Fluid Mechanics
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Lecture No. – 07
Fluid Dynamics: Reynolds Transport Theorem

Welcome you to this lecture on fluid mechanics. As we discussed in the last classes, the fluid at rest, which are very simplified cases, we discussed, and today we are going to start very interesting subject on fluid mechanics which is called fluid dynamics. In very simple way we will try to understand the complex fluid flow problems using Reynolds transport theorem and control volume concept.

So, in today's lecture I will focus on systems versus control volume concept as well as I will talk about the Reynolds transport theorem which is a very unique theorem, simplifies very complex problems, and we will try to solve the fluid mechanics problems that we get from this.

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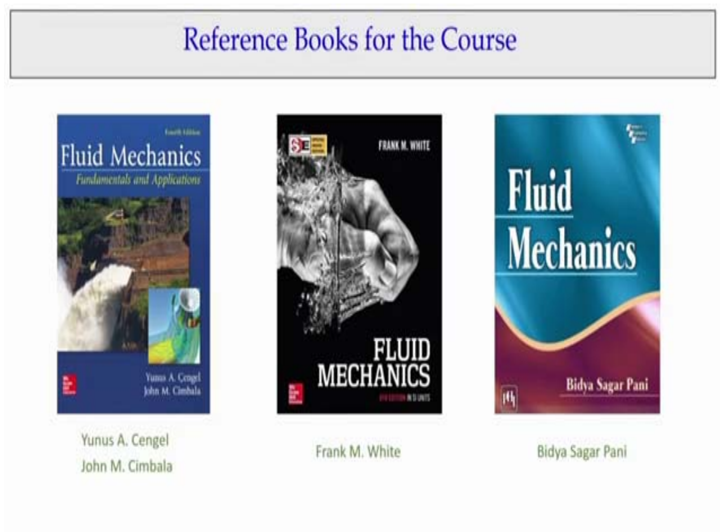

Let us have again the reference books. We discussed this also earlier. The Cengel Cimbala book which is having a lot of illustrations of real practical problems. Apart from giving mathematical treatment of fluid mechanics, also it indicates how complex fluid flow problems happens in natural conditions. And the F.M. White book which is concise mathematics driven with few illustrations. And another Indian book on fluid mechanics by Prof. Bidya Sagar Pani.

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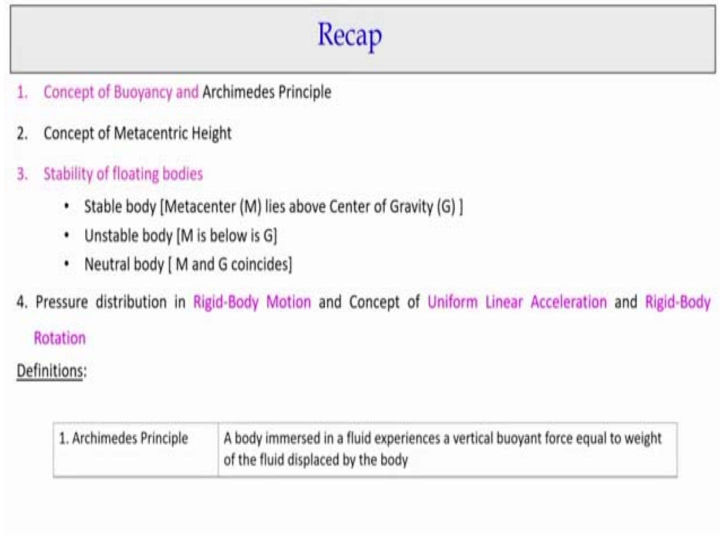

Let us start, in the last class, as I told you, we discussed about fluid at rest which is very simplified, that we know how the buoyancy forces happens and the basic concept of the Archimedes' principle as you know. Again I can repeat. A body immersed in a fluid experiences a vertical buoyant force which is equal to the weight of the fluid displayed by the body. That is what you experience when you dip in a swimming pool or a ship floating in a river or the ocean.

So, this Archimedes' principle holds good for simple day-to-day life that we experience. But if you go to micro scales like water vapour transport process which is the major process in the global climatic circulation pattern, that also follows simple buoyancy flow of water vapour transport mechanisms. Also we discussed what is the stability of floating of bodies, unstable, stable, or naturally stable conditions.

That, with respect to relationship between metacentric height and the CG, the center of gravity, based on that we can tell floating body is at what condition, whether it is stable or unstable or natural equilibrium condition. Then, we also simplified the conditions like, if you have a uniform rotation of a half-filled liquid container, what could be the pressure diagram, what could be the height of the liquid that will be formed when you rotate with uniform angular rotation.

Also, we discussed about control volume moving with a constant acceleration which is rigid body motion of oil tanker or if you can simplify that condition to find out the sloshing behavior of any water tank during the earthquake motions. So, those concepts, even if it is a simple concept, but those concepts also sometimes we can use to derive the force component due to pressure and the gravity force component.

So, as of now, when the fluid is at rest we talk about two force components, those are force due to pressure and gravity force. Now, as we are moving to the next level, we will talk about fluid in motion and to simplify it. I will today cover first what are the real challenges that we have been facing giving a simple example. Then, again, I will talk about why do we need virtual fluid balls concept as earlier I said.

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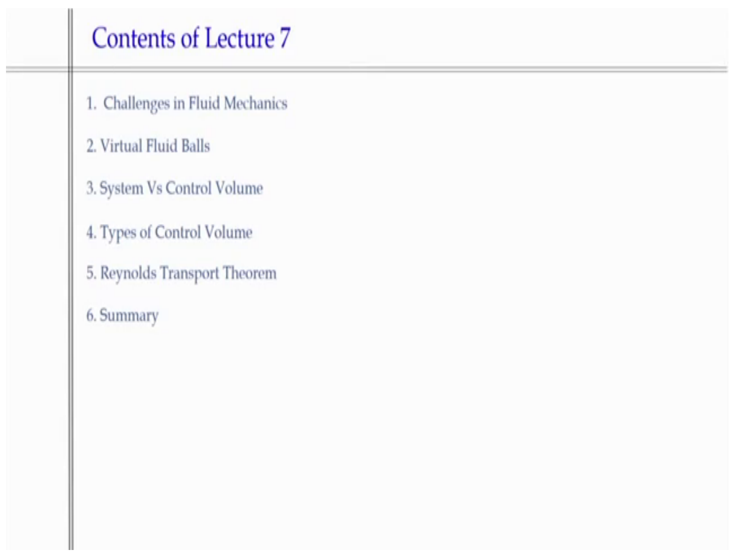

This is a new concept. I brought it for the students to visualise the flow. Once they visualise the flow, then can solve the problems. That again I am going to repeat, the virtual fluid balls concept, because that is what is necessary to choose appropriate control volume to solve the problems. So, that is the idea behind virtual fluid balls concept. That is what I will repeat in this lecture also.

I will discuss, as I have given earlier, the definitions of system and control volume. Today, we will discuss in more detail on systems and control volume. Then, I will talk about the types of control volume and we will go the Reynolds transport theorem which establishes the relationship of physical equations in system level and the physical equations at the control volume level, which is a great equation, that is a great simplified equation.

We integrate all types of fluid flow problems through the Reynolds transport theorem, we can solve it. And then I will summarise the content of today's lecture. Let us come to challenges in fluid mechanics.

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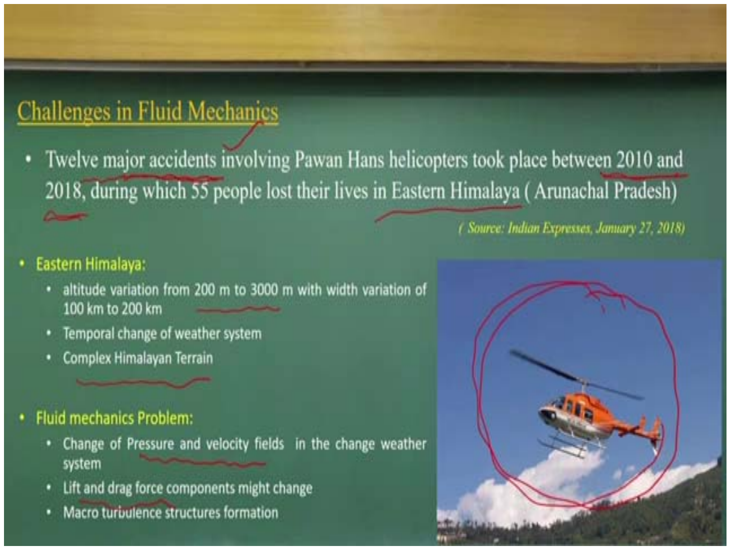

If you look at the report in Indian Express of January 27, 2018, it says that it has something like twelve major accidents happened to helicopters that took place between almost eight years it in the eastern Himalayas particular to Arunachal Pradesh which which lost 55 people. Now, the problems that is coming is why do we have so many major helicopter accidents over Himalayas. Let us understand what is Eastern Himalayas.

