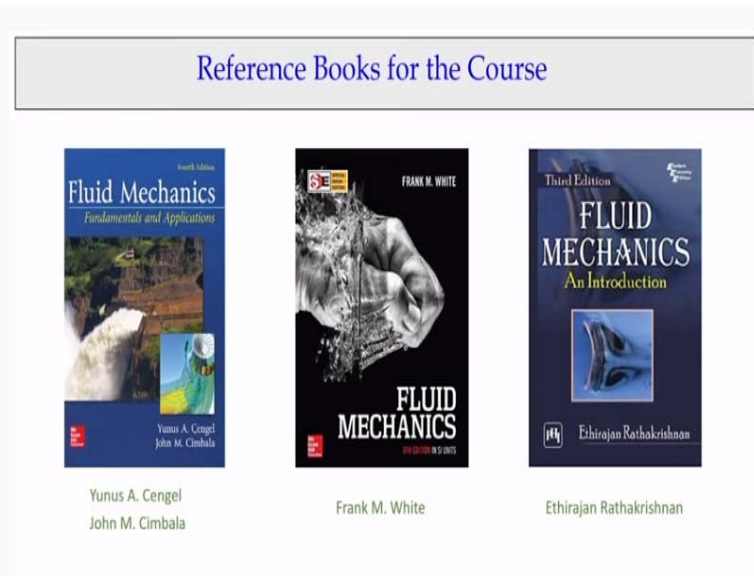


**Fluid Mechanics**  
**Prof. Subashisa Dutta**  
**Department of Civil Engineering**  
**Indian Institute of Technology – Guwahati**

**Lecture – 19**  
**Dimensional Homogeneity**

Welcome all of you to this lectures on dimensional analysis. This very interesting lectures what today I will cover it talking about how to do the experiments in fluid mechanics to characterize the probable. So very interesting topic and this topic I will cover it in 2 lectures.

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So today lectures and the next lectures what I will cover it? Again I am following these 3 books which already we discussed earlier Cengel, Cimbala, FM.White and Ethirajan Radhakrishnan.

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## Contents of Lecture

1. Dimensionless groups
2. Dimensional Homogeneity Principle
3. Buckingham's  $\pi$ -theorem
4. Dimensionless groups in fluid dynamics
5. Example problems on Buckingham's  $\pi$ -theorem
6. Summary

So today I will start with lots of good examples what we have gained? And we discussed that so I will talk about Dimensionless groups, Dimensional Homogeneity principles, Buckingham's  $\pi$  theorem, Dimensionless groups what is used in fluid flow that is what I will introduce to you then we will discuss some example problems based on Buckingham's- $\pi$  theorem and we will have a summary. So as I stated earlier let us start with very interesting examples.

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What we observed in May 3rd, 2019 you can see this these overturning bus overturning in these because of the super cyclonic what it happened in May 3rd, 2019 and this is what we conducted the experiment in wind tunnel in IIT Guwahati. The question rise that as you move it is a full scale

models okay prototype and this is a model this is a full scale that is what it got it in the cyclonic storm centre this is what I conducted in it a wind tunnels and we are getting the similar trend.

Are they enough for a study to conduct or we need to do some sort of a similarity analysis the flow pattern analysis how do we do it? How do we design these experiments? This is what I am just demonstrating when you conduct the fluid experiments you have to first design the experiment how we do that. So those things I will today I will discuss it before that.

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**Drag force measure experiment in IIT Guwahati**

Calculation of Drag for a Cylinder

The Pressure Coefficient ( $C_p$ )

$$C_p = \frac{p - p_0}{\frac{1}{2} \rho U^2}$$

$$C_d = \int_0^{2\pi} C_p \cos\theta$$

$p$  - Surface Pressure  
 $p_0$  - Free Stream Static Pressure  
 $U$  - Free Stream Air Velocity

I am to just show you that there was an instrument which can measure the drag force measurement experiment set up like these where you can compute the pressure co efficient and if you integrate it in get the drag co efficient and that way when you have put a cylinders you can compute the what could be the drag the force acting on these cylinders with the varying the fluid flow the air flow conditions here.

