Fluid Mechanics
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Lecture - 04
Concepts of Hydrostatic

Welcome all of you for this lecture on fluid mechanics. Today we will discuss about fluid statics that means fluid at rest.

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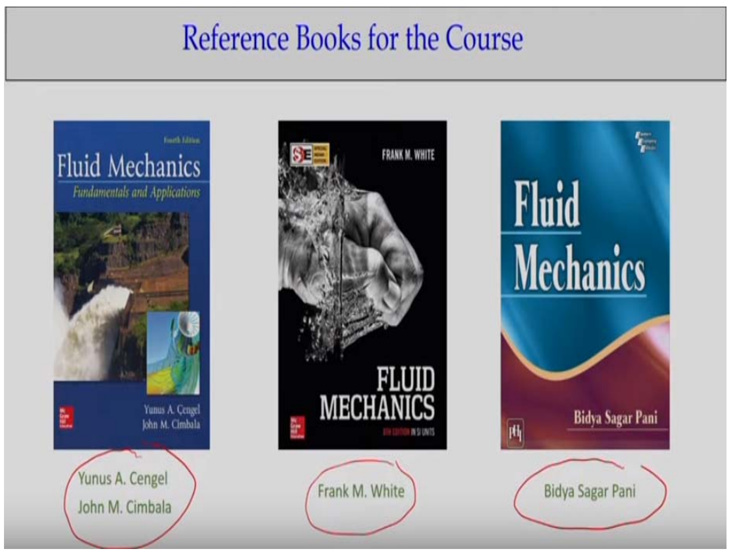

Before starting this lecture, again I want to repeat the reference books, like the most illustrations books like the Cengel Cimbala book which is very good book in terms of lot of illustrations, the examples, and the exercise what is given in Cengel Cimbala book. And who needs to very concise they can read it F.M. White book which gives mathematical forms with very classical problem solvings at exercise levels also the in examples problems.

And another books by Bidya Sagar Pani on fluid mechanics is concise which in Indian context book is very concise to read it. So please refer this reference books, the Fluid Mechanics the Fundamental and Applications by Cengel Cimbala or Fluid Mechanics F.M. White, or Fluid Mechanics by the Bidya Sagar Pani. So these are what the reference books I have been following it. So please follow these books for additional knowledge on this fluid mechanics course.

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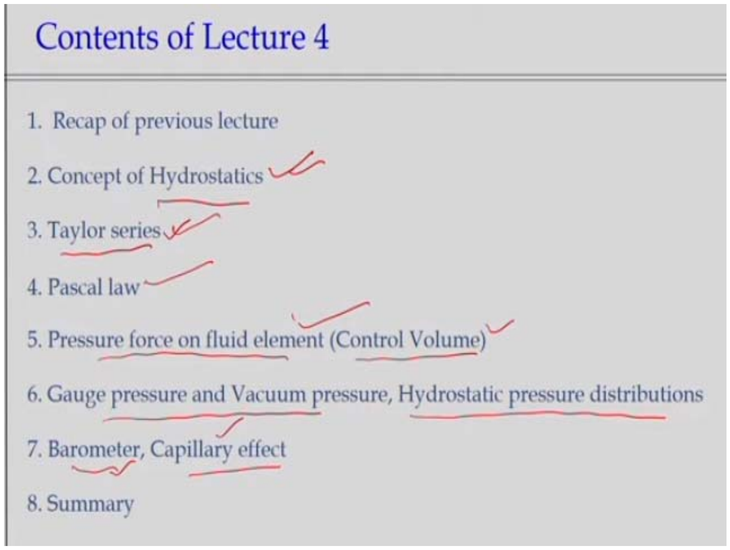

Now let us come to the contents of the today lectures. The first I will discuss what we so far in the last three lectures we discussed it. Then we will talk about concept of hydrostatics okay that means the fluid at the rest and most of the as you know it that when you consider a function okay you can approximate the function using a Taylor series, that the concept we will talk about. The Pascal law we will talk about.

Then the major components like the pressure force on fluid element, that is what is the control volumes. That what we will also discuss it. Then we will discuss about what is a gauge pressure, what is a vapor pressure, and hydrostatic pressure distributions. And there are two applications of this hydrostatic pressure distributions. One is barometer another is capillary effect. Then I will conclude today lectures by summarizing the lecture content.

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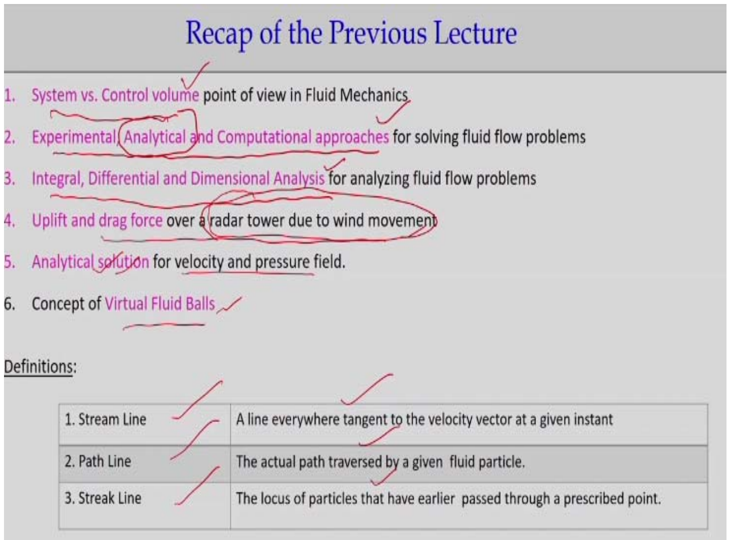

Now let us recap it, as of now what we learnt it. We already know it we talk about a either a system approach or the control volume approach to solve the fluid mechanics problems. And whenever you solve the fluid mechanics problems as I said it in the last class, we generally look for three velocities three fields, velocity field, pressure field and the density field.

But when fluid is incompressible, then we just look for the pressure field and the velocity field. So these two fields we can get it using this three different approaches as I discussed earlier. One is experimental method, conducting the experiment in a wind tunnel, computational approach, which is a computational fluid dynamic now extensively used for a very complex fluid flow problems or this analytical approach with a very simplified problem we can solve analytically to get the gross approximations of this pressure field the and the velocity field.

So that is what we look it for the flow which is considered incompressible flow. And then we already discussed about that to adopt these three different approaches we have to follow integral approach, differential approach and the dimensional analysis. And very interesting examples what I have given in the last class is that the how the force, very complex flow fields we get it in a radar tower due to this wind movement.

That is what we discuss it that and there is a option is there, we can do analytical solutions for that to get a velocity and pressure fields for a simple problem which can you solve it. And as again I can repeat it that the concept of virtual fluid balls we discussed lot and that what we will apply when you go for fluid kinematics and the fluid dynamics part which is later on I will discuss it that.