The Eastern Himalayas if you experience, if you can see it, really it has different altitudinal variations. That is altitude varies from 200 meters to 3000 meters and a width variation of 100 km to 200 km. So large variation of altitude happens with aerial distance of 100 km to 200 km., and that is a complex Himalayan terrain. So, today, you can visualise how three-dimensional Himalayan terrains are very complex, we can visualise it.

When you have very complex Himalayan terrain you expect that the weather will drastically from time to time. As I visited the Tawang region in Eastern Himalayas, I saw personally the weather changes within a few hours. So, when you have a weather system that is so dynamic, it changes from sunny weather to snow falls, then thunderstorm type of formation happens, so you have a lot of system changes in pressure and the velocity fields.

That is what we do not understand. In a complex terrain when you have energy exchange and the vegetation interaction, how this weather is changing it. That knowledge we have also is limited. When you have a very drastic change of pressure and velocity distribution about this helicopter system which was never tested for Himalayan terrain, it can experience the lift and drag force which was not used to design this.

Not only that, we can expect that there could be macro turbulence structures, the formation happening which we do not have a knowledge, and those combinations of macro turbulence structure and change in this pressure and velocity field are causing the different direction of lift and drag forces. Because of that most of the pilots they fail to control the helicopters, and because of that we had these twelve major accidents within eight years that was caused in Eastern Himalayas.

So, if you look at this problem, it is a fluid mechanics problem, and we have not studied or we do not have a knowledge of so complex weather systems. It is a part of fluid mechanics. And even if you have a very robust helicopter system it crashes usually. So, if you look at this problems, it is a really challenging problems. Still we have the challenge of fluid mechanics problems.

As I said, it is a simple example, that the crashing of the helicopters in the Himalayan region is because of the weather system which changes very fast and that weather system changes the pressure and velocity field so drastically and there might be major macro turbulence structures, that is the region. If you heard about the fighter plane crashes over the Eastern Himalayas in World War II, also we can have the same assumptions.

It is that change of the climate systems would have happened during World War II, that is what is never understood. That might be one of the causes. There was a lot of crashed weapon during World War II in these regions. So, fluid mechanics is an interesting subject. No doubt it is a basic course. We will talk about it in a basic level but my suggestion to you it to read fluid mechanics to solve what is real life, at present, that we have been facing.

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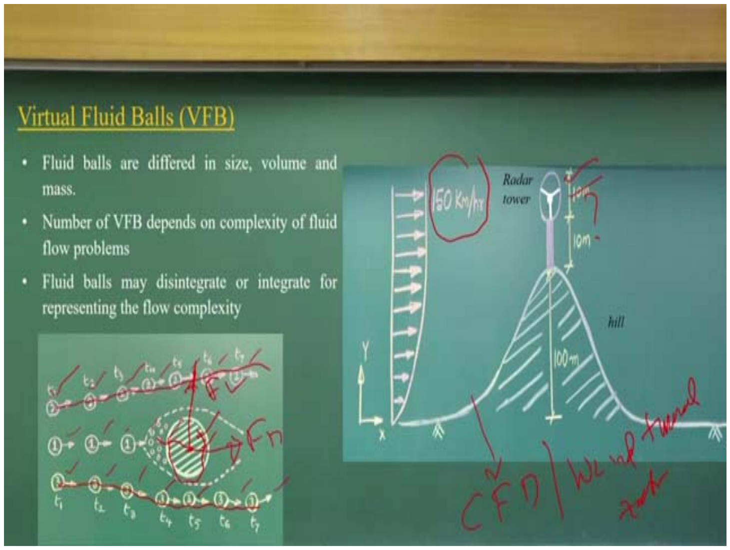

Now, as I said it earlier, to understand the fluid mechanics we should have the concept of virtual fluid balls, which is a new concept as I said it earlier. If you look considering these, I will discuss with you the difference between system and control panel. So, as I told earlier, I am just looking at virtual fluid ball concept where this type I have the hill, I have the radar towers, and this could be expected velocity distributions having 150 kmph speed, okay?

So, you can understand it, this is a very simplified case, as I gave example of helicopter crash, but this is quite simplified case that I have only one direction velocity distribution which could be higher in the order of 150 km per hour, and we need to design this radar tower. So, if you look at that, this is a very challenging fluid mechanics problems. But analytically, in a simplified way, if I have to compute it, what I will try to do is that this tower I can represent as a small ball, a spherical ball.

And I will have virtual fluid ball concept to find out what could be the diagram of streamlines, okay? If I draw the streamlines, then I can tentatively know what could be the force that is going to happen, the lift force and the drag force, the FD and FL, this lifting force mechanisms and the dragging. This is quite simplified one, okay? To find out just through link the virtual fluid balls and try to find out what could be the streamlines.

Because if we know the streamlines, we can tentatively know the pressure distribution and velocity distribution. As we know the velocity and pressure distribution, we can integrate the pressure distribution over this object to compute what will be the lift force, what will be the drag force. Again, I tell it, it is a simplified version. But for real field we can solve this problem by either computational fluid dynamics or conducting wind tunnel test, we need to do that part.

But in this lecture we will just talk about how we can simplify the complex problems and use analytical methods to solve the problem, that is what my responsibility for this course, visualise the fluid flow problems. Now, I can understand any of the students have a lot of resources now to visualize the flow, a lot of materials are available in YouTube and all, you can visualize the flow and visualization of flow is important as well as to know the analytical approach of solving fluid mechanics problems.

Now, let us discuss about the system versus control volume, what is a system and what is a control volume and how you simplify the fluid mechanics problems.

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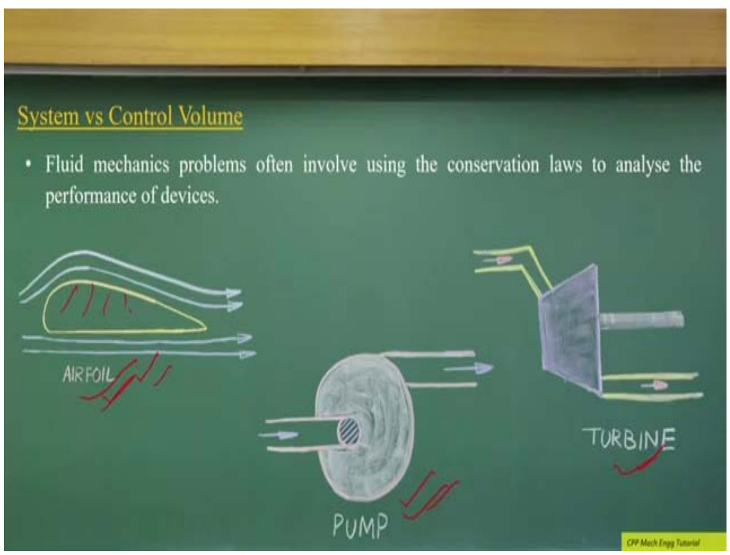

Now, if you look, there is airfoil, this is pumping system, and the turbine system, okay? We can define it, a system and control problem and solve this problems, okay? To know the performance of the pumping systems or the turbine system on the airfoil. You know it is very basic concept of the airfoil used for designing any aircraft. So, the aircraft wings looks, the cross sections, like this.

The streamline flow and the pressure distributions gives us what could be the drag force and the lift force. Based on that we design a fuel efficient aircraft. So, designing an airfoil is a very interesting thing. How to design the shape of the airfoil so that we can have maximum uplift and also the drag force will the less. That is the way we tested the performance of the airfoil or the pumping systems.

If you look at this pump belt or the turbine blade, those also work in a similar concept what we have seen in airfoil conditions.

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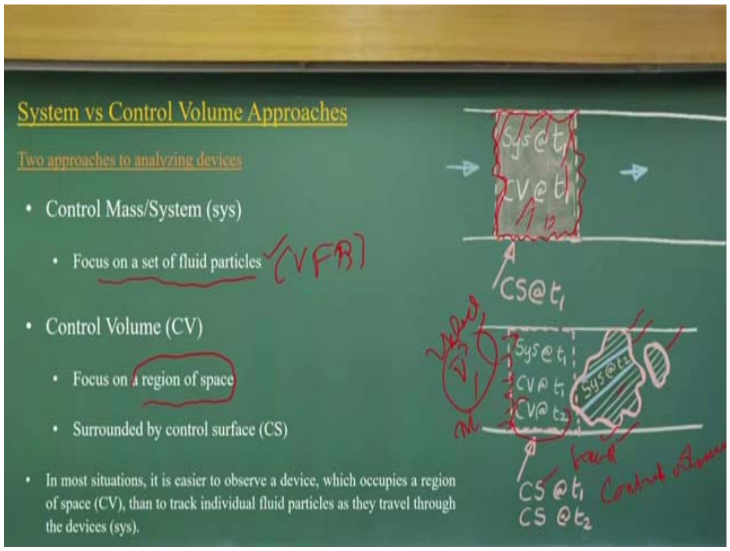

Now, let us come to what is a system and what is a control volume, okay? I will just now talk about the same virtual fluid ball concept. But if you look at this, when you talk about systems, that means we define it as the set of fluid particles. That is what is the common definition of what is control mass or the systems, that means it is what we consider as set of the fluid particles, but now I tell you it is not a set of the fluid particles.

