Fluid Mechanics
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Lecture No. – 08
Conservation of Mass

Welcome all of you to this fluid mechanics course. Today, we are going to plan about conversion of mass. As you could remember it, then, in the last class we discussed about Reynolds transport theorem. So, the same Reynolds transport theorem will be used to derive mass conservation equation which is an important equation for any fluid flow problems.

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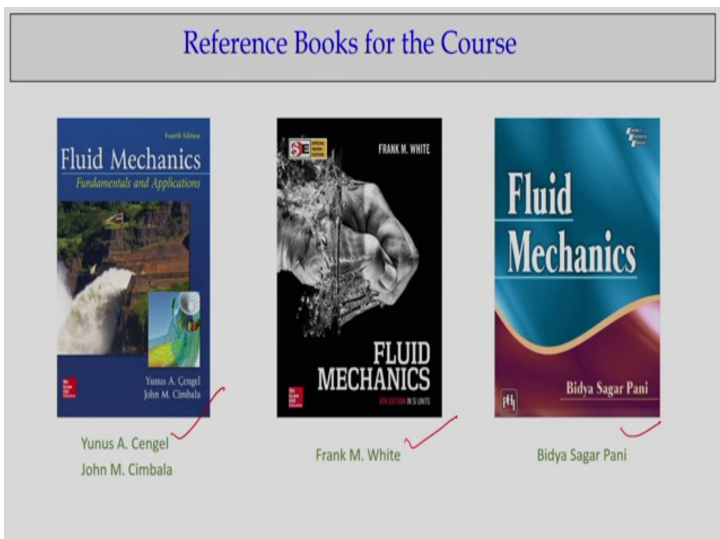

Now, again, I can repeat that you can have this reference books. Cengel and Cimbala, Fluid Mechanics, Fundamentals and Applications. Then, F.M. White, Fluid Mechanics, mostly the derivations and partly we have been following this book. And this Fluid Mechanics by Prof. Bidya Sagar Pani.

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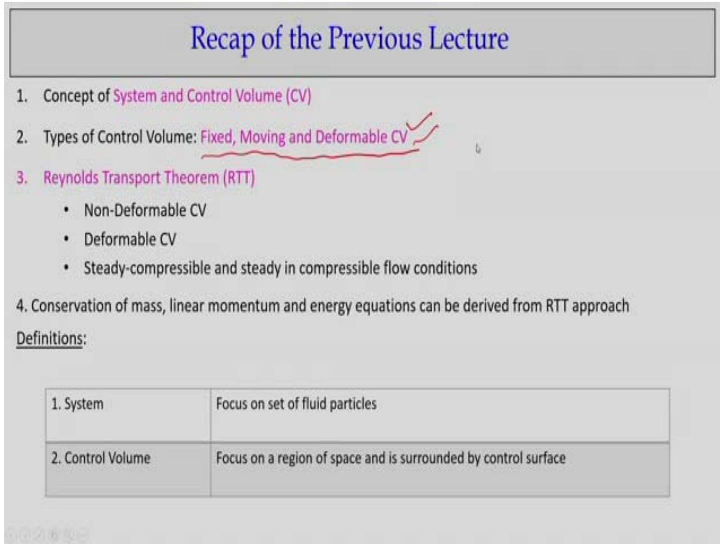

Let us come back to what we learned in the last class. If you remember it, I discussed very thoroughly what is the difference between systems and the control volume. Mostly in fluid mechanics we follow the control volume aspect. That is the reason we need a relationship between the system and the control volume. The Reynolds transport theorems establish the relationship of conversions of mass from system to control volume.

So, that is the strength of the Reynolds transport theorems and, as already I discussed, you can have three type of control volumes, fixed control volume, that means with reference to the space it remains at the same location. As the time goes, it does not move it. That is what fixed control volume is. It has definite control surfaces. The other one is moving control volume, like the ship movement in a sea or the river. The ship and the adjacent fluid can be considered as moving control volume.

And we solved many of the problems with moving control volume, considering this control volume is moving with a particular velocity. Not only that, the shape of the control volume can change. That was the deformable control volume. The shape of the control volume can change with respect to time, then we call it deformable control volume. So, there are three types of control volume, fixed control volume, moving control volume, and deformable control volume.

So, in deformable control volumes, the shape of the control volume changes with time. In case of moving control volume, the control volume moves with a particular velocity. That is what is moving control volume. And fixed control volume, as you can see it remains stationary at that space regions. Today, we will talk about how we can derive the conservation of mass.

As I said earlier, when you have the control volume, through the control surface there is, mass influx coming into the control volume, going out from the control volume. Similar way, momentum flux comes into the control volume, also goes through the other surface as momentum flux going out from that. Similar way, we can think this energy flux comes into the control volume and goes out of this thing.

So, that way the conservation of mass, linear momentum, and energy equations, that is what we will discuss in more detail. Today, I will focus on mass conservations only and that is the point we will discuss.

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So, as I have given very briefly Reynolds transport theorems and moving and deformable control volume, conservation of mass we will derive for simple case to complex problems, how we can write conservation of mass. Then, we will talk about some examples, real life examples we will take and how the appropriate choosing of control volume and the control surface can solve many of our problems.

That is what I will demonstrate, and the chosen examples we will tell to you, and end of the day, I will summarise whatever the lecture component today I will deliver.

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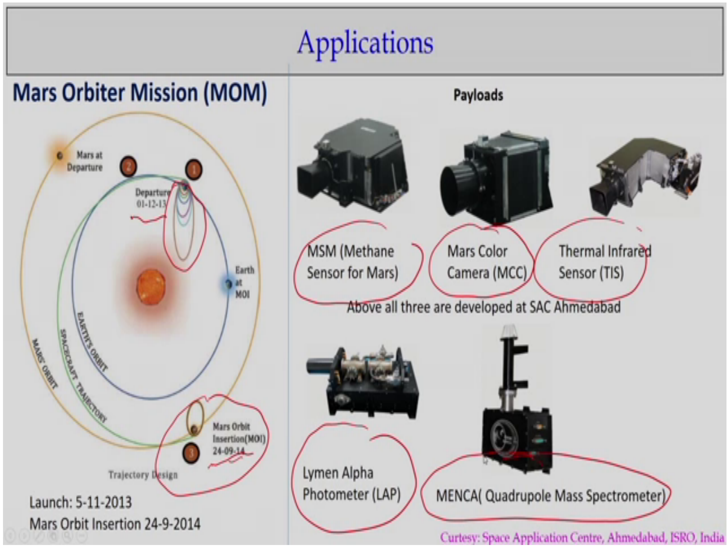

Now, let us go to very interesting applications. As you know it Indian Space Research Organisation launched the satellite which is called MOM programmes, that means Mars Orbiter Mission programme. But if you look at this, it is the fluid flow problem. If we look at the trajectory which is started from the earth, and if you look at the trajectory part here, how this trajectory goes on changing, which was launched on December 1, 2013, okay?

It is to reach the Mars orbit September 24, 2014. If you look that all satellites moving in the atmosphere, the space, so accurately, considering your drag force and all the things, so accurately the trajectory was designed to reach Mars orbit after almost more than one and half years. So, the fluid mechanics knowledge what we have today we can design the trajectory from earth to the Mars orbit. What I am trying to say is that fluid mechanics today is expanded in a very, very high applications way.

So, what we are to study in this course is introductory levels. And, if you have interest, then really we can solve these problems which we have done for Mars orbiter missions, launching the satellites from earth to reach up to Mars orbiter locations. And you look at the payload or the sensors, whatever was there in the Mars orbiter missions, starting for methane sensors, the colour cameras, infrared sensors, or alpha photometers, and quadrupole mass spectrometer.

I am not going to discuss each sensor what it does, okay? If you are interested, you can get a lot of literatures available in the internet but what I am trying to say is that India has capability or Indian scientists have the capability to design a satellite tracking systems with the sensors, starting with launching from the earth's surface and it can be reach up to the Mars orbiter.

Not only that, these sensor, whatever design it is, that also can get the data about Mars and that is what is the target, that is what succeed. So, all these knowledge of fluid mechanics, the advanced mathematics, all this help us to reach this mission's programmes, what is done by Indian Space Research Organization. With this note, let us talk about the fluid mechanics part.

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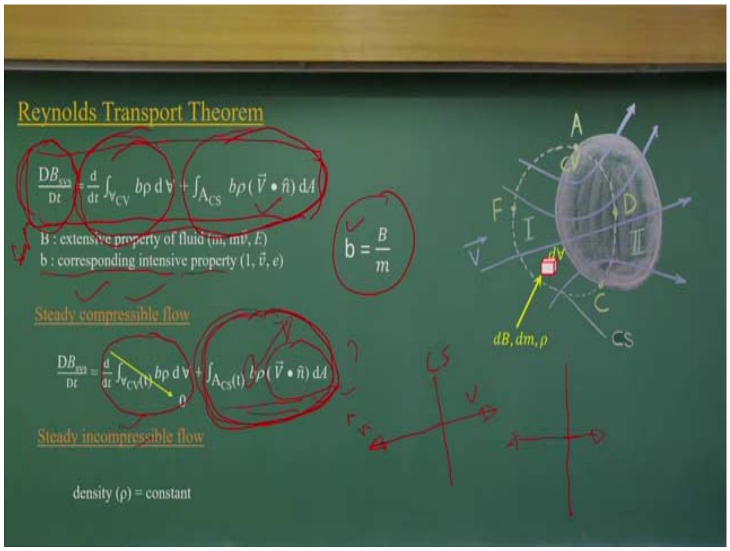

Coming back to the Reynolds transport theorems, as I derived in the last class, one is the system level equations and the other is the control volume level equations.

$$\frac{\mathrm{D}B_{\mathrm{sys}}}{\mathrm{D}t} = d/dt \int_{\forall_{\mathrm{CV}}} b\rho \, \mathrm{d}\forall \; + \int_{A_{\mathrm{CS}}} b\rho \, (\vec{V} \bullet \hat{n}) \, dA$$

B : extensive property of fluid (m, m$\vec{v}$, E)

b : corresponding intensive property (1, $\vec{v}$, e)

b = $\frac{B}{m}$

This is intensive property. Extensive properties are mass, momentum, and energy.

