Fluid Mechanics
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Lecture - 23
Flow in Noncircular Conduits and Multiple Path Pipeflow

Welcome all of you to this fluid mechanics lectures as I am to say is this is the last lectures for this series of the lectures for these 8 weeks and 20 hours lecture series on the fluid mechanics. So let us have a talking about the books. Again I am to repeat you that let us follow the some of the books okay which is highlighted here.

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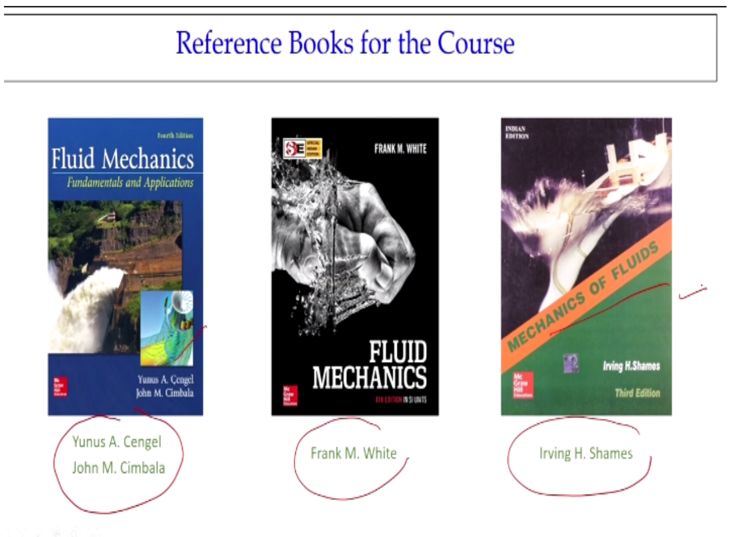

Mostly again I am to repeat it, if you want to have the basic concept and understanding the fluid mechanics as a illustrations wise the fluid flow characteristics, please read the books in written by Cengel Cimbala, Fluid Mechanics Fundamentals and Applications, F. M. White books and I. H. Shames books, Mechanics of the Fluids. Please go through these books which are really quite interesting way written the fluid mechanics from introductory level to the advance level.

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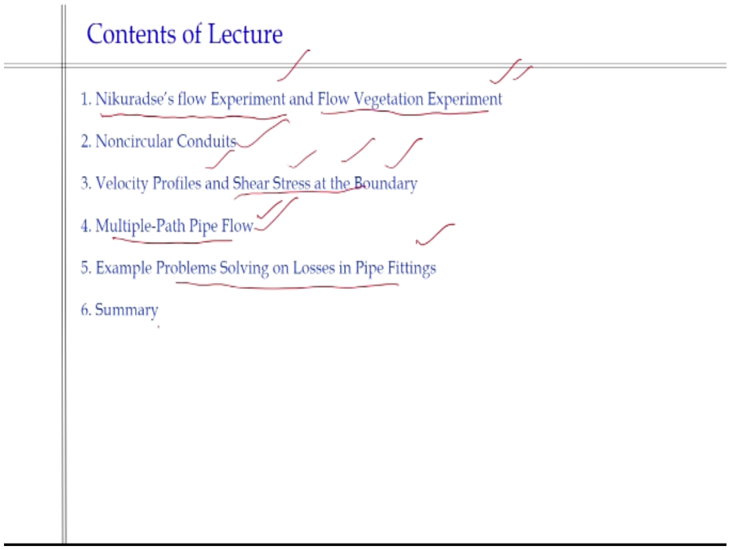

Now let us come back to today's contents what I am going to deliver to you. Very interesting experiments was conducted almost 70 years back. That is what we will be discuss here. And the new experiment what we are conducting at IIT Guwahati is as a glimpse I will show to you. Then we will talk about three things noncircular conduits, how we can apply the same equations for noncircular conduits.

And we also will talk about how does velocity vary in a pipe flow and how to compute wall shear stress at the boundary. And also we will talk about how to solve the multi-path pipe flows. And then we will solve some of the GATE questions on the fluid flow through pipes and we will have the summary.

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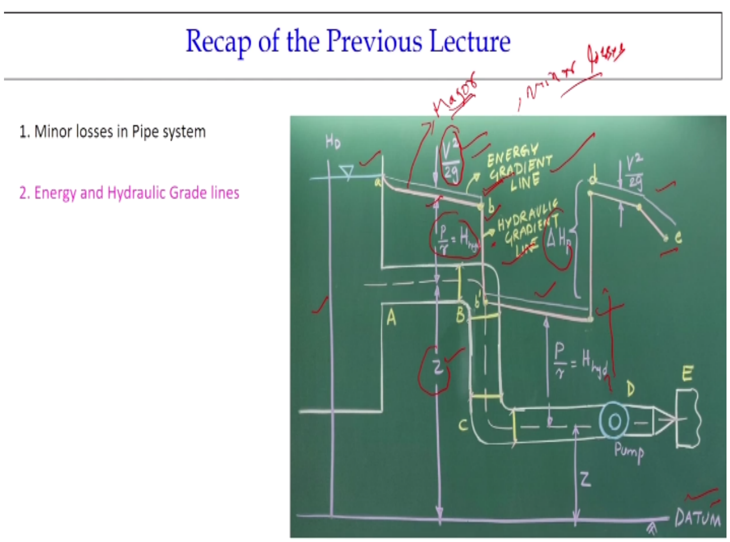

Now let us coming to the previous lectures, the recap of the previous lectures, as soon in these figures. Whenever you have any pipe flow components, we like a pumping systems, the reservoirs, please draw energy gradient line and the hydraulic gradient line. Those two lines will indicates us where the energy loss is happening it and where the energy gaining is happening.

For example, if you talk about the pump, because of these pump resistance we have a getting a extra energy to the fluid flow through the pipe systems. So that is the reasons again I am to repeat it to tell it first you draw a data and you have the reservoirs and from that reservoir you start drawing the energy gradient line, also the hydraulic gradient line.

$$H_{hyd} = \frac{P}{\gamma}$$

The hydraulic gradient line energy gradient line the difference will be the velocity head. That is what the $\frac{V^2}{2g}$, the velocity head. So any datum I can have a z is potential head then I have the pressure head, then I have the velocity head. So these three components, I can draw the lines like this. The for example, if you look at these diagrams from the reservoirs you do not have any velocity components.

That is the reasons you have a hydraulic gradient and energy gradients, both coincide at that locations. As you go, there is an energy losses and through this energy losses what it happens in pipe, that is what we call major losses and the bends entry point and exit point, we call minor losses, minor losses. And these losses already we quantified in the last class.

And as you know it, this the hydraulic gradient lines, drop of this hydraulic gradient line or the energy granted line because of the dropping of the potential energy from this pipe orientations. Again if you look it there is a major losses, this pumping systems is there which is giving extra energy or energy head to these. That is the reasons again you have velocity head and this.

So whenever you have a pipe flow problems try to sketch approximately energy gradient line and the velocity hydraulic gradient line. As you draw the energy gradient line and the hydraulic gradient line you can try to understand how the flow taking place, where the energy losses are happening it, whether the major losses, the minor losses or adding the energy to the system, all you should identified it when you drawing this energy gradient line, hydraulic gradient line.

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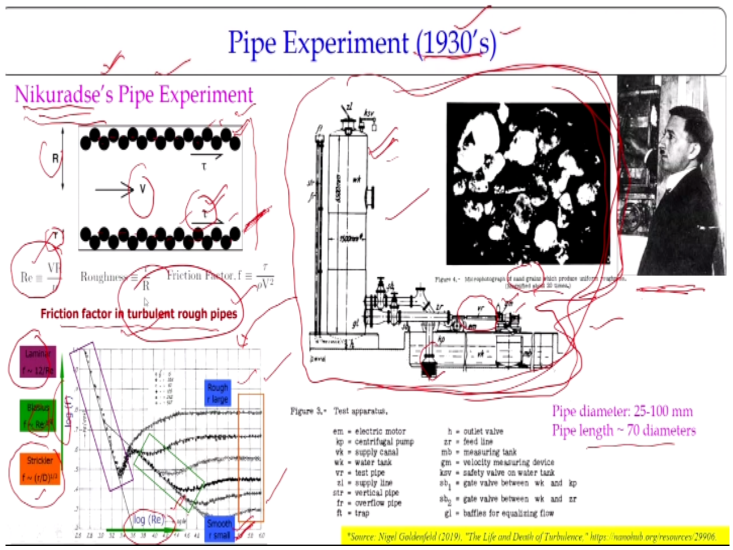

Now let us come back to very interesting experiments what it happened in 1930s, okay much before the World War II okay. The Germans the professor used to do a simple experiments okay which one of classical experiment conducted in a pipe flow and which helped to make a this complex flow to a simpler empirical based energy loss quantification, velocity distributions quantification as well what could be this wall shear stress.

If you look at this any of the pipe flow we can have a laminar flow, the turbulent flow or the transitional in between the laminar and the turbulent flow. What the experiment is conducted by the Nikuradse is that with having very simple concept that the pipe put it with a roughness, this equivalent roughness is from the sand grains. This figure is shown it with a zoomed with a 20 times.

That means you can see there is quite random the sand grains are roughened it or put it in here such a way that it can create a roughness on the wall of the pipe. If you have a that roughness which representing is R, the roughness site representing as R and R is the radius of the pipe flow. So you can know it the because of the roughness, your flow characteristic you can change it.

The also we know very well the Reynolds numbers also control the flow behaviors. So that way to know it what is will be the wall stress the tau value, what could be the velocity distributions, what could be the energy losses when you have a pipe length of L with the informations about the roughness, informations about flow Reynolds numbers.