We are talking about set of virtual fluid balls, VFB, okay? Fluid particles are the fluid balls. That means we define there are n number of virtual fluid balls as a system. And as they are in motion, they change the position, they change the velocity vectors, and they change the pressure vectors, okay? So, that means you can see that there is a system, it means it is the collection of n number of virtual fluid balls, or the other way around, a set of fluid particles.

That is what we are defining. But when you talk about the control volume we talk about the region of space, that is the basic difference. It is space definite, it is not related to virtual fluid ball which is having motion. The space can have motion but we define the space which is the control volume. That means we are at the space level, that is particles level. When you have defined the space, that means you have the control surface, you can see this figure, you will have the control surface.

Through this control surface the fluid balls will come in and go out. So, they are the control surface, there is no exchange of the fluid balls or the fluid particles. Same way, there will be a control surface through which we will have fluid, it will come into this control volume, there will be the surface and from that the fluid will go out. That will be the outlet. So, influx and outflux. So, that means you can understand the system and the control volume.

In control volume we talk about the region of the space, whereas in control mass we talk about fluid mechanics or virtual fluid balls. They are coming in and a group of the fluid balls have considered it. That is what this interesting figures is. What it is showing is that if you take the system at T_1 time and control volume at the same time, that means whatever the number of the fluid particles are there, that is what is my control volume at $t = t_1$ time.

But if you look at $t = t_2$ if it is a fixed control volume, then the control surface remains the same. The control volume will be there, but the system which is composed of n number of fluid particles or virtual fluid ball, they will be at time t_2 in this place. May be they are disintegrated, like two pieces. So, the system is dynamic. Giving example of n number of fluid balls, virtual fluid balls are coming in, after t_2 they will be in a different space. They may disintegrate, they may have a different location.

But the control volume, what we talked about is that if it is a fixed control volume, it remains at that space and whatever coming into and going out at the fluid particles level we do not look. But what we look is, because of this fluid particles coming into the control volume, what is the change in the velocity field here, okay, velocity vectors, and the mass flux, how much of mass coming through this surface, mass flux, the pressure gradient, momentum flux, and energy flux.

So, we do not bother about the fluid particles, the moment of the fluid particles. We bother about that because of this fluid particle the transport mechanisms happening through this control surface, how the velocity distribution changes, the pressure distribution changes, mass flux changes, the momentum flux has changes, and energy flux has changes. So, the problem is quite easy as compared to at the system level.

That is the reason that at the control volume level where we can define the surface through which there will be mass exchange, momentum exchange, and energy exchange, and because of that there will be change of velocity distribution and pressure distribution, and temperature distribution as I do not focus more on compressible flow here in this basic levels. All the field variations of the density, temperature, the pressure, and velocity, that effect we find out because of the transport mechanism of the fluid particles or the n number of virtual fluid balls which is going through this walls.

So, try to understand this fluid system. When you use the control volume we try to find out not to tract individual fluid particles what is happening, but to get it, because of this fluid particle's movement, what could be the change in pressure distortions, in velocity distortions, and also the mass flux and momentum flux and energy fluxes. That is the concept. We should build it and we should know how to define the control surface.

No doubt, I will deliver this lectures of the fluid dynamics which is there in F.M. White book or Cengel Cimbala book, more than five to six lectures, try to convince you the basic concepts of system control volume. If you understand system and control volume approach and if you are use the control volume concept like free body diagrams in solid mechanics, you can solve many problems in very easy way.

What I am to say is as you are trained to use free body diagrams to simplify a complex solid motions and the force components exactly. The knowledge of the control volume, velocity, pressure diagrams approximation, and the streamline patterns will help us to solve the complex problems in systematic order. Considering that I will emphasize more on control volume concept.

That is my objective and I have designed this course in such a way that you should have more exposure to control volume concept. And if you understand control volume concept, really you can solve actual problems that we have been facing in natural sciences, the designing of any hydraulic systems and all, especially the problem that we have been facing in 21st century. Let us come back to the topic, okay, we should have the types of control volume.

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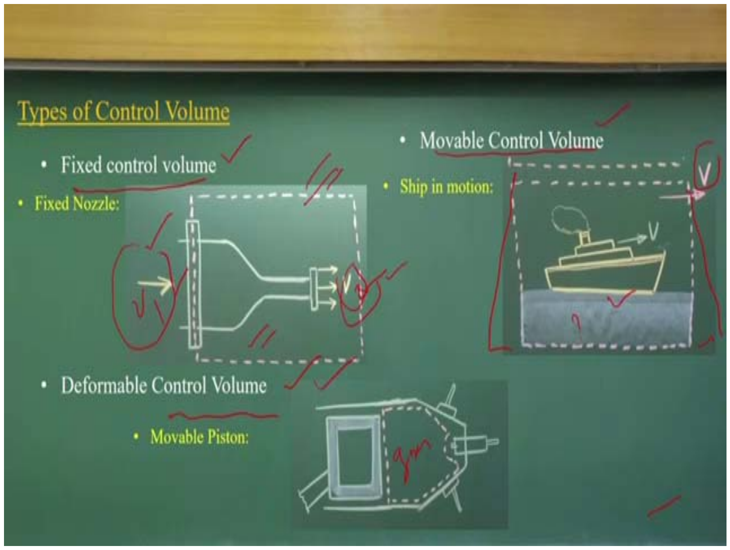

As I have given you firstly that if I have a nozzle, okay, the flow is coming in and going out. If I have V_1 velocity, definitely the velocity V_2 will be different. There will be a change in energy, change in the momentum flux, that will give you a force on this system. So, to find out what could be the force acting on this system, we can use a fixed control volume having influx of velocity of out flux of the velocity V to find out what could be the forces, what could be the energy losses.

So, we use many of the time fixed control volume. That means the control volume space does not change with time. The control volume remains at the same locations. Fluid are coming in and going out. So, that is what we do when we consider fixed control volume. But if you consider that I have a movable control volume, like for example, I have the ship which is moving with velocity V. So, we can consider a moving control volume.

That means I have the velocity V and I try to find out what could be the drag force acting on this or the lift forces acting on this, how the turbulence structure is created by this ship, and what that effect is going to do on ecology of the river which is the next level of fluid mechanic problem that we have been discussing in this present era. We follow simply a movable control volume.

That means the control volume is moving with a velocity V and you try to find out for this control volume, what could be the velocity distribution at the relative level because V is moving it. So, you have to use relative velocity component to find out the velocity field and all. Similar way, we can talk about a deformable control volume which is generally used for fluid mechanics problems in mechanical engineering like there is piston movement that is happening and within there is a gas.

And if I consider the space occupied by this gas will be deformed, also movable. So this very interesting concept that we can have the control volume three types, fixed control volume, movable control volume, or deformable control volume. How to define that, that is the problem that we have considered, whether a ship in motion or fixed nozzle, or movable piston case, that is what we will have.

But we have used control volume concept to solve this problem. Again what I am going to highlight is try to understand any of the problems. I have just given three examples. We can solve it if you can use control volume concept. And the control volume can be fixed, that means it remains at the same region, does not change its position, or we consider a control volume which is moving with velocity V, or a control volume which is expanding, deformable, okay.

Volume is changing with respect to time, then it is a deformable control volume. So, we have fixed control volume, movable control volume, and deformable control volume. Let us have just a few examples to show the difference between system and control volume.

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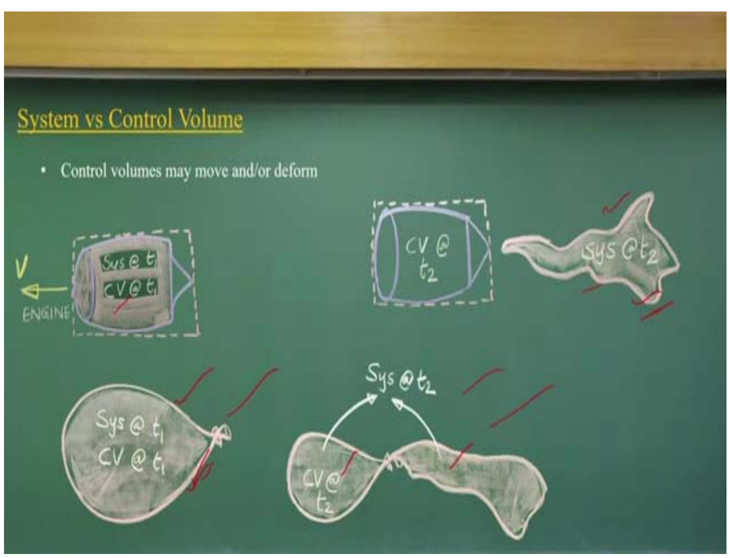

As you look, if you have a fluid which is there inside this engine at time t_1, it can be considered a system. That means, whatever the fluid particles are there they are representing a system and that space we define as a control volume at time t. But after t_2 time your control volume remains at the same place. The system has gone at different locations, can have different shapes. That is a very interesting figure that is there.