And as a definition already we said that what is the streamline, the pathline and the streaklines which is considered to define what type of flow is happening it and understanding or visualizing the fluid flow problems because that what using the streamlines, pathline and the streaklines.

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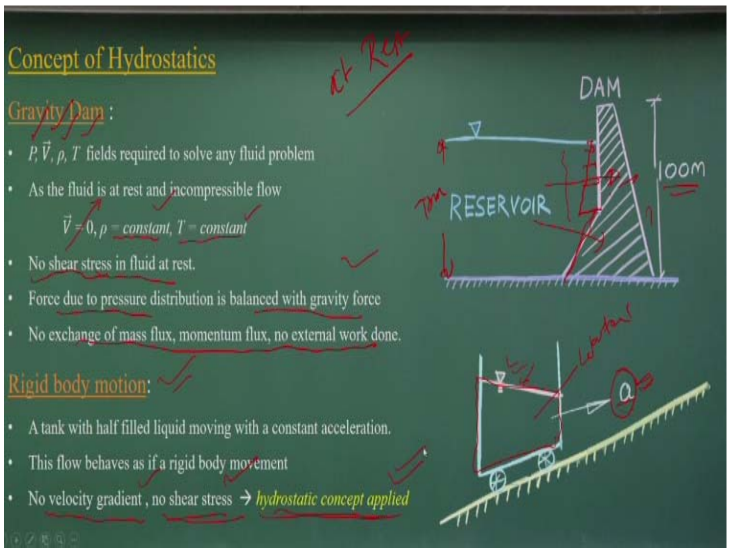

Now let us come to the very basic concept what we are talking about the fluid at rest. So the basically we are talking about now, the fluid at rest, okay? If it is a fluid is at rest, it is a very simplified problem now. Like as I said it any fluid flow problems we look at either the pressure field, the velocity field, or the density and temperature field.

When the fluid is rest now, very simply way the velocity vectors becomes zero and if I consider incomprehensible the density is a constant and if the temperature is not very much I need not need a thermodynamics first laws to define the problems. Then only the left is that the pressure field. That means when fluid is at rest conditions only we need to know it how the pressure variations is going over these fluid domains. That is what the simplified problems.

Since there is no velocity, there is no velocity gradient, definitely as Newton's second law says that there is no velocity gradients that means no shear stress. So when a fluid is at rest, there is no shear stress acting on that. So you can take a surface or take a control volume. Over that control surface you can define it the shear stress components become zero.

That is very simplified now that any control volume you consider it over that control volume surface as the fluid is at rest conditions, there is no shear stress acting on the control surface. So that is the very easy problems what we have now. And what the two forces we have? The gravity force and the force due to the pressure distribution. So the whatever the pressure distribution forces that what is equate with the gravity force.

Very simple things now, and since is a fluid is at the rest, so you can say that there is no mass flux is coming into the any control volumes or the momentum flux or no external work done it. So this is the what the simplified case. Like for example, if you take this dam, which is 100-meter-high and we have a reservoir, let you consider this is what 90 meter height from the bottom.

That is what the water levels is 90 meter from the bottom. And you can understand it because of these in the reservoirs the fluid is at the rest conditions and that rest condition exerting the pressures on these surface. So there is a vertical surface, there is a inclined surface. So we need to know it what is the fluid pressure is going to act on this dam, on these vertical surface and also the inclined surface.

Based on these the pressure load we can design this dam structure. That is what the examples for fluid at the rest. Similar way if you have an oil tanker, so we need to know it what is the pressure, the fluid oils with are in a rest how much of pressure is exerting on the wall of the tank.

So it is the problems what we use to design a tank, big reservoir tank or the natural reservoirs like a dam, we try to know it how much of pressure is going to act because of this fluid rest, fluid is at rest condition and how much of pressure is required. Because of this pressure and what is the pressure force is acting on that. Similar way what is of force is going to act on that.

So that is the problems what we will solve it. Second thing is that fluid act as a rigid body motions. Let us take an example here that I have a tank which is a half filled liquid is there. Let us consider it may have water and this tank is accelerated with a constant acceleration a. As you start accelerating this tank with a constant acceleration a, you can understand it, this free surface is going to change it.

And coming to an equilibrium surface such a way that after that this surface will not change it. That means at that time these control volume is moving as a rigid body and traveling with a constant accelerations of a. So we can consider is a very simplified case now, that this fluid control volume what we have here, which is moving with a constant acceleration a, and the with these free surfaces and that what will be there.

And as it is moving it there is no velocity gradient. So no shear stress, so we can adopt the hydrostatic concept. So let me summarize what I am talking it to you that one case we have a gravity dams where we know the fluid is at the rest. That means only the pressure force is going to act on the surface of the dam. In that case we can understand it that there is no velocity, no shear stress, only the pressure what is going to act it.

But in case when you have a rigid body motion, the fluids work as a rigid body motions like the tank is going moving with a constant acceleration of a, the similar conditions can be considered it any liquid field containers moving with the constant accelerations a, and what could be the free surface, what could be the height of the free surface all we can compute it that what in a later on I will tell it.

But the in conceptually in this case what it happens it the control volumes moves at a, constant accelerations. Because of that there is no velocity gradient as that of that there is no shear stress acting it. So we can apply the hydrostatic concept. So one case there is no velocity field at all because fluid is at rest. Another case we have a no velocity gradient. So that is the reasons there is no shear stress.

So we can apply the hydrostatic conditions. That the as equivalent to fluid at the rest conditions.

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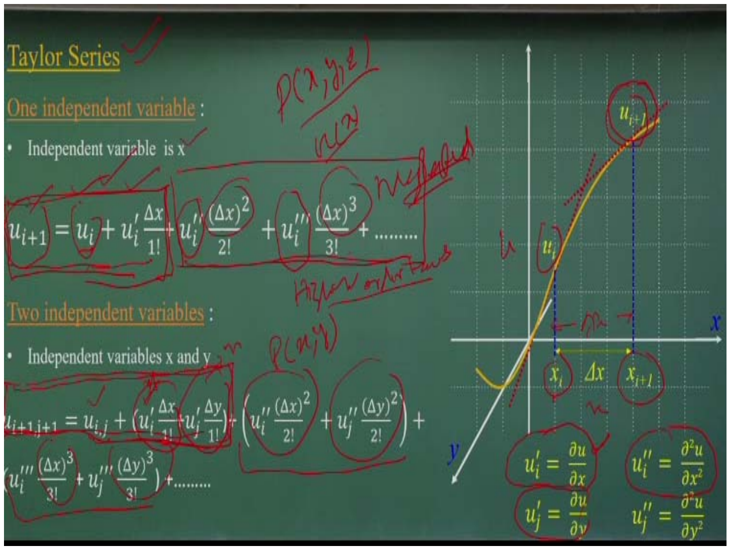

Now whenever I am talking about that I am looking for a pressure field as the fluid is at rest. So basically I am looking at the pressure is a function of the positions $P = P(x, y, z)$. Time is not there, as the fluid is at rest condition. The many of the times when you consider the pressure field, you try to look it from one point to other point. What is that value could be okay?