Intensive properties are unit value for the mass, for the momentum, the velocity vector for energy is also e, the specific energy, energy for unit mass. So, if you look at this equation, let us look it. This is the system level equations. This is what is at the control volume level. So, one talks about how this particular mass or momentum flux is crossing through the control surface. The net outflow is going through this control surface.

That is what it is and this is what it is talking about. The change of the extensive properties within the control volume with respect to time, one is with respect to control surface, one other is with respect to storage within the control panel, how that extended property is changing with time, that is related, the system and the control volume level. Now, let us have two simplifications or assumptions which is quite valid for the fluid flow problems for most of the times. That is steady and compressible flow.

$$\frac{\mathrm{D}B_{\mathrm{sys}}}{\mathrm{D}t} = d/dt \cancelto{0}{\int_{\forall_{\mathrm{CV}}} b\rho \, \mathrm{d}\forall} + \int_{A_{\mathrm{CS}}} b\rho\,(\vec{V} \bullet \hat{n})\, dA$$

That means the density vary but it is steady. So, once it is steady, if you look at this term, it becomes 0. Only what remains is the surface integration part which is quite easy now. If you take a steady problem or if you can visualize the problems, it can be solved as a steady problem. Then you need to go for complicated volume integrals and partial derivative or total derivative of these things.

You can focus on only the control surface and over this control surface we can have surface integrals and we can equate this equation. But when steady and incomprehensible, that means this density becomes a constant quantity. So, density can come out from this surface integral. That means only the b and velocity, the scalar products, the velocity and unit vectors, that part will do the scalar part and surface integrals.

Now, if you look if I have the velocity vectors, okay? Let us consider I have a control surface like this. If the n vector is like this, the perpendicular vector to the control surface is like that, if I consider one simple case, the velocity is either parallel or opposite, if the same direction or opposite direction, then my dot product is very easy, either v positive or v negative values.

What it indicates is that if you consider the control surface in such a way that your normal vectors and your velocity vectors either will have the same direction or opposite direction of that. So, if that is the condition, your scalar products will give it v positive or v negative depending upon $\cos\theta, 0^{\circ}$ or 180° degrees. So, it is very easy to choose appropriate control surface and find out what could be appropriate normal vector to that control surface.

If I know that, that means that should be aligned with my velocity vectors. So, once I know these velocity vectors I can have a approach, make my control surface in such a way it should be perpendicular to that or the unit vectors or perpendicular vectors that should equally have the same line or the opposite direction. The tricks what we follow is to simplify the problems instead of doing the scalar product.

We can do it, it is not a difficult task, but we can simplify that by taking appropriate control surface. That is to be remembered. Appropriate control surface taken such a way that this scalar product can be simplified. So, once it is simplified, only we need to do surface integrals of the velocity and the dA and the b value. Now the problem is quite simple for steady incomprehensible flow.

density (ρ) = constant

Mostly my lectures will talk about this part only.

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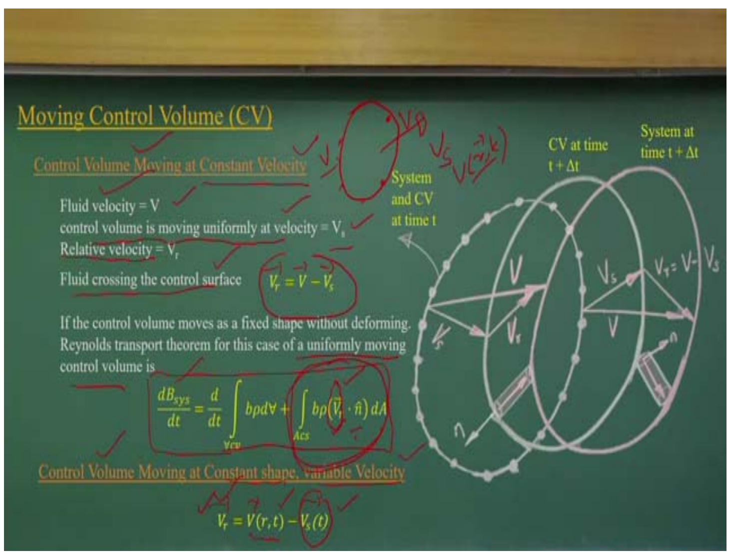

Now coming to a very interesting **(()) (13:15)** if I consider moving control volume like a ship which is moving with a velocity V. So, then, the concept will be the same. Only here we will talk about the relative velocity component. That means there will be fluid velocity V and control volume is moving with a velocity, let it be V_s. So, we need to know it, what will be the relative velocity V_r crossing through this control surface.

Fluid velocity = V

control volume is moving uniformly at velocity = V_s

Relative velocity = V_r

Fluid crossing the control surface

$$V_r = V - V_s$$

For example, if the V is 5 meter per second and V_s is moving in the same direction with 3 meter per second, the relative velocity will be 2 meter per second. So, that way find out what will be the relative velocities coming into a moving control volume. That is what generally you do in any physics problem. The same way where the control volume is moving with velocity V.

That means the control volume is moving with a velocity V s and the fluid velocity is V. So, we have the relative velocity which is $V_r = V - V_s$. You can represent it as velocity vector form. Then, this can come as vector form which will be complicated or you make it only one directional velocity. So, that way you can make $V - V_s$ or we can use a vector form. So, if that is the condition, if it is moving at this, the Reynolds transport theorem for uniform moving control volume will be the same. Here, we are using this V_r, the relative velocity.

So, this will be the vector form, the subtraction of V and V_s. We are using this relative velocity component and dA, that is what is coming from this part. Please take care of these things that when you consider the control volume moving with a constant velocity we use the relative velocity things. And the relative velocity, if it is one dimensional velocity field, it is very easy, but if not, you do this vector subtraction and that vector subtraction can be used to find out the scalar product of this and that is what is represented here.

Your Reynolds transport theorem has just modified it considering instead of the V, the fluid velocity, we use the relative velocity here, where V stands for velocity of the control volume. Now, let us consider another case. Like the control volume moving with a constant shape with a variable velocity, okay? So, that means what we are talking about is that the control volume is moving, it is not a constant velocity it is moving with.

At each point when it is moving it has the velocity which is the function of position vector and the time, okay, this position vectors with time, if the velocity of the flow is coming into this system is positions vector t and V is moving with a velocity t, varies with time. That means, you are not a fixed velocity, varying the velocity. Like you are accelerating control volume.

If the control volume moves as a fixed shape without deforming. Reynolds transport theorem for this case of a uniformly moving control volume is

$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{\forall cv} b\rho d\forall + \int_{Acs} b\rho\left(\vec{V}_r \cdot \hat{n}\right) dA$$

Initially it may have less velocity, as you go the velocity is changing, okay, and V_s is changing, or you are deaccelerating, V_s is changing, and velocity vector position you know it what will that be. In similar way, you can get the relative velocity. Again, vector subtraction of these two velocity fields you can get the V_r value. Substitute the V in Reynolds transport theorem, again you will get the equations for the control volume moving at constant shape with a variable velocity.

$V_r = V(r,t) - V_s(t)$

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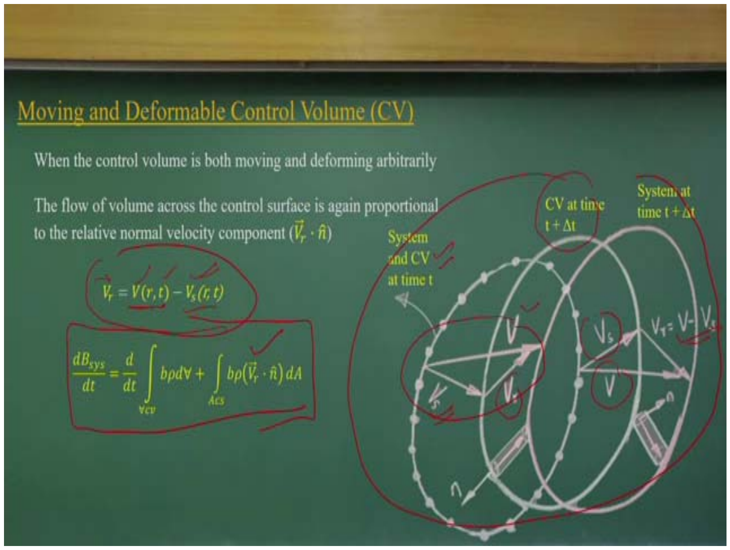

But in case you have a deformable control volume and also moving control volume, all the things you have are complicated now. That means you have V which varies with position as well as time, okay? So, like, for example, here it is given system control volume t equal to time, $t + \Delta t$ time. So, your V_s and V, fluid vectors have a function of position and the time. Similarly, because it is deformable control volume we consider your V also varies with position and time. So, it is r and t dependent, so position and time dependent.