The same way, if you talk about a balloon filled with gas, and whatever the gas particles are there, that part you consider it as a system and the space is what we consider as the control volume, and just open up the balloon and we can have a system that will be in two places, okay? In t_2 time the system can have it but you have a deformable control volume which is representing this part.

So, you can understand now with these two figures that we can simplify very complex problems with a deformable control volume and also the moving control volume concept. You should try to understand how we can simplify it when you consider the deformable or the moving control volume concept. That is the art of fluid mechanics portions, to define it, with what respect you can define the flow systems using control volume concept, either a fixed control volume or the control volume which will be moving with a velocity V.

The V could be in vertical form. It can move in any direction, any magnitude. Similar way, it can have a deformable control volume as it is given in the example here. So, if you can understand it, you can make complex problems using these three tools type of control volume, fixed control volume, movable control volume, and deformable control volume.

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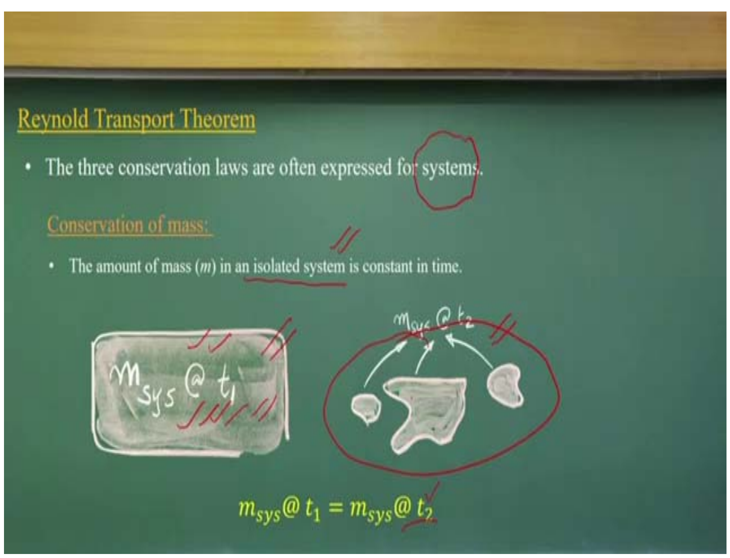

Now, let us come back to, as you know from your twelfth level of knowledge in fluid mechanics as well as solid mechanics, the three basic conservation laws we follow which are mass conservation, momentum conservation which is part of Newton's second law of motion, then we have the third law which talks about energy conservation, that is the first law of thermodynamics.

Now, when you consider a system that means these three conservation principles they are quite valid. When we consider an isolated system like, as I have given examples, the virtual fluid balls, they are isolated systems, okay? They can move at different times. Like, for example, if I consider a system of a set of fluid particles or a set of virtual fluid balls, as you can understand it, there is no change, no other type of mass decaying function is there. The mass will remain constant.

That means what will happen is mass at the system level at t_1 is equal to mass at the system level at t_2 will be the same, but they can have simple space at this location, but when it comes to t_2 level they can fragment and they can occupy very complex surface at t_2 times. So, mass conservation is a very simple thing. You just know it, the mass remains constant at t_1 and t_2 if you are considering an isolated system where you have n number of fluid particles or virtual fluid balls, that number will remain the same whether you have t_1 time or t_2 time.

So, that implies that the conservation of mass holds good. So, that very basic concept is what we will use for solving the fluid flow problems, when you consider a fluid system.

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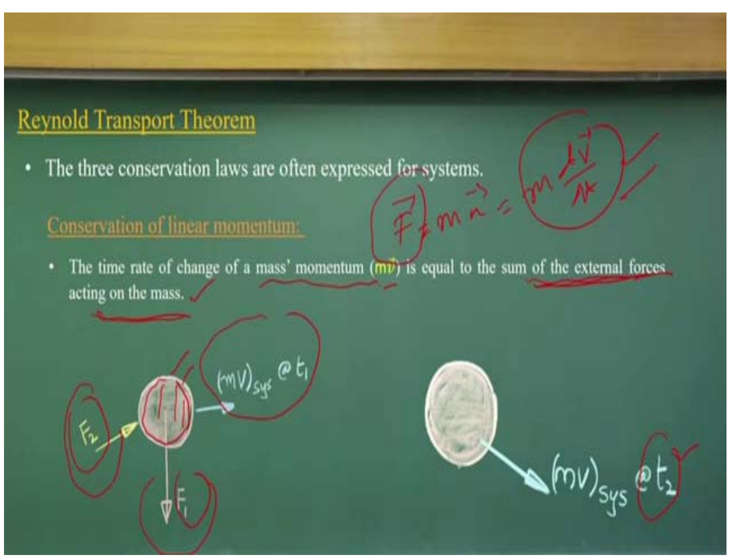

Now, as you might have solved a lot of physics problems on solid body translations, rotations and all, the basic concept of linear momentum equations you apply, which is, you know,

$$\vec{F} = m\frac{dv}{dt}$$

force is rate of change of momentum flux. That is the Newton's second law of motion and you have been using that to solve many problems in twelfth level of the class in physics or fluid mechanics type of problems.

Now, let us come to what happens when you consider as a system. If I consider a system here which may be containing a set of the fluid particles and it has two force components there, and the system is moving with mass with a velocity V, after t_2 time it can come to here. The change of the displacement of this system from one location to other location, it all depends upon how the net force component is working on that, what net resultant force is acting on that.

That is the time rate of change of mass' momentum is equal to the sum of external forces. Mathematically, what I have shown is that sum of the external forces which is going to act on it, that is what is mass into acceleration. That is the time rate of change of mass' momentum, m into v, okay? Mass momentum is equal to the sum of external forces acting on the mass. We know it and just repeat it to relate this when you talk about at the control volume level.

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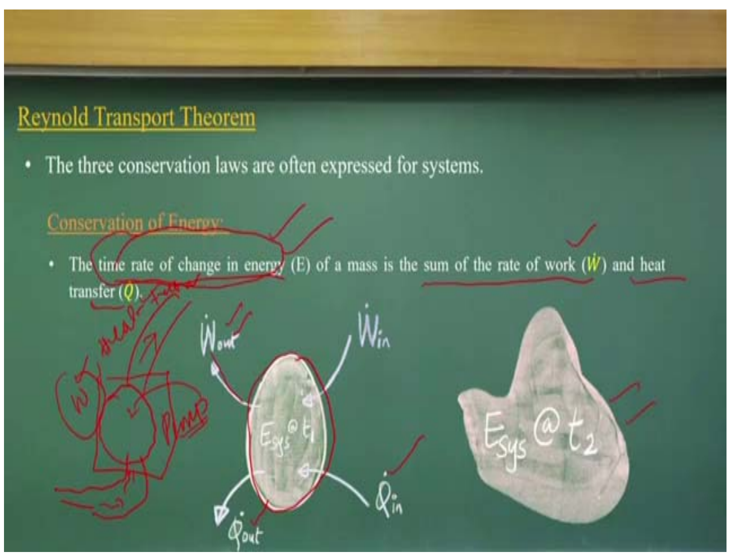

Now coming to the energy conservation equation, see, if you look at the energy conservation, you know very well, energy is conserved unless otherwise you have very particular cases. That means, if I have a system like, for example, let us have a pump system here, okay. And it is connected to the inflow, also it is connected to the outflow. That means two pipes we have, connected to that I have a pump.

So, if you look at that, the pump is giving energy to this fluid, okay. We convert from mechanical energy to potential energy. The pump is giving additional energy or work done by the mechanical to the fluid, okay. So, we have exchange of work to this system. Also, as you feel it, if you touch any pumping system, just the outlet level and the inlet pipe, you can see there will be a temperature different. If there is a temperature difference, that means heat transfer is happening.

So, work done is there and also heat transfer is there. So, any system you consider, pump or the turbine, in a fluid flow system you can say that the time rate change of the energy or fixed mass of fluid particles will be sum of rate of work done and the heat transfer. I have given the examples. That means if you have a system t_1, you can have work done components coming in, going out, the flow is coming, heat is coming out, going out, the net, the sum of these ones, rate of work and heat will change the time rate change in energy.