Like for examples, if I take a very renowned series like a Taylor series, for one independent variables, that means here the u is only a function of x, only the function of the x and u is the variable. And we know what is the value at the x i the u i value, we know that. We want to compute it, what could be the value u_{i+1}which is a Δx distance from x_i and that what is x_{i+1}

So this is a known to us when to compute what could be value u_{i+1}which is having a on the x direction a distance of Δx. The Taylor series gives a infinite series like if you look

$$u_{i+1} = u_i + u_i' \frac{\Delta x}{1!} + u_i'' \frac{(\Delta x)^2}{2!} \quad + u_i''' \frac{(\Delta x)^3}{3!} \; + \cdots$$

But most of the times what we will look it in a fluid flow problem, we need not need this higher order terms. This is what the higher order terms, okay? We need not need it because these values will be much smaller than this the first term.

So we neglect this high order term, only we consider this part, okay? That is what so this is the higher order terms which are much less than these points. As you can see that $\Delta x^2$is there, Δx^3 is there, and these value becomes much smaller as we go further more higher orders. So we neglect that part. Only we consider any functions if I am approximating at u_{i+1} locations that what will be u_iu i the first gradient of $\frac{\partial u}{\partial x}$, Δx by this value.

That is what I am to say that. So we can approximate either a pressure field the scalar component of velocity field or the density if it is we are considering as the variable in a place and the time component like independent variable of x and y or z or the t. We can define them in a Taylor series, very simple way as if I y_i at a one point y_{i+1} will be the first gradient into Δx. That is the this is what we consider.

This is what we neglect it. This is what we neglected because these terms are very less as compared to the first term. But when you consider two independent variables okay that means the pressure is a functions of x and y for example. In that case we can have the Taylor series is similar way if you can see it that there we will have the i and j : Independent variables x and y,

$$u_{i+1,j+1} = u_{i,j} + \left(u_i' \frac{\Delta x}{1!} + u_j' \frac{\Delta y}{1!}\right) + \left(u_i'' \frac{(\Delta x)^2}{2!} + u_j'' \frac{(\Delta y)^2}{2!}\right) + (u_i''' \frac{(\Delta x)^3}{3!} + u_j''' \frac{(\Delta y)^3}{3!}) + \dots$$

So $y_{i\,j}$ will be again the first order terms of x directions and the y directions. Then again second order term which will be the in x direction and y directions like this we go it. As I already told it, these terms are much lesser because Δx^2we have $\Delta x^3$is there with value as it a square as it a small value this becomes a very less.

So whenever you consider two independent variable case we approximate the function

$$u_{i+1,j+1} = u_{i,j} + \left(u_i' \frac{\Delta x}{1!} + u_j' \frac{\Delta y}{1!}\right)$$

That what we consider when we are applying the Taylor series to approximate a function if I know at initial conditions and the next conditions which is a Δx or Δy from that positions we can approximate the functions like this.

If you are more interested on Taylor series, you can read any mathematical books to know how the Taylor series used for approximating the functions.

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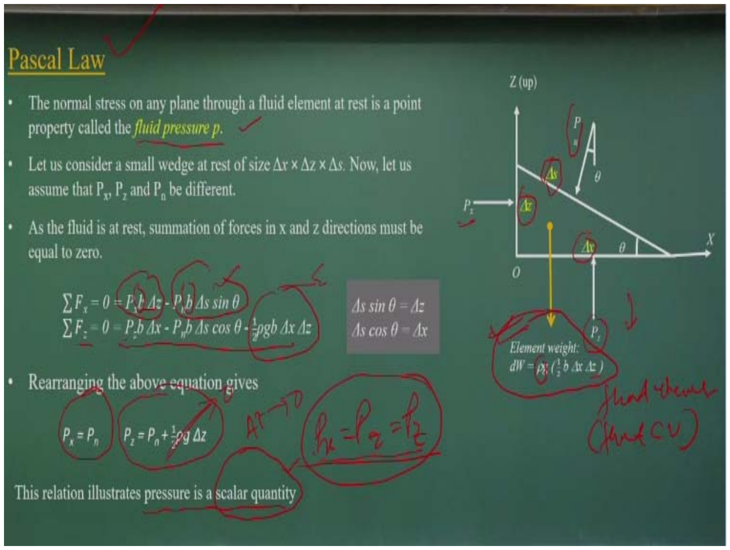

Now let come it to very basic law is called Pascal. As you when you fluid is rest let us consider is that the there will be a normal stress acting on any plane. Okay, that is what we are considering is that that what is the fluid pressure. Here I am considering a fluid element or you can say it the fluid control volume element or fluid control volume.

Now if you consider this is what my control volumes and along this perpendicular to this surface that is what my unit value. That means one unit I have considered perpendicular. This is a three dimensional control volume I am representing as a two dimensional thing. And I am defining the pressure P_x is acting on the surface, the P_j is acting on this surface and P_n is the pressure components acting on this inclined surface. And each one is a Δx, Δz and Δs.

So if it is that when the fluid is rest we can find out some of the forces acting on this control volume should equal to zero. So then we can look it force in a scalar component in x direction and the y direction. That means the force component in x direction should be equal to zero. Force component in the z direction should equal to zero. The force is what? The pressure into the area.

As the fluid is at rest, summation of forces in x and z directions must be equal to zero.

$$\sum F_x = 0 = P_x b\, \Delta z - P_n b\, \Delta s \sin\theta$$

$$\sum F_z = 0 = P_z b\, \Delta x - P_n b\, \Delta s \cos\theta - \tfrac{1}{2}\rho g b\, \Delta x\, \Delta z$$

$\Delta s \sin\theta = \Delta z$ $\qquad\qquad$ $\Delta s \cos\theta = \Delta x$

What is the weight of these control volumes will be the volume multiplication of unit weight. That is the multiplications of density and acceleration due to the gravity. So you get the element weight that the components if you can see it, that components are here. And we have the force component what you have result. So we have just considered this fluid element or the control volumes at the equilibrium conditions.

Similar way

Rearranging the above equation gives

$$P_x = P_n \qquad P_z = P_n + \tfrac{1}{2}\rho g\, \Delta z$$

That means what you are considering is that your control volume become smaller and smaller, smaller and it could be infinitely smaller value, okay? That what if you consider it z equal to zero then these components become zero. So what will get it?

The $P_x = P_n = P_z$

That means the pressure in the x direction, z direction, and the any direction of normal to the surface will be the same okay? That is what is the Pascal law. And since pressure is same in all the directions, so we can consider is a pressure is a scalar quantity and that what they must stated by Pascal's law.