$V_r = V(r,t) - V_s(r,\, t)$

And that vectors difference, if you see this graphically, the resultant velocity V_s and V and V_r is relative velocity. Similar way, for this surface, you know the V_s, you know the V, you know the V_r. So, this is what is represented for you, how the moving and deformable control volumes as it is moving, how the velocity fields changes, how the velocity with respect to the control volume changes from each point to point. Each point to point you can have the difference because it is depending on position vectors and also the time.

So, if you look at the similar equations only, the V_r, we are deriving this relative velocity vectors which is different for different cases like moving control volume, moving control volume with deformable which will be coming like this. So, again, I am going to repeat it, please do not be afraid of the surface integrals and the volume integral things because most in of the engineering problems we simplify these two integrality in such a way that we need not do the integration.

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{\forall cv} b\rho d\forall + \int_{Acs} b\rho\left(\vec{V}_r \cdot \hat{n}\right) dA$$

We try to have just area multiplication or constant velocity **(()) (19:41)** concept we use it. But for any complex problems where the geometry of the control volume can have complex natures, at that time we need to surface integral and volume integrals. But again I am going to repeat, do not be afraid. We have a lot of mathematical softwares which does the surface and volume integrals for us.

So, how to do the integrations for the surface level or in a volume control level, it is not a difficult task at the present area, but I will simplify the problems which is coming for your competitive exams.

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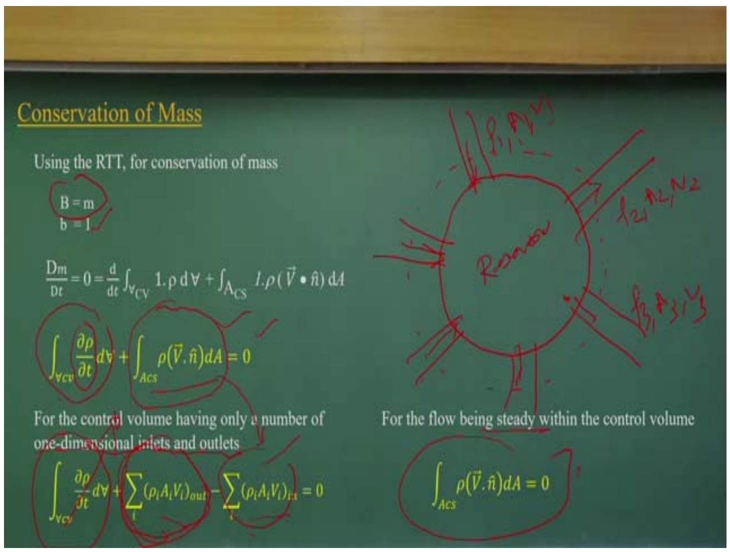

Now, let us come to the, as I said it, we need to have three equations. One is conservation of mass, conservation of linear momentum or the angular momentum. And third is, as you know it, conservation of energy. So, most of the times you remember whenever you solve the fluid mechanics problems we have three helping hands, it is not two, it is three helping hands. One is conservation of mass, the other is momentum, and another is energy.

But the problem is that many of the times we solve only the pressure and the velocity, okay? Many of the problems we are going to solve in this introductory fluid mechanics. Only need to solve for the pressure and the velocity. So, you may not need three equations. You just need two equations. In that two equations first and foremost requirement is always mass conservation will be there.

Whatever the problems you solve it, the first thing is that mass conservation will be there. Only the option left with us is whether you have to consider conservation of momentum or the energy, thus the difference, because we have only two solutions we are looking at, we have three equations. That is why it is quite interesting in fluid mechanics, sometimes we use the linear momentum equation, angular momentum equation, or the energy equations depending upon the problems. So, how easy to solve the problems.

That is what you try to get. So, let us come in to the mass conservation equation which is very easy equation to be solve and the conservation of mass. At the Reynolds transport levels if I put

B = m

b = 1

and no doubt that $\frac{\mathrm{d}}{\mathrm{d}t}$ or $\frac{\mathrm{D}m}{\mathrm{D}t}$ for any systems, that is 0, unless otherwise you have nuclear energy or additional energies coming to the system, what in general you do not consider in this case.

No chemical process, only we are talking about the process where the additional mass is not coming to that or mass is decaying from the system. If you look at the things, substituting b equal to 1. Now simplify the problems.

$\frac{\mathrm{D}m}{\mathrm{D}t} = 0 = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\forall \mathrm{cv}} 1.\rho\,\mathrm{d}\forall + \int_{\mathrm{Acs}} 1.\rho\,(\vec{V} \bullet \hat{n})\,\mathrm{d}A$

If you substitute b equal to 1, if you look that the density will be a function which we can put, and you have this part. So, the volume integrals and surface integrals here.

$$\int_{\forall cv} \frac{\partial \rho}{\partial t} d\forall + \int_{Acs} \rho(\vec{V}.\hat{n}) dA = 0$$

And this is a very interesting equation. Now, if you look that, this is talking about the net outflow of the mass through this control surface. That means, if I consider a control surface, there will the regions, which are the inflow reflow and there will regions which will be the outflow. And there will be regions where there is no exchange of mass, no flow region. So, that means what you are looking at? You are looking at inflow mass and outflow mass.

$$\int_{\forall cv} \frac{\partial \rho}{\partial t} d\forall + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0$$

The net change of the mass in this control volume should be equal to the change of mass storage within the control volume. That is what the concept is you are look. If I put as a unit level, the density Kg/m^3. Multiple with the volume which is the m^3. So, what it indicates for me is that Kg per unit time. So, this is how much of Kg of mass storage is happening per unit time. That is what we are getting from this within the control volume.

$$\int_{Acs} \rho(\vec{V}.\hat{n})dA = 0$$

This is the net outflow of the mass. If you look at the V into d A into the d A into the rho. V is the velocity, if you just look at the dimensions, just I am putting the dimension part which is easy to remember or whenever you write the equation, by mistake something, always check the dimension of the problem, okay? The dimension of any equation component should be the same. For example, the density, again I can say that it is Kg/m^3.

Velocity is m/s into area is m^2. So, finally it is coming Kg/s. So, these two equation component is Kg/s. This is mass flux. The net outflow of the mass within this control volume will be the net change in the mass within the control volume. How much of crossing through it, crossing things and the net flux. So, that is what is the mass thing.

But many of the times we simplify the problems. We make it one dimensional inlet and outlet problem. What is that? Let us have the problem of the pipe network connecting to one reservoir, okay? Like you have a big reservoir here. There are the pipes that are connected. A series of pipes are connected. Some are inflow, some are outflow, okay. If this is the reservoir, let this be from the same liquid.

So, you have ρ_1, A_1 and V_1. Here could be ρ_2, A_2, V_2; ρ_3, A_3, V_3. Like this we can have some are inflow, some are outflow if you consider one dimensional flow. Second dimensions we are not talking or third dimensions we are not talking. We are assuming that flow is one direction. I know the average velocity of this things. If it is that, then very easy problem now.

I can, instead of doing this surface integrals, okay, if I take this is my control volume, so I can just get the mass flux per unit time, it will be the density multiplications of area into the average velocity, ρ_1 A_1 V_1, out, in. So, there will be in and there will be the out. So, here, we remember out is a positive, that is because of scalar product, okay. That will be the positive sign here and you will have a negative sign.

So, if you look very tactically, if the flow is one dimensional nature if we present it, in terms of average velocity V_1, V_2, and V_3, ρ_1, ρ_2, ρ_3, and A_1, A_2, A_3, as flow is only one direction, instead of doing the surface integrals we make it a simple summation of each inflow and outflow. That net out mass influx is equal to change in the mass storage within the control volume.

But it is steady, as you know it, this becomes 0. So, you have very simple problems. Only the summations is what you will do for inflow and outflow. So, that means what happens is that, like, for example, your bank account, okay. If the money what you put in and what you spend they are equal, your bank account balance will remain the same. That is what happens in fluid flow problems.

If the problem is steady, there is no change in the storage within the control volume. What it indicates is that net outflux of this control volume becomes zero. What it says is that the sum of influx of mass is equal to some of mass outflux going through the control surface. That is the examples I have given. Some are influx, some are outflux. You just find out at the mathematical level this or as a simple summation for one-dimensional flow.