That is how you can have a system of energy at t_2. So, you can imagine that you have flow pipe systems in your hostels. So, at each point this energy of the flow systems are changing it. So, you have an overhead tank, you have pumping systems, you have pipes, there is energy loss in the pipes, you have outlet points like sink basins. It is just simplified if you look at what pipe networks you have to supply water to your hostels, all follow the energy conservation equation.

But sometimes some components are not that significant. For example, in a pipe flow system I have incompressible flow. The heat energy loss may not be that significant as compared to the friction losses in a pipe. So, that type of simplification we do and that simplification helps to solve the problem. But try to understand that whenever you have a system, to conserve the mass we will have sum of rate of work and the heat transfer.

There is heat transfer process happening. May be the magnitude is more or less, that is what we consider. And because of that there will be a change in energy with respect to time which is the first law of thermodynamics as you know it. That is what the first law of thermodynamics is. We are introducing new laws. The conservation of mass for the fluid particles level, as you know it, is not important, because always it is conserved.

The conservation of momentum which is required for you to know it, how the fluid particles are moving it, what could be the force exerting on that, what could be the velocity. Similar way, we can understand the energy conservation which plays a major role for us when the fluid comes from one location to other locations, how much of work is done by the fluid or into the fluid.

Similar way, whether there is heat transfers happening which you can feel it, if there is a temperature gradient there will be heat transfer either to the surrendering of the systems or into the systems or out of the system, that is what we can do. So, to summarise this, that means, we all know that there are three energy conservation principles that we follow in solid mechanics when you consider as a system.

Same concept also we can use at the system levels to solve the problems, conservation of mass, conservation of linear momentum which is Newton's law, and the conservation of energy which is the first law of thermodynamics. As I discussed, there is a system and control volume. Let us understand the Reynolds transport theorem which establish the relationship between the conservation law at the system level and the conservation at the control volume level.

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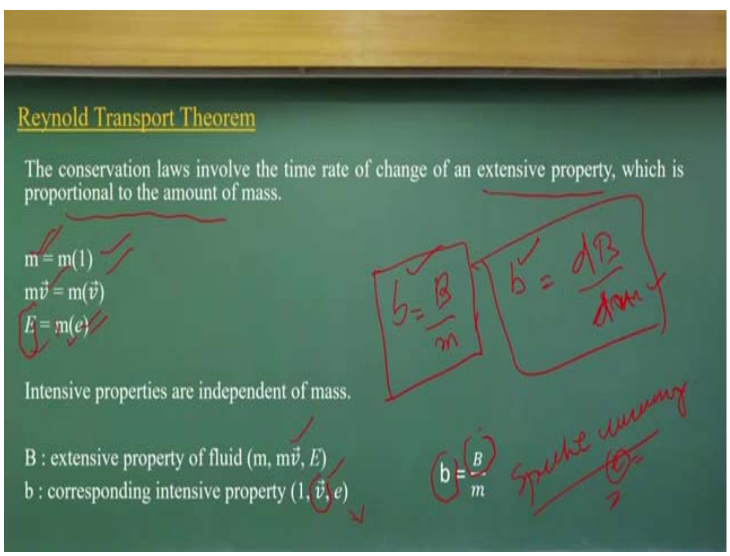

Now, let me define two types of properties that we have; one is called extensive property and the other is the intensive property. The extensive property which is considered as proportional to the amount of mass. When you apply extensive properties, that means you are the properties which are proportional to the amount of mass. That means, as mass increases you will have extensive properties going to increase.

Mass decreases, the extensive property decreases. It is proportional to the mass. For example, as we discussed it in three basic laws, we talked about mass conservation, momentum conservation, and energy conservation. So, m will be the mass conservation part, momentum and energy conservation. But when you look at the intensive properties, that means it is independent of mass, that means, which is denoted as

$$b = \frac{B}{m} = \frac{dB}{dm}$$

So, if you look it that way, there are two properties, extensive property and intensive property. In intensive property independent to mass or $\frac{B}{m}$, per unit mass what we are talking about. For example,

$$m = m(1)$$
$$m\vec{v} = m(\vec{v})$$
$$E = m(e)$$

for energy conservation the extensive property will be the one, but in the case of the momentum, but intensive property will be the velocity vectors. Similar way, if you look for energy conservation, if you look at extensive properties, that is energy.

But intensive property is e which is the specific energy. That means energy per unit mass. So, e is independent to the amount of mass in the control volume or system level. So, we define the difference between extensive property and intensive property. Extensive property we define as B, intensive property we define as b. They have the relationship, simple relation like this, mathematically dB by dm. That is the relationship that is there.

$$b = \frac{B}{m} = \frac{dB}{dm}$$

And we define the difference between the extensive properties in three cases, mass, momentum, and energy, but correspondingly for intensive properties which is independent of mass will be one velocity vectors and e stands for specific energy. Now, we will go to derive Reynolds transport theorem. The derivation of the Reynolds transport theorem are available in almost all the fluid mechanics books.

The idea for me is to introduce the Reynolds transport theorem so that you can easily understand it. But the step wise derivations, if you are not understanding it, I could suggest you to follow any of the fluid mechanics books, F.M. White, Cengel Cimbala, or any other advanced fluid mechanics books, you can see the derivations of Reynolds transport theorem. Only the symbol of representation of extensive properties, intensive properties, either B or b or β are used in different books in different forms.

Otherwise, the Reynolds transport theorem which is the basic equations, the derivation of this equations is available almost in all fluid mechanics books.

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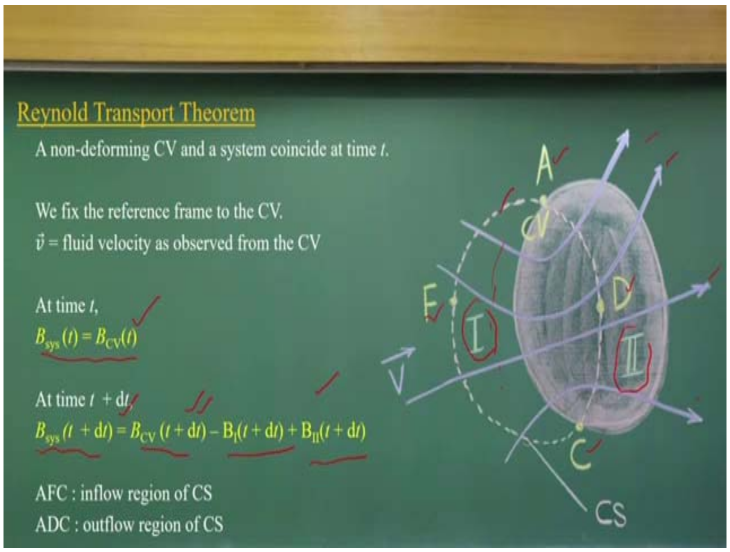

Now, let us come to the derivations which I will highlight as I say it while derivation, which are the major components, not line by line. So, first, what we are considering is a non-deforming control volume. And this is my non-deformable control volume and also I have drawn the streamlines representing the flow that is coming in and coming out. So, if this is my control volume, I can define there will be a control surface defined by A, D, C, and F.

$\vec{v}$ = fluid velocity as observed from the CV

This is the gas part that is the control surface and this part is indicating, these lines or the streamlines are indicating how the influx is coming into the control volume and going out from the control volume, through this surface. That is what is my control volume. That means, whatever the fluid particles present at time t, that will represent the control volume and the reason is these are fixed. Let me have a very simplified case.

I will consider that is the system for me at time t. That means I consider at time t whatever the fluid particles are there within this control volume that is what the system is. As already I illustrated, at $t + dt$, at the next instant of the time, definitely the fluid particles there will move, as we can see from the velocity vectors, they will move out from this and they can occupy the space, okay. This can occupy this space.

At time t,

$B_{sys}(t) = B_{CV}(t)$

At time $t + dt$,

$B_{sys}(t + dt) = B_{CV}(t + dt) - B_{I}(t + dt) + B_{II}(t + dt)$

Now, I can define this 3 into 3 different spaces, like the space defined by this part I can give it as I and this can be II and this can be used as the control volume space, the space is occupied at the system at t time as well as system at $t + dt$ time. So, I have defined these regions into three parts, one is the influx region, the other is outflux region, another is the common region which is there when the system at t time and also t plus delta time.

So, there is influx region, there is outflux regions. So, at the system level $t + dt$ I can define it at control volume level of $t + dt$. The positive and negative you can understand. We are defining in terms of in or out. That is the sign convention that you can try to understand when I talk about the velocity and area dot products, that is influx and outflux will have different signs, that is what we will discuss.

