \text{ m}$ $t = 60 \text{ s}$. If you substitute it, you will get this q_L . So this is a quite interesting problems where q is given a function of x, area is also function of x and t and we want to find out what will be the lateral flow if $x = 30$ meters and $t = 60$ second if it follow this mass conservation principle. So this is basic just applying this mass conservation equation.

With this, let us have completing today's lecture. Again, we will discuss a few examples as well as the momentum equations in the next class.