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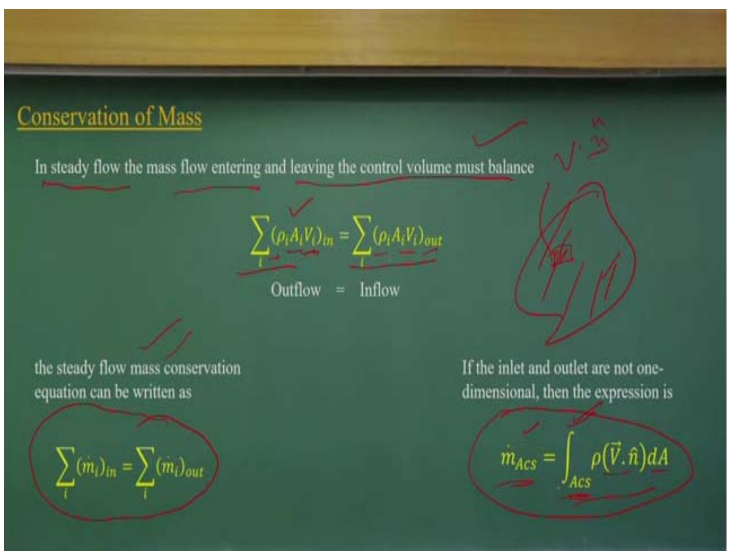

That is what again I am going to repeat it to tell you that further steady flow, mass flow entering the leveling control must be balanced. It is too packed, there is nothing else. Any system, if it is steady, it does not change with time, with respect to time. That means, what it indicates is that there is no change in the storage, mass flow, there is no change in the mass storage within this control volume. So, inflow should be equal to the outflow.

That is what you can do the summations of the number of inflow, the number of the outflow. We should know density, we should know the area of the flow, we should know the average velocity. So, similar way, we can have in and out, we can do the summation of that. If you have same steady flow mass conservation mass level or this one, the same concept, only here we are putting it.

$$\sum_i (\rho_i A_i V_i)_{in} = \sum_i (\rho_i A_i V_i)_{out}$$

And here I am talking about one-dimensional flow. If control surface and velocity vectors they are not aligning, they are not parallel, then you need to do scalar products, then do surface integrals to find out what could be the mass flow going through that control surface. These are surface integration, the scalar product. Then, you do the integrations if the density is also a function of the space and the time, then you will have more difficulty.

the steady flow mass conservation equation can be written as

$$\sum_i (\dot{m}_i)_{in} = \sum_i (\dot{m}_i)_{out}$$

If the inlet and outlet are not one-dimensional, then the expression is

$$\dot{m}_{Acs} = \int_{Acs} \rho(\vec{V}.\hat{n})dA$$

Otherwise, if the density is constant you can take it out, only this velocity scalar product and the d A we can do a surface integral to compute what will be the mass flux. That means if you consider it is very complex control surface as you like it to make it complex things. Then, there will be no problems, you can solve the fluid problems. You need to have a leverage or you need to use high-end mathematic tools.

In today's world what is available like MATLAB, Mathematica and all to integrate it. Considering that for each reach, the velocity varies and each point also your end vectors vary. Do the scalar product, integrate with d A and get it, what will be there. That is possible, but as I say, various ways, because tools are available, only you have to use that tools. For example, for the academic point of view we will not go to doing surface integrals in exams or any competitive.

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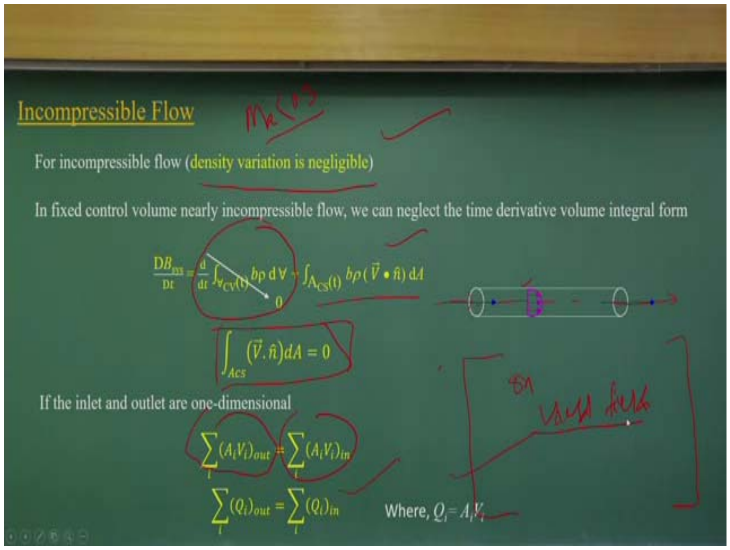

Now, let me come in to the, if the flow is incompressible. Again, we have a lot of simplifications. Again, I can repeat it. The flow systems when you have mac number less than 0.3, okay, whether it is gas, whether it is a liquid or any flow system, if you think that the within the flow system the flow becomes less than the mac number less than 0.3, then there will be density variation, but that variation of density is much much negligible comparing to other components.

So, we can assume the flow is incompressible nature. Again, I am going to summarise that. When you have any flow systems, mac number is less than 0.3, so we can use flow as incomprehensible flow, density does not vary significantly. So, density becomes constant, as density becomes constant, as you know it, it is very simplified problem what we are going to solve.

So, density varies negligible, as the density variation is not significant and beta equal to 1, so only this equ ation is left for us. Simple thing. This is very simple equation.

$$\frac{DB_{sys}}{Dt} = \frac{d}{dt}\int_{\forall CV(t)} \cancelto{0}{b\rho \, d\forall} + \int_{Acs(t)} b\rho\,(\vec{V} \bullet \hat{n})\, dA$$

$$\int_{Acs} (\vec{V}.\hat{n})dA = 0$$

Now, there, the scalar product of V and n and d A, okay, and density can come out. So, instead of the mass flux we are now talking about volumetric flux. That means, if you multiply the velocity into area, then what you get is unitm^3/s, volume per unit height, volumetric flux, okay? So, please do not confuse, this is a different equation.

If the inlet and outlet are one-dimensional

$$\sum_i (A_i V_i)_{out} = \sum_i (A_i V_i)_{in}$$

$$\sum_i (Q_i)_{out} = \sum_i (Q_i)_{in}$$

Where, $Q_i = A_i V_i$

Only that we are not showing the density multiplication. If you multiply with a Q, the V into A is Q is discharge. So, Q = V $\times A$, is the discharge. So, most of the conservation of mass you write it, since density is a constant, you make it come out from that equation.

So, it looks like volumetric level we are comparing but all are mass conservation equations. We talked about mass flux is coming in or going out from the control volume or mass flux is changing within the control volume. That is the concept to that. But as it is simplified, in case the flow is incomprehensible, density is a constant, that density component comes out from Reynolds transport theorem which helps us look like we are looking at equating the volumetric thing, but it is not that.

Please remember we are still doing the mass conservation equation. The volumetric form has come in because you have taken out the density. That is what if you look it A_1 V you have these things. That means, if I have a pipe flow like this, you can anticipate it. As we discussed earlier, the velocity will be 0 near the wall, velocity will be maximum at the center and so there will be velocity distribution, there will be velocity distribution from this side.

The flow coming and going out. If this is simple in and out system, you can know this velocity distribution area, find out the discharge from inflow and outflow, equate it, then you can solve the problem. So, what I am looking at is again summarised here. To solve this mass conservation equation I should have knowledge on velocity field. I should know how the velocity varies or I should know whether the velocity is a constant or the velocity varies. If I know the velocity variations on this control surface, then I can solve the problem.

So, basically, when you apply the mass conservation equation your knowledge of velocity variation is important to you, how you are simplifying the velocity field on the control surface. That way we solve the problem.

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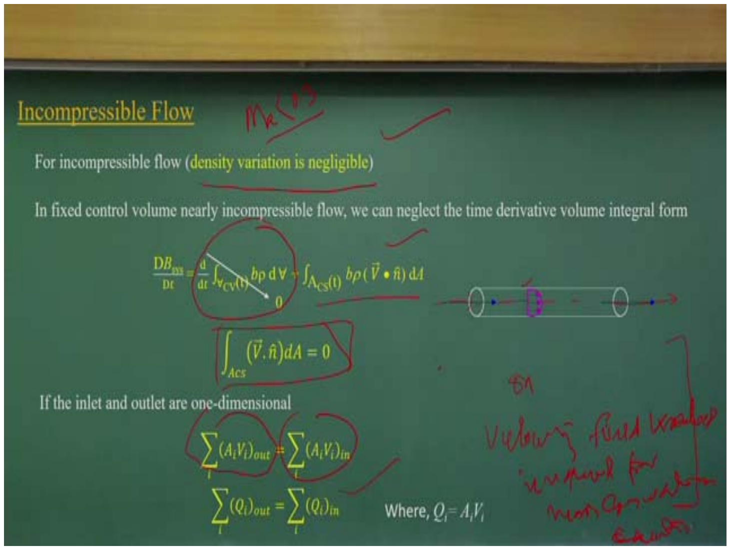

So, the velocity field knowledge is required for mass conservation equations.