Those that was the basic question was there before starting the experiment by Nikuradse experiment setup, which is very basic experiment setup, you can see it this from this. Very basic experiment set up having the big water tank connected with a pump and having a test sections here, which measure the velocities and all. And if you look at the dimensions of these water tank, which is 6.5 meter tall 1.5 meters diameters.

So you can look it that how precisely experiment we are conducted to quantify what could be the energy losses and what could be the velocity distributions and what could be the relationship of velocity distributions with wall shear stress. As you can understand from this experiment what was conducted in 1930s, it is quite remarkable way we got it with relationships for the turbulent rough pipes.

That you have a friction factors and you have the flow Reynolds numbers which gives us a very three reasons. One is if you have a R is too large and the Reynolds number is too large you can have a friction factor is a function of R by V. That means roughness factors, R by V is roughness factor and when you have flow is laminar, it is a functions of the Reynolds numbers.

And if you have a smooth pipe where the roughness is very, very small, roughness is the factor is much slow, very small, then you can have a the smooth R values. The R is a very small then you can follow this Blasius equations which is a functions of the Reynolds numbers.

So now if you look it that by conducting a series of experiments, by Nikuradse finally bring a the friction factors relations with the Reynolds numbers which is Moody chart and we have been using that for designing all this pipe flow networks for the industry, the water supplier, sewage treatment plant all where we are using the same equations, what was developed way back in 1930s conducting a series of experiment for turbulent rough pipes. This is very complex till now to do in a shape simulations.

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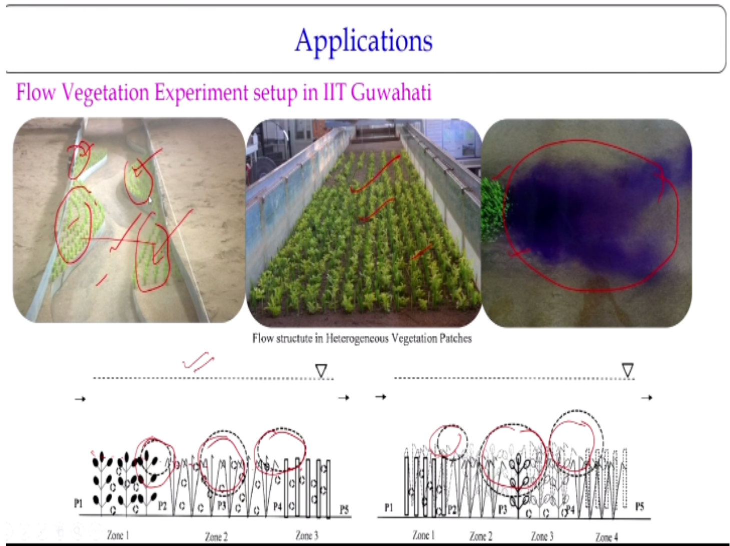

Now if you come back to that what we have been doing at IIT Guwahati, we also do similar sort of creating the roughness in open channel flow. That is what you can see this vegetations are there. Some degree of we are creating the roughness and we try to measure the velocity distribution. Try to quantify the wall shear stress what could be there, bed shear stress could be there.

All we are quantifying, measuring the velocity distributions, measuring the water flow and what we have? We have introduced the roughness and you can see this roughness behaviors and how the flow dispersions are happening it with a putting the color dyes. I am not going much detail on that.

But if you look it for this case, if the flow is submerged conditions, and you have a the roughness like this, there are the vortex the eddies formations and the strength of eddies formations you can get it and that is what we got it conducting a series of experiments in this flow as well this flow.

So what I am to tell it that the experiments gives us a lot of confidence of a very complex problems like a turbulent flow in a rough pipes which is very difficult to model as of now also in the softwares, but we can conduct a series of experiment and we can establish the empirical relationship and those empirical relationship is used in generally in industry to design the pipe networks.

Now coming to the other case, many of the times also we do not go for only the circular conduits or the circular pipes from one point to other points okay.

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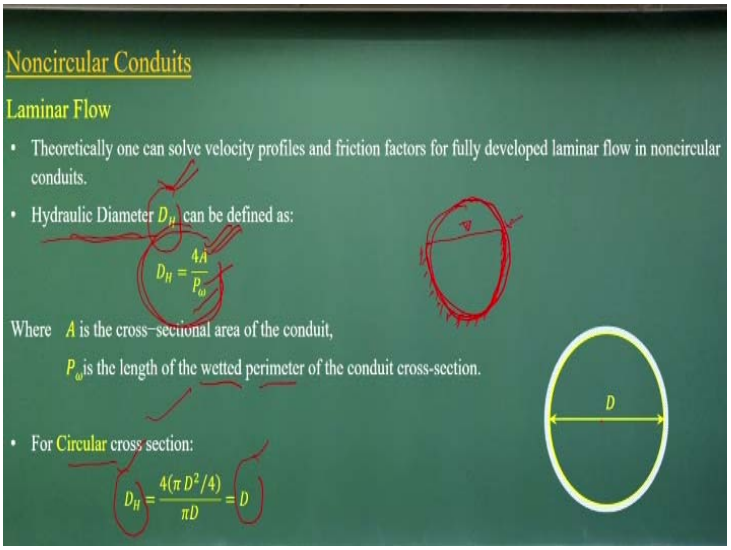

But when you go for a noncircular case you need to define as a equivalent flow okay. So that means we introduce a hydraulic diameters okay, it is called as hydraulic diameters,

$$D_H = \frac{4A}{P_\omega}$$

which is a functions of area and wetted perimeter. The part of the perimeter which is under wetted conditions, wetted perimeters. Like for example, if I have the flow pipes, it is going through whole, so that way the total perimeter I can consider as wetted perimeter.

But if I have the flow half filled the pipe flow is going it, then I will consider the wetted part of this ones only. I will consider the wetted perimeter. I am just highlighting that. We need to consider wetted perimeters the area, then we can compute the hydraulic diameters, okay as equivalent diameter we can get it. That is what is for the circular sections.

$$D_H = \frac{4(\pi D^2/4)}{\pi D} = D$$

If we just substitute if the D is a diameters and the perimeter for the whole flow of these pipe systems, you will get it hydraulic diameter in a circular pipe when it contains full of water, okay full of liquid what is flowing through that. Then you can get it the hydraulic diameter as equal to the geometric diameters. So that what is equal components which comes for that. Please do not be confused. The diameter, hydraulic diameter as a equivalent diameter representing for other noncircular conduits.

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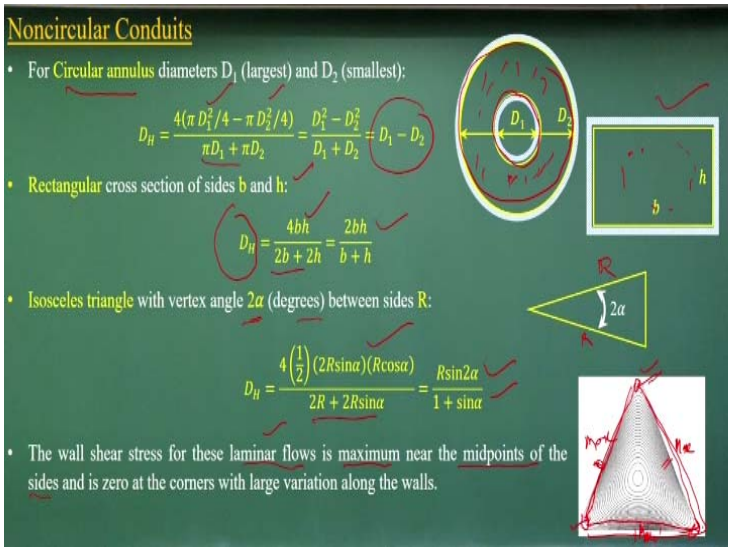

Now come back to the noncircular conduit like you may have a conditions where you have a only flow through this ones okay. That is called circular annulus diameter.

$$D_H = \frac{4(\pi\, D_1^2/4 - \pi\, D_2^2/4)}{\pi D_1 + \pi D_2} = \frac{D_1^2 - D_2^2}{D_1 + D_2} = D_1 - D_2$$

You have a D_1, D_2 and you have a flow through these ones. So you can easily find out what will be the flow area and what will be the wetted parameters for both the cases. That is what will give you this one.

Similar way if you have a rectangular cross-sections and flow through these systems, I can define it what will be the area and the wetted perimeters. That is what will give me the hydraulics diameter. If you have a isosceles triangles with a two alpha degrees between radius R, also I can put it numerically what could be the area what could be the perimeters and I can get it for these ones.

Rectangular cross section of sides b and h

$$D_H = \frac{4bh}{2b + 2h} = \frac{2bh}{b + h}$$

Now let us come it with that when you have a noncircular pipe like if you have a pipe like a triangular shape okay. So in this case what will happen the if you have a laminar flow you will have a wall stress will be maximum near the mid points of the sides. It will have a maximum at the midpoint of the side, wall shear stress. Becomes will be zero at this point, zero at this point, okay.

Isosceles triangle with vertex angle 2α (degrees) between sides R

$$D_H = \frac{4\left(\frac{1}{2}\right)(2R\sin\alpha)(R\cos\alpha)}{2R + 2R\sin\alpha} = \frac{R\sin2\alpha}{1+\sin\alpha}$$

So more details I am not going it because it is quite undergraduate level and we did not discuss much detail of the flow patterns okay what supposed to be there when you have a the complex geometry like a isosceles triangles that you can see that the more maximum wall stress will develop at this point, well it will be zero at these three points.