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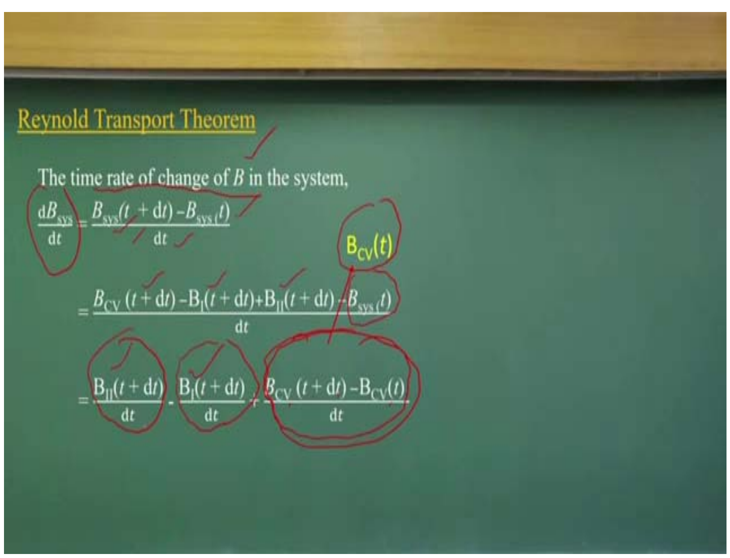

Now, if I look at simple definitions, the calculus is that the time rate of change of B in the system as a definition $t + dt$ minus the system at the t level, and I just apply what I have derived, the three components at the three regions, replacing these values. Again, I know, the B value of extensive property at t is equal to the B value of extensive property of control volume at t.

The time rate of change of B in the system,

$$\frac{dB_{sys}}{dt} = \frac{B_{sys}(t + dt) - B_{sys}(t)}{dt}$$

So, I replace this value at the system to the control volume level. If I just do a rearrangement of this equation, I will get one part, you can understand it, at the control volume level which is showing B control volume at $t + dt$ time, B control volume at t time by dt. That means, what is the time rate of change happening at the control volume level, that is the definition what we will get.

$$\frac{dB_{sys}}{dt} = \frac{B_{II}(t + dt)}{dt} + \frac{B_{I}(t + dt)}{dt} + \frac{B_{CV}(t + dt) - B_{CV}(t)}{dt}$$

And we have other two parts which is representing influx and outflux of these regions, of region 2 and 1, $t + dt$ by dt and $t + dt$ by dt for B_I and B_{II} which are the different regions, the influx and the ouflux regions. So, you can understand it. We get dB by dt at the system level can be composed of three parts. Now, I will discuss these three parts to simplify it.

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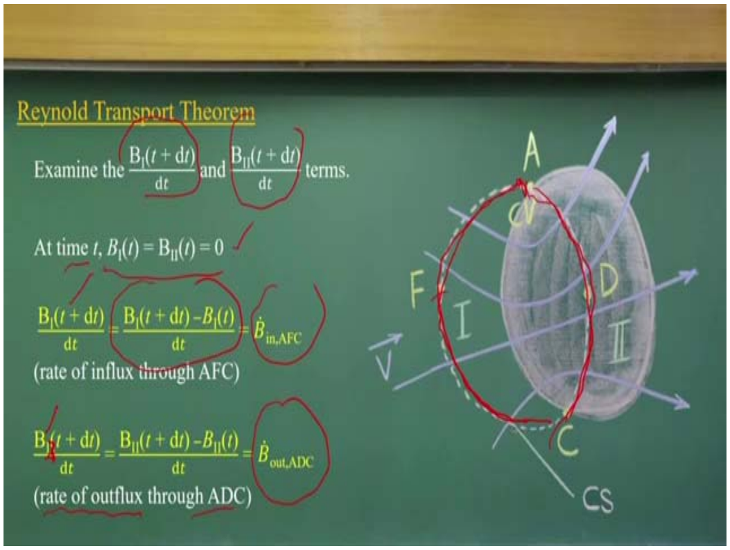

Let us examine this part, okay? Examine the $\frac{B_I(t + dt)}{dt}$ and $\frac{B_{II}(t + dt)}{dt}$ terms

What is the time rate of change of B_I at $t + dt$ time or B_{II} at $t + dt$ time? As you know it, At time t, $B_I(t) = B_{II}(t) = 0$

$$\frac{B_I(t + dt)}{dt} = \frac{B_I(t + dt) - B_I(t)}{dt} = \dot{B}_{in,AFC}$$

what is representing this? That is representing rate influx of B through the surface of A, F and C. Similar way, you can find out,

$$\frac{B_{II}(t + dt)}{dt} = \frac{B_{II}(t + dt) - B_{II}(t)}{dt} = \dot{B}_{out,ADC}$$

which will be again the rate of outflux through this A, D, and C. So, the two components we define it, one is representing influx and the other is representing the outflux, how much of rate is happening through a surface of A, F, C or through the forces of A, D, C. That is what we represent.

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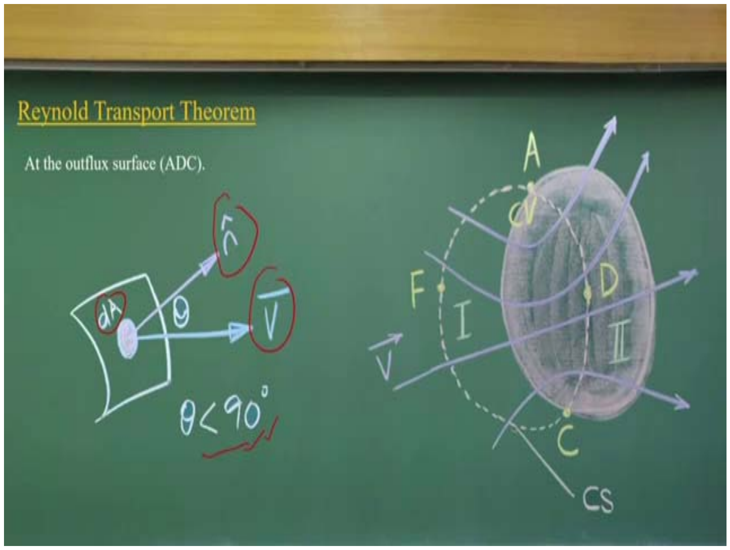

Now, how to compute the outflux surface, okay? That means, assuming it is a three-dimensional control volume, and over that surface I want to integrate it, I want to know how much of influx is coming into the control volume or going out from control volume. That means I can take a small area dA and I can have a normal vector and I can have the velocity vector to that.

So, if theta is less than 90˚ which is representing the outflux, the flow is going out from this control surface. If I have this condition of dA area and I try to compute what could be the flux going out from this surface during the time dt.

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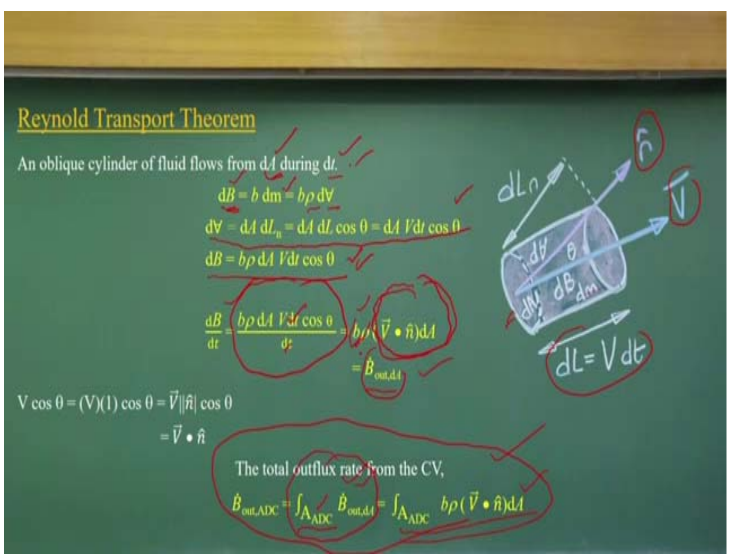

That part if I look, very simple thing, the same element area, I have the velocity vectors, n is the normal unit vector of these things. And if we are in dt time, if you know this velocity V, then the length of this imaginary volume will be velocity and time will be the length, that means what is the volume of the space because of this outflux. That will be V into dt, that is what is the length and we know this area. Now, we have to compute how much is coming through this oblique cylindrical surface which has dA area and during time dt. You can have

$$dB = b\,dm = b\rho\,d\forall$$

dm is representing the elemental mass, the mass will be the density in times of the volume of the surface, and we can use a simple geometry to find out the volume which will be area into length, then we can find out the length projections, then we can convert the length dL equal to V into dt. So, we will get dB will be this part.

$$d\forall = dA\,dL_n = dA\,dL\cos\theta = dA\,V dt\cos\theta$$

$$dB = b\rho\,dA\,V dt\cos\theta$$

If you have time rate of the dB, if you look it, you get this part where dt is cancelled out, okay, and if you look, $V dt\cos\theta$ is nothing else, it is a dot product of the velocity vector and the unit vector into dA which is the outflux representation.

