Fluid Mechanics
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Lecture - 05
Measurement of Pressure and Hydrostatic Forces

Welcome to this lecture on fluid mechanics. As we discussed in the last class fluid at rest and fluid statics we have derived basic equations of fluid statics. That is what the pressure equations with related to gravity field. So now, just we will have a two applications of the fluid at rest or hydrostatic pressure distributions that what I will address as a applications to two cases.

One is manometer. Another is for a inclined surface, a somewhat inclined surface what could be the pressure distributions, what could be the total pressure force acting on that plane, also the center of pressure.

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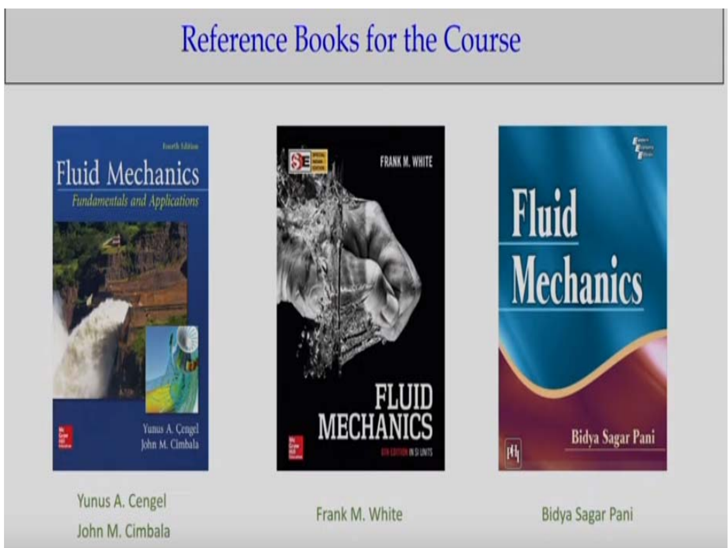

As again I am to highlight it that you just refer these books starting from Cengel Cimbala, F. M. White, and the Fluid Mechanics by Bidya Sagar Pani.

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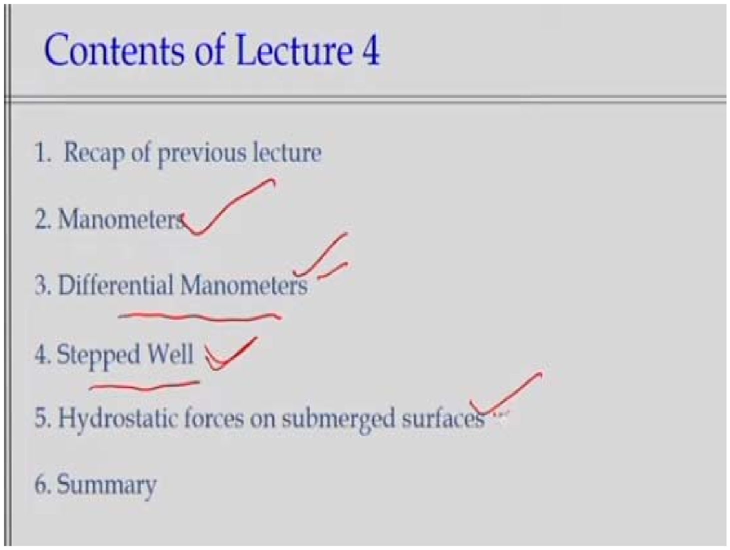

In the last class as you know it, we discussed about fluid at the rest. That is what I will have a recap. Then I will talk about the manometers. And then we will talk about differential manometer. And I will give you a very interesting examples of the stepped well, which was designed long back by ancient architect and how they have designed the stepped well and that what we will discuss it.

Then we will have the applications of hydrostatics that to find out the forces on the submerged surfaces and we will have the summary of this.

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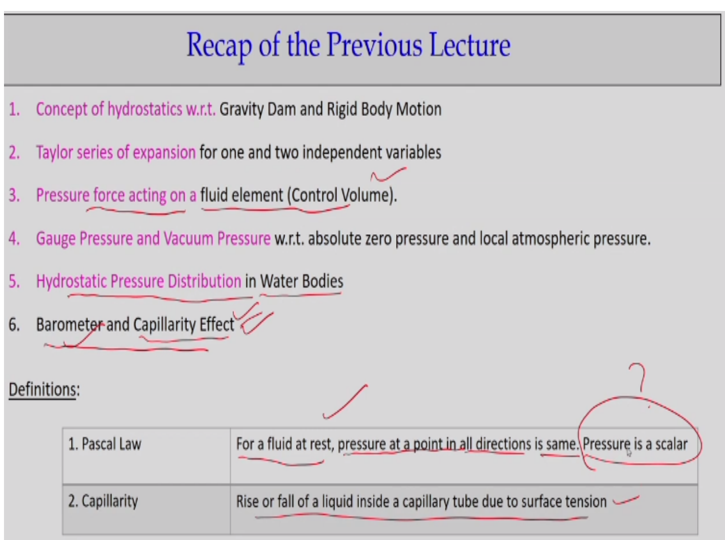

In the last class just to have these things that we derived the pressure force acting on the fluid and or the control volume. And also we know it how the pressure, the hydrostatic pressure distributions it just varies along the z direction when you simplify the problems and on a horizontal surface, the pressure becomes constant. So those the hypothesis we will use it now for derive the equations for manometer or the inclined submerged surfaces.

And also we have told you that how simple capillarity effect is considered for a mercury and the tube to measure the atmospheric pressure that is a barometer and how this capillarity effect also we computed using the control volume concept not the simple equating the force components. That is what we discuss it and the Pascal's law as is things that pressure at all the point in all the directions is the same when fluid is at the rest.

That means pressure is a scalar quantity, does not have the directions and the capillarity concept what we discuss it with respect to the surface tension.