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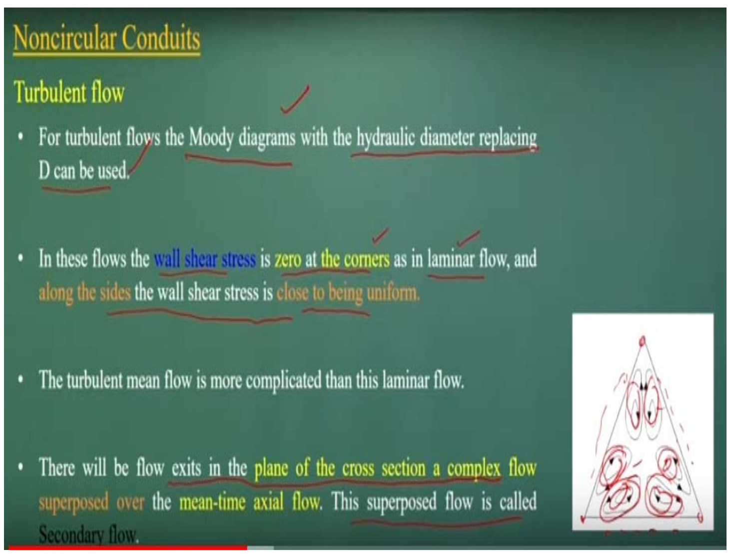

Now if you look it that if you have the turbulent flow same case you have a turbulent flow the velocity distributions as well as the wall shear stress distributions exchanges it. What we do it in any case of the turbulent flow we also use the Moody's diagrams. Hydraulic diameters replacing with a D. That is what is used to quantify what could be the energy losses.

The wall shear stress is zero at the corner as already we tell it in laminar cases. So we will have a wall shear stress at the corners is zero. But along the side the wall shear stress is close to being the uniform. This side you will have uniform, not the maximum at the midpoint as you got in case of laminar flow. So that is the flow behavior so what is observed by conducting a series of experiment quantifying the velocity distributions knowing it what could be the wall shear stress.

Now if you look at that, when you have this type of flow, always you can have this type of vortex formations, okay. This type of vortex formations you can see it and these vortex formations we call the secondary flow generations what it happens it and these are responsible for the mass actions, momentum actions through these the flow systems.

So as you go from away from this circular pipes for this case like a triangular flow zones you have a more complicated the velocity distribution shear stress distribution as compared to the circular pipes as a symmetrical problems what we try to understand it. Now coming back to the turbulent flow again we can follow the same head loss equations.

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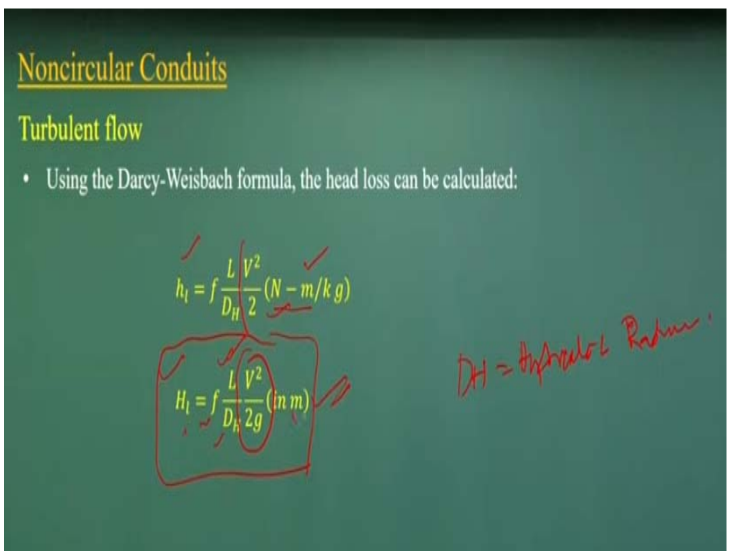

The head loss is a functions of the velocity head which is

$$h_l = f\frac{L}{D_H}\frac{V^2}{2}(N - m/k\ g)$$

$$H_l = f\frac{L}{D_H}\frac{V^2}{2g}(in\ m)$$

Most often we use these because this is what is gives the energy losses in terms of meter, but we can use these functions which gives us in terms of Newton meter per kg. So divide by the acceleration due to gravity will get in terms of meter.

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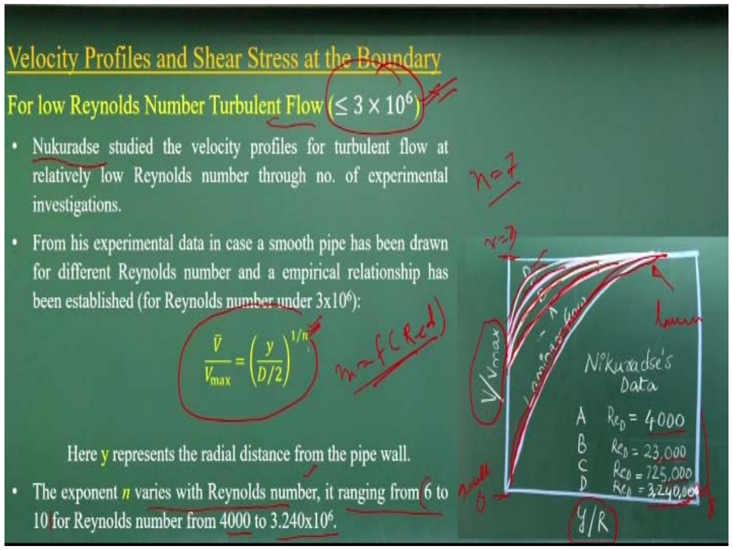

Now coming back to the velocity profile, or the shear stress at the boundary. Here we divide into two zones as suggested by Nikuradse by conducting a series of experiment, finding out velocity profiles for the turbulent flow where you have the flow Reynolds number less than three millions okay. If you look at this Reynolds numbers, okay it is too large okay.

But because as I said it earlier the experiment setups was designed in such a way that he could conduct the experiment which will have a flow Reynolds numbers in the range of three millions more than that. By conducting this experiment what he got it that the non-dimensional velocity distribution

$$\frac{\bar{V}}{V_{\max}} = \left(\frac{y}{D/2}\right)^{1/n}$$

And this is the and R equal to zero that means it is a center points.

If you plot y by R okay and $\frac{\bar{V}}{V_{\max}}$ no doubt the velocity distribution for laminar flow this follows this ones. This is the laminar flow. But as increase the Reynolds numbers, okay, as increase the Reynolds number it goes off to these you can go to up to 3.24 order of million scales. If you look it that the velocity distributions as going more flatter to flatter okay. That is the very uniqueness it comes it.

In case of the laminar flow you will have the velocity distribution like this but in case of the turbulent flow you will change the velocity distributions as we increase this flow Reynolds numbers. So to quantify the what could be the relationship for these type of velocity distribution $\frac{\bar{V}}{V_{\max}}$, it establish a relationship with $\left(\frac{y}{D/2}\right)^{1/n}$.

$$\frac{\bar{V}}{V_{\max}} = \left(\frac{y}{D/2}\right)^{1/n}$$

It is fitted with a data curve and finally find out the n varies with a Reynolds numbers. Its ranging from 6 to 10 for the Reynolds numbers from 4000 to the 3.2 million. So in this case and varies considering from the 6 to 10. So this n is a function of Reynolds numbers okay. But many of the case also n is considered is 7. So many of the time is called 1 by 7 for describing the velocity distributions in a pipe flow for turbulent regions.

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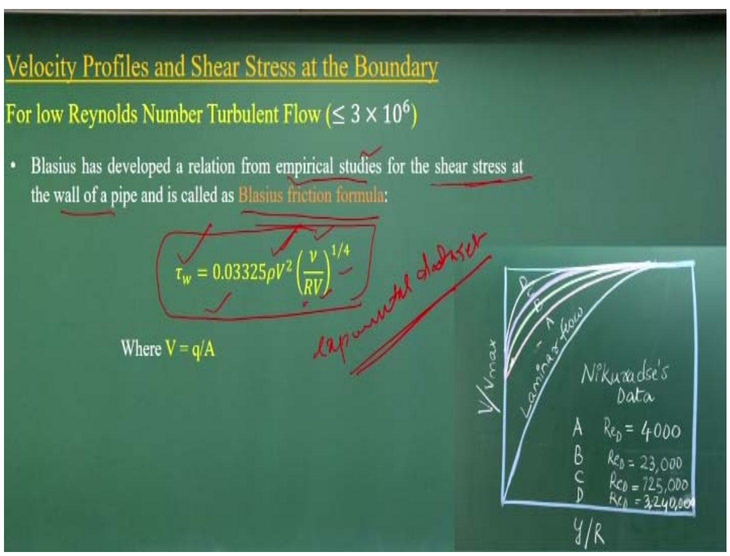

But when you look it what could be the shear stress that is what is the wall shear stress, shear stress at the wall of the pipe. That is what is also empirically established with the functions of wall stress, the average velocities and hydraulic radius and V. That is what is develop it to compute it what could be the wall stress from experimental data set, from experimental data set it derive.

$$\tau_w = 0.03325\rho V^2 \left(\frac{\nu}{RV}\right)^{1/4}$$

Where V = q/A

That is the reason is called empirical studies, the wall stress the relations with the average velocities and the kinematic viscosities and R value. That is giving the what could be the wall stress for a circular pipe.