Very simple way as I consider it a fluid element, fluid at the rest conditions and just equate the pressure due to the pressure the force component and also the gravity component equate in x direction and y direction and consider that Δz tends to the zero. That means your control volumes become smaller and smaller and infinitely smaller. At that periods, you will have a P x, P z and P n are equal.

That is the Pascal law, the mathematically with all its pressure becomes in a fluid at rest condition is a scalar quantity. So fluid flow what we are considering here all the cases the pressure is a scalar quantity. That is what the Pascal's law.

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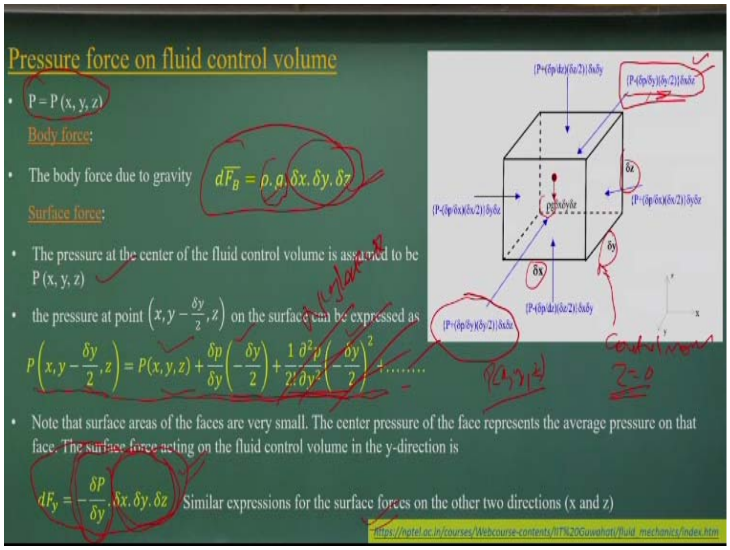

Now if I go for the next ones that how to get the pressure field when fluid is at the rest. That means I am just looking the what could be the functions of the P, P = P (x, y, z). If it that it is now again I am considering a very simple case, let us consider a control volume like this okay. This is what my control volume. As I said it when the fluid is at rest the shear stress become zero.

So there is no shear stress component on this fluid plane. Only this normal stress which is as equivalent to the pressure will act over this control surface. Over the control surface only the pressure is going to act it because the shear stress is zero okay. That is very simplified case. And as you have considered is a simple parallelepiped control volumes with having a $\delta x.\delta y.\delta z$.

There are three Cartesian directions of x, y and z directions and I am considering a point which is just a centroid of the parallelepiped is this thing. The pressure at the centre of the fluid control volume is assumed to be P (x, y, z). At that point, the gravity force is acting it which is the body force component. The gravity force what will be there? It will be,

$$d\overline{F_B} = \rho.g.\delta x.\delta y.\delta z$$

So we can have a two force components. One is the surface force component and other is body force. The body force components if you look it that it will be unit weight multiply the volume of the control volume, which is $\delta x.\delta y.\delta z$ which is very simple things. The control volumes, the volume is this much and the unit is $\rho.g$ and ρ stands for here the density and g stands for accelerations due to gravity.

Now if you look it over this control volume, all these pressure is going to act it as the Pascal law says that pressure acts normal to the surface. So that what we can consider pressures act there normal to the surface. Then we need to define now as a pressure as a x, y, z is a pressure field what we are looking it which is a function of x, y, z I can use of Taylor series to approximate $\frac{\delta y}{2}$ distance of this surface which is a far away from $\frac{\delta y}{2}$. That what is first order approximation of Taylor series.

The pressure at point $\left(x, y - \frac{\delta y}{2}, z\right)$ on the surface can be expressed as

$$P\left(x, y - \frac{\delta y}{2}, z\right) = P(x, y, z) + \frac{\delta p}{\delta y}\left(-\frac{\delta y}{2}\right) + \frac{1}{2!}\frac{\partial^2 p}{\partial y^2}\left(-\frac{\delta y}{2}\right)^2 + \ldots\ldots.$$

If I know P(x, y, z) what will be the pressure at this $y - \frac{\delta y}{2}$ will have a this function. You can look at this negative part is there because this is what $\frac{\delta y}{2}$ distance far away or negative side of or the opposite of the y direction.

So that the $\frac{\delta y}{2}$will be there and we have the expressions like this. Now I neglect this component. This is what we are neglecting, all these high order component here. So it is simplified to like this stuff. If this is the pressure is acting on the surface, similar way I can get it what will be pressure is acting. I know this pressure multiply the area over these simplified control volumes, I will get the force component. That what is acting along the y directions.

That what if you equate it finally, we will get this one. The gradient and the volume of the control panel. The negative gradient and the volume of control volume is what we will get. This is the force component acting along the y directions. More detail derivations of these simplified cases please refer to web course what I developed a long back on NPTEL. That is the link what is available here.

So that is what please go through that. So what I am talking about that the force components acting along the y directions because of the pressures we can get,

$$dF_y = -\frac{\delta P}{\delta y}.\delta x.\delta y.\delta z$$

So similar way, we can derive the pressure component, the force component due to the pressures in x directions and the z directions respectively.

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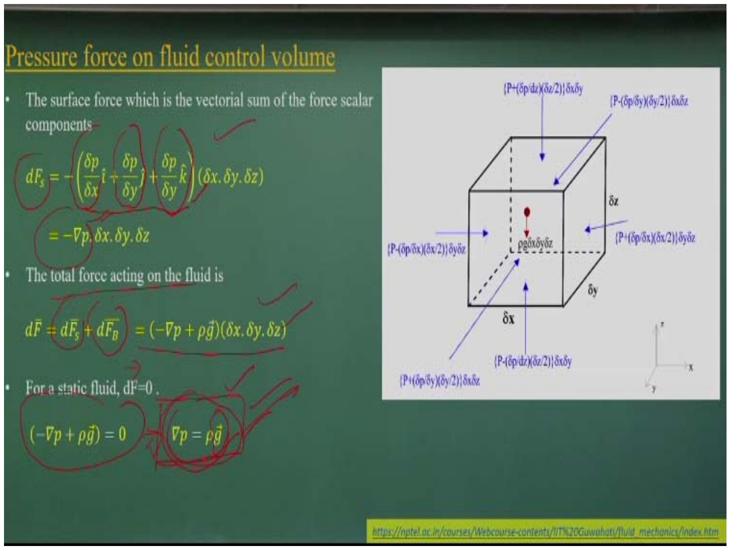

If I do that, that what as a vector form I can write it this way. This is the force component along the x directions. This is the force component along the y direction. This is a force component acting in the z direction. So this is a gradient vector. So I can have a gradient vector field of dF. As I said it earlier only two forces is there, one is a gravity force and another is force due to pressure distributions.

So total force acting we can equate it dF dy. That what in terms of vectors will have, this minus gradient and $\Delta\rho g$. As the fluid is rest, the sum of the force should be equal to zero. That the vector is equal to zero and that field what we will get it this part and is finally the $\nabla\rho g$. So here please note it that we have not considered the x, y, z in any g, g can be any directions, g can have a vector okay.