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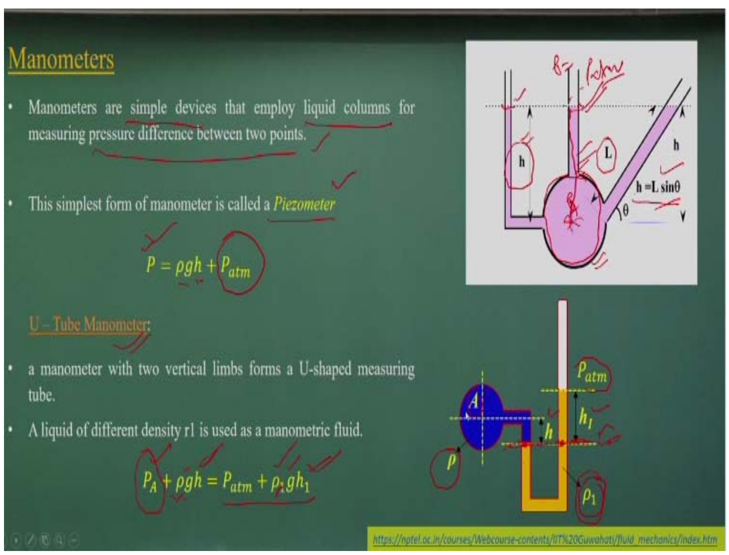

Now let us come to the manometer. Very simple devices are used to measure the pressure like you have a, let have these type of conduit, the pipes the carrying any liquids and you want to measure it what could be the pressure on that pipe. So we can just insert the tubes like you can see can have the tube like this or can have a tube like this with inclined and all or you can have a inclined like this.

So each one having the merit and also disadvantage, like for examples, the manometer is simple device that is what use a liquid column to measure the pressure between two points okay. So in this case, we are measuring the pressures at for these the conduit the pipes using just a simple manometer or liquid columns to measure the pressure. So you can see that when a simplest form is a piezometer.

What is showing here is a piezometer and to find out the pressure. If you know it pressure at this point is P_{atm}. P = P_{atm}. If I taking a point P here and this dimension is very small as compared to the length of height of this piezometer column. So if it that this P_A is the pressure on this conduit will be the atmospheric pressures then weight of this column. That what will be the per unit area.

That what become a $\rho g h$. If its height of the column what we got it h, then the pressure at this conduit will be the atmospheric pressure. That is what is acting on this and the $\rho g h$will be the pressure what we will get. So if we just measure the height you can find out what will be the pressure on this pipe what is going through that.

$$P = \rho g h + P_{atm}$$

Similar way you can have a inclined one or just a vertical one or the having this concept of the half rectangular case.

See if you look it that in case of the inclined one that if I have the inclined length is l that means the height of the liquid rest is

$$h = L \sin \theta$$

simple projections okay. $\sin \theta$ will be the projections part of this incline. That what we have a $h = L \sin \theta$. But sometimes we use the U tube manometers. That means we do not insert just a column. We make a U-shaped manometer; we make it U shape.

And we also put different fluid on this. So like for example we will have a different the liquid like a heavier or the lighter. Then this the fluid what is carrying by these pipes. So what we have a we have a manometric liquid and we use a U tube concept okay. Is just a U shape we consider as a U tube. And then if it this is the conditions if you look it that I am to find out what is a pressure is acting on this surface which is a P_A pressure at the centroid of the pipe.

See if it is that conditions and this is what is carrying a liquid which is having the density rho and the manometric liquid having a density ρ_1, if I take it this surface, if you look it, if I take this surface, as the fluids are at rest, you can understand it the pressure at this point and this points should be equal. As the when fluid is at the rest if you take a horizontal surface, on that horizontal surface, any point you consider that pressure will be equal.

So the same concept we are using it that if I take these surface which is interface between these two liquids I will have the pressures will be the same. So if the pressure at these two points are same, I can just equate it in this case like if you are looking a pressure at this point will be,

$$P_{atm} + \rho_1 g h_1$$

That is what the pressure at this point.

Similar way pressure at this point will be the $P_A + \rho g h$, ρ is a density of the water. This is density of the liquid what is carrying by this pipe and g and the height what would be observed in this case. So you have a h here, h_1 here. So if I know measurement of these two height and the density of the liquid, but carrying by the pipe also the density of the liquid, which we use as a manometric liquid, then I can compute it, what will be the pressure P_A at this.

$$P_A + \rho g h = P_{atm} + \rho_1 g h_1$$

So most of the times we use a simple device of this type of U tube manometers to measure what will be the h and h_1 with a graduated scales. And measuring this h and h_1 and using the simple hydrostatic equations, we can compute the P_A value.

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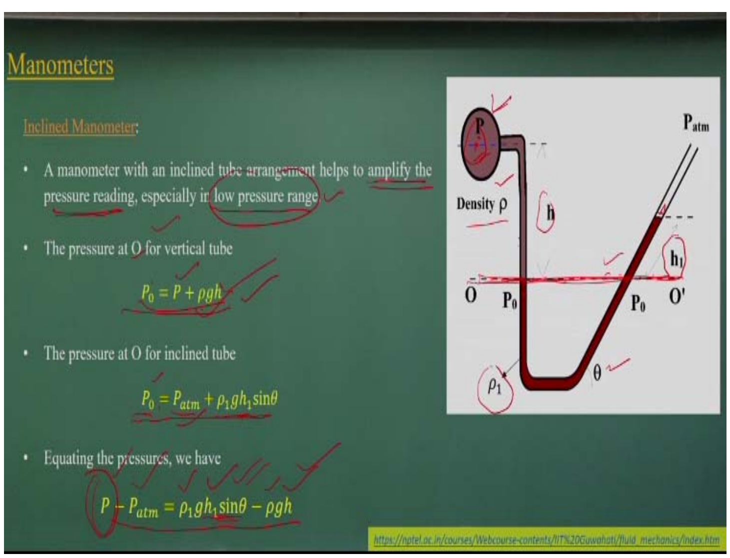

Another case let we have an inclined manometer which is have a quite advantage in the sense that it amplifies the pressure reading as compared when you talking about low pressure ranges. When you are measuring the low pressure ranges it being inclined so it is a give a easier for us to measure the low pressure as compared to the vertical one.