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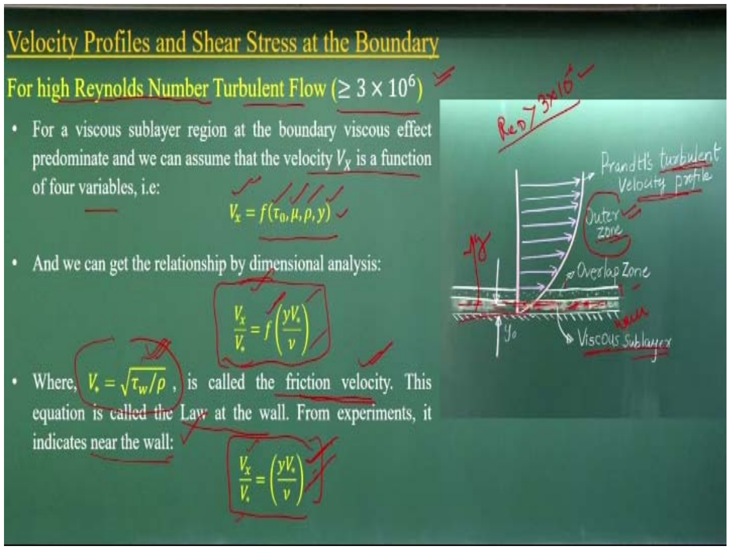

Now if you look it that when you go for higher Reynolds numbers turbulent flow, higher Reynolds number turbulent flow, which is more than 3 million, the Reynolds number is more than 3 millions. In this case what it actually happens is I am not going more details. When you have the Reynolds numbers more than 3 millions, then there is certain zone of the velocity profile develops.

Like at the wall you can see there will be a flow which behave like the laminar flow, which behave like viscous sublayers. And the you go outer of zones you will see that will be a velocity profile will be develop it, which is not the linear relationship between. In laminar flow there is a linear relationship between the shear stress and the velocity distribution. But that does not happen it when you go for the outer zones.

$$V_x = f(\tau_0, \mu, \rho, y)$$

And in between there is a overlap zones formations. In this lecture, we are not going more details, because I have not introduced you boundary layer concept, but try to understand it when you go for higher Reynolds numbers flow, you will have a very small thickness, near to the wall, where the flow behaves like a layer, a way the viscous is dominated too much and that is the reasons you will have a viscous sublayer zones.

But as go far away from the wall, you will have it where it will have a turbulent velocity profile zones, which experimentally established that change of reasons will happen as we will go from the wall. Most of the times this type of things as I said it earlier we do a non-dimensional analysis and then we try to establish the relationship with dependent variable or independent variable.

For this case if velocity distributions is considered is a functions of shear stress, wall shear stress, mu, rho and the y, y is the distance from the wall. And if you do a simple dimensional analysis we can compute like this. This is a simple dimensional analysis to rearrange the terms. You will get a friction velocity or the shear velocity in terms of wall stress. In terms of wall shear stress we will get it this value.

$$\frac{V_x}{V_*} = f\left(\frac{yV_*}{\nu}\right)$$

Where, $V_* = \sqrt{\tau_w/\rho}$

So this is the relationship. We do not know this f value and from the experiment we found that f value is just a linear functions okay, just a linear function. It does not have any, so we can find out the Vx, V_* is this way in the regions which is near to the wall okay. This is the regions near to the, the law at the wall which is very simple as a $\frac{V_x}{V_*}$. V_* is representing this friction velocity is a simple $\frac{yV_*}{\nu}$.

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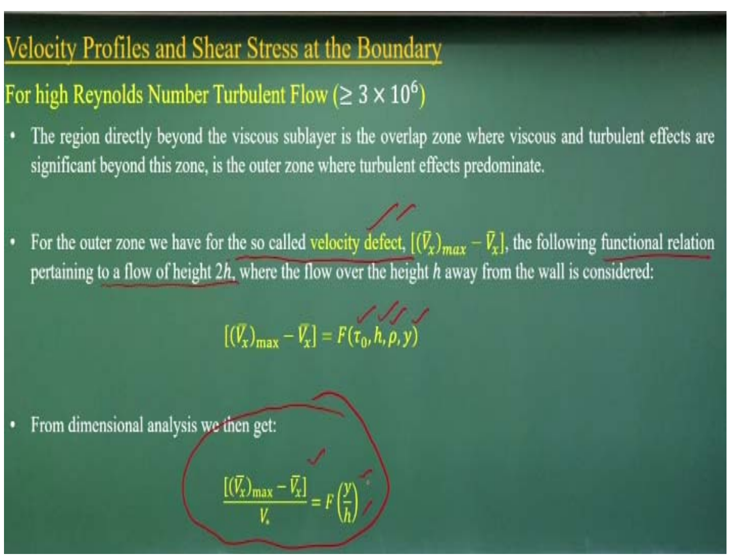

But if you go to the outer layers where we look it that a velocity defect concept, how far the velocity from average velocity, that the defect means how much deviations how much difference between that; if you look it that and looking,

$$[(\overline{V}_x)_{\max} - \overline{V}_x] = F(\tau_0, h, \rho, y)$$

We again we get the similar functions from the dimensional analysis between V and y and h,

$$\frac{[(\overline{V}_x)_{\max} - \overline{V}_x]}{V_*} = F\left(\frac{y}{h}\right)$$

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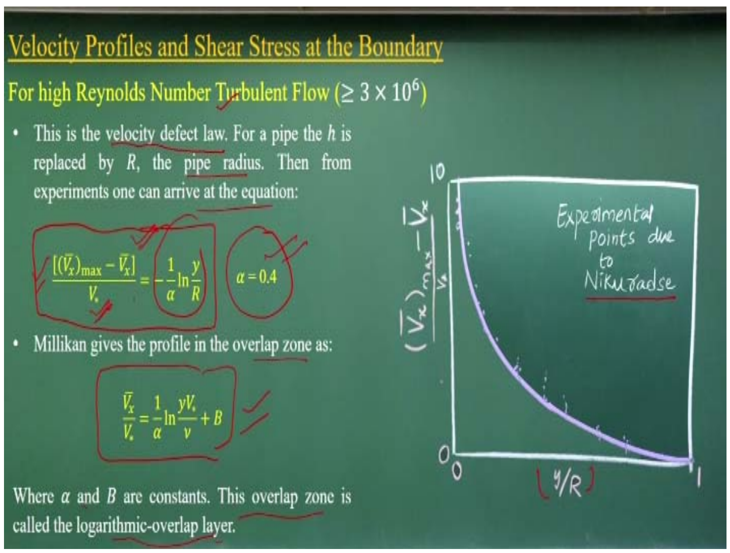

Now if you look it that, if you put it high turbulence flow and the velocity the reasons is very this is called velocity deflect law and h is replaced by the R the pipe radius then you can have this experimentally derived components and this alpha will represent a equal to the 0.4 and this is what the average velocity or time average velocity components and this is a special average velocity component how they are fluctuating with shear velocity.

$$\frac{[(\overline{V}_x)_{\max} - \overline{V}_x]}{V_*} = -\frac{1}{\alpha}\ln\frac{y}{R}$$

$\alpha = 0.4$

And more details if in a overlap zones you will have a this equation. So now if you look it from the experiment and the dimensional analysis using this Nikuradse experiment data set it was found what could be the alpha value okay which is here is 0.4 and for the overlapped zones alpha, beta as a different value and here I am not talking much more. It is called the logarithmic overlap layers okay.

Millikan gives the profile in the overlap zone as

$$\frac{\overline{V}_x}{V_*} = \frac{1}{\alpha}\ln\frac{yV_*}{\nu} + B$$

In between these two regions we can locate how the velocity distributions is taking it. Now we coming back to the very simple examples okay. And that is what is in your text book is necessary to for. If you look it that, many of the times you have the pipes in series, pipes in parallel or three reservoir junction problems okay. Pipe in series is a very simple problems like electric circuits, okay. You can have a series of wires you have from point A to B, okay.

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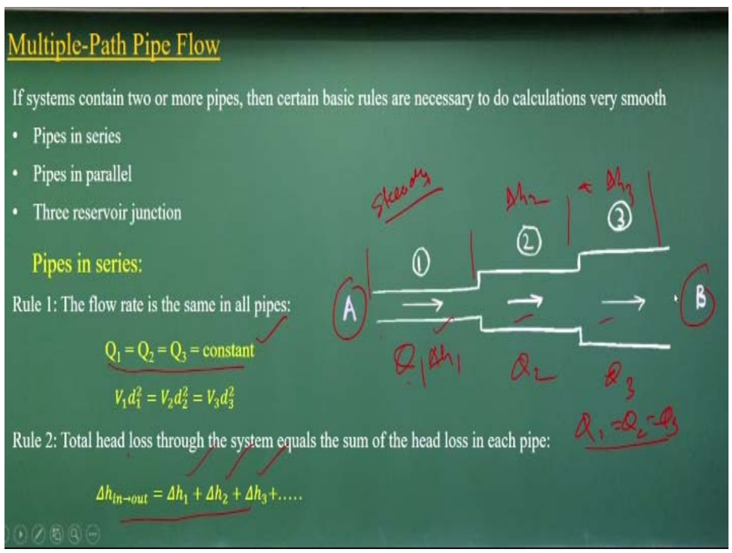

If you have definitely the discharge will be for a steady state conditions for steady flow conditions. So discharge at the Q_1, Q_2, Q_3 that should be equal because this is a steady state.

$Q_1 = Q_2 = Q_3 =$ constant

$V_1 d_1^2 = V_2 d_2^2 = V_3 d_3^2$

But if you are having a energy losses between delta h 1 is energy losses for this regions. This is

$$\Delta h_{in\rightarrow out} = \Delta h_1 + \Delta h_2 + \Delta h_3 + \ldots..$$

So this pipe in series a very simple problems, in which your flow is a constant okay. But the total head loss is a sum of the head losses of individual pipe is connected in a series, very simple problems.