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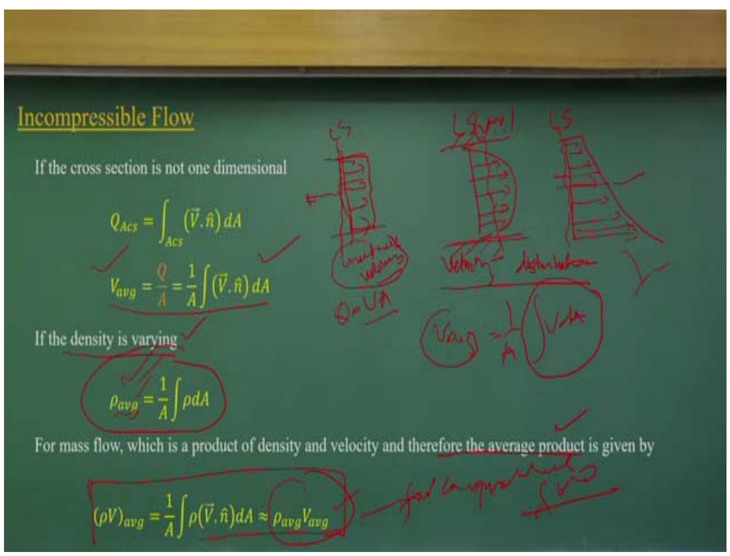

So, that way, whenever you take fluid mechanics problems, first you think what could be approximate velocity field, what could be the velocity direction. If I assume the flow is one dimensional, is it enough for me that one dimensional flow is okay for us or not, for that problem or not, or you need to have two-dimensional velocity fields, or what could be the direction, whether it is a direction with respect to the control surface normal vectors.

All the knowledge you should have when you are applying any real life fluid problems, but as I said earlier, this academic problem is simplified and they do not tell the velocity component, whether the velocity what they talk about that is perpendicular to the control surface, that way it is considered. So, that is the easier way it is there for knowledge about the velocity fields, that is what is necessary.

And where you have velocity fields are not known, I do not think you can apply the conservation of mass equation properly. So, let us come into the incomprehensible flow. Let us look at these figures, you can assume it uniform velocity distribution, okay? That means velocity does not vary with respect to the position. It is a constant velocity. If it is a constant velocity, V into area will give me the Q value.

$$Q_{Acs} = \int_{Acs} \left(\vec{V}.\hat{n}\right) dA$$

It is very easy, V into area will give its value. Only I need to have the control surface where the normal vector should have either velocity vector directions and this normal vector, either 0° or 180° to find out whether it will be Q positive or Q negative. So, when you have uniform velocity, the problem is quite simplified. But the case is you do not have uniform velocity as you know it when you have a pipe flow.

You cannot have uniform velocity. You do not expect that you will have uniform velocity for that. So, you will have 0 velocity near the boundary of the wall of the pipe and you will have velocity like this. So, in that case, many of the problems will give you average velocity. The average velocity what we get,

$$V_{avg} = \frac{Q}{A} = \frac{1}{A}\int \left(\vec{V}.\hat{n}\right) dA$$

So, it represents average velocity. It considered the velocity distribution to compute the average velocity. So, somewhere the average velocity will come like this okay. So, some of the case, the problems give the average velocity. That means it considers the velocity distribution, after that it has given the average velocity. By doing surface integrals average velocity is given.

Once you know the average velocity you multiply with the area you will get the volumetric flux or the discharge. But in some of the cases, if you know the velocity distributions, how it varies. You do surface integrals of that and find out what is average velocity. Multiply with the density and the area will get the mass flux. We will get mass flux if you have the product of density, velocity, and the area. Velocity will be the average velocity.

In some cases like this, you may have the condition where the velocity variations may be very complicated. Then, you need to do the integration to compute the average velocity for that. So, we need to do surface integrals to compute it, how the velocity is varying it and this is my control surface. So, that way we can quantify the V average which is by integrating velocity.

Similar way, consider if the flow is comprehension, you assume the density is varying it, you can use average density concept, okay? Again, you can integrate density with respect to area, then you compute the average things and you can multiply that. Please remember in this case it cannot be if a density and the velocity, the multiple functions will not be separate functions, okay?

They will be depending on each other. Then, this simplification cannot be done. But assuming the density and the velocity does not depend on each other, they are independent, then you may follow this concept to do this averaging to find out for compressible flow, okay? For comprehensible you can do this average product. But the assumption is that the density does not have dependency with the velocity vector.
If the density is varying

$$\rho_{avg} = \frac{1}{A}\int \rho dA$$

If it is that, then you need to do the product of the velocity and the density and do the surface integral to solve the problem.
For mass flow, which is a product of density and velocity and therefore the average product is given by

$$(\rho V)_{avg} = \frac{1}{A}\int \rho\left(\vec{V}.\hat{n}\right)dA \approx \rho_{avg}V_{avg}$$

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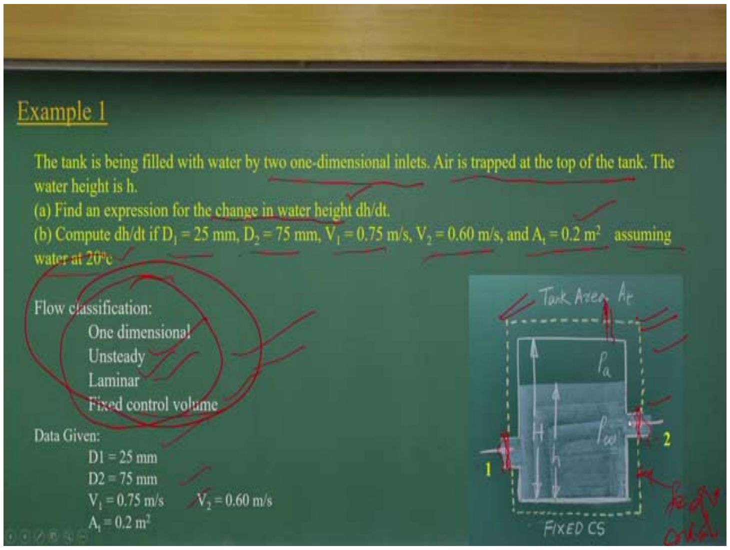

Now, let us come to the very interesting problem which is there in text book of F.M. White book. What is there, there is a tank. If you look at this figure, there is a tank, two inflows are there, the tank is being filled with waters. Within the tank there is air and there is water, liquid and gas form of water. The problem is not given here but I can put it there should be air valve here, okay?

The tank is being filled with water by two one-dimensional inlets. Air is trapped at the top of the tank. The water height is h.

(a) Find an expression for the change in water height dh/dt.

(b) Compute dh/dt if D_1 = 25 mm, D_2 = 75 mm, V_1 = 0.75 m/s, V_2 = 0.60 m/s, and A_t = 0.2 m^2 assuming water at 20^{o}c

As the liquid expands it, there should be air valve for air to come out from that. Otherwise, this will be very complicated problem. As you put more and more liquid, but the air cannot be compressed, that type of problem we are solving. Having said that there is air valve, as the liquid increases the space and the gases are not able to fit there, it will come out, okay? That is very simplification we have to do it for this problem.

Flow classification:

One dimensional

Unsteady

Laminar

Fixed control volume

So, this is one-dimensional inlet. Air is trapped at the top of the tank with air valve, what I have included here, and this water height is h, find the expression change in the water level with respect to time. How water level is changed?

As you know it, the density also changes with temperature and the pressure, but mostly for the liquid like water it is temperature dependent. So, that is what we are doing here. So, given data is here. But again I need to tell you when you solve the problem, first you do the flow classification. The problem here is one-dimensional in nature. Unsteady because we are finding out the storage varies with respect to time.

Flow can be laminar or turbulent, we do not know it, okay, and we have a fixed control volume. This is what we consider is fixed control volume, okay? First, whenever you solve the problem, you classify the problem. Once you classify problems it gives indirectly that these are the assumptions that are valid for this problem that we are solving. So, classify the problem. Give the data given.

Data Given:

D1 = 25 mm

D2 = 75 mm

$V_1 = 0.75$ m/s $V_2 = 0.60$ m/s

$A_t = 0.2\ m^2$

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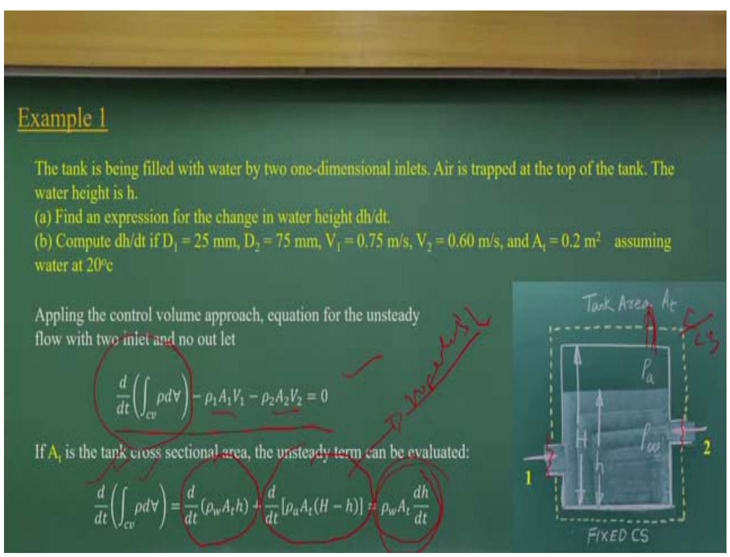

If it is that, apply the Reynolds transport theorems, okay? You apply the Reynolds transport theorems. You have the inflow and the outflow. There is no outflow in this case. In both the case you have inflow, which is negative here. So, rho 1 A1 A2 and rho 2 A2 V2 and what is changing the storage of the water inside this control volume. This is the control volume and this is control surface. The flow is coming in, okay?