So now if you look it if I have this arrangement, this is the conduit where the pressure on to measure it which is having pressure P and this is the conduit is carrying the liquid which is having density ρ and this is a manometric liquid which is having a density $\rho_1$and because of this pressures we are getting the h_1 the incline distance along this directions and the h is the distance from this P point.

And this manometric look it the interface between these monometric liquid and the carrying liquid by these pipes. If that is the conditions along again in this locations I have to equate the pressure. So as you know it the pressure at this point and this point will be the same as it is a horizontal plane. So pressure at O if you take it the vertical tube will be,

$$P_0 = P + \rho g h$$

The very simple things has come up here. For pressure for incline tube we have considered $h_1 \sin\theta$, the vertical projections part as its inclined at θ and ρ_1 is a manometric liquid g h and at this point we have the pressure equal to the atmospheric pressure so we get the pressure P_0.

$$P_0 = P_{atm} + \rho_1 g h_1 \sin\theta$$

Equating these two we will get these equations which we use to compute what will be the pressure at this point.

$$P - P_{atm} = \rho_1 g h_1 \sin\theta - \rho g h$$

So in this case because you are measuring h_1 which is inclined so that is the reason the low pressure we can measure it because we will have a more the length graduated as compared to the vertical scale. Otherwise this is similar way to measure the things but inclined manometer help us to measure the low pressure regions because it is amplified pressure reading as it is amplifying the case.

As you remember it the wind tunnel what I have shown to you in the third class that is what the inclined manometers used in tunnel to measure the pressures. So this is the same concept of the inclined manometer. That is what we use to measure the low pressure things and which is very easy. Just we are considering the horizontal surface which is a interface between the two liquids.

And then you are just equating the pressure at these two point to get it what will be the pressure at the P point. That what we are looking it. This is very easy. Just you have remembered this concept, considering this, the surface, horizontal surface then equates the pressure part. But only you have to look it that, when you are going down use a positive sign or if you are going up use a negative sign.

That the very basic concept here that if we have a pressure P here, my point is here, which a distance between these, the height difference is h. So I will use a positive thing plus ρgh to compute this. If it is below that, I will use a negative thing or h be considered as a negative value and that what will we use it to this one.

So only you have to remember it to consider a horizontal surface and most of the times you consider horizontal surface that is the interface between two liquids and at that locations surface, you just equate the pressures at the two points to derive the pressure equations. And once you know the pressure equations, as you will have a measurement of h_1 and h as a graduated scale if you have then you know the $\sin\theta$ you know the density and you know the atmospheric pressure you can measure what will be the pressures on this pipe carrying any oil or the water.

So you can measure it what will be the pressure at that point. Now there is another type of manometer which is called the differential manometers. As the name says that here, we do not measure a particular point pressure measurements where you consider two points. Because many of our applications, we are not worried about absolute pressure.

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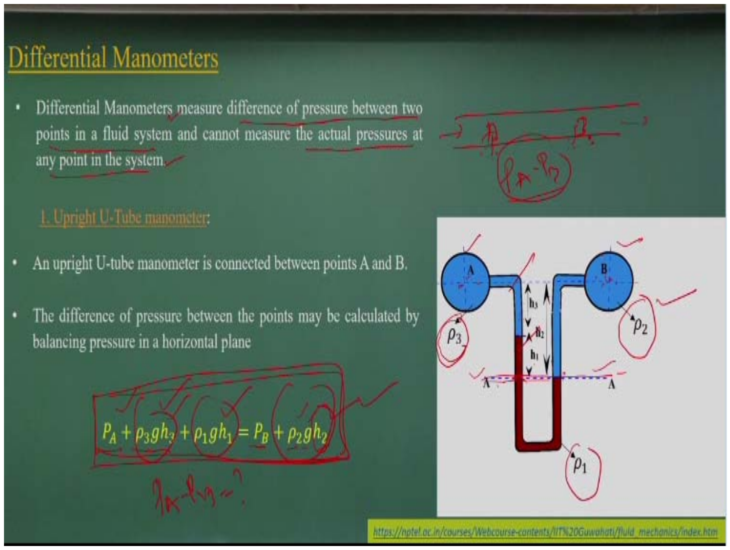

Mostly we are looking it at the two points like I have the pipe and I have the point A and B. So this is a pipe flow, I have the point A and B. At this A and B locations, I want to know it what is the pressure difference between these two point of A and B in this pipe flow. I am not worried about the absolute pressure at the A or the absolute pressure B.

I want to looking for a relative pressure between these two difference between these two points. That means I am looking the difference between which is driving the flow from one side to the other sides. I am just looking it what is the difference pressure between that. So that means we use differential manometers which measure the difference of pressure between two points in a fluid system.

That is what I said earlier it does not measure the actual pressures of any point of the systems. It is a differential pressure. Any two point of a system we can attach this manometer to measure the pressure difference between $P_A$and P_B. That the difference we are measuring. It has also a very simple U tube connected to these two locations like this case, I have the two pipes let be a P_A and B and we are to locate the pressure difference between the P and P_B.

And this is what the manometer is attached to these two flow systems. If it is that the conditions and consider these tubes are carrying the same the different liquids okay. They are carrying the different liquid having the different density like ρ_2 and ρ_3 and ρ_1 is the manometric density of the manometric liquid okay. If this is the arrangement and you measure the distance from this flow systems and this manometer becomes equilibrium after this certain flow that.