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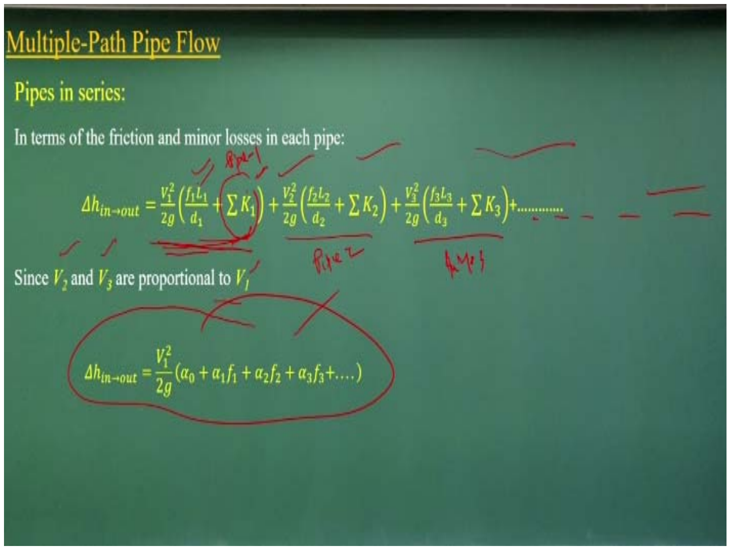

But when you have a pipe in a series please remember it that you always should consider whether there is a minor losses. There is major losses which is the frictional losses component for the pipe 1. Also there will be a minor losses because the change of the diameter of the pipe okay there will be expansion or the contraction exit loss, entry losses all the loss component you have to look it which way the flow is happening it and you can quantify what could be the minor losses.

So this is the major loss plus minor loss. That is what is represented here for the pipe 1. This is the representing for the pipe 2, and this is the representing for pipe 3. So similarly, if are more number of pipes, you can have a just summations of the head loss, the head energy losses along the pipe from pipe 1, pipe 2, pipe 3. Most of the times if you look it because the same discharge is going through that, you will have the V 2 and V 3 which is a functions with a discharge is same.

$$\Delta h_{in\to out} = \frac{V_1^2}{2g}\left(\frac{f_1L_1}{d_1}+\sum K_1\right)+\frac{V_2^2}{2g}\left(\frac{f_2L_2}{d_2}+\sum K_2\right)+\frac{V_3^2}{2g}\left(\frac{f_3L_3}{d_3}+\sum K_3\right)+\ldots\ldots\ldots\ldots$$

It will have a proportionality with V_1. So you can write a simple formulae like this okay So these are very shortcut way to do it, but I always encourage you please follow these the energy losses computations for major losses minor losses major losses minor losses and compute total.

Then find out what could be the energy loss is happening in a pipe in series conditions. Now if you look it another very simple problems that pipes in parallel.

$$\Delta h_{in\rightarrow out} = \frac{V_1^2}{2g}(\alpha_0 + \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3 + \ldots.)$$

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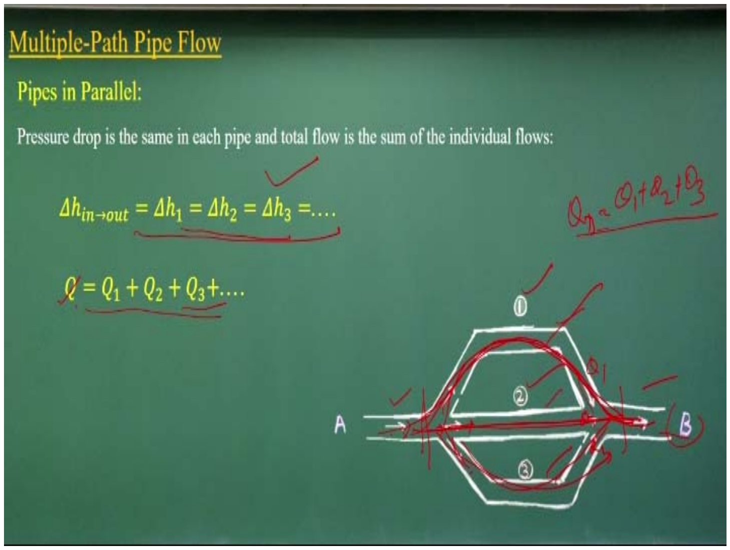

When you have a pipe in a parallels, you can understand it see if I have the pipe in the parallels, there are three pipes are connected here. This is the A is entry point, B is exit point. From the this point to this point the total energy losses passing through this path A or path B or path C that should be equal. So energy losses should be equal, whether it follows a path A, path B, or path C, all the energy losses should be equal.

$$\Delta h_{in\rightarrow out} = \Delta h_1 = \Delta h_2 = \Delta h_3 = \ldots.$$

That is the conditions. And as the flow is coming and divide into three part, we can always write the Q what is coming it will be distributed into the three part in this figure. So as you have a branching out. As you are converging it, you can write

$$Q = Q_1 + Q_2 + Q_3 + \ldots.$$

So sum of the discharge will give us the discharge what is passing through the A or B and the energy losses whether following the 1 path 2 path or 3 path should be equal. Then the energy at these two point will be come to a the same values. Otherwise it will not be possible to have a different energy losses. That cannot be happen it in a flow systems.

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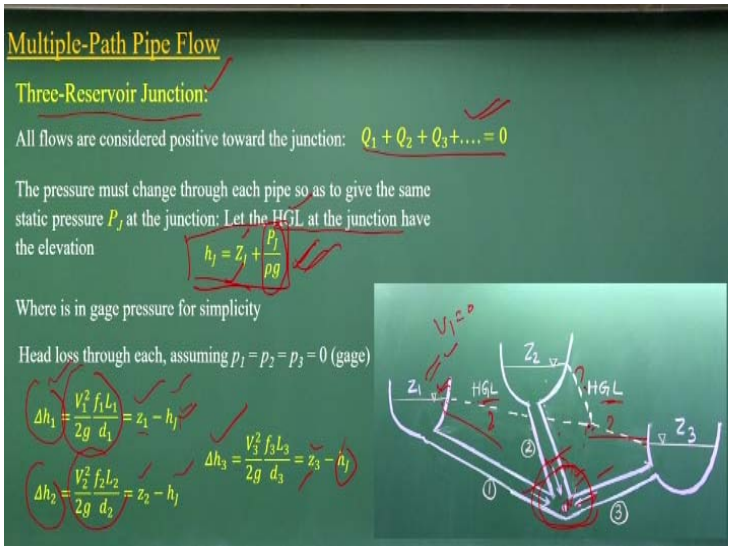

Now if you look it the three reservoir junction problems which many of the time it is given that you have a multiple reservoirs okay. You may have the multiple water tanks are there and connected to the pipe flow systems and you have a junctions where you have three are connected here. Out of these three flow one could be also outflow. We do not know that which direction it will be, outflow it.

But we can say that some of the Q discharge should be equal to zero at this point.

$$Q_1 + Q_2 + Q_3 + \ldots = 0$$

There is no outlet, okay. So at the junction point, the continuity equations are mass conservations at these equations should be equal to zero. But another one what you need to compute it how much of energy or the head is what is the elevations of high gradient line.

$$h_J = Z_J + \frac{P_J}{\rho g}$$

These hydraulic gradient is supposed to be equal to yours the change of we have this gradient line at this point like this. And I know this hydraulic gradient lines at these points. If I looking that how much of head loss is happening because the pressure head is equal to zero here, the velocity head equal to zero here.

assuming $p_1 = p_2 = p_3 = 0$ (gage)

So this is energy gradient line, also the hydraulic gradient line. So that way if I look at these the line of hydraulic gradient line, not the energy gradient line, line of hydraulic gradient line because I am not looking the velocity point. That should be equal at this point after having the energy losses.

$$\Delta h_1 = \frac{V_1^2}{2g}\frac{f_1 L_1}{d_1} = z_1 - h_J$$

$$\Delta h_2 = \frac{V_2^2}{2g}\frac{f_2 L_2}{d_2} = z_2 - h_J$$

$$\Delta h_3 = \frac{V_3^2}{2g}\frac{f_3 L_3}{d_3} = z_3 - h_J$$

So how much of energy losses will happen in such a way that we will have a hydraulic gradient at the junctions will be this ones. That is what we apply with a loss and compute what is the head losses should be there in this direction in this direction and in this direction, okay. And based on that we can compute that what will be the flow contributions Q 1 or Q 2 and Q 3 directions.

So you have a three equations as well as this equations help us to find out the flow directions and the head losses of a three reservoir junctions problems okay. This quite interesting problems but try to understand that we are computing the hydraulic gradient line and what is the hydraulic gradient locations of at the junctions and based on that, we are just equating the energy losses to compute that part.

That is what the basic (()) (36:49). Now come back to a example one which is a GATE 2014 question paper which looks like very lengthy but is a very simple problems okay.

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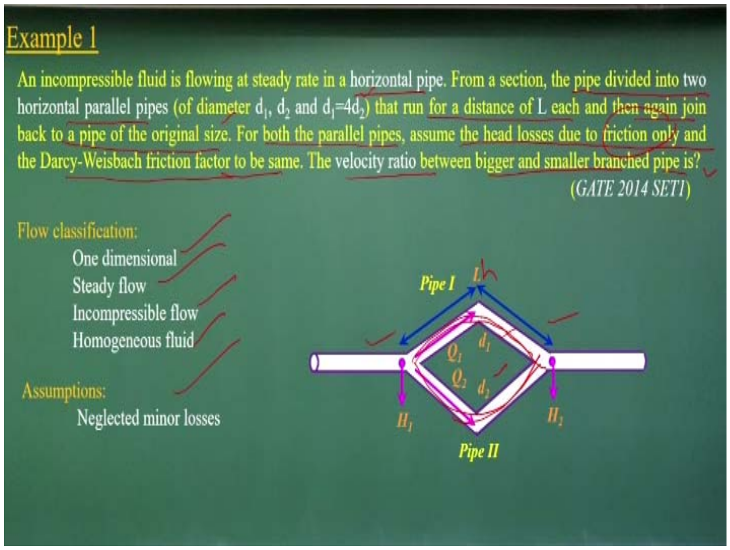

An incompressible fluid flowing through a steady rate in a horizontal pipe, okay. From a section, pipe is divided into two horizontal pipes as given the d 1 and d 2 okay and d 1 is four times of d 2 run for a distance l each other. Then again join back to pipe of the original size okay. We will just sketch the problem, okay? Many of the times please sketch the problems then you can understand it.