The surface force which is the vectorial sum of the force scalar components

$$dF_s = -\left(\frac{\delta p}{\delta x}\hat{\imath} + \frac{\delta p}{\delta y}\hat{\jmath} + \frac{\delta p}{\delta y}\hat{k}\right)(\delta x.\delta y.\delta z)$$

$$= -\nabla p.\delta x.\delta y.\delta z$$

The total force acting on the fluid is

$$d\bar{F} = d\bar{F_s} + d\overline{F_B}$$

$$= (-\nabla p + \rho\vec{g})(\delta x.\delta y.\delta z)$$

g not necessarily that g should be the just opposite of the z directions. Here we have considered g can be any directions like you are solving a problem where the control volume we have considered very complex control volumes, your g may not exactly match with a z directions, then you can g you can use as a vector and you can solve these problems with a $\nabla p = \rho \vec{g}$.

So this is a in a vector forms the hydrostatic equations which will be simplified in later on which is very simplified case, what we will get it in later on. But the general equations is this ones,

$$(-\nabla p + \rho \vec{g}) = 0$$

$$\nabla p = \rho \vec{g}$$

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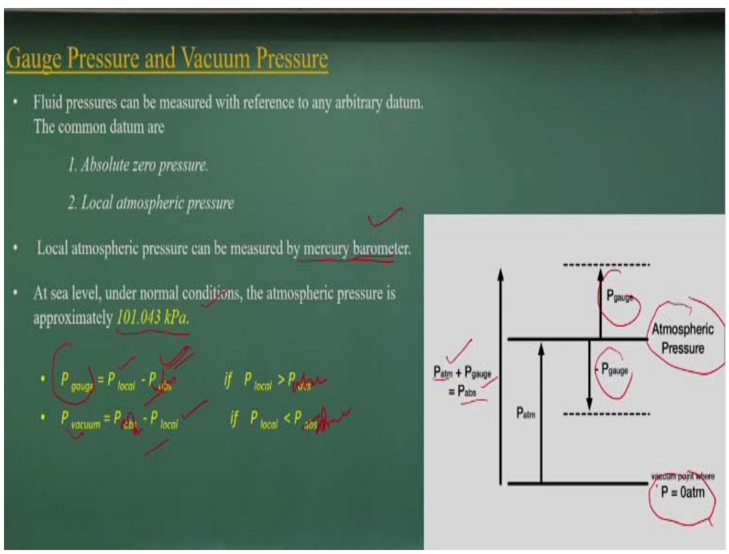

Now the point is what we are going to discuss is that gauge pressure and vacuum pressure. Components now is coming it what is your datum to measure the pressure. Whether you have to make a absolute zero pressure, that means you have a vacuum. From there you are measuring the pressure, or you consider as local atmosphere, to measure the pressure.

There is two conditions from where you have to measure the pressure. It could be absolute vacuum point where the pressure is equal to zero okay, theoretically it is a zero pressure. You have a vacuum where the zero pressure and you are measuring it okay. That what will come an absolute pressure. That means you are absolute zero pressure you are measuring it.

But if you most of the case as you know it when you consider the fluid all the fluid will have a surrounding of a part of atmosphere. Most of the cases what we consider here, we are not solving the problems in a space or area. We are just solving or any other planets, we are just solving the very simplified problems in and around the earth. So we consider the atmospheric pressure is the datum to measure the pressure.

So when you measure the atmospheric pressure is a datum. There is a two conditions is comes it. One will be the pressure above the atmospheric pressure, another one will be the pressure below the atmospheric pressure. So when you have the pressure above the atmospheric pressure then we call the gauge pressure. That is that we differentiate between the local pressure and the absolute best.

So that is what will be the atmospheric pressure and this atmospheric pressure can be measured with mercury barometers okay. As you know it, it could be from the 101.043 kPa, the Pascal is Newton per mm^2. So that what we measure it when you have a difference between P_{local} and $P_{atmosphere}$ okay. That what $P_{atmosphere}$ and here is also $P_{atmosphere}$.

- $P_{gauge} = P_{local} - P_{abs} \quad if \quad P_{local} > P_{abs}$
- $P_{vacuum} = P_{abs} - P_{local} \quad if \quad P_{local} < P_{abs}$

So we can have a pressure measurement either from the vacuum or from the atmospheric pressure.

If you need to compute it what will be the absolute pressure, then it is a very easy. You just use whether the gauge pressure or this the vacuum pressure that difference what we can get it that what you just add with atmospheric pressure to get the absolute pressure. There is nothing else. Where your datum is there, whether at the absolute levels or at the atmospheric level.

Like many of the problems what you solve it the fluids having a boundary at the atmospheric pressure levels. So as that time, we can measure the pressure difference between that. The positive side we call the gauge pressure and the negative side we call vacuum or the suction pressure which is below then the atmospheric pressure. Never we go below the pressure equal to atmospheric absolute pressure which is a zero atmosphere or no pressure is there which is at the vacuum levels.

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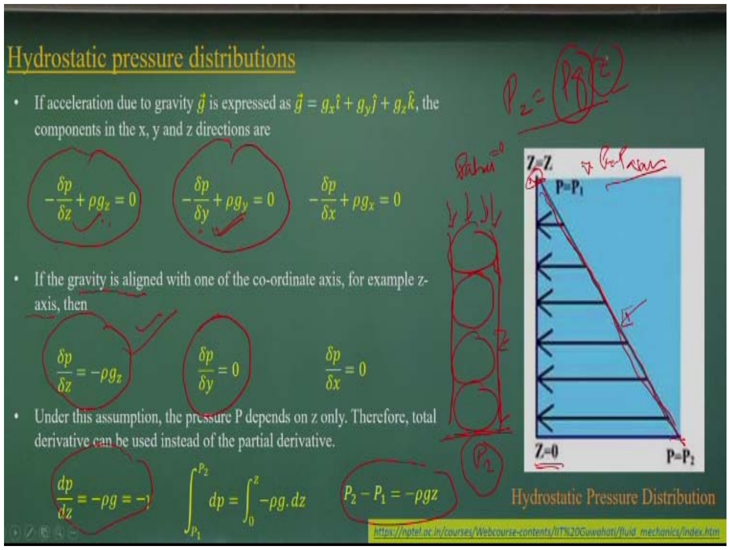

Now as we derive pressure distribution equations which in vector forms and let we simplify that equations which earlier we consider the acceleration due to gravity is a vector which will have,

$\vec{g} = g_x\hat{\imath} + g_y\hat{\jmath} + g_z\hat{k}$

If you make it at the scalar level as you can just split that equations, you will have a three equations. One will be the z direction, another will be the y direction, another will be the x direction.