Then we measure the distance h_2, h_3, and h_1. Now, I can write on the surface of AA okay, on the surface of AA, I can equate the pressure. Because I already told you that pressure along the surface will be the same. So if I start that ones from this side will be the $P_B + \rho_2 g h_2$. That is very clear cut. The pressure will be this way. If I go it P_B is the pressure at this point, then difference will be the $\rho_2 g h_2$.

$$P_A + \rho_3 g h_3 + \rho_1 g h_1 = P_B + \rho_2 g h_2$$

If I look it at this point the pressure which is connected to A part will be the P_A pressure at that point then because of the fluid what is carried by these conduit, you have a h_3 is the height which will be $\rho_3 g h_3$. Then you have a manometric height liquid the difference which will be a $\rho_1 g h_1$. So you will get the this very basic equations to compute the pressure difference.

So you can finally make it what is a $P_A - P_B$that what you can simplify it or you need to remember any things. You take appropriate horizontal surface at the interface then just equate it the pressure as I go down from $P_B + \rho g h$, the weight of the fluid will come it. Similar way I start from P_A the weight of the fluid of this part the weight of the fluid of this part if I combine it the pressure will come it here.

When you are talking that where per unit area we are considering so that is the reason we have $\rho_3 g h_3 + \rho_1 g h_1$. So this two terms comes here and this is the terms what is comes from here. So you can write the simple equations. You need not to remember it. Only to know it that as I am going down the weight of the liquid we have to consider it when you are going down and that way it will come it here to per unit area.

That way it will be as equivalent to pressure will come it. That what you will add it to at this surface. So this very easy way. So whenever you get a manometric applications problems first you choose which ones would be appropriate, the surface where you can write the pressure equations. So if you know that one then you can solve these problems easily, it is not that difficult ones.

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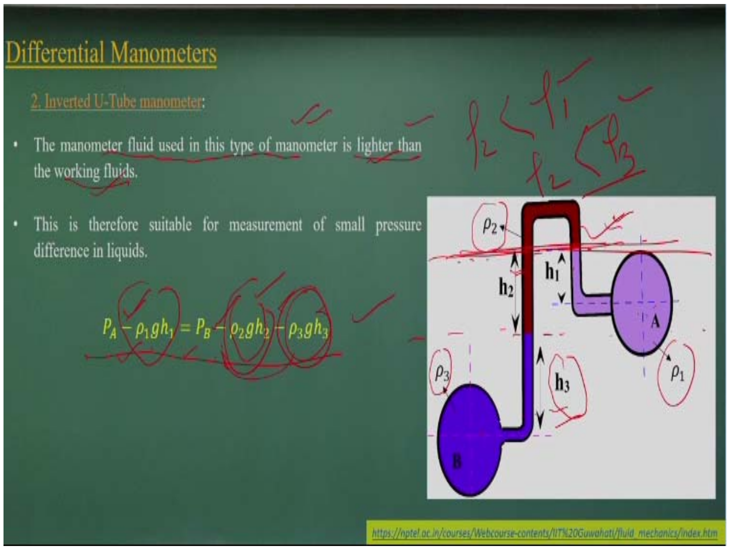

Now if you look it if I have a inverted U tube okay that is inverted cases okay, this is the meter. When you use the manometer fluid use in this type of manometer is lighter than the working fluids. So these are two are the working fluid, which is having this ρ_1 and ρ_3 at two different density of the working fluid. But ρ_2 is the density of the manometric fluid.

And here this manometric fluids having the lighter than the working fluid. That means the $\rho_2$is less than ρ_1 and ρ_2 is less than the ρ_3, okay. So these are very less as compared to this, then we need to have a inverted case. So as it is inverted case, and if I consider a surface, like I am considering this is what the surface for me, okay. Here if you look it, we have to consider negative surface.

Because when you have the P_A here, you can use $\rho_1 g h_1$but it is a negative because it is going up as compared weight is we are losing that weight which is against of that the pressure part okay. So that way P_A $\rho_1 g h_1$. This negative is coming it. Similar way if you start,

$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_3 g h_3$$

So either you can put it here or here it does not matter it.

So you can write this pressure equations for a inverted U tube manometers the considering this is the surface where you are equating the pressure which is the you can consider any other location that does not matter it. Only you have to a sign conversion whether these working fluid, the positive and negative sign of h you can consider to solve the problems.

So where you have to consider this point it does not matter it, you can solve the problems. But you can know it which one will give a simplified case like the surface what you consider it and you can. So in case of a U tube manometer which is inverted one then we use the same equations. Only these negative terms are comes it the because it is a U tube inverted one.

We go off to measure the pressures on that surface as compared to the working fluid which is a lower than that. So that is the reason is a lighter fluid as compared to the working fluid. So manometric fluids are lighter which is a inverted sets then you have this. So we have to measure like this.

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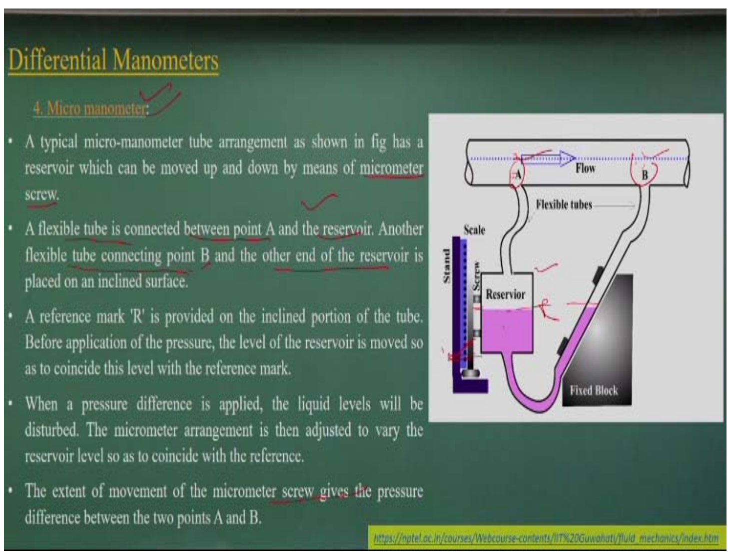

Now there are a simplified micro manometers are there which is a measure the pressure difference between A and B and you have the flow. This flow may be carrying oil or maybe carrying slurry. It maybe carrying any industrial if you look it the any industry is a lot of pipe networks and we have to measure the pressure difference between two points, which is required to measure the discharge, to measure the frictional losses and all.