For both the parallel pipe assume the head loss due to the frictions only. Darcy-Weisbach friction factor to be same. So f is a constant, f is constant. What could be the velocity ratio between bigger and smaller branched pipe, okay. If you look at this problems, as we solved the earlier we follow this basic concept of one dimensional steady flow incompressible this neglecting the minor losses only the friction losses to be consider it.

As is the parallel pipe flow you know these head losses of this path pipe 1 and pipe 2 should be equal. This is very simple; f is a constant and the head losses between either the pipe 1 or pipe 2 should be equal. Based on that you can compute what could be the d 1, d 2. Ratio is known to us. We have to compute the velocity ratio.

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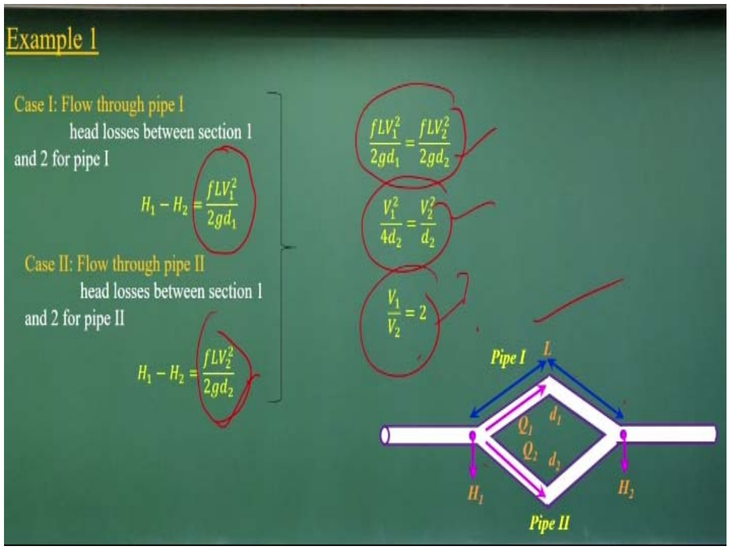

Now if you look at this way, is very simple things, that you know it head losses between section 1 and 2 for this. These two if I equate it I will get it the ratio of the velocity. That is what is the problem, okay. Even if I do not need continuity equations to show it. Only with alpha the head losses between section a and 2 being a parallel pipe that should be equal and that is what if you equate it I will get the ratio between the velocity V 1 and V 2 okay. That is the point what we derived.

Case I: Flow through pipe I

head losses between section 1 and 2 for pipe I

$$H_1 - H_2 = \frac{fLV_1^2}{2gd_1}$$

Case II: Flow through pipe II

head losses between section 1 and 2 for pipe II

$$H_1 - H_2 = \frac{fLV_2^2}{2gd_2}$$

$$\frac{fLV_1^2}{2gd_1} = \frac{fLV_2^2}{2gd_2}$$

$$\frac{V_1^2}{4d_2} = \frac{V_2^2}{d_2}$$

$$\frac{V_1}{V_2} = 2$$

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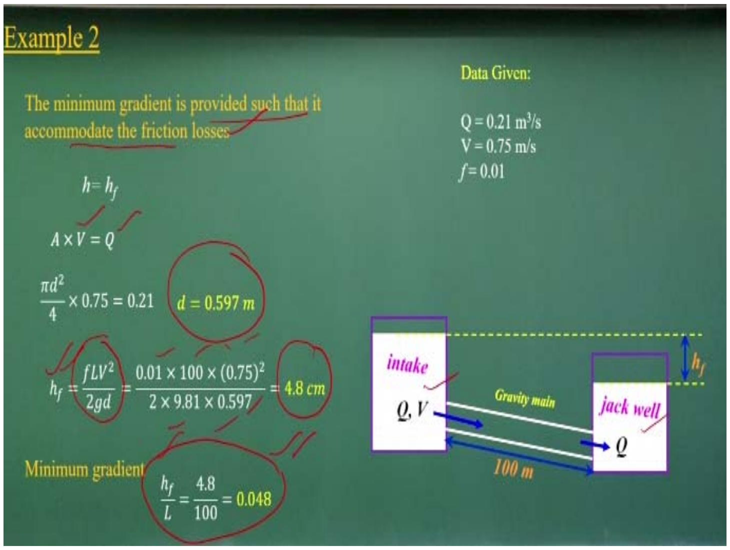

Another problems which is GATE 2014 questions also. A straight 100 meter long raw water, okay this only waters gravity main is carry the water from intake to a jack well okay of a water treatment plant okay. So basically there is a intake, to take the water from the rivers or ground waters and go to a well which is called the jack well where basic screening and all things turn it and then go for the water treatment plant.

Required flow through the water main is given to us. The allowable velocity is given to us 0.75 meter per second; f is given g is given to us. The minimum gradient what could be the minimum gradient to be given to the main so that it require amount of flow without any difficulties okay. We just sketch the problems. The problem is the 100 meters, there is a intake. This is the jack, there the Q is given to us.

Data Given:

$Q = 0.21\ m^3/s$

$V = 0.75\ m/s$

$f = 0.01$

The Q is given to us and also it is given the allowable velocity. If this is the maximum velocity can go through this pipe. What is looking it what could be the minimum gradient to given to this pipe so that the required amount of water can go to the jack well from the intake well. The problems if you look it, it looks very difficult but it is not that.

What is looking it that how much of energy losses is happening it when flow is going from A to B because of frictional losses, how much of energy loss is happening it. Whether we can put a gradient such a way that it can make a flow maintain it. That means you compute the energy losses multiply it with the length. That what will gives us totally energy loss what is happening it.

And if I know the total energy losses, then I can know it what could be the gradient or the potential energy head I have to provide it to have this flow system. Again I am to repeat these things to tell it that the problem is frame it in different way but try to understand it. What is looking it that what could be the minimum gradient to be given to these flow, so that water can flow from this point intake to the jack well.

$h = h_f$

That means, when flow is going through this pipe there will be a frictional losses. The energy losses due to the pipe frictions. Those frictions should be energy losses should be overtaken by the potential energy gradient what we have to keep it. That is the problem. So potential energy gradient to be provided with respect to the energy losses what is going to happen it.

If you do not provide minimum gradient, then water will not flow through that. That is the very basic concept here. The minimum gradient is to provide such as that it accommodates the frictional losses, okay. That is the trick of the problem. Since the velocity is given, the Q is given, we know what could be the diameters using the continuity equations, very simple things.

$$A \times V = Q$$

$$\frac{\pi d^2}{4} \times 0.75 = 0.21$$

$$d = 0.597\ m$$

$$h_f = \frac{fLV^2}{2gd} = \frac{0.01 \times 100 \times (0.75)^2}{2 \times 9.81 \times 0.597} = 4.8\ cm$$

Minimum gradient

$$\frac{h_f}{L} = \frac{4.8}{100} = 0.048$$

And you also know it what could be the energy losses in terms of meter. Just substituting these values okay you can get it, what could be the energy losses. Then the minimum gradient what you were looking. This the gradient is necessary to have flow from intake to jack well.

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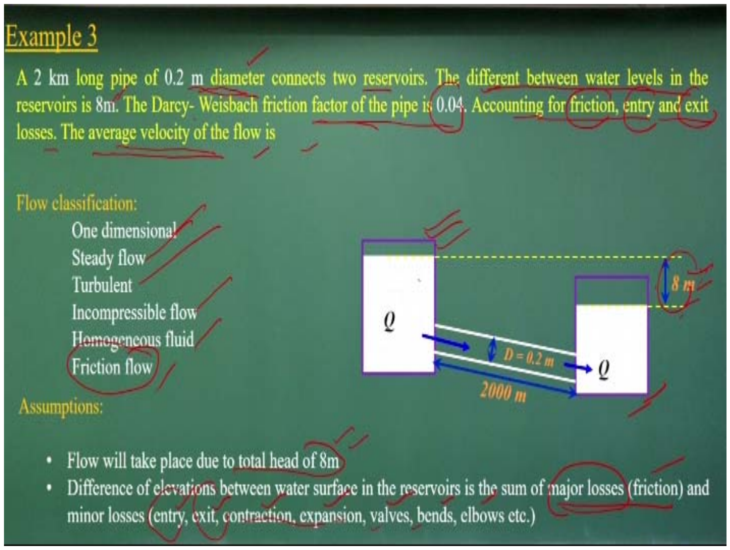

Let us start this third examples which is gives that 2 kilometer long pipe with a diameter of 0.2 meter diameter connects two reservoirs as given in the figures. The difference between the water levels in the reservoir is 8 meters. That means the energy losses between two reservoir is 8 meters. Darcy-Weisbach friction factor is given to us which is 0.04.

Accounting friction, entry and the exit losses we have to estimate the average velocity of the flow. So that way we can say it the flow will take place with having total energy losses of 8 meters and difference between these energy losses will account for major losses, which is a friction losses. The minor losses here is the entry and the exit losses. That is what the additional things what will consider it, but other losses we will not consider this case as given in the problems.