Appling the control volume approach, equation for the unsteady flow with two inlet and no out let

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) - \rho_1 A_1 V_1 - \rho_2 A_2 V_2 = 0$$

What is happening is there is change of the mass of water here, change of the air here. This part we are neglecting it because we are not putting this air valve here. We are neglecting that part, how the air part is changing, otherwise we will go for incompressible and all. Do not go for that, that is complicating this problem. So, we are just talking about how this part is changing it. So, you can simplify it.

If A_t is the tank cross sectional area, the unsteady term can be evaluated

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) = \frac{d}{dt}(\rho_w A_t h) + \frac{d}{dt}[\rho_a A_t (H-h)] = \rho_w A_t \frac{dh}{dt}$$

The density does not change; it is incompressible flow. We know the area of the tank. So, dh by dt, that is how this integral and surface integral will come to this one. Now, we will equate that.

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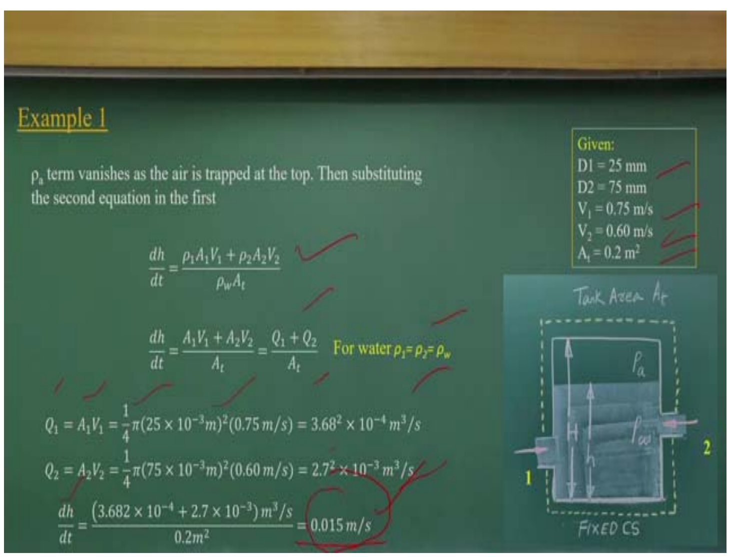

ρ_a term vanishes as the air is trapped at the top. Then substituting the second equation in the first

$$\frac{dh}{dt} = \frac{\rho_1 A_1 V_1 + \rho_2 A_2 V_2}{\rho_w A_t}$$

Again, I am going to repeat it. First, you classify the problem. Assume the appropriate control volume and the control surface and each control surface you identify which is the inflow and outflow. Then, you talk about what is the change of the storage and what is happening.

So, if you can understand properly, applying this Reynolds transport theorem and the simplification and put the numerical value, it does not take much time or much problem to solve any problem. The first is the classification and application of appropriate control volumes. Find out the influx what is coming, mass influx, from where and from which part of the cross section, like this part is inflow, this part is also inflow, and the change in the storage is here. And the change in storage what we can represent.

And if we do that, the problems you can solve it. So, first how to use the classification. Again, I am going to repeat it. Please do the classification accurately. Then, choose appropriate control volume. The control surface should be perpendicular to the velocity vector. That is what you see there. I can have any shape, then I have to do surface integrals. To avoid the surface integrals we have to make control software so that velocity should be perpendicular to that, okay.

With respect to normal vector it should have 0˚or 180˚. So, in that case, you need not do surface integrals. So, your control surface you chose in such a way that you should have the normal vector of the surface and the velocity they should be collinear. That is the concept to be considered. Then, it is very simple simplification, you just substitute the values;

$$\frac{dh}{dt} = \frac{A_1V_1 + A_2V_2}{A_t} = \frac{Q_1 + Q_2}{A_t}$$

For water $\rho_1 = \rho_2 = \rho_w$

$$Q_1 = A_1V_1 = \frac{1}{4}\pi(25 \times 10^{-3} m)^2(0.75\, m/s) = 3.68^2 \times 10^{-4}\, m^3/s$$

$$Q_2 = A_2V_2 = \frac{1}{4}\pi(75 \times 10^{-3} m)^2(0.60\, m/s) = 2.7^2 \times 10^{-3}\, m^3/s$$

$$\frac{dh}{dt} = \frac{(3.682 \times 10^{-4} + 2.7 \times 10^{-3})\, m^3/s}{0.2 m^2} = 0.015\, m/s$$

Finally, you will get the rate of change of the height, that means in this the height will change 0.015 meter per second. The storing within the tank will be there like this, okay, 0.015 meter per second. Let us come to the second example which is very interesting example, showing you the facilities what we have in IIT Guwahati, in the department of civil engineering.

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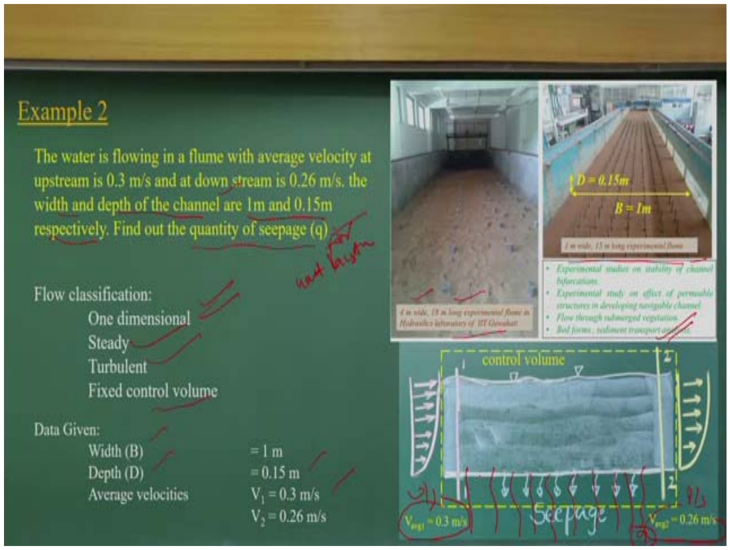

The water is flowing in a flume with a downward seepage average velocity at upstream is 0.3 m/s and at down stream is 0.26 m/s. the width and depth of the channel are 1m and 0.15m respectively. Find out the quantity of seepage (q).

That is experimental flume 4 meter wide and 18 meter long. The photograph you can see. Similar way, we have the experimental facility to do hydraulic studies, having 1 meter wide and 15 meter width. Here we have the seepage arrangement which is unique in this way. So, that seepage arrangement means water from the surface can go to the ground water. That means, from this control volume water can seep downwards.

That is the seepage facility what we have here. Considering that is what the control volume is, we are now going to solve this problem that if I have the velocity measurement at the upstream and the downstream, can I compute what will be the seepage rate per unit length, okay? That means I know the average velocities at the upstream, I know this downstream velocity.

Also I know the width and the depth of the channels respectively, then can I compute it how much of water goes out from this control volume at seepage as a downward movement. Flow classification:

One dimensional

Steady

Turbulent

Fixed control volume

So, this is how we have a control volume. So, this is the upstream direction and this is the downstream direction. This is my control volume. Some of the water seepage out from this control volume.

Data Given:

Width (B) = 1 m

Depth (D) = 0.15 m

Average velocities, V_1 = 0.3 m/s

V_2 = 0.26 m/s

Mass influx, mass outflux since it is a steady problem, and density is constant. So, at volumetric level I just compare it, how much of water is coming to this control volume, how much of water goes as seepage, as a downstream, outside from this control volume.

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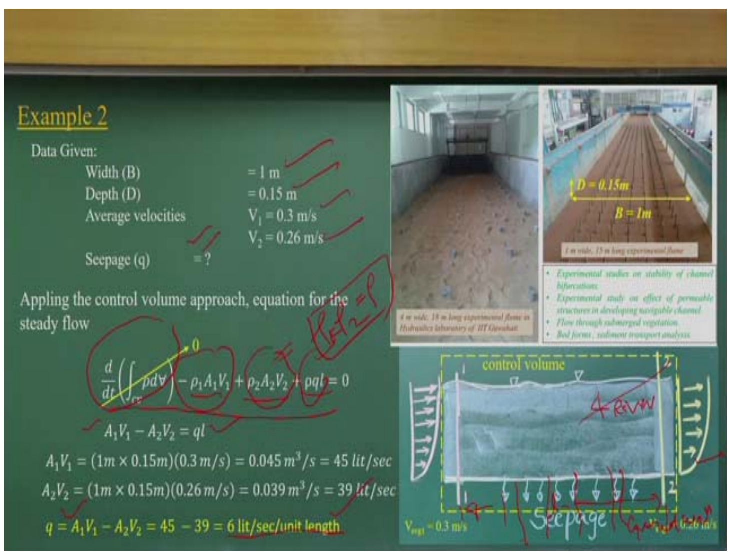

Now, we are getting this width and depth. We have the velocity, then we are looking at the seepage rate. You can see I apply the mass conservation equations. Data Given:

Width (B) = 1 m

Depth (D) = 0.15 m

Average velocities, V_1 = 0.3 m/s

V_2 = 0.26 m/s

Appling the control volume approach, equation for the steady flow

$$\frac{d}{dt}\left(\cancelto{0}{\int_{cv}\rho d\forall}\right)-\rho_1 A_1 V_1+\rho_2 A_2 V_2+\rho q l=0$$