Since I have a big control volume and the surface is irregular, then I can integrate that through a surface of A, D, C to get total outflux rate that is going from this control volume. So, if you can try to look at it with a simple geometry, we can find out how much total outflux rate is going out from this control volume doing a surface integration.

$$\frac{dB}{dt} = \frac{b\rho dA\ V dt\cos\theta}{dt} = b\rho(\vec{V} \bullet \hat{n})dA$$

$$= \dot{B}_{out,dA}$$

If I do the integration, then I can find out what is the amount of the total outflux rate that is going out from this control volume, either mass conservation, mass and momentum flux are energy flux. That is what we will discuss more as I proceed.

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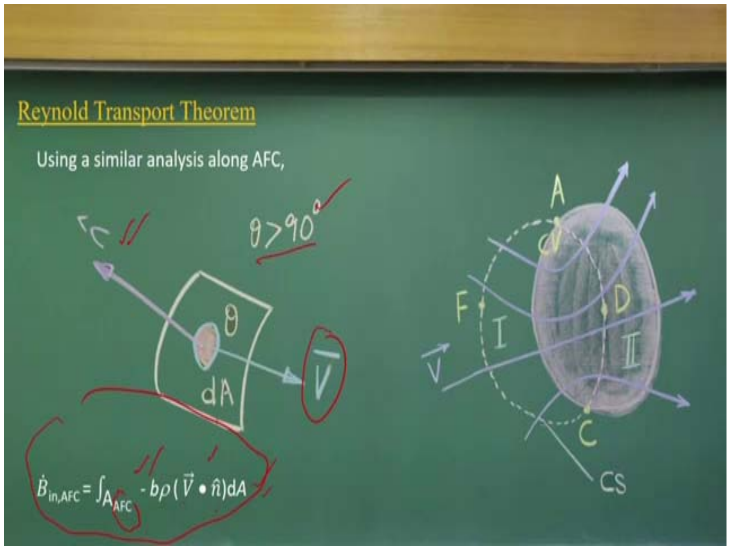

Now, we look at the similar way, if I am to compute how much influx is coming in, so you can understand the velocity will be inward and the surface unit vector will be outwards and your theta will be that. And the same way you can get it, only this sign convention will be different, nothing else, okay. The same way, find out the total influx coming in through this control surface.

It will be a surface integral with respect to AFC, the sign convention indicates which direction it is going on with a dot product of the velocity and the normal vectors of the surface area and then you get the dA part.

$$B^{\cdot}_{in,AFC} = \int_{A_{AFC}} - b\rho(\vec{V} \bullet \hat{n})dA$$

So, the same derivations as I said it. You try to understand the derivation what I have been talking. If you have any doubt you can follow up any of the fluid mechanics book in the chapter of Reynolds transport theorem.

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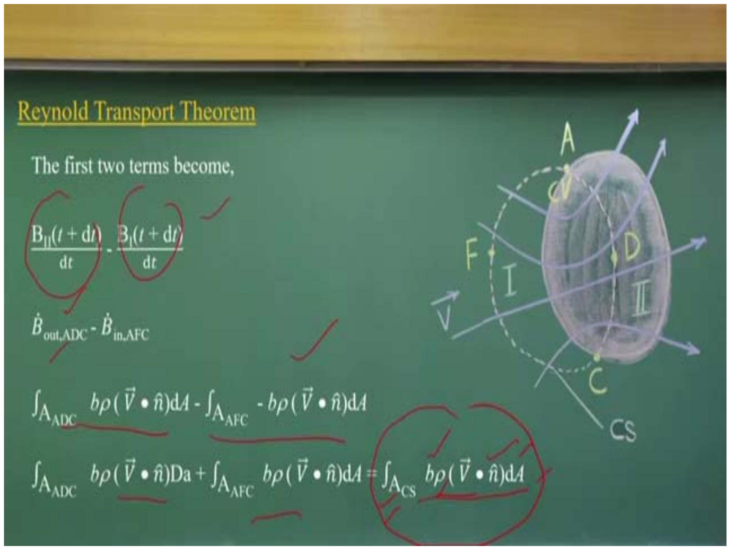

Now, coming to the third part. The first two terms we can write it as earlier you will have the influx and outflux. That part can be written as integrals of influx and outflux and if you combine it next what you are getting? This is total cross section, okay? That means net outflux of mass, momentum, or the energy flux going through the system, either influx or the outflux but as you integrate it it represents the net outflux that is going through this control surface.

The first two terms become,

$$\frac{B_{II}(t+dt)}{dt} - \frac{B_I(t+dt)}{dt}$$

$$B^{\cdot}_{out,ADC} - B^{\cdot}_{in,AFC}$$

$$\int_{A_{ADC}} b\rho(\vec{V} \bullet \hat{n})dA - \int_{A_{AFC}} -b\rho(\vec{V} \bullet \hat{n})dA$$

$$\int_{A_{ADC}} b\rho(\vec{V} \bullet \hat{n})dA - \int_{A_{AFC}} -b\rho(\vec{V} \bullet \hat{n})dA = \int_{A_{CS}} b\rho(\vec{V} \bullet \hat{n})dA$$

If I know the velocity and the unit vectors, if I know the b and if I know the density, how it varies, then we can find out what could be the net outflux that is coming out if we are doing surface integrals of these functions. It looks easy but it is not that easy, that is what my idea is for you to understand the problems in a better way.

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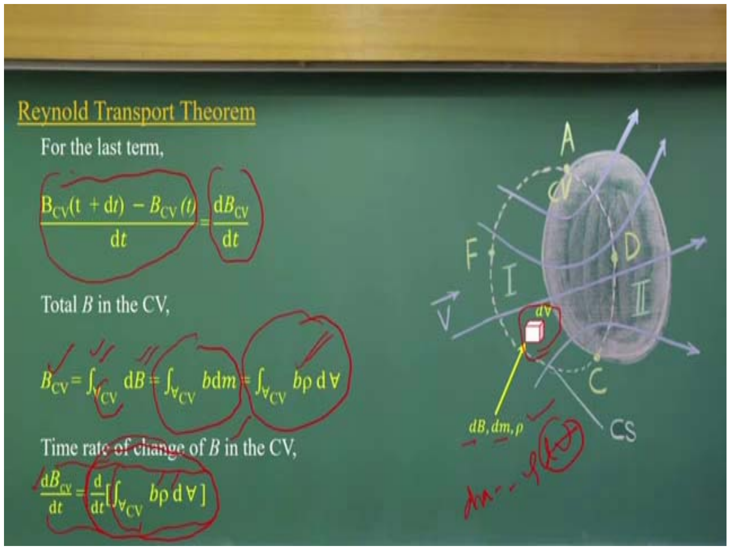

Now, if you look at the last term which is very simplified form, if you look at the definition wise, what is this, the change of the B value

$$\frac{B_{CV}(t + dt) - B_{CV}(t)}{dt} = \frac{dB_{CV}}{dt}$$

How to compute the B_{CV}? What is the total extensive property in the control volume? That means you can do volume integrals of the small control volume what we have considered here.

If ρ is the density, the dm and dv is the mass and this, that means we can just integrate this part and we can have this part.

$$B_{CV} = \int_{\forall_{CV}} dB = \int_{\forall_{CV}} b\,dm = \int_{\forall_{CV}} b\rho\, d\forall$$

Very simple definition, density times volume is the mass. If I integrate the volume integrals over this control volume, then I will know what will be the B_{CV}?

Time rate of change of B in the CV,

$$\frac{dBCV}{dt} = \frac{d}{dt}\left[\int_{\forall_{CV}} b\rho\, d\forall\right]$$

That means the time rate change of the control volume mathematically you can write it as this. Just substituting B CV at this point, it is a time rate change of the control volume. This is the volume integral of B density and dv. If I do it, that is what will represent the **(()) (61:35).** Now, I will just use these three terms to form Reynolds transport theorem which we are just discussing.

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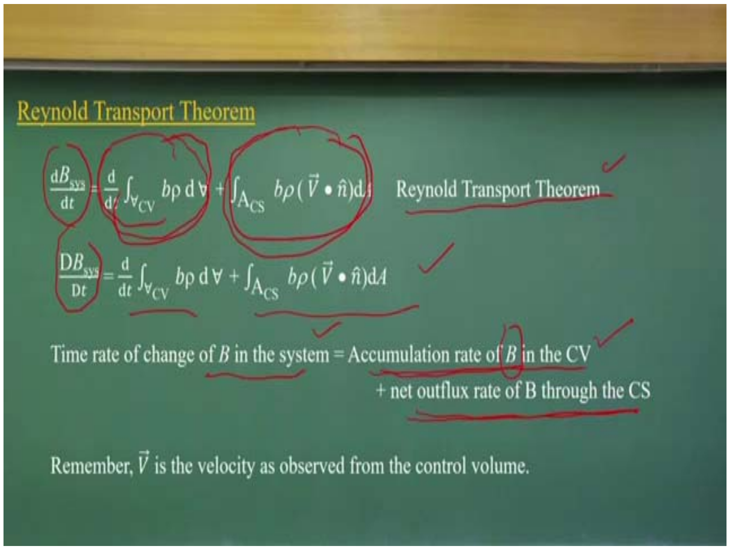

That means the system level of the change of the B value at the time rate of the change of B value at the left side is equal to the, what is the accumulation rate of change of B value at this control volume level which is indicating as volume integrals of B ρ dB. How much change is happening at the control volume levels, how much net outflux of the B through this control surface, that is what is indicating here, okay?