Just you are getting the component of minus of the pressure gradient is equal to the gravity component on that into the rho times which is as equivalent to a unit weight on that directions. Similar way this is the y directions and this is what the x directions. Now if you align the gravity in one of the coordinate axis most times that we do along the z directions.

$$-\frac{\delta p}{\delta z} + \rho g_z = 0$$

$$-\frac{\delta p}{\delta y} + \rho g_y = 0$$

$$-\frac{\delta p}{\delta x} + \rho g_x = 0$$

If the gravity is aligned with one of the co-ordinate axis, for example z- axis, then

$$\frac{\delta p}{\delta z} = -\rho g_z$$

$$\frac{\delta p}{\delta y} = 0$$

$$\frac{\delta p}{\delta x} = 0$$

That means, if you consider z is vertical direction and x and y is on the surface. So these equations what it indicates is that in that case you will not have a any pressure gradient. If I consider a horizontal plane, there will be no pressure gradient in x and y. So there will be no pressure gradient. Only the pressure will vary along the z direction.

$$\frac{dp}{dz} = -\rho g = -\gamma$$

So if you consider a horizontal plane, at that horizontal plane you will be pressure will be the constant. That what these equations indicate for us. Now if you pressure only varies along the z directions, that means if I take from this point and if I just integrate this because pressure become now only a function subject okay. Pressure is only functions of the z, only in the vertical directions.

$$\int_{P_1}^{P_2} dp = \int_0^z -\rho g.dz$$

$$P_2 - P_1 = -\rho g z$$

We remember we are considering z location from a particular level, so that negative component you will look it whether you are going up or down. That is what the reference point what you will look it. So if you look it that if I am a z equal to zero here and I am putting it the z value like this the pressures the P_2 and pressure distribution will come a linear function with respect to the z.

So pressure will vary, this is the free surface. At this point the pressure will be the atmospheric pressure, which most often we neglected that which is very less as compared to the liquid what we have generally we consider it. We neglect that pressure.

Then after that we draw the pressure distributions as it is a relative pressure difference is only a function of z of a linear function only.

So we can have a linear variation of pressure from these free surface to this one and that what we get it here, the linear variations of the pressure and you can look it that what is a unit weight into the z value. That what the pressure. It is a very simplified thing okay. We have derived it but if I consider the very simplified the concept of virtual fluid balls.

We have derived the equations very complex way theoretically then followed simplified it then what we got it the pressure does not varies along a horizontal plane. Only the pressure varies only in the z directions and that the pressure variations is a linear function with respect to the z. But if you consider the virtual fluid ball concept that means I have a condition where the balls as equivalent at the rest conditions.

And pressure let be acting on these $P_{atmosphere}$ okay. I want to know it what will be pressure at this point let be the P_2. Nothing else. These pressure will be the because of this atmosphere pressure if I make it zero, then what will be this? It is the weight of whatever the weight of the liquid that what will be the pressure on this. Weight of the liquid divide by area, surface area.

If I consider the unit surface area that means unit weight of the waters or multiplied by the z that what will give a pressure to this. That means I will have a pressure will be rho g weight of the water or weight of the virtual fluid balls whatever is starting here into z the column height. So these are very simple way.

We can find out if having a problem now or not getting the equations which are very simple equations you can just think it that we have just stacking of the fluid balls, virtual balls and just find out what will be the weight acting on a unit surface area and that what will the pressure is going to act it. Just weight of the fluid balls and divide by the area. That what will give it.

The weight will be the unit weight multiplied with the elevations z into the area as we consider area is equal to 1 so ρ g z that is the very simple things we can as the z varies the pressure distributions varies linearly. That the simple way if you consider virtual fluid balls not bothering much about whether is a what type of equations, how the particle will be simplified for the z directions.

If you have the fluid in the rest conditions just take the control volume just consider the virtual balls all are in the rest. And this virtual fluid ball can have a different way. Let me explain that considering a simple example here.

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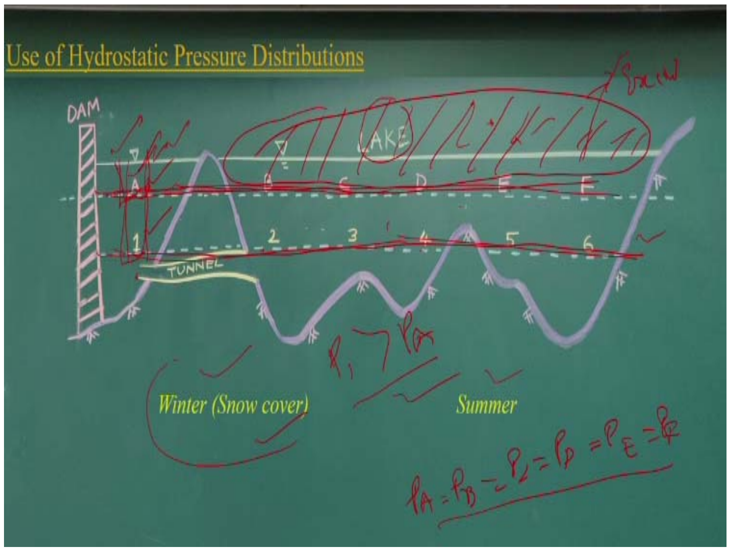

Very interesting examples what we have consider it here like you have high altitude lake, okay? And that what is connected through a tunnel of a reservoir and the damn you have. There is a winter snow cover will be there and the summer there is no snow cover. We want to find out what will be the pressure is going to act on the P, A, B, C, D, E, and F. This is the locations we can consider a horizontal surface.

As we have known it that along the horizontal surface the pressure will be the same when the fluid at the rest. Otherwise, fluid could not be rest if there will be the pressure difference. That what the fluid will start motioning. So any horizontal surface if you consider it when the fluid is rest, the pressure on that horizontal surface becomes equal.

So that means, in mathematically that will be P_A, P_B, P_C, P_D all will be the equal whatever way the as the connected surface. It may happen it similar way, if I take 1, 2, 3, 4, 5, 6 which are laying on a horizontal surface you can consider pressure will be equal. That is the pressure will be the equal. The P_1, P_2, P_3, P_4 will be equal. Let us consider there is the in a winter seasons there are lot of snow formations here.

Okay, lot of snow formations here okay additional the snow is here. If you again you consider the pressures along this P_A, P_B, P_C, P_D, P_E,P_F you will have the same pressure. There is no doubt there will be adjustment of the water levels because of this change of here as it is connected through the tunnels. But the pressure of any horizontal surface will be the same.

So either you have a snow cover or no snow cover it does not matter it. The as the fluid is rest and it has a connected through a tunnel when is the fluid is rest the basic general velocity is here along any horizontal surface you will have the pressure is equal as the pressure does not vary along the x and y direction. Only pressure vary along the z direction.

As the snow formations happens here that what will exist the water level from these two in the region in such a way that any horizontal surface if you consider it you will have the pressure difference will be the same. Similar way if we are adding a different fluid, let we consider we are adding a different fluid with a different density, not the water here.

