

Remote Sensing and GIS
Rishikesh Bharti
Department of Civil Engineering
Indian Institute of Technology – Guwahati

Lecture - 09
Digital Image Processing-III

Today is the third lecture on Digital Image Processing. So in this lecture we will see some of the very important concepts like which we use in digital image enhancement. So let us see first of all before we go into the method part let us understand what do we understand by edge because in all the images we have several edges and that we need to enhance right.

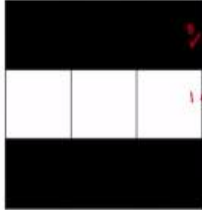
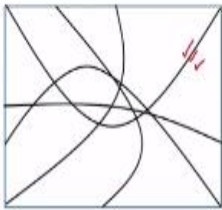
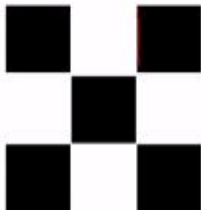
(Refer Slide Time: 01:00)

Edge Enhancement Methods

भारतीय प्रौद्योगिकी
INDIAN INSTITUTE OF T

What is an edge?

- ✓ Boundary where brightness values significantly differ among neighbors...
- ✓ Brightness value appears to abruptly jump up or down...



So let us see what do we mean by an edge. So these are the boundaries where brightness values significantly differ among neighbors. So here I always refer this neighborhood right why because whenever you have any object you are basically working on highlighting this with respect to surrounding. So if you want to highlight or if you want to identify this particular edge let us consider this is the edge then what you have to do.

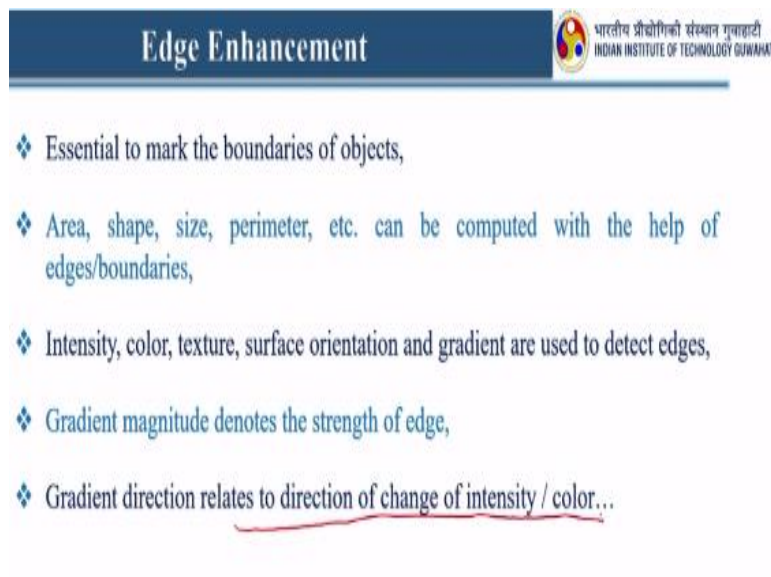
You have to identify the changes in the intensity with respect to neighborhood. So here also we will have the similar concept. So the next point is brightness value appears to abruptly jump up or down. So basically whenever we say this is an edge or this is where that object is changing from one to another we basically identify the corner pixel and based on that we say it is basically different object.

So let us see some of the example in case of Boolean. You can see clearly these edges right why because intensity is changing abruptly either upside or downside. This is another example where you have lines and lines also basically if you identify the value of this let us say 1 value of this line will be 0 and value of this white will be 1. So basically here you can say that intensity is changing abruptly 0 to 1, 1 to 0.

And because of that you could able to see all these lines. In case of this check box you can easily find the intensity difference. So that is why we are able to perceive this difference or this object right. Now if you see the next example here also I have put this is 1, this is another so here it is 0, here it is 1. So basically why I am saying why it is 1 because white is higher side, 0 is lower side. So lower is darker.

So here 0101 right. Here 1 here this line is 0 and then 1. So this is the reason we are able to see these edges right.

(Refer Slide Time: 03:39)



Edge Enhancement

- ❖ Essential to mark the boundaries of objects,
- ❖ Area, shape, size, perimeter, etc. can be computed with the help of edges/boundaries,
- ❖ Intensity, color, texture, surface orientation and gradient are used to detect edges,
- ❖ Gradient magnitude denotes the strength of edge,
- ❖ Gradient direction relates to direction of change of intensity / color...

It is very essential to mark the boundaries of object why because our ultimate aim of this remotely sensed image to interpret to have an input for our objective or for our model. So how we can do that by identifying the object or by identifying the material. Let us say here I have written mark the boundaries of objects. So if you want to highlight or if you want to estimate area of forest right how will you do that.

Because you have a image and here you have different objects. So let us take some example this is forest, this is water body, this is barren land and this is urban area and if you want to

calculate what is the area of this forest. So you need to identify number of pixels in this particular class. So but how will you define that these are the number of pixels only belonging to this particular class by identifying this edge.

If you want to highlight or if you want to estimate what is the area of this lake. So again you need to find out this boundary then only you will be able to calculate how many pixels are lying in this right. So similarly for all other categories edge is an important parameter. So we have to be very careful or we have to use very sensible logic to enhance all these edges and that helps to identify or compute the area right.

Next is area, shape, size, perimeter etc. can be computed with the help of edge or boundaries right. We have already seen this example so this is very clear to you. Now intensity, color, texture, surface orientation and gradient are used to detect edges. So how we can use this intensity color texture so intensity is like how within a color range whether it is dark or light.

So let us say in red color so you have a color bar and here this is very light color and here it is increasing. So I cannot draw this gradient. So from here to here it is very dark, here it is medium range here it is very light. So if you have such intensity difference that also tells you like this is the change in the intensity so there could be change in the object. So we have seen how intensity can help to identify this edges. Now let us see how this color can help.

So let us assume that you have a image and here you have different sections. So here you have blue color, red, green, yellow. So you can easily find out what is this edge and where it is lying. So exactly you can pinpoint the edge pixel right and likewise you can always identify these edges with texture. So texture means suppose if this is the area then it is filled with this kind of texture.

This is the texture in this particular portion this is the texture in this particular portion and here you have different type of textures right. So this is the different way how we represent our maps, but in nature we always find some natural textures which help us to identify this edges right and then same time surface orientation and gradient. So gradient is one very important concept because we will be using some gradient detector.

So that detector will help us to identify these edges right. So what do we mean by gradient?

Gradient magnitude denotes the strength of edge right. So here we will use gradient as an indicator of strength of edge. So here gradient direction relates to direction of change of intensity or color. So here we will use gradient to identify change in intensity or either change in color.

(Refer Slide Time: 08:51)



- ❖ A set of connected pixels that lie on the boundary between two regions/objects,
- ❖ The pixels located at the edges are called edge points,
- ❖ Gray level, color, and texture discontinuity around the edges causes edge perception,
- ❖ Position and orientation of edge are key properties...

So a set of connected pixel that lie on the boundary between two regions or objects that is the edge. The pixel located at edge are called edge points. So suppose if this is area and this is the edge right and here it is uniform. So this is the enlarge view. Now you can find out these are the pixels like this is one pixel, this is another pixel this is third pixel. So these pixels are nothing but the edge point right. So these are edge point. So I hope this is clear to you.

Gray level color and texture discontinuity around the edge cause edge perception. So because of the gray level changes or any color level changes so intensity of the color or intensity of white to black that will give you grey levels and texture discontinuity. Discontinuity means that is not continuous. So that will give you the perception like edge is present in this particular portion of this image. Position and orientation of edge are key properties.

(Refer Slide Time: 10:17)



So let us see some example here I have put one very basic image and we all are familiar with this right. So here you can see different types of vegetables with different color and texture. So can you see all these vegetables separately and can you identify them it is very easy. Why because we are able to see these colors and we are able to count them why because we have the edges.