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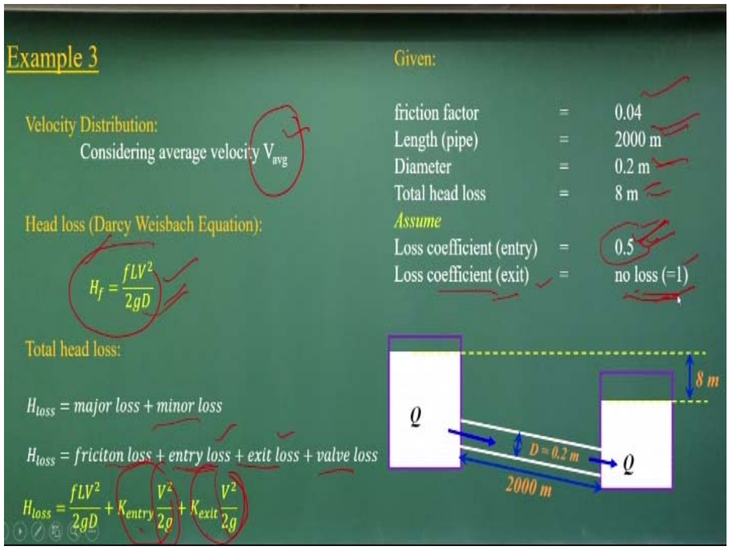

So the friction factors data what is given it length of the pipe the diameters and total head losses. The loss coefficient in terms of velocity head as you know it at the exit we consider as 1, at the entry we consider 0.5. The half of the velocity head losses at entry levels and at the exit level total velocity head what we lost it at the exit level. This is 1, this is 1.5.

Given:

friction factor = 0.04

Length (pipe) = 2000 m

Diameter = 0.2 m

Total head loss = 8 m

Assume

Loss coefficient (entry) = 0.5

Loss coefficient (exit) = no loss (=1)

Now we apply the Darcy Weisbach equations to compute what is energy losses for the major losses or the pipe flow because of the frictions components and the minor losses.

Head loss (Darcy Weisbach Equation

$$H_f = \frac{fLV^2}{2gD}$$

Total head loss:

$$H_{loss} = major\ loss + minor\ loss$$

$$H_{loss} = friciton\ loss + entry\ loss + exit\ loss + valve\ loss$$

$$H_{loss} = \frac{fLV^2}{2gD} + K_{entry}\frac{V^2}{2g} + K_{exit}\frac{V^2}{2g}$$

That is what is a coefficients we use in terms of velocity head and here we have considered 0.5 and the 1.

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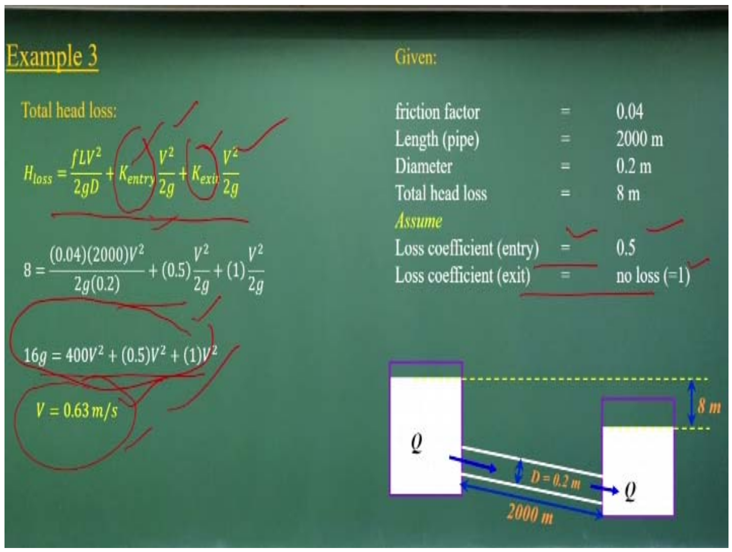

By substituting these and we can get it the series of equations like this and substituting the value you will get a quadratic functions and solving that you will get the velocity. So if you look at these problems is quite easy. Only you have to remember about the coefficients, the factors what we use it, the loss coefficient or factors what we use for computing the entry loss and the exit loss.

$$H_{loss} = \frac{fLV^2}{2gD} + K_{entry}\frac{V^2}{2g} + K_{exit}\frac{V^2}{2g}$$

$$8 = \frac{(0.04)(2000)V^2}{2g(0.2)} + (0.5)\frac{V^2}{2g} + (1)\frac{V^2}{2g}$$

$$16g = 400V^2 + (0.5)V^2 + (1)V^2$$

$$V = 0.63\, m/s$$

The exit loss is total velocity head. Half of the velocity head we use it to as a loss at the entry level. That is the things, otherwise these problems quite a numerical problems to solve these ones.

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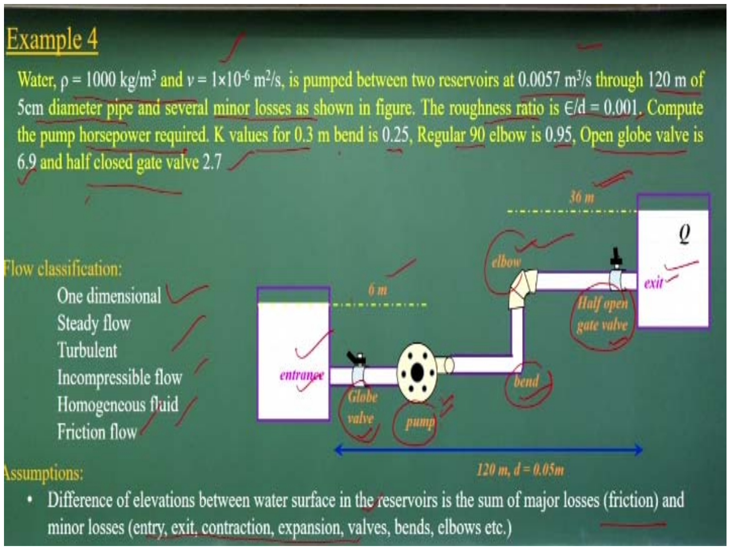

The last examples, let us have a it is slight bit a design problems could be consider it where you have the two reservoirs okay you have a entrance and exit. The water density and the kinematic viscosity is given to us. The two reservoirs having the discharge 120 meter long, 5 centimeter diameter pipe several minor losses can happen it like a valve losses.

There is a pumping systems here, there is a bend, there is a elbow, there is a half open valve gate, the exit and the entrance. All the losses can be there. If a roughness ratio of this pipe is given it is this part. Compute the pump horsepower required and the K value given for the bend is 0.25. Regular 90 degree elbow is 0.95. Open globe valve is 6.9. Half closed gate valve is 2.7.

Entry and exit already we discussed what could be the loss coefficients. So that way we need to compute how much of energy is required, the pumping at the pump so that we can have a these conditions which is a 6 meter, this is a 36 meters. The difference between them is 30 meters. So here we have a major losses, we have a minor losses. So as usual you will have a flow classifications.

In this case one dimensional flow and all and here you have a frictional losses and you have a losses what you have considered for the entrance valve, bend, elbow, half and exit valves. So these all the losses components are given it and we are looking it what could be the pumping requirement.

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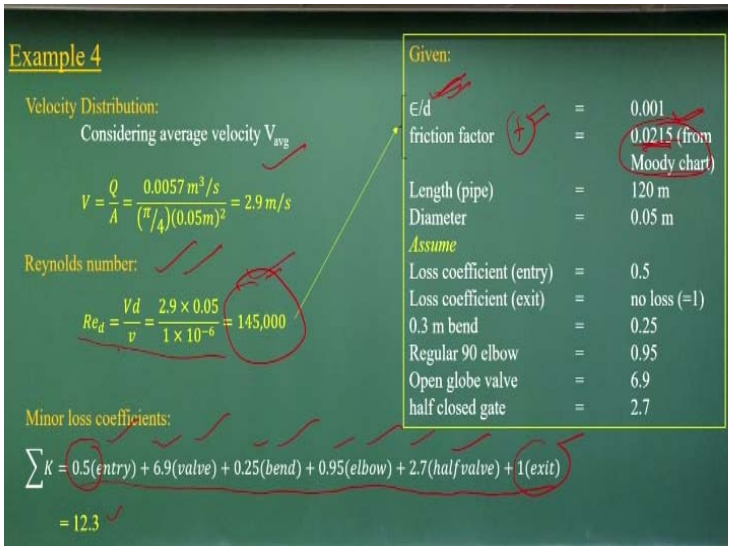

Since here the roughness factor is given to us, we need to compute the friction factors from Moody's chart which needs Reynolds numbers. So first let us compute what will be the average velocity and what could be the Reynolds numbers for this flow. Diameter is given, velocities, then we can get this. So these Reynolds numbers with these roughness factors we will get it from Moody's charts, the value of friction factor.

$$V = \frac{Q}{A} = \frac{0.0057\,m^3/s}{(\pi/4)(0.05m)^2} = 2.9\,m/s$$

$$Re_d = \frac{Vd}{v} = \frac{2.9 \times 0.05}{1 \times 10^{-6}} = 145{,}000$$

$$\sum K = 0.5(entry) + 6.9(valve) + 0.25(bend) + 0.95(elbow) + 2.7(halfvalve) + 1(exit)$$

= 12.3

This is what in general design we do it to compute the Reynolds numbers. Then we for a given roughness factors, we can compute it what will be the R. Get this from chart what will be the friction factors value. Then we can account for all the loss components in terms of velocity value. That entry, the valve, bend, elbow, half valve and exit which will be 12.3 in terms of velocity head.

This is the factors, the sum of the factors for the energy losses, factors we are adding it starting from entry is 0.5, exit is 1 and this value is given to us and mostly in a tabular forms are available to us. By conducting a series of experiment any of the industry they give these value what is the range of the energy loss coefficients for the bend, valve, elbow it is given generally in a any pipe manufacturing company.