That what we will discuss when a later on classes end of these courses. So the basically we want to measure the pressure between these two point P_A and the P_B. What we do it, we use a reservoir here okay. It is bigger, then a inclined pipe. Then we have a flexible tube which connected to between point A and the reservoir. Another flexible tube is connecting the point B other end of the reservoir.

So you have the tubes, you connected it that ones and there will be a reference mark R will be there here okay. And this level of the pressure moves so that this level will be the reference mark. You can make put it the reference mark. Here what will be there, when the flow will be there, there will be a pressure difference. That pressure difference will cause the difference in the height of this case.

And these mechanical screw arrangements with a pressure reading as equivalent as the screw goes up and down the what could be the as equivalent to pressure that what is written on this graduated scale on the stand. So we need to read this the manometric height level. Here just to we rotate the screw till make it the reference level and that lifting up this reservoir and the screw arrangement will give us a reading at this point which is the pressure difference between these two points.

So it is a simple arrangement with having a reservoir, the inclined one. We can use the pressure equations to solve it or here to adjust the level of reservoir with help of the screw and the screw is calibrated with a pressure difference what is a graduated here and as you move it as you make a screw adjustment for a particular level, then you will have a parallel reading what would be the pressure on this.

So a simple mechanical arrangement with a manometer, with a reservoir and inclined plane we can measure the pressures with this type of screw arrangement. See this what is called micro manometers. That what we use to measure with a micrometer screw, the screw which is the micrometer levels that what is used for measuring the pressure difference to point P and P_B locations.

Let us come to a very interesting topic on pressure acting on a submerged plane surface. Before going to that let me give a very interesting examples what we have the stepped wells in the western part of our countries. That what is used for rainwater harvesting and the wells are designed such a way that even if rarest drought period also the water is available in that period.

So the well was built long way maybe 2000 year olds or more than that and the depth of the well it goes beyond 50 meters, 100 meters and the well is not serving just for a water, also the wells are used many of the times to have a social gathering, the programs also they conduct it. So how they have designed so beautiful well systems in western part of our country.

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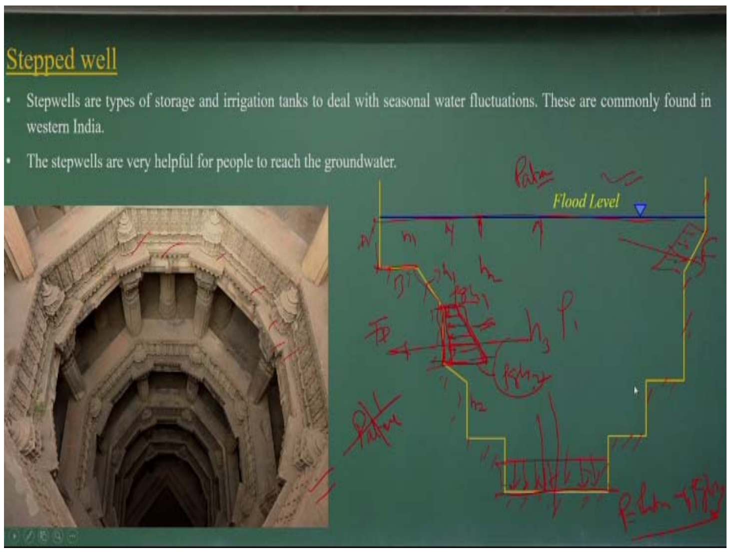

The one of the photographs, what I am showing it the stepwells if you can see it this is a plan view of the step view and there are lot of architecture part and there are the peripheral part. So that means in during these rainy seasons when you have a high rainfalls these well get totally filled up. As it dries off during the summer seasons it goes down and down and down.

So if you look at these the constructions how they have considered the hydrostatic pressure when you have a extreme flow conditions or when you have this well are at the full filled conditions. Like for examples this well is considered let be the flood level here. Now, I am not considering so complex geometry what is there.

If I consider very simplified geometry what I had sketching it here these are there are inclined surface, there are vertical surface, there are may be curved surface are there and there are horizontal surface and all. And these water levels if you can understand it fluctuates okay. During the summer seasons it goes down. During the rainy season it goes up. So it goes up and down with a seasonal variability.

And how they have designed it. What is the water pressure is acting on this surface, working on these surface it is surface A, surface B, one is a horizontal, another is vertical, another is inclined and it could you have the curve nature what is there? There are the curve, the projections and all. How much of water pressure acted when you have a totally the tank while well is in a filled condition.

Very interestingly when design it that means we need to determine what is the pressure force acting on vertical surface, horizontal surface, and the inclined surface. For a horizontal surface like what here it is a very easy. If I have the fluid ρ_1 okay and height is h_3 pressure at this point is P atmosphere you know it the pressure at this point will be the P P atmosphere plus rho g h 3.

And the pressure distributions will be the uniform as it is the horizontal surface. As it is a horizontal surface you will have a pressure distributions will be uniform and as the uniform pressure distributions. So it will be at the maximum force will be act at the centroid of these surface. As the pressure distribution is uniform the center pressure are where the pressure force is acting on that center the that what will be the match with the centroid of the these surface.

The surface can be a circular, can be elliptical shape, but that what will act on that. So you have the results of that. When you have these vertical surface let we consider this case okay. You can find out that as you know if it is h_1 and this is the h_2. So the pressure distributions will be the trapezoidal distributions. Because at this point the pressure will come it as $\rho g h_1$. At the point pressure will be $\rho g h_2$.