This is the outflux going out from this. This is outflow, seepage, downward path. So, if you rearrange it, you get the ql is this as the density is a constant. The water density does not change in this flow system. So, you get what will be the q value liter per second per meter. That is what you compute. Only we have to visualise that. There is the flow, inflow and outflow, the seepage.

$$A_1V_1 - A_2V_2 = ql$$

This is the same way in a river. If this is the river, this is the groundwater zone. There is exchange, river recharges to the groundwater. This same process happens, or reverse also is true, ground water can give flux into the river. That means this q will be positive or negative sign. Otherwise, with the same control volume we can apply mass conservations for a river and groundwater interaction study. That is what you learned in hydrology course.

$$A_1V_1 = (1m \times 0.15m)(0.3\, m/s) = 0.045\, m^3/s = 45\ lit/sec$$

$$A_2V_2 = (1m \times 0.15m)(0.26\, m/s) = 0.039\, m^3/s = 39\ lit/sec$$

$q = A_1V_1 - A_2V_2 = 45 - 39 = 6$ lit/sec/unit length

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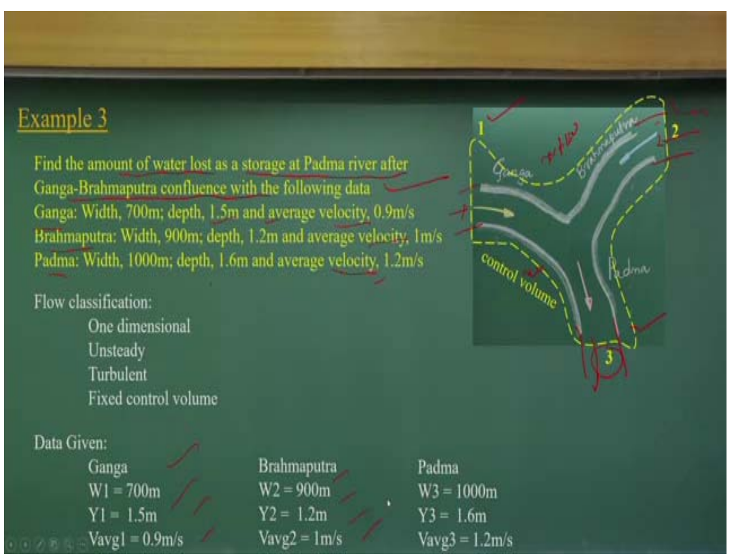

Now, take another example, problem which is with Ganga-Brahmaputra confluence which is not in India, it is in Bangladesh, okay. Let us consider Ganga and Brahmaputra is meeting, okay? We have Padma river systems, and with these rivers we have all the measurements available of these systems. At 1, 2, and 3, when Ganga and Brahmaputra and Padma meets there we have a cross section 3 here.

Find the amount of water lost as a storage at Padma river after Ganga-Brahmaputra confluence with the following data

Ganga: Width, 700m; depth, 1.5m and average velocity, 0.9m/s

Brahmaputra: Width, 900m; depth, 1.2m and average velocity, 1m/s

Padma: Width, 1000m; depth, 1.6m and average velocity, 1.2m/s

So, we have a control volume like this. So, this is a no flow, this is a no flow. The flow will be only this, here and here and here. So, this is inflow, this is inflow, this is outflow. So, the average velocity is given, width is given, depth is given. So, we have to compute the q_1, q_2, and q_3. Based on that we have to find out whether change in storage is there or not. That means we have to find out amount of water lost as storage in Padma river after the Ganga-Brahmaputra confluence with the following data.

Flow classification:

One dimensional

Unsteady

Turbulent

Fixed control volume

Data Given:

Ganga

W1 = 700m

Y1 = 1.5m

Vavg1 = 0.9m/s

Brahmaputra

W2 = 900m

Y2 = 1.2m

Vavg2 = 1m/s

Padma

W3 = 1000m

Y3 = 1.6m

Vavg3 = 1.2m/s

In Ganga, Brahmaputra, and Padma which is are confluencing, after that we call Padma. How much the flow depth is there, width is there, average velocity is given which is more or less average velocity as we do a lot of river survey in Ganga and Brahmaputra systems in our country.

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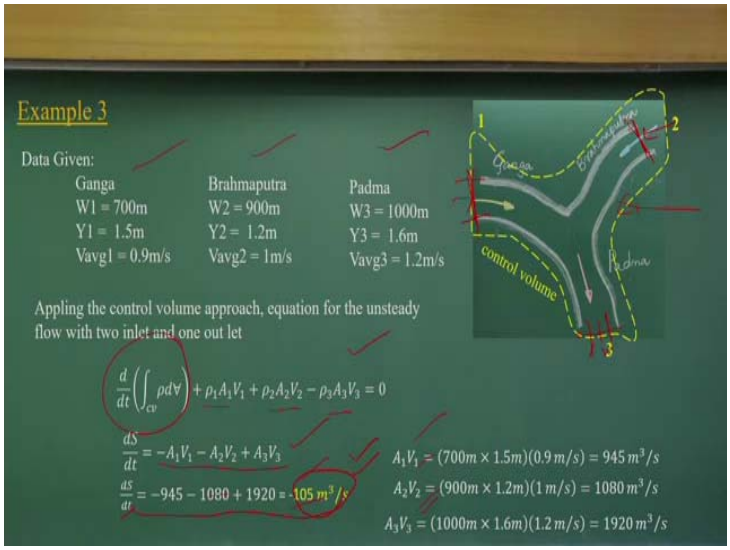
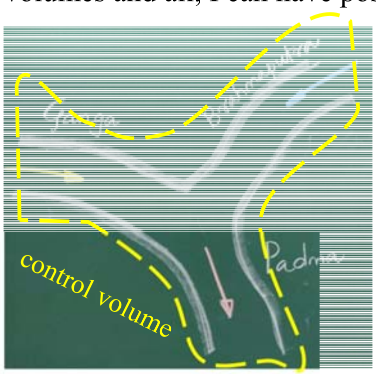


So, if I have width and the depth and velocity like this, so we can apply this continuity equations for this control volume, okay? For Ganga, Brahmaputra, Padma systems I have control volumes and all, I can have positive and the negative, okay?



Appling the control volume approach, equation for the unsteady flow with two inlet and one out let

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) + \rho_1 A_1 V_1 + \rho_2 A_2 V_2 - \rho_3 A_3 V_3 = 0$$

So, substituting this value with have a negative or a positive of change in the storage. So, simple way we can just multiply it to get the q1, q2, and q3. If you look it is a very complex problem, but with the help of the control volume we just look at mass flux coming in from this surface and going out this surface. That will be change in the mass within this control volume. That is what it says, how much storage of mass is changing within control volume.

$$\frac{dS}{dt} = -A_1V_1 - A_2V_2 + A_3V_3$$

Considering the measurement of the velocity and the flow depth and the area at these three points we can judge how much of water we are losing or gaining in a stretch of river systems. You can understand these type of problems we can solve, okay? Whether river is gaining from other locations or the water is losing from the river. That type of study with a simple type of control volume we can do it.

$$A_1V_1 = (700m \times 1.5m)(0.9\,m/s) = 945\,m^3/s$$

$$A_2V_2 = (900m \times 1.2m)(1\,m/s) = 1080\,m^3/s$$

$$A_3V_3 = (1000m \times 1.6m)(1.2\,m/s) = 1920\,m^3/s$$

$\frac{dS}{dt} = -945 - 1080 + 1920 = -105\ m^3/s$

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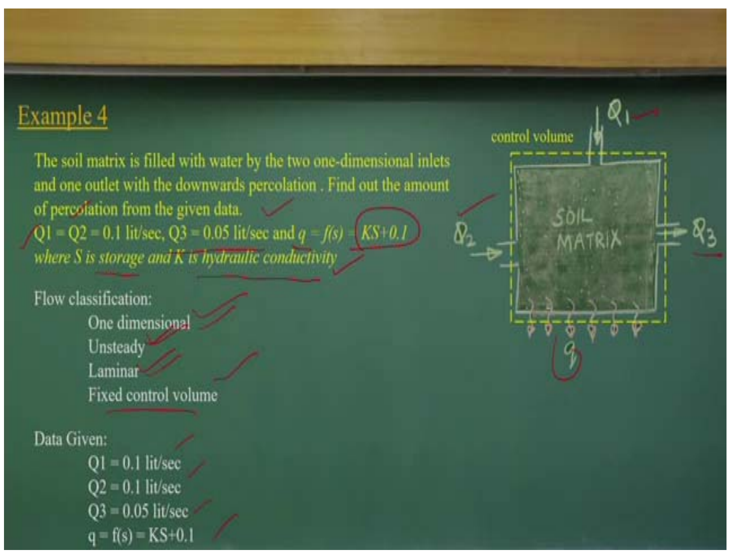

Let us have the last example, okay? It is slightly a bit complicated. There is a soil matrix, let it have chambers. The flow is coming Q1, Q2, and Q3. There are two inflows. The discharge is coming, Q3 is outlet, and there are the seepage flow or download percolations are happening which depends upon the storage within the system. That is why Q is a function of storage.

[The soil matrix is filled with water by the two one-dimensional inlets and one outlet with the downwards percolation . Find out the amount of percolation from the given data.