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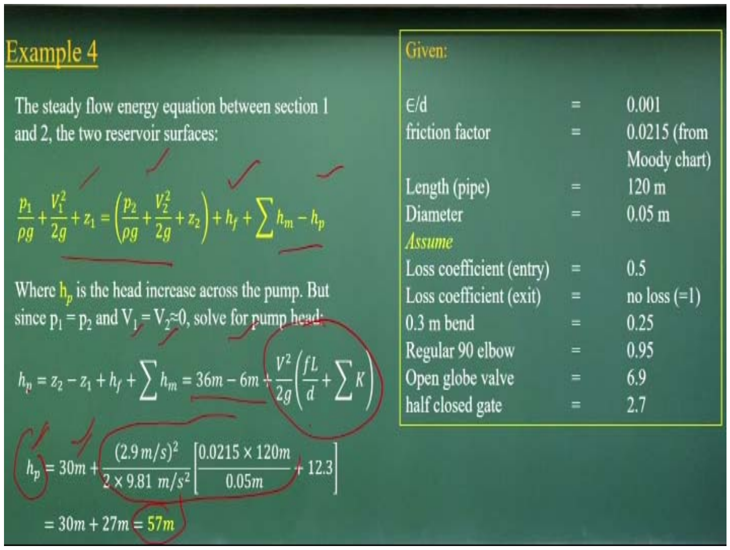

Now we will go to the next what we will do it is very simple things that we are applying this energy equations head loss and this part since two part V_1, V_2, if I put the energy losses V_1, V_2 is zero, and solving this pumping head, okay which is will give it the elevation difference and the major and the minor losses component. Then I will get it what is the head requirement at the pump which is coming out to be 57 meter, okay.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) + h_f + \sum h_m - h_p$$

Where h_p is the head increase across the pump. But since $p_1 = p_2$ and $V_1 = V_2 \approx 0$, solve for pump head:

$$h_p = z_2 - z_1 + h_f + \sum h_m = 36m - 6m + \frac{V^2}{2g}\left(\frac{fL}{d} + \sum K\right)$$

$$h_p = 30m + \frac{(2.9\,m/s)^2}{2 \times 9.81\ m/s^2}\left[\frac{0.0215 \times 120m}{0.05m} + 12.3\right]$$

$$= 30m + 27m = 57m$$

So this is the loss component and this is what the elevation, the potential head what we need it from 6 meter to 36 meters. That what is coming of. So for the for lifting 57 meters, how much of power we requirement.

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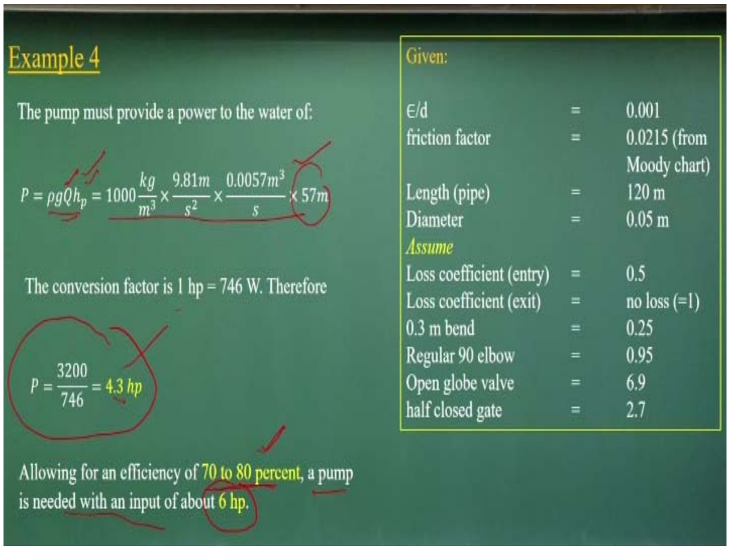

It is there which is very simple things for us to compete is that,

$$P = \rho g Q h_p = 1000\frac{kg}{m^3} \times \frac{9.81m}{s^2} \times \frac{0.0057m^3}{s} \times 57m$$

So you know it what is a power requirement to lift this much of mass of the flow. That is weight mass flux into the h_p. That what will give us the power.

The conversion factor is 1 hp = 746 W. Therefore

$$P = \frac{3200}{746} = 4.3\ hp$$

In terms of hp we will get it 4.3 hp power requirement is necessary to lift the water up 57 meters and if I consider any pumping centers we have this efficient with how much of efficiency 70 to 80. But you cannot have a 100% efficiency.

Considering that this 4.3 hp power and designing that we can consider the 6 hp pipe pump is enough this pipe flow, this system of two reservoir connecting with a 30 meter difference and energy losses what it will be accounted for. So this way, we generally use a design problem to solve the problem, do the designing of the piping system, additional energy like a pumping or in hydropower projects, we put it turbines and all.

The basic concept what you use is the energy gradient lines, the Moody's charts, and the Darcy Weisbach equations to solve this problem. With this, let me summarize this problems lectures what I discussed today.

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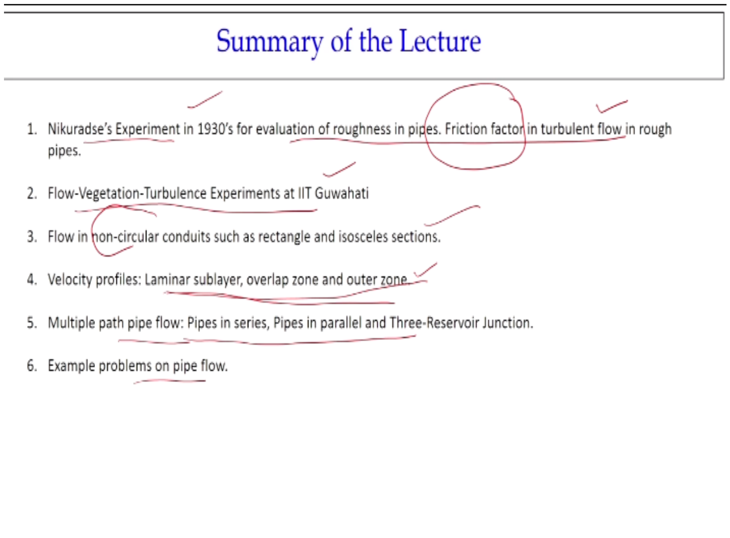

Starting from the 1930s Nikuradse's experiment is one of the finest experiments conducted way back in 1930s. And that was given us very complex problems like a turbulent flow in a rough pipes and as equivalent a energy losses in terms of friction factors and established the relations, which is quite interesting and quite inspiring to us to know that the series of experiment can help us to understand very complex flow turbulent flow in the pipes.

We also shown this just examples that similar type of experiment we have been conducting at IIT Guwahati. I discussed about noncircular pipes. We also discussed about very introductory levels because we did not discuss much about boundary layer concept or turbulent flow much details in these eight week classes.

So I just give you a introductory levels to know it, how the velocity profile is there and at the last we talked about the multipath pipe flow, pipe in series parallel and three junctions. And we solved the four examples. So with this I wish to concludes this 8 weeks and 20 hours lectures on the fluid mechanics. We cover all these topics and very beautiful illustrations, the ppt and all the blogs what you were doing it all they are helping by three PhD students, okay.

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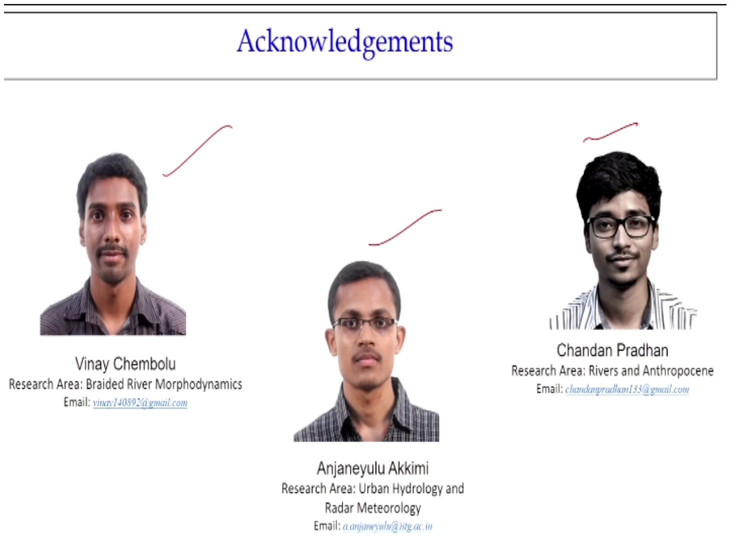

And I do acknowledge their effort for developing this course seamlessly, and ending this part let me conclude with this quote.

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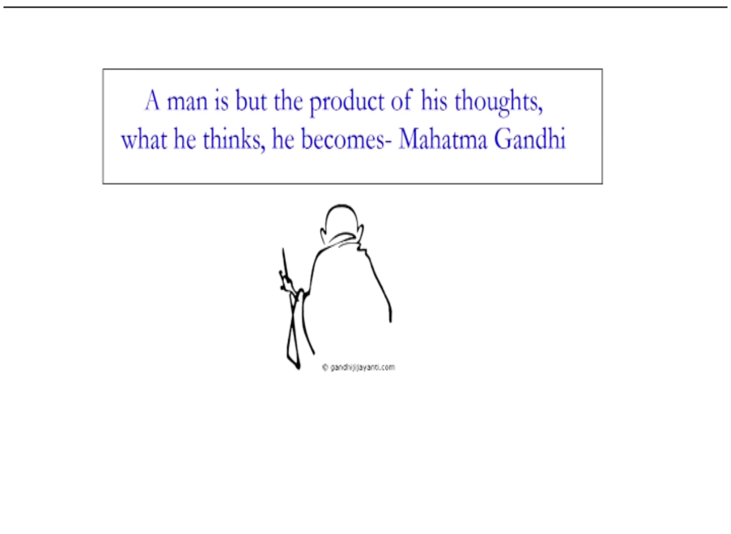

A man is but the product of his thoughts, what he thinks, he becomes. With this note, I can say that NPTEL course is given you opportunity to things beyond what you thought, okay? And that is what should look it that this quote will tell you, will inspire you to for the next level. With this, thank you lot to have this almost 23 hours lectures hearing from us. Thank you lot.