So this is the what pressure diagram. This is what the pressure diagram as the vertically the $\rho g h_1$ is a pressure at this point and this h_2 distance from the top will be $\rho g h_1$and I am neglecting the atmospheric pressure. That what we are neglecting as compared to the pressure what is getting from the water which is you know it much higher value of density as compared to the density of atmosphere, pressure at the atmosphere as compared to that.

So we can get the pressure diagram which will be trapezoidal forms and if I know this pressure diagram, I can compute what will be the total force acting due to this pressure diagram which is a very simplified thing to find out the area of these pressure diagrams, okay multiplied with the perpendicular distance or that what is many of the time we consider unit distance.

Then the area of these diagrams will tell me that what is the force is acting because of these pressures. Now another point is coming it where does this force act, the locations of this force acting is the center of pressure. That the center of pressures we can compute it very easily. That what I will be demonstrate it to you, but when you have an inclined surface also we can determine the pressure diagrams.

As you can find out the pressure will come it as a similar way trapezoidal part and the pressure will be normal to the surface, normal to this inclined surface and that pressure diagram again you can find out the area of the pressure diagrams or you do the integrations that is the same things that you are integrating the pressure diagrams to find out what will be the force acting due to the pressure distributions.

Or again you look it where is the center of pressure at which point this pressure force acts. That is what we look it as a central pressure and many of the times if you look at this beautiful the well, stepwells, our ancient people they well known about these hydrostatic pressures and all and they designed the structure which is existing still now without any damage. So that what indicate us the knowledge on hydrostatics, our knowledge on the fluids.

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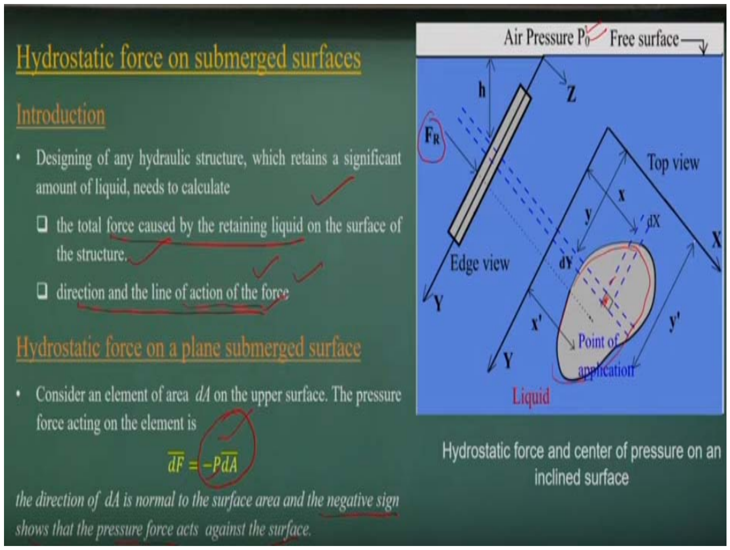

Now if you look it that, let us coming to the derivations of the hydrostatic pressures on a submerged surface. That means surface is submerged in a liquid. So we have the atmospheric pressure acting on that. We have the free surface and I am just putting a surface which is the x and y projections in these directions is it look it from this point you will have an inclined surface which is having this x and y.

This is what very complex shape we have considered it okay. And that is what inclined and is having the h height and we are looking it what could be the force due to this pressure distribution and where does it act. That is what we are looking it. If these two things we know it then we can design these things that what type of material to be used, what type of lifting arrangements will be there, if this is as equal to a get conditions. The submerged plane can be considered as a get conditions.

So total force caused by retaining liquid that what we are looking and directions and lineup for actions as the directions will be always perpendicular to the surface that what the directions and the line of actions what of the force is the center of pressure. Now to derive this part let me consider a small element okay, let me consider a small element. In this small element of the dA if the pressure is P Then P into dA will be the force, pressure into area will be force.

$$\overline{dF} = -P\overline{dA}$$

The negative sign we have considered because considering the normal vectors. If I considering the area that normal vectors and the pressure they have opposite directions. That is the reasons we use the negative sign. The pressure force acts against the surface because surface normal vectors and pressures they are in opposite direction. Because of that we use the negative signs.

Otherwise the pressure acting on this small area of dA will be the force P into dA is a small elementary area what we have considered to derive what could be the total force x acting on this as well as the center of pressure.

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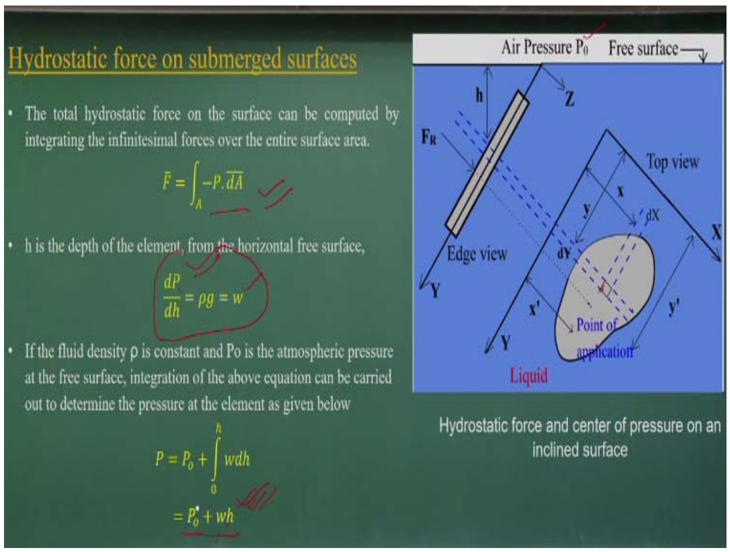

If you look it that, that means to get a total hydrostatic pressure we have to do integrations, the area integration, surface integrations of this quantity okay.

$$\bar{F} = \int_A -P.\overline{dA}$$

So if h is the depth, we know it. The depth is a functions with the unit weight $\frac{dP}{dh}$. Substituting that any point the pressure will come it as we discuss in manometric case the pressure at this point will come as pressure atmospheric pressure plus unit weight of the waters or any liquid what we are considering.