Water plus another density of water, low density water where fluids we are adding to that, definitely it will have response to here. So when is you have any connected systems or fluid is a rest conditions, you take a horizontal surface, on that horizontal surface the pressure is equal. As any connected system is going to change in one part that what is going to reflect in the other part.

That what I tried to tell that if something happens to this lake, it affects to this reservoir. Still it rest to the rest condition. Still these coming to a conditions where you will have the pressures along the horizontal surface. So it should be equal. That the conditions prevail it. Either it is a snow formations or adding more water of or different liquid with a different pressure that what is reflected here.

That what will be reflected here. So these are very simple conditions what we are talking about, very complex problem if your fluid is in static conditions, we can simplify that problems as if it is a connected it like a through the tunnels or through any tubes okay. That is what we will discuss in a manometer. We can consider any horizontal surface and that horizontal surface as it indicates is that the pressure is equal.

Pressures are the equal all over that surface. That is the conditions we consider to solve many fluid flow problems in a hydrostatic and second thing is that you can easily know it if I consider a and one point, definitely the pressure at the one so do we call more than pressure at the air because if you consider the virtual fluid balls also the weight of the water and weight of the waters here that what is different.

The more water weight of water is holding per unit area at the air as compared to these ones. So pressure at the one will be more than this. Or as you go deeper and deeper you will have a linearly proportional more pressures will get it. That is what very basic things that as you go more differ and differ the z is more, the depth is more, the weight is more.

And you will have more pressure and the pressure is a linear function in a z direction and pressure does not vary in a x and y directions as it becomes constant. This is the very simplified consider we consider when the fluid it rest conditions like this lake problems, which is connected to a reservoir.

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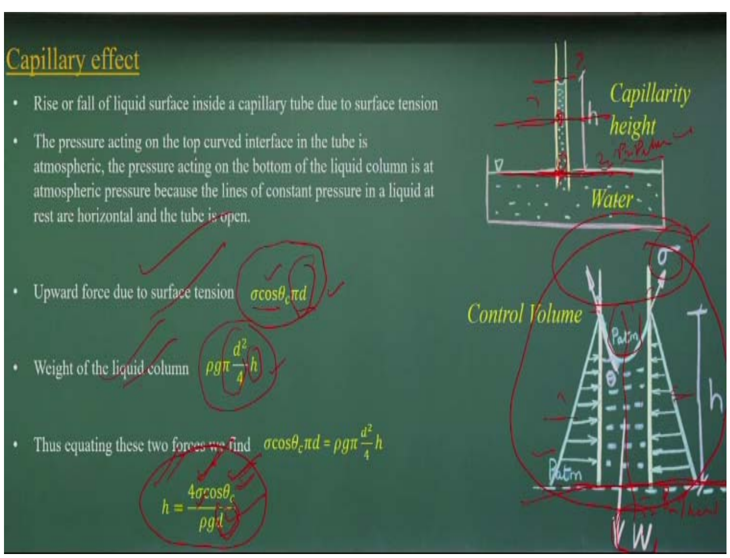

Now let me consider the capillary effect as you could have seen some of the books if any of the class 12th levels. What do we do it we just coming the two force components, the force due to the surface tensions and the gravity force of the capillary part. You know it what is the capillary. That means if use a small tube and insert it into a fluid, you can see there is a rise of water or the fall of mercury.

That what you can observe it and that height is called capillary height. That what is here. We have a water and a small tube, glass tubes, if you inserted it which can you can visualize that the water will go into that. This is what the capillary effect. Now, in this lecture what I am talking about that let us consider a control volume and solve the problems okay. That was is my point.

Let us consider a control volume and solve this capillary problem which is a very simplified problem, because here this two force component works. One is the gravity force; another is the surface tension force. These two force components works. Now if I consider this is the control volume. That means, I have considered this part as a control volume, okay.

This is what the surface tension and theta c will be angle of contact. As I say that pressure will have a linear variation. So you can have a pressure will be linear variations, pressure will be the linear variations. But when I consider this hydrostatic distribution the point at 1 which is inside and the point 2 which is the outside since I have considered a horizontal surface at the second point the pressure is equal to pressure atmosphere.

The definitely point which is the inside the tube also the pressure will be the atmospheric pressure. That is what the conditions. Since we consider the fluid is at rest, if I take a horizontal surface which is just matching with the free surface of water and air where the pressure is acting as atmospheric pressure, so inside the tube also in that horizontal surface pressure will be atmospheric pressure.

So my control here I have the pressure which equal to atmospheric pressure. At the top as water and the air is there we will have a atmospheric pressure. So at the top you have atmospheric pressure, below you have atmospheric pressure. And over this control surface with a radial control surface you will have a pressure distribution will be the hydrostatic pressure distribution starting from zero to the at the z level.

And what the force component is acting, weight of this part plus counterweight by the surface tension force, nothing else. Because we are looking the force component along these z directions, along these directions only. Only in this direction. So weight will come it, that what is counter balance by the force due to the surface tension. So upward force due to surface tension you can find out: $\sigma\cos\theta_c\pi d$

Weight of the liquid column $\rho g\pi\frac{d^2}{4}h$

So you have the weight of the fluid. Just equate these two part. The upward force due to the surface tension and weight of the fluid. What will get it;

$$\sigma\cos\theta_c\pi d = \rho g\pi\frac{d^2}{4}h$$

$$h = \frac{4\sigma\cos\theta_c}{\rho g d}$$

h will be this part okay as a function of surface tension, d is the diameter of the tube, theta c is the contact angles, we are getting this value. So the capillarity if you can see that it is proportionality to the surface tension.

Here we are talking about water and air and the glass. If you have used different fluid conditions, interaction conditions you will have a different surface tension also the contact angles, and the D which is a inversely proportional. That means if I make a smaller diameter of capillary tubes I will have a more height rise. That what it happens it.

As the d is become smaller and smaller h will be the more and more okay. So that is the conditions we will see it and the surface tension and θ_c it depends upon the fluid, fluid contact with another fluid, maybe water, maybe other things and the solid surface here is the glass. So surface tensions and the θ_c depends upon the where the interactions happening it and d is the diameter of the tube, the capillary tube what we use it which is that.

As the diameter of capillary tube it become smaller, definitely h is increases at that. Here we have considered the control volume to solve the problems. To define you how the pressure distributions happening it how I am equating all these force component. No doubt I have neglected, I have tactically I use it only the force component equating along these z directions, which is very easy for me to just look in the z direction because other forces balanced each other.

That what I am not worried about that. More things what I am looking it that as I consider a horizontal surface just connected or the atmospheric fluid is at rest. So any fluid inside the tube also will have the atmospheric pressure. That is what the major conditions we have consider it. That is what we have. So but you cannot consider any point above of that and equate it.