So we can easily find out this is the edge between these two, this is the edge between these two right. So why because of the change in the intensity and color and texture there are many parameters, surface orientation. So everything is involved together and then we could able to see these edges right. So this is a very good example to understand what do we mean by edge.

(Refer Slide Time: 11:23)



Here this is another example you can even count the number of grains present. How because

based on the edges. So once you identify the edge you will say next grain starts and why we are able to identify this because of the change in the intensity, orientation, texture. You can find out these leaves, you can count them whatever is visible to you right. So this is the benefit when you are able to identify the edges.

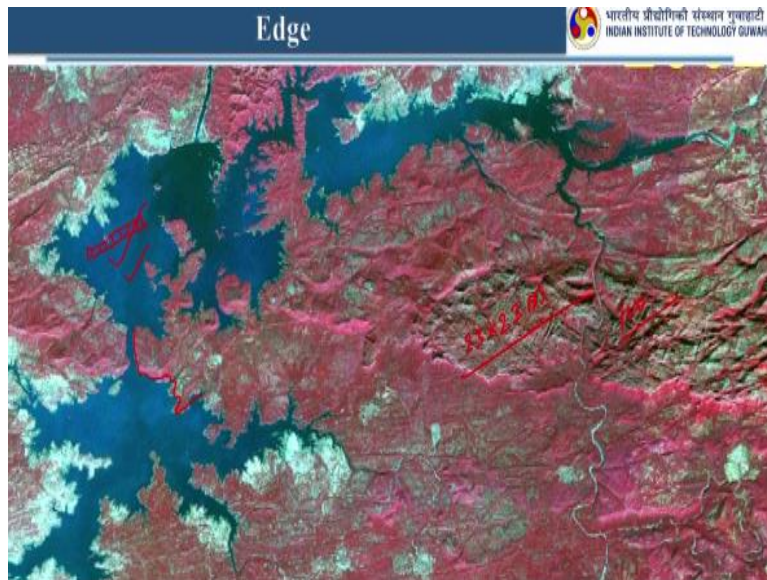
(Refer Slide Time: 11:58)



Now let us talk about some real example like remotely sense image. So here you have a satellite image and you can even count the number of trees like you can easily find out this is the tree right this is the house and all these things are basically from one house, this is stadium. So all these things we can easily count or we can easily compute the area. Why because we are able to perceive the edges.

So here on the visual analysis it is very easy when your data is of high resolution, high resolution means high spatial resolution I am talking about right.

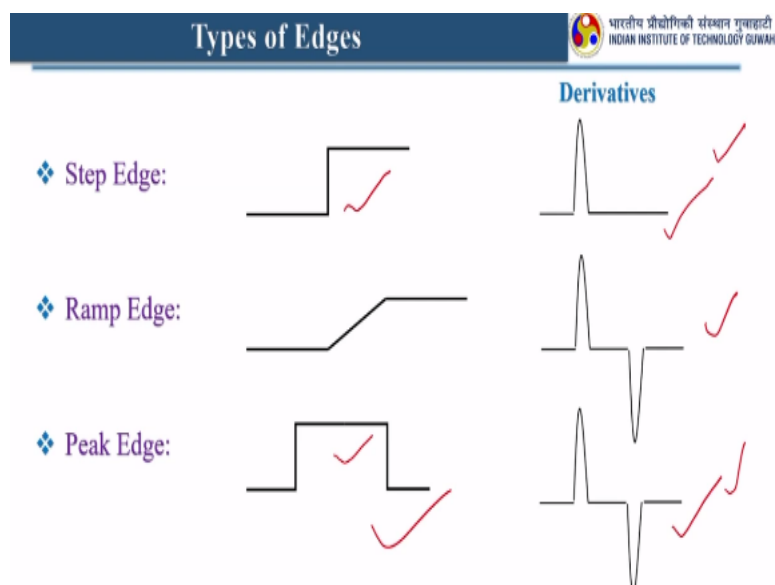
(Refer Slide Time: 12:49)



Let us see another example. Here in the low resolution data low spatial resolution data we can easily find out what is the area covered by this particular lake. Why because we are able to find out the edges and once you find out the edges you can easily find out how many numbers of pixel are lying in this particular area and then you can count and you can easily find out the area of this particular lake.

How we will do that? We know the spatial resolution that gives you the area covered by each pixel on the ground. So let us say if it is 23 by 23 meter of one pixel then suppose here there are 100 pixels belonging to this particular class then you can easily find out what will be the area of this lake. So this is very easy when you are able to make out the changes in the intensity and that will lead to identify the edges.

(Refer Slide Time: 14:08)



Now what are the different types of edges we have. So let us see first one is step edge. So in step edge this is the behavior of the change in the intensity and if you do the first derivative what will happen. In the first derivative this will be the behavior. In the second type when we have ramp edge and this will look like this and its derivative will be this. Now the third one is peak edge.

Here this is the behavior of peak edge or how we identify or how we classify edges into peak edge if their behavior is like this and if you calculate the first derivative this will be the result right. So you can see that in the normal behavior they are not able to make much difference, but if you can calculate the derivatives of these edges your change in the intensity will be very prominent.

So we will be discussing about this in next few slides. Let us see what do we mean by gradient operator. So here you can see there are different types of edges present and what is their behavior in derivatives. So here basically we are going to identify the change in the intensity with respect to gradient right. So let us see what we mean by gradient operator.

(Refer Slide Time: 15:50)

Edge Enhancement: Gradient Operator  भारतीय प्रौद्योगिकी संस्थान मुंबई
INDIAN INSTITUTE OF TECHNOLOGY

Estimates the intensity gradient in two perpendicular directions...

Gradient Directions



Vertical Gradient Horizontal Gradient Diagonal Gradient

- ✓ Two mutually perpendicular gradient detectors are required to detect edges...
- ✓ An edge can be resolved in terms of two orthogonal components...

So it estimates the intensity gradient into perpendicular direction. I will show you some photographs and then it will be very clear that how we are going to do this on the real image right. So what are the different types of gradient direction so one could be this vertical gradient where the intensity is changing in the vertical direction. So you can easily see this then next is horizontal gradient. In horizontal gradient change in the horizontal direction.

Then in diagonal gradient it can be either in this direction or this direction that is very easy to understand with these examples. So two mutually perpendicular gradient detectors are required to detect edges and edge can be resolved in terms of two orthogonal components. So this is very simple I hope you have understood this concept.

(Refer Slide Time: 16:59)

Roberts Operator

It operates on 2×2 neighborhood size to calculate gradient...

1 ✓ 0
0 -1 ✓

0 1 ✓
-1 0 ✓

A ✓ B ✓
C ✓ D ✓

Gradient Magnitude: $\sqrt{r_1^2 + r_2^2}$

r_1 and r_2 are the gradient outputs ...

$r_1 = f(A) - f(D)$, and $r_2 = f(B) - f(C)$

Direction = $\arctan(r_2/r_1)$

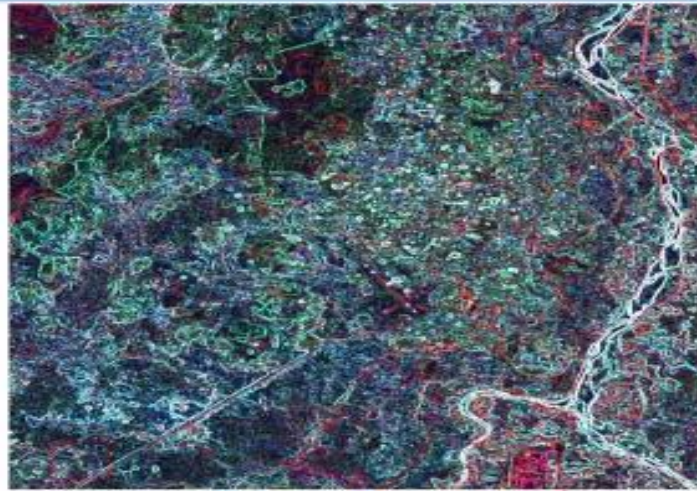
Let us see some of the gradient operators like Roberts operator. Here what it does is it operates on 2 by 2 neighborhood size to calculate gradient. So basically here we want to identify the gradient. So it is kernel sizes 2 by 2. So 2 by 2 neighborhood size I guess neighborhood is clear to you because in the last class we have covered the complete concept of this neighborhood. So here A, B, C, D are the image pixel.