$$\frac{dP}{dh} = \rho g = w$$

Then the h is the height is the vertical height what we are considering. That is what the pressure. If you just integrate these equations you will get it this part, that is what is there and which is we are integrating from zero to h to get it what is the pressure at this element and elemental area of dA.

$$P = P_o + \int_0^h w dh$$

$$= P_o + wh$$

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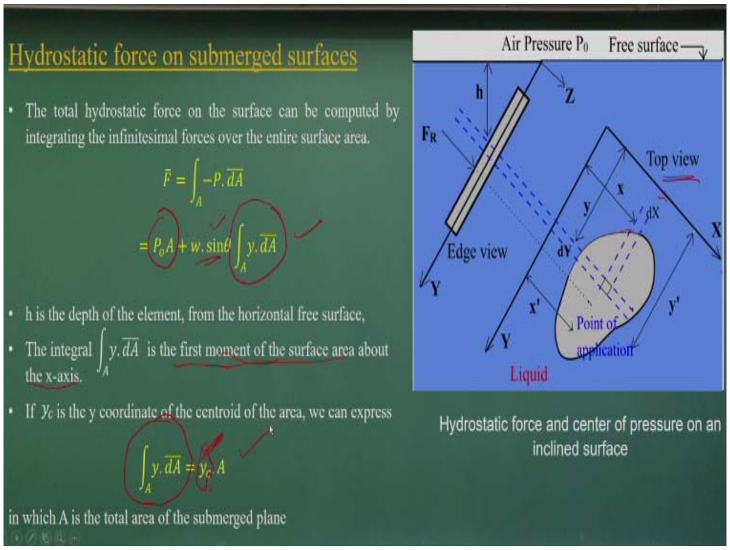

Now if I integrate this part if you know it one component will come it with atmospheric with a another component will be

$$\bar{F} = \int_A -P.\overline{dA}$$

$$= P_o A + w.\sin\theta \int_A y.\overline{dA}$$

What is the y dA? Very simple things is a first moment of the surface area about the x axis. If I have this x axis, if you look it this top view, you have the x axis this is the y axis.

If y_c is the y coordinate of the centroid of the area, we can express

$$\int_A y.\overline{dA} = y_c.A$$

in which A is the total area of the submerged plane. So that means if you integrate this first moment, that means it will become a centroid of y_c that y_c distance from x axis. That is the distance along the y directions. That is the centroid point what will be there if this surface have a centroid this y_c will come it.

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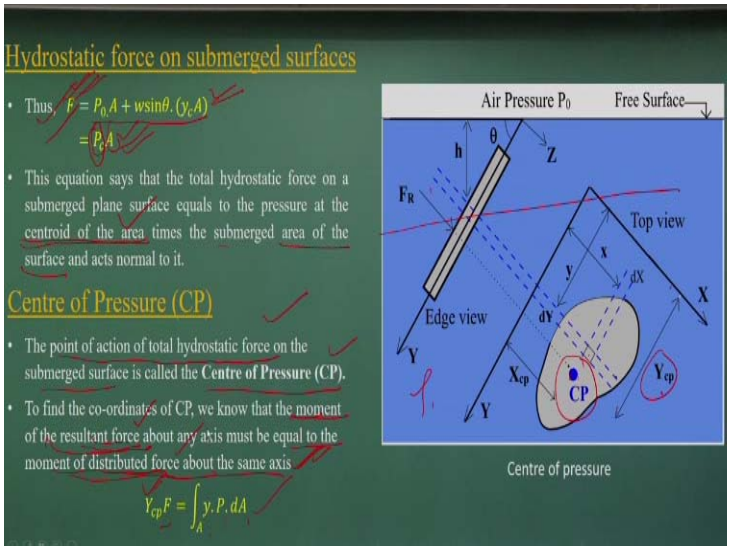

That means, if I just put all these values what I am getting this part, which is

$$F = P_0 . A + w\sin\theta . (y_c A)$$
$$= P_c A$$

If you look at this component and take the A out from these come on to this that is what at the center point, at the center gravity points CG point. At that point, we have what is the pressure that is the Pc multiply the area that what will come is the force. That means if you do integration it is okay.

But if you do not do the integrations for any surface, you can find out the total hydrostatic force on a submerged plane that equal to pressure at the centroid of the area that is what we have derived okay. Centroid of area multiply it with the area of the surface and as we have discussed earlier, it acts normal to the surface okay. One is that you can do an integrations and find out these things or if you know the CG the center of gravity of that submerged plane surface, at that CG point you get what is the pressure.

That is why easy to find out what is the pressure. And once you know that pressure, multiply with the area of the submerged space that what will give you the force component. Is that what you have just to remember it if you want to solve the problems at faster rate okay as compared to do a integrations and solve the problems. But be remember that when you have not a single liquids case like for this case we have a single liquid system.

That means I have the single liquid systems which is easy to do it, but let you have the plate which is having somewhat in a two liquid interfaces. In that case you have to do the integrations to find out where does it act it okay. So the like in this case what we have consider is a single liquid having the rho 1 constant that but if you have a multiple liquid the liquid 1 and liquid 2 they will have a different densities and find out what is pressure force is acting on this.

Then you need to do the integrations, find out the element area, integrate it from one liquid, another liquid, find the total force acting on that as the pressure diagram changes it when you have a two different densities, two different fluids of the different density. So you will have a different pressure diagrams and that what you have to consider and in that case you have to do integration.

But if you have a simple one fluid systems then the density is a constant. Then you can find out the CG. At that CG point you find out what is the pressure. That what you can just equate it atmospheric pressure then height, how what is the point below the from the free surface. Then you if you know that the pressure at this point multiply this area then what will you get it the force component.