Q1 = Q2 = 0.1 lit/sec, Q3 = 0.05 lit/sec and *q = f(s) = KS+0.1*

where S is storage and K is hydraulic conductivity]

How much of water in storage, that function with a K, K is linear coefficient that is where which may be considered sometimes as hydraulic conductivity in storage. I have Q1 and Q2, I have the Q3, how of litre per second, because there is very less quantity of water, not comparable with the Ganga and Brahmaputra systems. So, litres per second we are coming to that. So, in that case, flow can confine.

Flow classification:

One dimensional

Unsteady

Laminar

Fixed control volume

Data Given:

Q1 = 0.1 lit/sec

Q2 = 0.1 lit/sec

Q3 = 0.05 lit/sec

q = f(s) = KS+0.1

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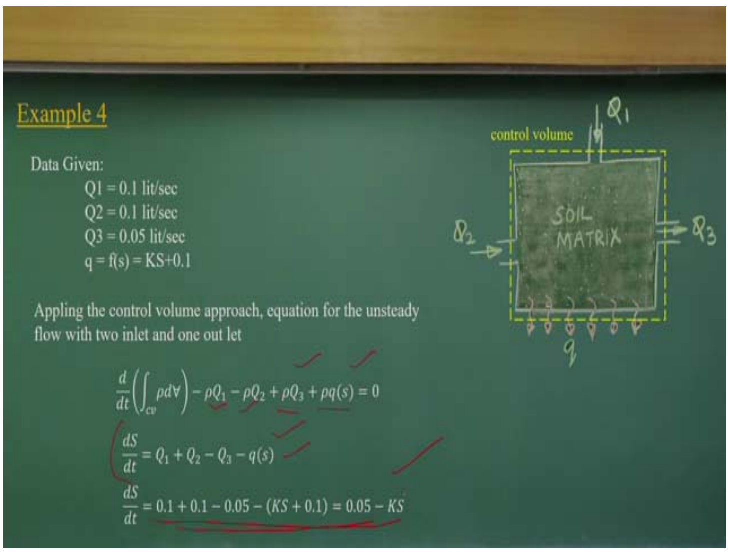

Now, if I substitute this equation, if you look at these inflows are the negative, outflows are positive, and Qx is the function of this. So, Appling the control volume approach, equation for the unsteady flow with two inlet and one out let

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) - \rho Q_1 - \rho Q_2 + \rho Q_3 + \rho q(s) = 0$$

$$\frac{dS}{dt} = Q_1 + Q_2 - Q_3 - q(s)$$

$$\frac{dS}{dt} = 0.1 + 0.1 - 0.05 - (KS + 0.1) = 0.05 - KS$$

This is in terms of s. K is a constant.

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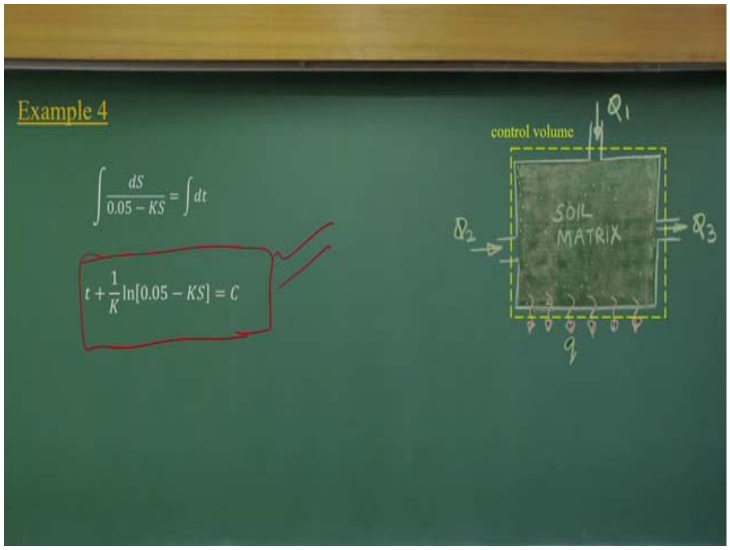

So, we can integrate it to solve these problems.

$$\int \frac{dS}{0.05 - KS} = \int dt$$

$$t + \frac{1}{K}\ln[0.05 - KS] = C$$

So, finally you get this equation. That is how s varies with respect to time. That is what is our problem. So, I have given three examples. One is a simple tank problem. Another is three river confluence point. Third is seepage problem. And fourth is the soil matrix problem. So, that way if you look at any of the Cengel, Cimbala, or F.M. White book, a lot of exercises are there, there are a lot of example problems which are also solved.

So, only this art of applying this control volume concept that you should learn it. May be very complex problems but use of appropriate control volume and the control section knowing the direction of the flow that will help us. By applying the Reynolds transport theorems, we can solve the problems. That is my idea and that what I need to convince.

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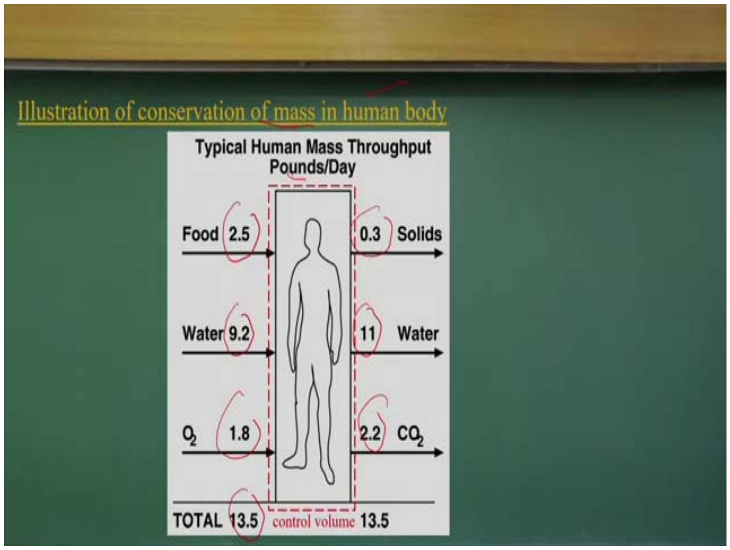

Before that let me give a warning to you or suggestions to you, that whatever human body I or you, all are having mass inflow and outflow systems, that is what it is saying, that the conservation of mass in a human body is pounds per day, 2.5, 9.2 pounds. This much of water and food and oxygen we take, which is 13.5 pounds, okay. And we release the solids, water, and CO_2, some of this 13.5.

Please do not disturb this inflow and outflow. If you disturb the inflow and outflow we do not know what we are doing to our system of body, okay? Either we are deteriorating the body, our health, we do not know it. What I am going to tell is the 9.2 pounds of water please drink it. Similar way, the 2.2 pounds per day of food you eat so that your systems would be perfectly okay for now and total youthful life.

You can enjoy it if you maintain the simple balancing the mass conservation principle is followed by the human body with different water, food, and this, with a slight bit variations. But overall this equation you should follow, with food 2.5, water 9.2, oxygen 1.8 pounds per day. And we release the same amount whatever we get it and release the same amount, only we vary the solids to 0.3, the water to 11, and 2.2.

So, many of the foods we convert to water and we convert the oxygen to carbon dioxide. That we do it and for a healthy life we should follow this equations and we should remember this equation. Whenever you wake up in the early morning we should also maintain this equation, then we will have a healthy life. It is more important to say this.

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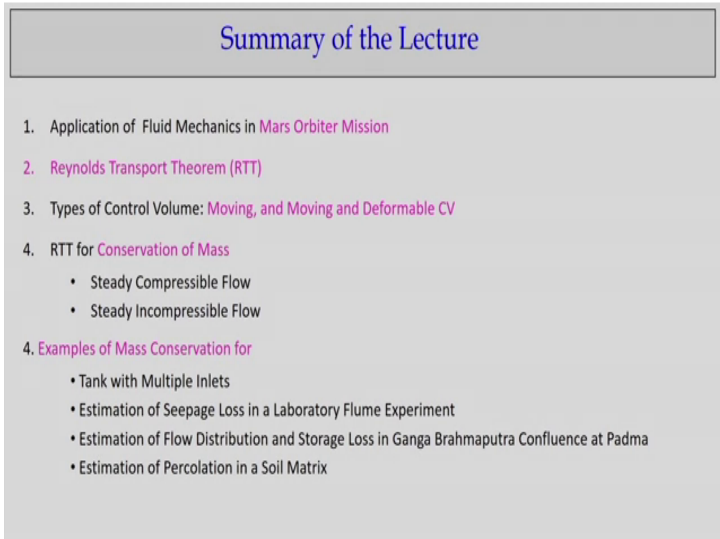

With this let me conclude this very interesting lecture. I do not know whether you enjoyed or not, but let us have a talk about that. We have applied Reynolds transport theorem which looks very difficult, surface integrals or volume integrals, but it can be simplified in many ways and when you simplify this complex equation and apply to real life problems like as I have given examples.

Similar way, real life problems if we can apply it really we can find out what will be change in the storage, what will be the change in mass inflows and outflows, that is a standard problem. So, please try to solve some of the problems which is given in Cengel Cimbala or F.M. White book in exercise and example problems. With this, let me conclude this lecture today. Thank you a lot.