And here we have this Roberts operator where in the diagonal direction it is 1 and -1. Again here it is 1 and -1 so this is the diagonal gradient right. So if you want to calculate the magnitude you can use this or here r_1 and r_2 are the gradient outputs where r_1 is $f(A) - f(D)$ and r_2 is $f(B) - f(C)$ and the direction you can calculate using arctan of r_2/r_1 right.

So this is how we are going to apply the logics that is why I had covered what we mean by edges and how we perceive these edges and these gradients are useful in identifying these edges. So Robert operator is one of them right.

(Refer Slide Time: 18:40)

Roberts Operator



Roberts Operator ✓

And this is the result of Robert operator if you apply on your satellite image or you can apply on any image right because these operator does not know that you are going to use satellite image or image captured by your phone or DSLR. It will give you edges whatever you give the input, but among this list of gradient operators you have to select appropriate one which is giving the good result.

So that depends on the analyst how efficient he or she is to identify the result or justify the results with respect to real features.

(Refer Slide Time: 19:22)

Prewitt Operator ✓

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

Prewitt 1

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Prewitt 2

Gradient Magnitude: $g = \sqrt{P_1^2 + P_2^2}$ ✓

P_1 and P_2 are the gradient outputs ... ✓

Gradient Direction: $\theta = \arctan(P_1/P_2)$ ✓

Now next operator is Prewitt. In Prewitt operator 3 by 3 neighborhood size and here you can see it is going to give you the changes in vertical direction and here it is going to give you changes in horizontal direction how because it is going to highlight this suppress this and

highlight this. So the direction you have to see how it is going to work on your image. So gradient magnitude is= this $g = \text{square root of } P_1^2 + P_2^2$.

And P_1 and P_2 are the gradient output and the direction is again same $\arctan$ of P_1/P_2 . So this is how we can use Prewitt edge detector and you can see some of the examples I have put.

(Refer Slide Time: 20:28)

Prewitt Edge Detector



More stable than Roberts, robust to noise in the image, and produces better edges.

Problem: More time consuming,

$$f'(x) = f(x+1) - f(x)$$

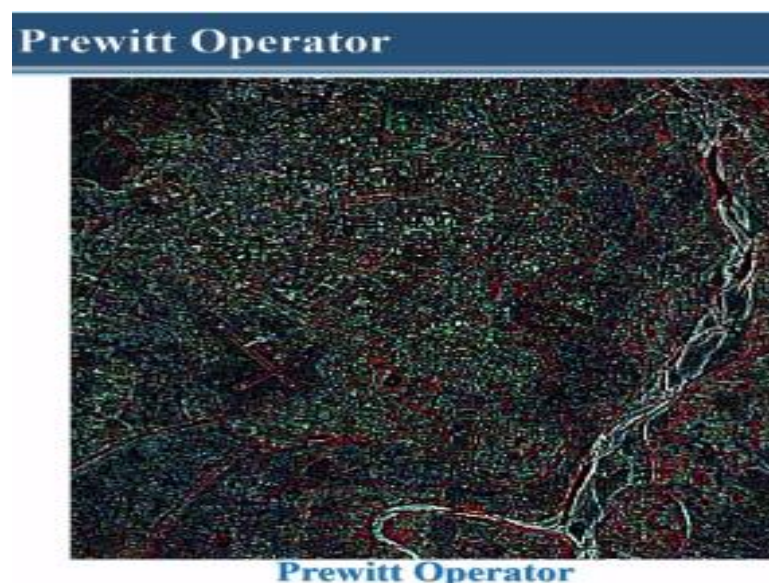
$$f'(x-1) = f(x) - f(x-1)$$

$$= f(x+1) - f(x-1)$$

X-1	X	X+1
-1	0	1
-1	0	1
-1	0	1

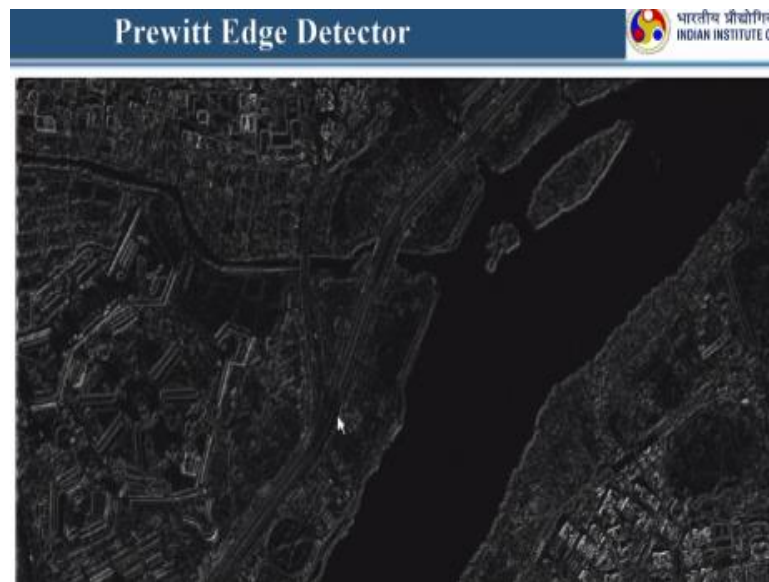
So before that I would like to tell you that it is more stable than Roberts, robust to noise in the image and produces better edges that you could able to see in the results. The problem with this Prewitt edge detector is it requires more time so that is the only problem with this edge detector. So if you do not have time then it is better to use others, but provided that should give you the good result.

(Refer Slide Time: 21:02)



So you can see here all the edges are better perceived than the previous operator.

(Refer Slide Time: 21:09)



This is another example here you can see this is the raw data and once you apply this Prewitt edge detector it will highlight all the edges very beautifully because in the previous one it was not very good because that was low spatial resolution image, but here in high spatial resolution image where you have sufficient pixel in the edges right so then it works very nice.

(Refer Slide Time: 21:38)

1	2	1
0	0	0
-1	-2	-1

-1	0	1
-2	0	2
-1	0	1

Sobel 1 (S1) Sobel 2 (S2)

Gradient Magnitude: $g = \sqrt{S_1^2 + S_2^2}$

Gradient Direction: $\theta = \arctan\left(\frac{S_1}{S_2}\right)$

Now the next detector is Sobel edge detector. In Sobel edge detector what we do is we use 3 by 3 kernel neighborhood size and then we use this magnitude and the direction. So this is the simple only thing is we only call Sobel detector when this neighborhood size is 3 by 3 and the weightage given in this particular detector is 1, 2, 1, 0, 0, 0, -1, -2, -1 right and for this Sobel 2 - 1, 0, 1, -2, 0, 2, -1, 0, 1 right.

If you have different numbers in this particular weightage, then you cannot call them Sobel edge detector because this is the advantage when you are having this particular weightage you will have more edges in the output right.

(Refer Slide Time: 22:45)

Laplacian Operator

❖ It is based on second derivatives along x and y directions...

❖ Highlight the regions of rapid intensity change...

❖ Very effective in edge detection...

❖ Two commonly used Laplacian filters are:

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

$-4 + 4$



Next operator is Laplacian and this is one of the very popular edge detector. So it is based on the second derivative along x and y direction. So here it works in x and y direction that means either it will enhance vertical or horizontal edges and it highlights the region of rapid intensity change because if you remember previous slides I have shown you if this is the type of edge and if you calculate the derivative this will enhance and this edges will be very better.