So force component due to the hydrostatic pressure distribution on a inclined submerged plane can easily obtain following these simple equations. Now let us compute to that we need to compute center of pressure. That means, the point of action of total hydrostatic force on this submerged surface. That what I tried to explain it that it is not that where his force acting.

That is what is also the plays a major role to design a structures okay, design a gate, design a any wall we need to know it where this force, the location of the force also matter it and that is what the center of pressures. That was the center of pressures. Now if you look it how to compute it.

This is a very simple case as you have done it in case of solid mechanics you take a moments of resultant forces the total pressure and the pressure distribution and take it from one point here we consider from a axis of axis point to compute it what will be the point where it acts it. So there is a simple problem like as you have done cantilever with different loading to find out where the point you have the force is acting it.

The same concept we use it to find out the moment of resultant forces about any axis you can x axis and y axis equal to the moment of distributed force about the same axis. As we do in a solid mechanic, same concept we are using. We know the pressure distributions. There you know the load distributions and you know it what is the total force acting in.

Here also you have a total force acting because of this pressure distribution. Then you try to find where it works it. That what to find out the moment of resultant force and moment of the distribution force. That what we do it. That means $Y_{cp}F$.

$$Y_{cp}F = \int_A y.P.dA$$

This is the point of CP, center of pressure will y into P into dA pressure into area force y dA we just integrate it to equate this part. So y multiplication is there. That what is the any point if you consider that what the moment about the distributed force what we compute it. This is simple from as you did in solid mechanics.

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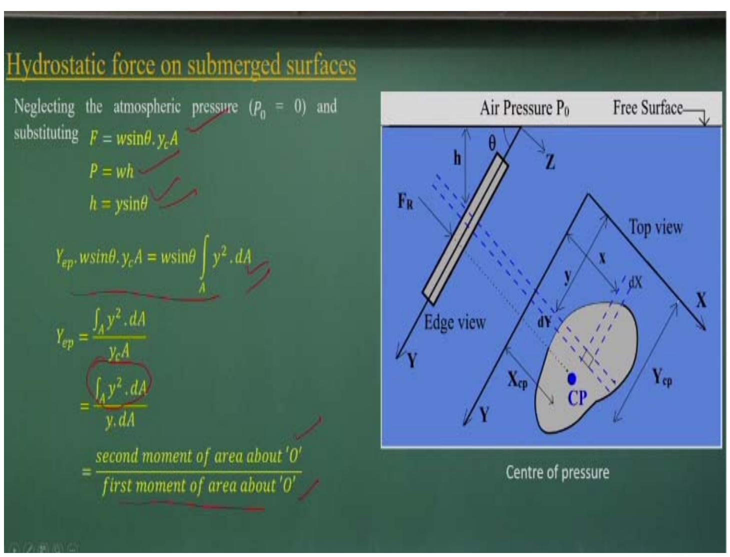

If I do that first, do the integrations and all what I will get it like this force F will be this component as we have derived it. Pressure will be unit weight and h;

$$F = w\sin\theta.y_cA$$

$$P = wh$$

$$h = y\sin\theta$$

Equating all then what we are getting it that,

$$Y_{ep}.\,w\sin\theta.\,y_c A = w\sin\theta \int_A y^2 .\,dA$$

$$Y_{ep} = \frac{\int_A y^2 .\,dA}{y_c A}$$

$$= \frac{\int_A y^2 .\,dA}{y.\,dA}$$

$$= \frac{second\ moment\ of\ area\ about\ 'O'}{first\ moment\ of\ area\ about\ 'O'}$$

That means second moment of area about zero okay. The first moment of area about O which is the y dA part.

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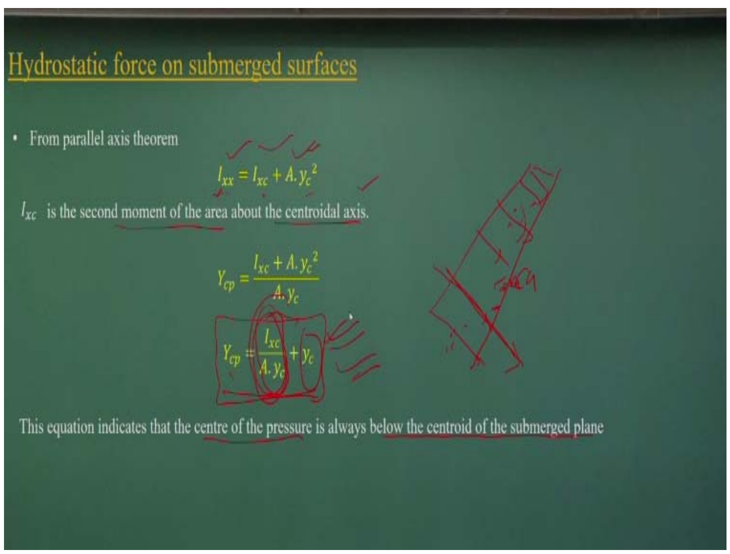

Now if you use the parallel axis theorems, then you can find out what could be the second moment of area about the centroidal axis.

$$I_{xx} = I_{xc} + A.\,{y_c}^2$$

Now if you look at these equations that any distributions you will have this component which is additional component and this is the centroid locations y value. This is Y cp.

$$Y_{cp} = \frac{I_{xc} + A.\,y_c 2}{A.\,y_c}$$

$$Y_{cp} = \frac{I_{xc}}{A.\,y_c} + y_c$$

That means the center of pressure is always below the center of a submerged plane. So if you consider a submerged plane, if you have a CG at this point, your CP is always be lesser than that because of this component mathematically.

And if you look at the pressure distributions you can find out if you have a linearly increasing the pressure distributions definitely it will not be the force, the resultant force will not act on the CG. It will act below that to make it the equal portion of the area, equal proportion of the area in both the sides. That what will happen it just below on that.

So that what the graphically you can understand it. Mathematically also we can see that with the integrations and all we can compute the Y_{cp} will be this component plus the y_c.