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{\forall_{CV}} b\rho\, d\forall + \int_{A_{CS}} b\rho(\vec{V}\bullet\hat{n})dA \quad \textit{Reynold Transport Theorem}$$

And sometimes we use total derivative to represent that one. So, if you look at that, dB system by dt we can have this system to define it. As summary to that I can say that time rate change of the B in the system is equal to, at the control volume level, accumulation rate of the B value in the control volume, net outflux of the B through the control surface. So, remember the velocity that you observed in the control volume level.

$$\frac{DB_{sys}}{Dt} = \frac{d}{dt}\int_{\forall_{CV}} b\rho\, d\forall + \int_{A_{CS}} b\rho(\vec{V}\bullet\hat{n})dA$$

I will talk about how you use the relative velocity component when you go for control volume moving with velocity V. Thus, we look at very complex problem.

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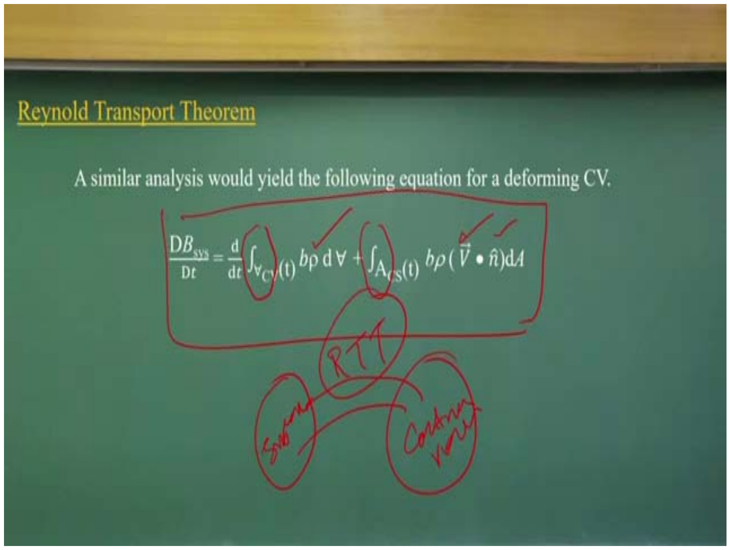

$$\frac{DB_{sys}}{Dt} = \frac{d}{dt}\int_{\forall_{CV}} b\rho \, d\forall + \int_{A_{CS}} b\rho(\vec{V} \bullet \hat{n})dA$$

So, this is what is Reynolds transport theorem, and the basic physics you can understand. It is that we are relating with a system level and the control volume levels. That means Reynolds transport theorem now, if you have a system, you have a control volume, and very simple representation is the relationship is developed by Reynolds transport theorem which looks like mathematically very complex problem.

Now, we have volume integrals, we have surface integrals, we do not know how density varies it, how the velocity varies it and how it is related with the system to control volume level. But this is what is my duty, to simplify this complex equation and solve many problems. That is what I will do in the coming five to six lectures.

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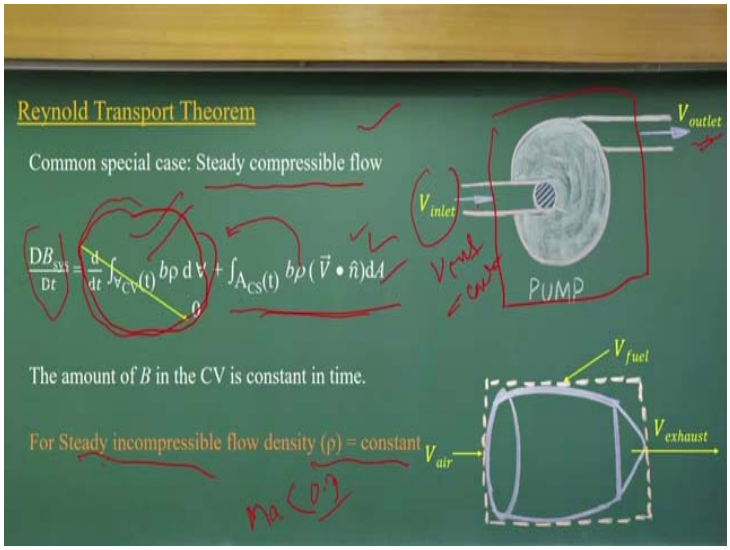

Let us come back to very simple case that we can do, the steady incompressible flow, okay. If you consider a steady incompressible flow, that means the flow does not change with time, the density changes in this case of the steady compressible flow.

$$\frac{DB_{sys}}{Dt} = \frac{d}{dt}\int_{\forall_{CV}} b\rho \, d\forall + \int_{A_{CS}} b\rho(\vec{V} \bullet \hat{n})dA$$

0

If that is the condition, your major part of this becomes 0, because if d by dt becomes 0. Let us consider this case. I have V inlet coming and V outlet is going out and I have a pumping system.

If I consider the V inlet is constant, you can observe it, your V outlet will be constant, steady state will come, it you make it, the inflow is constant. After a certain time if you look, the V out is constant. So, there will be no change. There will be flow but there is no change in the velocity influx and outflux, then what it indicates is that there is no net change in storage or accumulated mass momentum flux within the control volume.

That becomes 0. If it is that, then, dB by dt system is equal to this part. Just we are going to do surface integral to solve how the system is changing it at the control volume level. So, that is the simplification if you use steady compressible flow. But most of the time as you use the steady incompressible flow density is constant, the mac number what you consider the flow is less than 0.3, then your density will come out. So, the problem becomes more simple.

You just do integration of velocity and the unit vector product with the B value which could be specific energy, could be the velocity vectors, or could be 1. So, the problem becomes simpler when you consider steady incompressible flow. What I am to emphasise is that the student has to understand how to simplify the problems, whether he has to solve the problem as a steady compressible flow or steady incompressible flow or you make it total unsteady compressible flow, which again you have to do volume integral to solve this.

The simplification matters a lot to solve the problem as compared to using the advanced mathematics, try to understand this.

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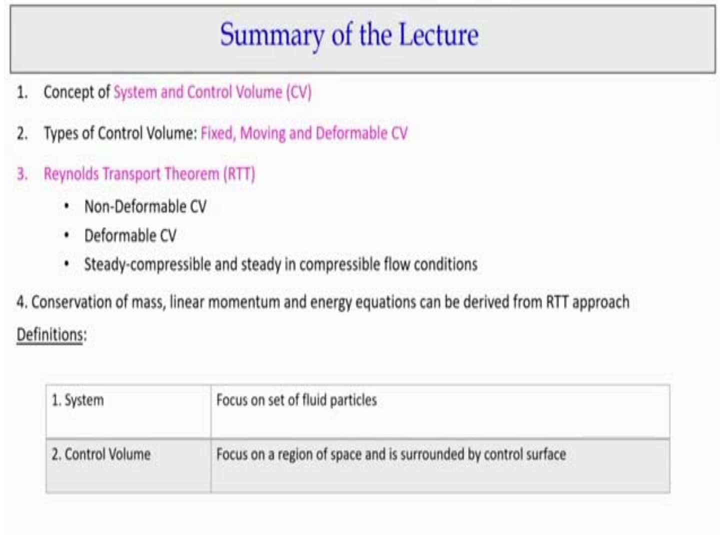

That is what I will try to give a lecture on that, how you develop an art to simplify a complex problem using the control volume concept. With these things let me summarise today's lecture. We discussed about system and control volume. We talked about fixed and deformable and moving control volume concept. The more important thing is that we derived the Reynolds transport theorem which can be used for fixed control volume, deformable control volume, moving control volume.

And we also demonstrated the use of the simplification of the steady problems. We simplified the problems as compared to go for unsteady incompressible or unsteady compressible flow which are complex problems where we need to integrate surface integration and volume integrations and solve this problem which is more complicated than these things.

So, we can have conservation of mass and linear momentum energy equations. We can derive it from RTT. That is what we will do in the next class, and as system wise I can again put this focus on the set of the fluid particles. We talked about the region of the space which is bounded or surrounded by the control surface. We will discuss in more detail about how RTT will be used to derive this mass linear momentum energy equations in the next class. Thank you a lot for this.