So here in case if you are using Laplacian it is using the second derivative and then because of that it highlights the rapidly changing intensity then the third point is very effective in edge detection. It is better than the previous one, but again I will say that the results of all operator you should be able to analyze and you should be able to justify then because sometime what happens for my application Laplacian works well for your application maybe Sobel will give you better results.

So it depends on what is your area what is the quality of your data and what is your objective. So there are different types of Laplacian filter and here I have listed two different types of Laplacian filter. So first one is this where there is the arrangement of the values. Second one is this where you have arrangement of value and which is fixed. So if you see -1, -1, -1, -1 so it will become -4.

And in the center it is 4. So if you see effectively we are not adding or subtracting any value for this particular neighborhood. Similarly, here this is 8 and if you sum all other values it will be -8. So -8, 8, 0 right. So this is the beauty of this Laplacian filter so you can design your own Laplacian filter, but provided that sum of that should come to 0.

(Refer Slide Time: 25:21)

Laplacian Operator



भारतीय प्रौद्योगिकी संस्थान
INDIAN INSTITUTE OF TECHNOLOGY

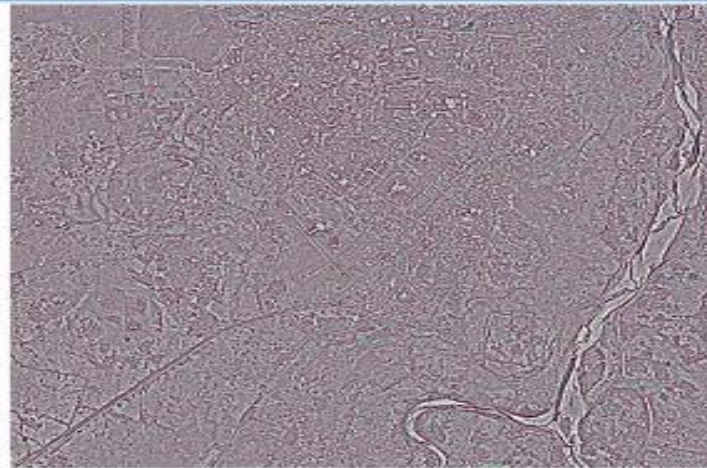
- ❖ Isotropic operator: cannot give orientation information...
- ❖ Any noise in image gets amplified...
- ❖ Faster since only one filter mask involved...
- ❖ Smoothing the image first prior to Laplace operator is often needed for reliable edges...

Laplacian operator is isotropic in nature and it cannot give orientation information. Any noise in the image gets amplified because you are using 8, 4 and those values and suppose if the noise pixel get that value that will get amplified and it is faster since only one mask involves and smoothing the image first prior to Laplace operator is often needed for reliable edges. So in case if you want to or if you are going to use this Laplacian filter or Laplacian operator first what you have to do.

You have to smooth your image so that the results of Laplacian operator will be more sensible.

(Refer Slide Time: 26:15)

Laplace Operator



Laplace Operator

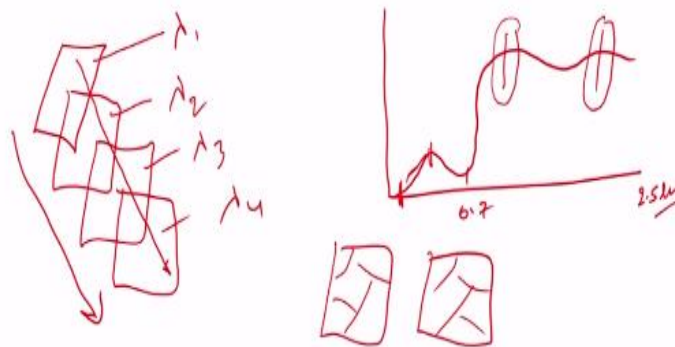
This is the example of this Laplacian operator you can see all the features, edges are enhanced and it is very clear to you right.

(Refer Slide Time: 26:24)

Principal Component Analysis (PCA)



"A statistical method which uses an orthogonal transformation to convert a set of dataset into a set of values of linearly uncorrelated variables"



Now let us see principal component analysis. So this is one of the very important method or technique which is used in image processing and here we use this principal component analysis to get the uncorrelated datasets. So what do we mean uncorrelated datasets that I will explain you slowly. So let us see what is the definition of principal component analysis. So it is a statistical method which uses an orthogonal transformation to convert a set of datasets into a set of values of linearly uncorrelated variables.

So here what it says it is a orthogonal transformation and on a set of dataset and the results will be linearly uncorrelated variables. So it is very important to understand first like what do

we mean by a set of dataset here in terms of remote sensing right. So let us see you have different images. So these images are produced by a sensor and where each band correspond to some wavelength right. So wavelength 1 maybe 2, 3 and this is 4.

So these are basically different types of images captured by a sensor in different wavelength region. So what happens if you have a vegetation you might have seen this spectra for vegetation this is 0.7 and this is 2.5 micrometer right. So if you have a band here and here and if you are talking about vegetation these two will be not be correlated, but if you see if you have a band here and here.

And if you see the information about the vegetation will be as good as which is available in the previous band. So what happens you have double information about the single object in two or three different bands. So in PCA we want to reduce the dimensionality of the data so that the output value will have very uncorrelated information. So out of this 4 maybe let us say if you have only 2 images which can give you the unique information about the objects.

So one image will give you about some x, y and z and here a, b, c. So you can easily find out what is the a, b, c, d x, y, z in your area, but in case if you do not have this kind of flexibility you will end up of analyzing all these bands. So here I have given you only 4 band example, but let us assume that you have a hyper spectral image. So in that case you have 100 or 200 bands so then imagine how much time you need, how much effort you have to put to analyze that particular image.

(Refer Slide Time: 30:00)

Principal Component Analysis (PCA)

भारतीय प्रौद्योगिकी संस्थान ७
INDIAN INSTITUTE OF TECHNOLOGY

"A statistical method which uses an orthogonal transformation to convert a set of dataset into a set of values of linearly uncorrelated variables"

- ❖ Principal component analysis (PCA), also known as Karhunen-Loeve analysis,
- ❖ Data reduction through decorrelation,
- ❖ In general, bands acquired in different wavelengths are correlated,
- ❖ Highlights/reduce the data redundancy,
- ❖ The number of output principal components images will be less than or equal to the number of input bands,
- ❖ Highlights specific features using their responses in different bands...

So principal component analysis is used for the data reduction through the decorrelation. This is very, very important. Data reduction through decorrelation. In general band acquired in different wavelength are correlated I hope you have understood this because I have covered just now. It highlights or reduces the data redundancy. Data redundancy means if one information is coming in all the bands then what happens it is redundant information is coming again and again.

So you want to reduce that frequency of the information in different bands. So we will be using different PCA. The number of output principal component will be less than or equal to the number of inputs bands. This is very important when you are going to apply this principle component analysis technique on your image to find out the unique information the output image or the number of output image that can be less than or equal to your number of input bands.

It cannot exceed that number input bands. This is very important we will see it highlight specific feature using their responses in different bands. So what happens so what happens here it highlights specific feature that is the flexibility you have when you are applying this PCA and how it will highlight because it will identify the similar information available in different bands and then it will identify and it will highlight those features.

(Refer Slide Time: 31:51)

Principal Component Analysis (PCA)	
	
Assumptions	Advantages
❖ Linearity,	❖ Reduced complexity in images,
❖ Sufficiency of mean and variance,	❖ Low noise sensitivity,
❖ Orthogonality of principal components,	❖ Makes data easy to interpret,
❖ Large variances have important dynamics.	