You cannot do it that because the pressure acting on this point is not equal to the atmospheric pressure. There is no liquid connected on that. So that will be the difference. But when you have a common fluid is connected to the surface and you take that horizontal surface and that horizontal surface definitely will have a the atmospheric pressure what is acting there. That what will be the atmospheric pressure inside the capillary tube.

So that the things what I just repeat you to look it at the control volume approach, which is we can easily solve our very complex problems also we can solve it if you follow control volume approach as compared to the just simply balancing force component what we generally do in other cases.

But my point is here, please look at the control volume level, draw the pressure diagrams on the control surface, equate the force components, then you solve the problems which is much more a systematic approach to solve the problem as compared to just equate the force without drawing the control volume, without drawing the control surface, without drawing the pressure diagrams and solving this thing.

That please do not follow that one which is sometimes it will be very difficult when you have a very really complex problem not the academic problems what we do it this thing.

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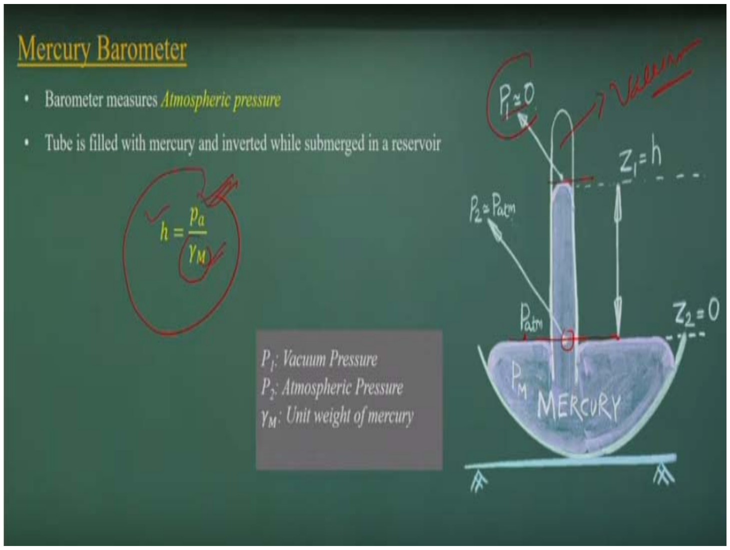

Now let me come it to the mercury barometer case with a very simplified same capability concept what we used to measure the atmospheric pressure. Here we use the mercury and just insert a tube such a way that the tubes would contain the vacuum. So we can have a system to remove the air and to have the vacuum and you just inserted in that tube on that one. Same concept again is coming it.

Here the P_1 will be the zero because this case is the vacuum. Same capillary tube effects nothing else and same control volume concept we will use it to solve the problems. Here if I consider the point as the surface I will have atmospheric pressure here also will have a P_2 with atmospheric pressure as this thing. So again I will wait the same way if I do the control volumes and all I will get it the height will be $P_{atmospherics}$ divide by the unit weight of the mercury.

So if I know the height I can compute what will be the atmospheric pressure. This is what you use very simplified concept for mercury barometers. Just measure the height with a graduated scale. As you know the height you know the unit weight of the mercury. So you can compute it will be the atmosphere pressure. As you know it you go for higher altitudes like go to top of the Himalayas.

Definitely the atmospheric pressure is less and that is the reasons you feel dizziness when you go for the higher altitudes and there are other problems will cause it when go for the higher altitude. Because of that many army problems when army goes to higher altitudes they go step by step. They stay for a different altitude then accustomed with that pressure then they go for the higher altitudes.

As we know it we go for a higher altitude your pressure decreases and that is to (56:20) customize with that pressure difference army when moves from the lower attitude to higher altitudes it goes phase wise. Like for example if somebody goes to the higher altitudes like Himalayas which is as on average is 5000 meter above the sea. So pace wise you go it, first 2000 meters above the mean sea level, 3000 meter above the mean sea level, then we go for 4000 meter about the mean sea level.

So that the body customize the army persons' soldiers body customize at the different level and he can that the low pressure zones at the high altitude he can take care of when you go for the higher altitudes. So that is the reasons we measure the atmospheric pressure and the different and that what we can use this mercury barometer. Nowadays there are the electronics versions of pressure measurement device are available. Tube is filled with mercury and inverted while submerged in a reservoir,

$$h = \frac{p_a}{\gamma_M}$$

But mercury barometer works in very simple concept using the capillarity concept only. Here we use the mercury as you know it the unit weight is much larger 13.6 times than the water. So h becomes lesser. So we can measure the height and we can find out what will be the atmospheric pressure.

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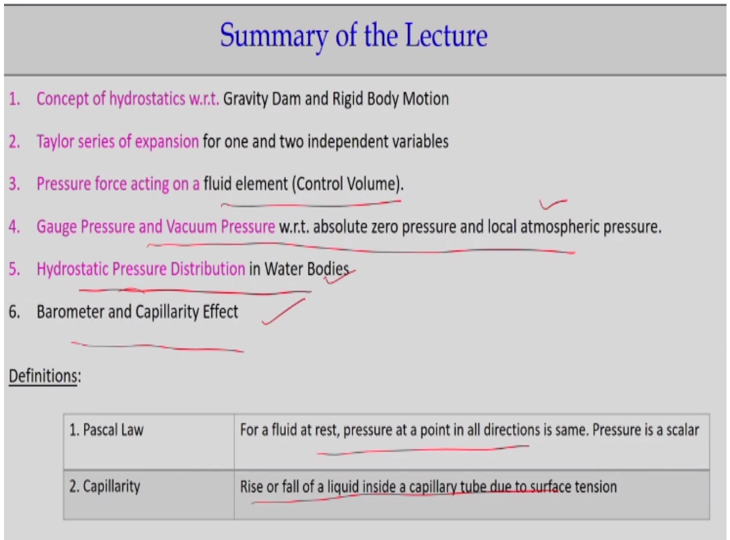

With this note let me summarize today lecture that we started with the examples of gravity dam and liquid working as a rigid body motions which we will discuss much later more that. Again I am to highlight is we use the Taylor series expressions for approximating any continuous functions like pressure, velocity, density and temperature.

And that is what is necessary when you use a control volume, which becomes a infinitely small delta x and delta y, delta z. Then you consider what could be the pressure distributions if your pressure P is at the centroid, what would be the pressure at the different surface and as you know the pressure distribution and just equate the force component to get the equation, that is all. It is not a big thing.

The basically we look at that. Then we discussed about the what is the datum we consider it to measure the pressure, whether the pressure from vacuum levels or the pressure from the atmospheric level. So that atmospheric pressure or the vacuum pressure what you consider it. Hydrostatic distributions we discuss it the barometer and the capillarity height how to deduce that what we discuss is.

And as is again I can repeat it that the pressure is becomes a scalar quantity as it does not have a direction. That means the any point if you consider it pressure can be considered is what that is normal to that. That the concept we will use it later on and you also know this capillarity. With this I conclude this lecture. Thank you.