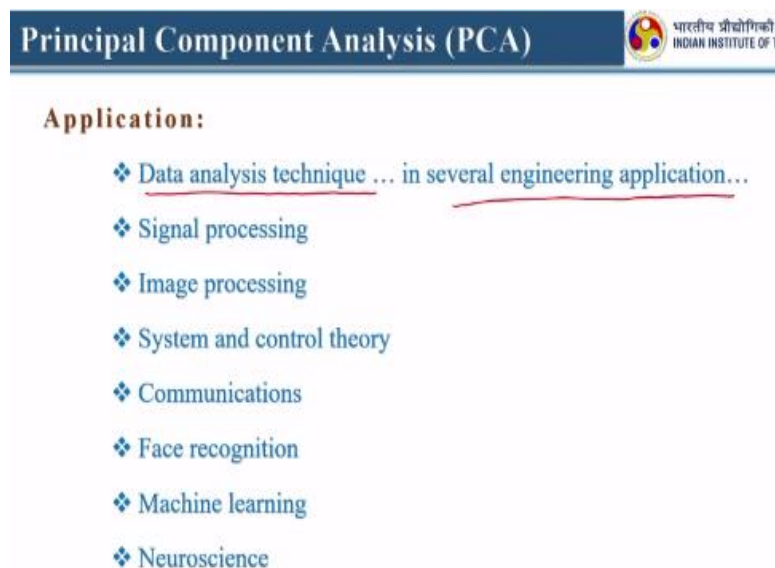
So what are the different assumptions and advantages with PCA let us see. So it assume the datasets have linearity or correlation, sufficiency of mean and variance, orthogonality of principal component that you will understand later and large variance have important

dynamic because here we are talking about the variance. Variance means if variance is high among two dataset that means both the datasets are not correlated and they are having unique information of their own.

And the advantage with this particular PCA method is to reduce the complexity in image because I told you if you have 4 band image then it is very easy to interpret, but if you have 100 band image or 200 band image it is very difficult and you have to put lots of effort and the low noise sensitivity. So what happens when you apply this PCA and when you get the results your output image will have very low noise which was available in the input data.

And it makes very easy to understand the output datasets or to interpret the features in the output image. So these are the advantages you have when you apply PCA on your remotely sensed images.

(Refer Slide Time: 33:28)



The slide features a dark blue header with the title 'Principal Component Analysis (PCA)' in white. To the right of the title is the logo of the Indian Institute of Technology (IIT) and the text 'भारतीय प्रौद्योगिकी' and 'INDIAN INSTITUTE OF T'. Below the header, the word 'Application:' is written in brown. A list of applications follows, each preceded by a blue diamond icon. The first item, 'Data analysis technique ... in several engineering application...', is underlined in red. The other items are: 'Signal processing', 'Image processing', 'System and control theory', 'Communications', 'Face recognition', 'Machine learning', and 'Neuroscience'.

Principal Component Analysis (PCA)

Application:

- ❖ Data analysis technique ... in several engineering application...
- ❖ Signal processing
- ❖ Image processing
- ❖ System and control theory
- ❖ Communications
- ❖ Face recognition
- ❖ Machine learning
- ❖ Neuroscience

So let us see what are the different application of this principal component analysis. So here this is the first one so this is basically used in data analysis right and you can name the application where you can use this. So let us talk about civil engineering in civil engineering, water resources, earth sciences everywhere you can use transportation, everywhere where you want to mine the data you can use this principal component analysis.

So that is why in several engineering application you can easily find out some examples where PCA have been used and in signal processing, image processing, system and control theory communications, face recognition, machine learning everywhere PCA has been used

like very efficiently. So these are few examples where PCA have been used. Now let us see how we can apply this PCA on our image right.

(Refer Slide Time: 34:39)

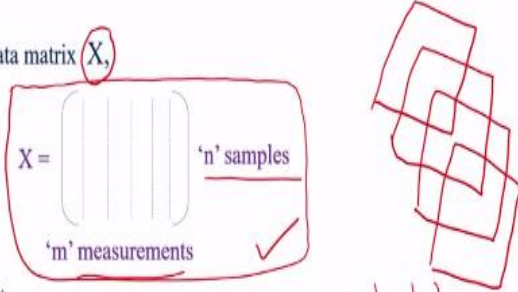
Principal Component Analysis (PCA)



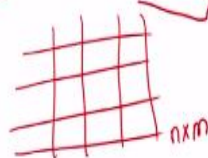
भारतीय प्रौद्योगिकी संस्थान गुवाहाटी
 INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

Background of PCA:

- ✓ Suppose we have a data matrix X ,



- ✓ Variance of an attribute:

$$\text{var}(A_i) = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n-1)}$$


So to understand this PCA let us assume we have a data matrix which is X right. So X matrix will have some samples and measurements. So here in this case we are using n sample and m measurements right and you can easily find out what is the variance of an attribute using this formula and if you have this let us understand how did you get this particular matrix. So you have a image again I am going to take this 4 band example.

So you have 4 bands captured by a sensor and which is in different wavelength region and we assume there is some correlation that is why we want to apply this PCA to segregate this repeated information and we get unique information in our output bands right. So here if you just import this into excel or maybe in ASCII format using MATLAB or C what you will have you will have a matrix right and matrix will have number of elements.

So here in this case we assume line n sample and m measurement. So your matrix will be n by m right and here we are calling it x . So this is what I wanted to say here right. So we assume that X is equal to matrix and where n samples and m measurements have been considered.

(Refer Slide Time: 36:28)

Principal Component Analysis (PCA)



- ❖ Remotely sensed images are high in dimension and difficult to analyze.
- ❖ Images are produced by sensors sensitive in different wavelength regions.
- ❖ Acquired Images are correlated (similar information in two or more bands) to each other.
- ❖ Not all data sets are independent of each other.

✓ Covariance of two attributes:

$$\text{cov}(A_1, A_2) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{(n-1)}$$

So remotely sensed images are high in dimension and difficult to analyze that already I have explained to you images are produced by sensors, sensitive in different wavelength region because all those 4 bands they were captured in different wavelength from a single sensor, but their information available with them is different right. So acquired images are correlated why because similar information in two or more bands can be present.

So this is what motivated us to apply some segregation technique so we can easily find out unique information from our image right and not all datasets are independent of each other. So if you plot band 1 versus band 2 you will find it is coming like this right. So DN value of band 1 and DN value of band 2 right. If you plot them so band 1 and band 2 basically we are starting from green and then red.

So if you have green and red information or let us assume that SWIR vegetation right. We targeted vegetation in the previous example where vegetation is having equal reflected energy in infrared region. So if you are having this and if your image is mostly covering the vegetation what will happen this correlation will be very, very good. So this is what I wanted to explain here not all datasets are independent of each other.

So the variance you can calculate for one variable, but covariance can be calculated for two different variables. So if you have covariance you can easily find out whether both the datasets are varying together and they are independent of each other so that is the importance of this covariance here.

(Refer Slide Time: 38:49)

Principal Component Analysis (PCA)



$$X = \begin{bmatrix} & \\ & \\ & \end{bmatrix}; n \text{ samples, and } m \text{ measurements}$$

Assumption: All the measurements are not independent (i.e. correlated).

Principal Component Analysis (PCA) is the Eigen decomposition of covariance matrix $(X^T X)$.

$$X^T X$$

$m \times m \text{ matrix}$

Eigen decomposition of covariance matrix $(X^T X) \rightarrow$ W (Eigenvectors)
 λ (Eigenvalues)

So again let us go back and let us start with matrix X when you have n and m measurement and here what we assume all the measurement are not independent. So that means we assume images captured in different wavelength are having some kind of correlation and that we want to remove. So principal component analysis is the Eigen decomposition of covariance matrix that is X transpose X.

So here X transpose X will have m into m size right and once you decompose this covariance matrix X transpose X into Eigen vector W and Eigen values lambda right.

(Refer Slide Time: 39:44)

Principal Component Analysis (PCA)



$$X \cdot W = T$$

$$(n \times m) \cdot (m \times m) = (n \times m)$$

- ✓ where, W is set of eigenvectors or loadings (column values) and T is score.
- ✓ Each column of W is a Principal Component (PC).
- ✓ Largest eigenvectors column (PC) corresponding to the eigenvalues will be the PC – 1.
- ✓ Ordered columns by the values of λ .
- ✓ Elements of eigenvector matrix (column) are the weightages assigned to the input bands in PCs.
- ✓ Once the data is transformed and ordered, PCs can be selected to represents the unique measurements..

So then you can write it like matrix X multiplied with W=T. The size of X is n into m size of W is m into m and T is basically n into m and T is the score. So each column of W here what we have is basically a principal component. So once you have the principal component and X

is basically let us n into m and W is m into m. Then the each column right so each column is basically your principal component that I will show you.

So the largest Eigen vector column PC corresponding to the Eigen value will be the PC1. So here the PC1 will be the first column right. So principal component 1 will be first column of W and this W will be ordered by this lambda. So the more lambda value will be in the first column so that will be called as principal component 1. Element of Eigenvector matrix are the weightage assigned to the input bands of PCs right.

So elements of eigenvector matrix so let us say you have this W matrix of m by m and first column is basically W1 and this is nothing but the PC1 and PC1 will have maximum eigenvalue and the elements of the PC1 or the W1 are nothing but the weightage assigned to the input band in PCs. So this is what we want to understand. Now once you have that you can easily find out what is the weightage of band 1, band 2, band 3, band 4 in my first principal component.

And because of that only we are getting this enhanced feature or the unique information in different principal components. Once the data is transformed and ordered PC can be selected to represents the unique measurement that we will see in the example.

(Refer Slide Time: 42:31)

Principal Component Analysis (PCA)



$$W = \begin{bmatrix} \vdots & \vdots & \vdots \\ W_1 & \cdots & W_r \\ \vdots & \vdots & \vdots \end{bmatrix}$$

- ❖ Principal component corresponding to the largest eigenvalues is the first one.
- ❖ PC-1 will have high eigenvalue.
- ❖ So a truncated W_r can be formed with first 'r' columns.
- ❖ Tr is the truncated representation of the data.

$$\begin{matrix} X \cdot W_r \\ (n \times m) \cdot (m \times r) \end{matrix} = \begin{matrix} T_r \\ (n \times r) \end{matrix}$$

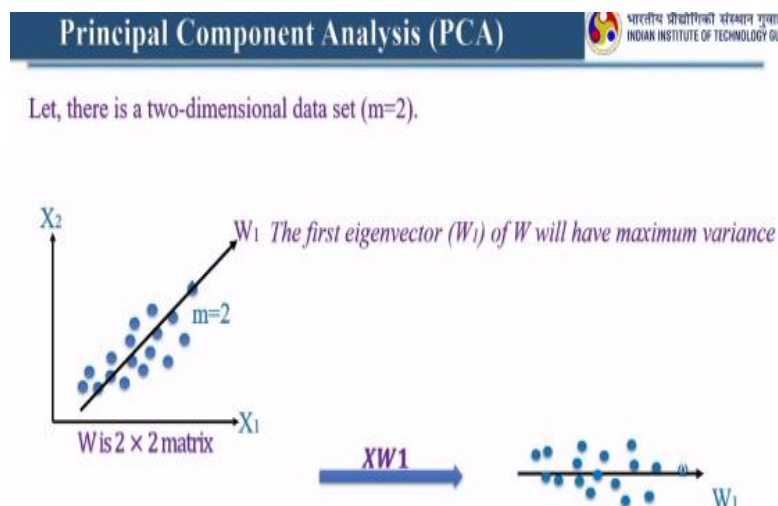
$$W_r = \begin{matrix} \text{[Diagram of a matrix with r columns]} \\ \text{r columns (which contains maximum Information)} \end{matrix}$$

So here you can see this is the W and this W1 to Wr and each element of this W right W1 are the weightage assigned to different bands. Principal component corresponding to largest Eigen value is the first one. PC1 will have high Eigen value that is clear to you. Now there is

one more concept called truncated W_r that means you want to reduce the number of output in output data right.

So once you have this W and it is starting from W_1 to let us say W_n , but you want in output only W_r . So what will happen so this is the W_r and this is r column of this W_r matrix right and Tr is the truncated representation of the data. So here $Tr = X$ into W_r and W_r here there is a modification that you will use m into m here you will use instead of m into m you will use m into r . So your output will be r column.

(Refer Slide Time: 44:04)



So let us assume there is a two dimensional dataset. So $m=2$ and here you can see that X_1 and X_2 are plotted here and it is having very good correlation which we do not want. We want bands or the images to have unique information which is very difficult from the raw data. So what we do we calculate the W_1 that is eigenvector and it will be like in this direction. So here what we do we will use some angle to rotate this information.

Because I hope you are familiar with this, this is positive correlation this is negative correlation and if your datasets are distributed then it does not have any correlation. So what we will do we will uncorrelate this dataset by transforming it or by changing the axis and rotating it.

(Refer Slide Time: 45:16)

Principal Component Analysis (PCA)



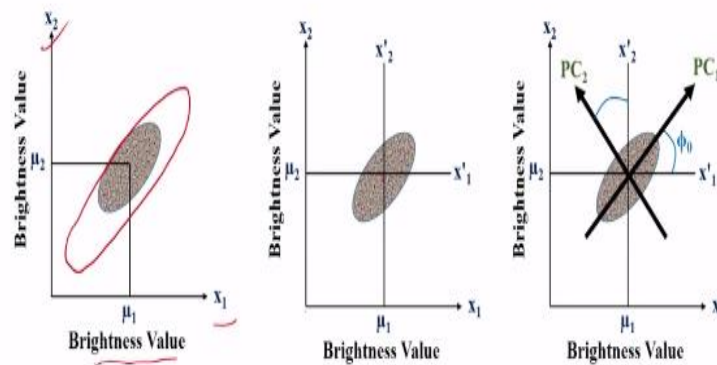
	W ₁ PC-1	W ₂ PC-2	W ₃ PC-3	W ₄ PC-4	W ₅ PC-5	W ₆ PC-6
Band-1	0.22	-0.06	-0.31	0.72	-0.57	0.1
Band-2	0.37	-0.06	-0.36	0.35	0.78	-0.05
Band-3	0.69	-0.28	-0.29	-0.55	-0.24	-0.05
Band-4	0.38	0.92	0.1	-0.03	-0.04	0.03
Band-5	0.42	-0.25	0.8	0.25	0.03	-0.24
Band-7	0.12	-0.1	0.19	-0.03	0.09	0.96
Variance (%)	80.56	16.42	2.41	0.52	0.08	0.03

So here once you apply this principal component analysis you will have this kind of tabular information once we save the Eigen value and eigenvector. So here this is W₁, W₂, W₃, W₄ and W₅ and W₆ and these are the bands. So as I told you that W₁ that means that is PC1. So PC1 elements are nothing but the weightage assigned to different bands right. So if you want to identify or if you want to calculate principal component 1 you can easily calculate based on the Eigenvalue and Eigenvector.

So what is the role of Eigen value here if you see this is maximum here. So the maximum Eigen value will be ranked first so this is the based on Eigen value this has been ordered this complete matrix has been ordered and this is the 1st PC this is 2nd PC then 3rd, 4th, 5th, 6th, but in case if you want to apply truncated W_r then you can have out of 6 you can have only 2 or 3 principal component in your output right.

So you have seen how it works right, but what is the physical meaning that you need to understand very carefully.

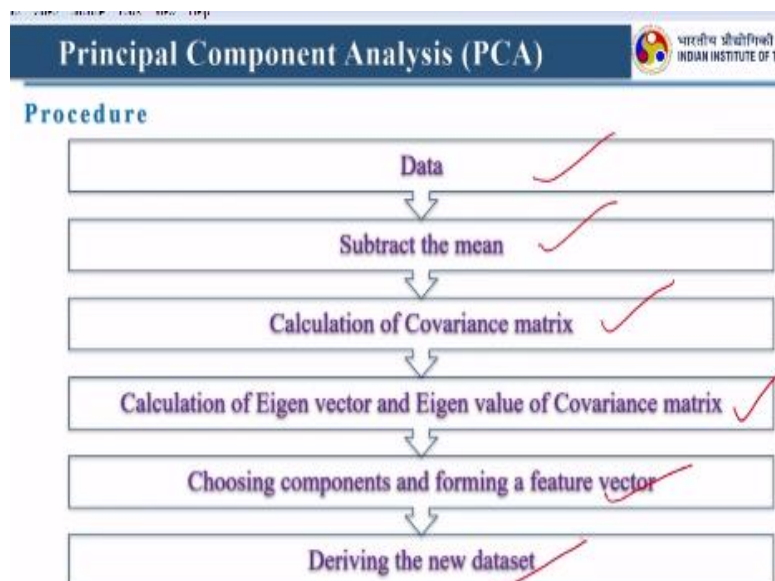
(Refer Slide Time: 46:48)



So let us see you have a brightness value of x_1 and x_2 and if you plot it is coming and it is giving you very good correlation right. So what we do we change the axis, we shift this axis to the x center of this particular plot and then we rotate so that it will be uncorrelated right. So this is what we are doing in principal component analysis and this is how we are deriving meaningful information in few bands from maybe 100s or 1000s of band right.

So this is very important not only in terms of remote sensing, but in any kind of data analysis you can easily apply this principal component analysis.

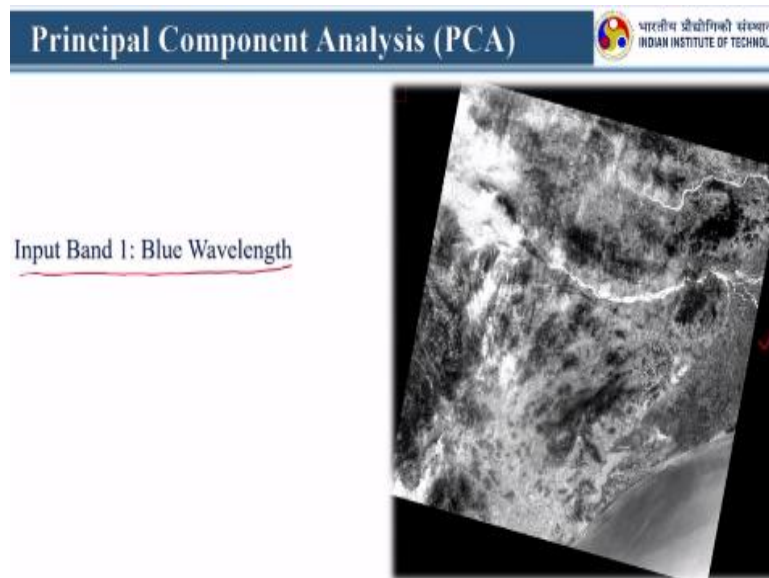
(Refer Slide Time: 47:43)



So this is the procedure how we apply. So we have a data then subtract the mean, calculate the covariance matrix, Eigen value, eigenvector and choosing components and forming a feature vector and deriving the new dataset. So this already we have seen. Now let us see

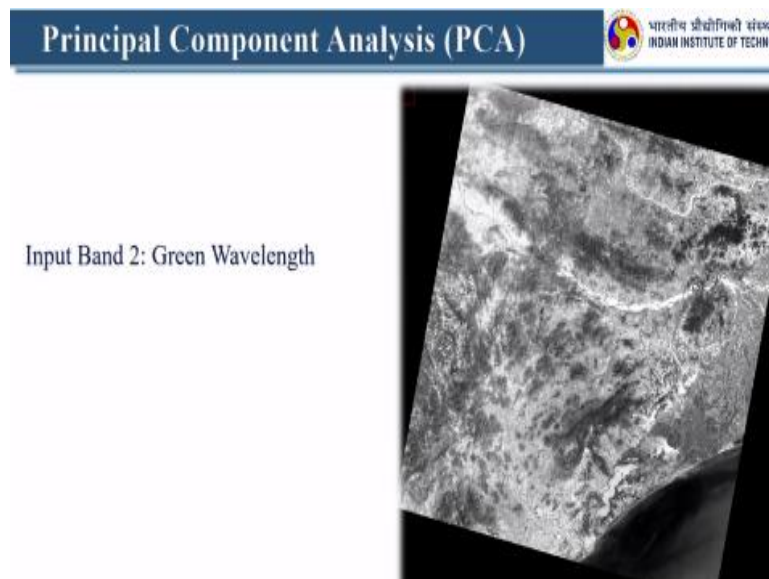
what is the effect of this PCA on the image right.

(Refer Slide Time: 48:10)



So in the previous example I have taken 4 band right, but here I will be using 6 band of Landsat data and then how this PCA is changing this input right. So here this is the first band of input which is blue band. Now you just see this particular image how it is going to change or the information is getting changed when you are having different bands captured in different wavelength. So blue band is this.

(Refer Slide Time: 48:46)



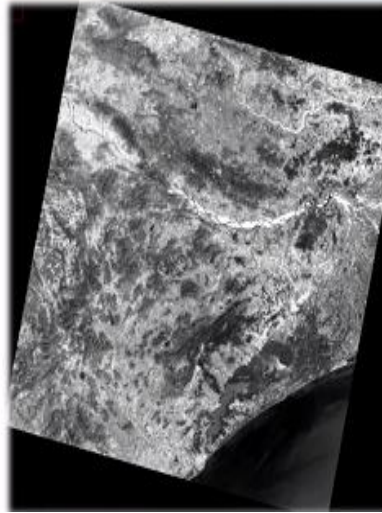
Green band.

(Refer Slide Time: 48:48)

Principal Component Analysis (PCA)



Input Band 3: Red Wavelength



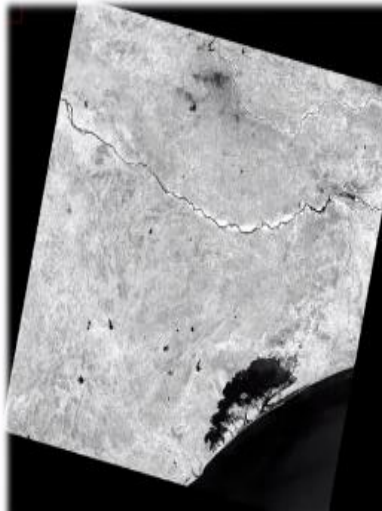
Then red band.

(Refer Slide Time: 48:50)

Principal Component Analysis (PCA)



Input Band 4: NIR Wavelength



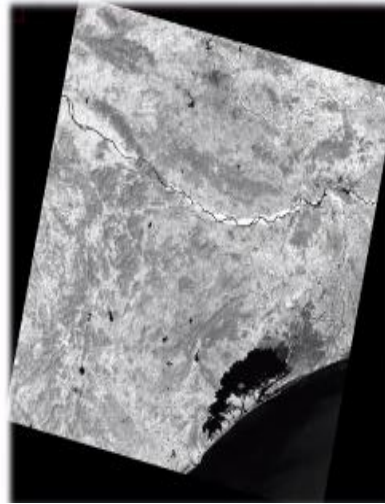
Then NIR band.

(Refer Slide Time: 48:52)

Principal Component Analysis (PCA)



Input Band 5: SWIR Wavelength



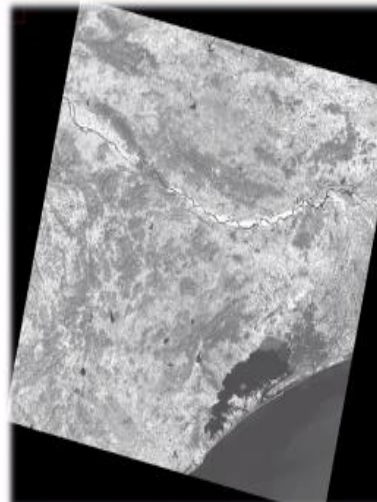
Then SWIR band.

(Refer Slide Time: 48:54)

Principal Component Analysis (PCA)



Input Band 6: SWIR Wavelength



Then this is another band in SWIR. So you might have noticed like there is a small change in this image right whenever we are changing the wavelength right. So what we will do we will consider them as matrix and we will apply this PCA and then we will see how it works.

(Refer Slide Time: 49:21)

Principal Component Analysis (PCA)

Input Image: FCC



So this is the false color composite of the input data this is how it look like the area. Now we have generated Eigen value and Eigenvector matrix and as we know that W_1 is PC1 and this is the composition of weightage for PC1 and you can see that there are different weightage assigned to different bands so somewhere it is negative, somewhere it is positive. So the elements of this W_r are known as loadings.

So these loadings are nothing but the weightage assigned to individual bands. So sometimes whenever you are reading book or maybe related to this some journal paper you may find that people are using loading as a indicator or to highlight this particular weightage. So once we have this principal component analysis of the input data and then by going through all this methods right.

(Refer Slide Time: 50:36)

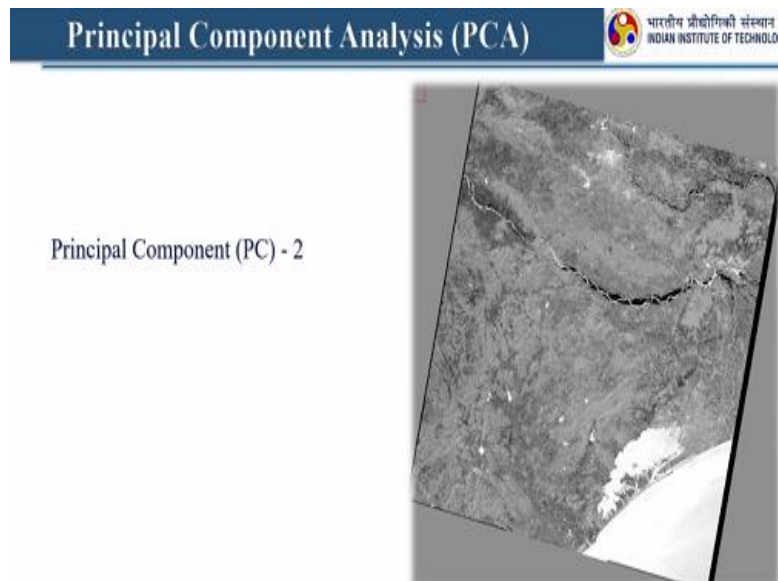
Principal Component Analysis (PCA)

Principal Component (PC) - 1



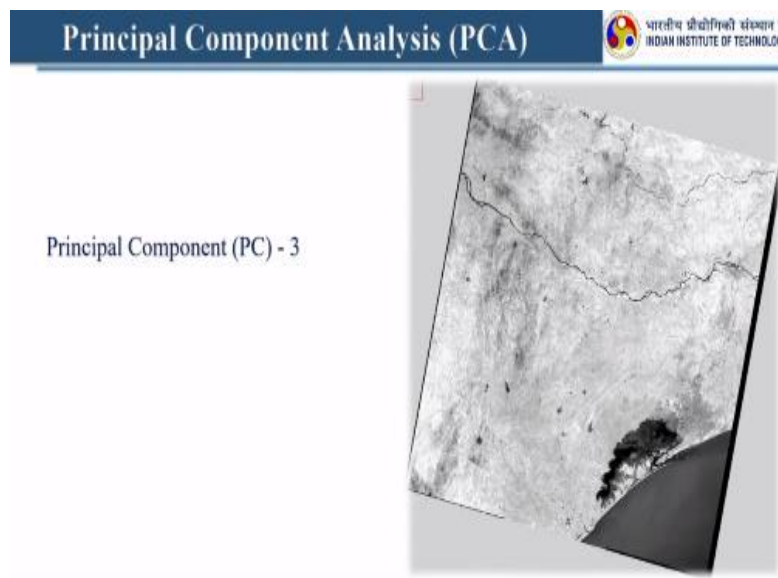
We will have our principal component 1. Then first principal component this is the information you can see.

(Refer Slide Time: 50:45)



This is the second principal component.

(Refer Slide Time: 50:47)



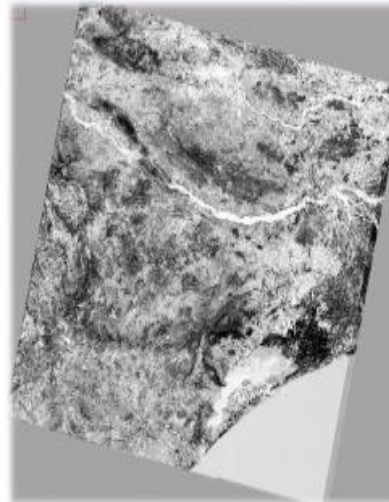
This is the third principal component.

(Refer Slide Time: 50:49)

Principal Component Analysis (PCA)



Principal Component (PC) - 4



This is the fourth one. So here I have given you only 4 principal component when I had 6 input data right. So what I did I use the truncated Wr. So what happened in output the number of output images are less than your input image, but it can be equal if you want you can generate all the principal components for 6 bands.

(Refer Slide Time: 51:20)

Principal Component Analysis (PCA)



	W_1 PC-1	W_2 PC-2	W_3 PC-3	W_4 PC-4	W_5 PC-5	W_6 PC-6
Band-1	0.22	-0.06	-0.31	0.72	-0.57	0.1
Band-2	0.37	-0.06	-0.36	0.35	0.78	-0.05
Band-3	0.69	-0.28	-0.29	-0.55	-0.24	-0.05
Band-4	0.38	0.92	0.1	-0.03	-0.04	0.03
Band-5	0.42	-0.25	0.8	0.25	0.03	-0.24
Band-7	0.12	-0.1	0.19	-0.03	0.09	0.96
Variance (%)	80.56	16.42	2.41	0.52	0.08	0.03

Feature oriented Principal Component Analysis (FPCA)

$$PC6 = 0.10(Band1) - 0.05(Band2) - 0.05(Band3) + 0.03(Band4) - 0.24(Band5) + 0.96(Band7)$$

But in the later stage you will find the information or the variance is very, very low. So the whole image will have similar information so there is no point of keeping those bands so you can truncate them right. Now there is a concept called feature oriented principal component analysis. In feature oriented principal component analysis let us assume that you have a 7 band input right.

And we assume that my x target is going to have or is having a very good absorption feature

in band 7 right. So I need to find out from this Eigen value eigenvector matrix which principal component or which W_1 , W_2 and W_7 right which one is going to have maximum or minimum weightage of band 7 so that particular component can be used to highlight my object.

So this is one example when I am having band 7 as input so 7 band as input and output will be PC7, PC1, PC2 like that we will have 7 output principal components and out of that principal component 6 is having this composition. So 10% weightage to band 1, 5% to band 2, 5% to 3, 3% to 4, 24% to 5 and 96% to band 7. So that means for me does not matter what is the variance.

But I will go with the principal component 6 because that is going my highlight my object because if you compare this weightage with all other bands it is quite high. So that means does not matter what is the variance, but if I want to highlight some object which is very sensitive in band 7 of my observed data then I have to find out accordingly which principal component is sensitive to my particular band that is all. So that is all for today.