The Pressure Coefficient ( $C_p$ )

$$C_p = \frac{p - p_0}{\frac{1}{2} \rho U^2}$$

$$C_d = \int_0^{2\pi} C_p \cos\theta$$

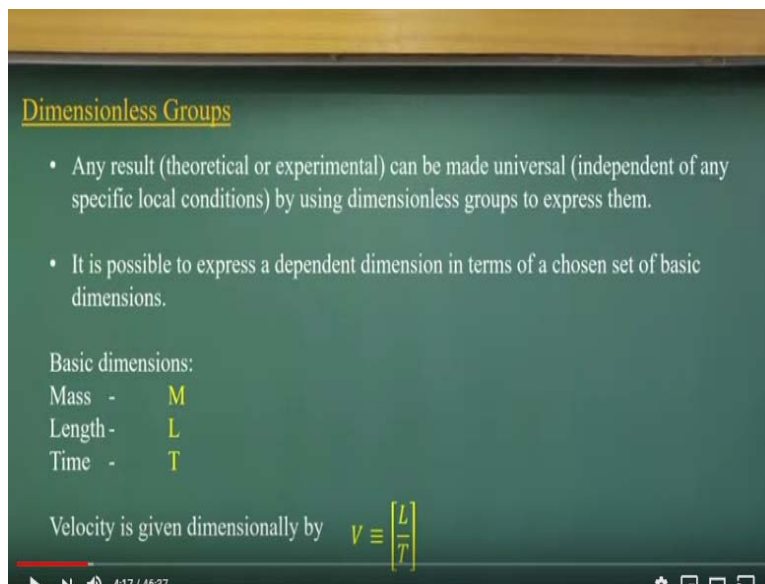
$p$  - Surface Pressure

$p_0$  - Free Stream Static Pressure

## U – Free Stream Air Velocity

So these type of experiments this is a possible now and we also have the facilities in department of Kinetic energy to visualize that how the drag force are hafting it one case I showed you the wind tunnel and other so case I have so neat that how you can measure the drag force measurement in IIT Guwahati set up what we have where you can measure the drag force okay as you know drag force is a very preliminary data for any experimental designing. So drag force we use for these case we are just talking about drag force for a cylinder.

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Dimensionless Groups

- Any result (theoretical or experimental) can be made universal (independent of any specific local conditions) by using dimensionless groups to express them.
- It is possible to express a dependent dimension in terms of a chosen set of basic dimensions.

Basic dimensions:

Mass	-	M
Length	-	L
Time	-	T

Velocity is given dimensionally by  $V \equiv \left[ \frac{L}{T} \right]$

Now we will come to the dimensionless groups I think these quite you know it. Even at the class 11th or 12th levels. So basic dimensions what we have mass length and the time these are 3 basic dimensions mass, length and time. So what we have when we would do a theoretic analysis or the experiment analysis can be made universal okay that is what exactly highlighting here independent of its specific locations using a diverse group to express them.

What I am talking is that we will discuss more in these we conduct the experiment but the experiment becomes inverse now when you make it a dimensionless group analysis. Otherwise it will be a particular case studies not the experiments what were we innovation study. So we have a very basic dimensions of any variables that mass length and the time as you know it what is the

velocity? The distance or in the time? The length by time so any of the fluid flow variables we can define in terms of these 3 basic dimensions mass length and time.

Basic dimensions:

Mass - M

Length - L

Time - T

Velocity is given dimensionally by  $V \equiv \left[ \frac{L}{T} \right]$

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Dimensions of Fluid Mechanics Properties			
• Length	$M^0 L^1 T^0$	• Pressure, stress	$M^1 L^{-1} T^{-2}$
• Area	$M^0 L^2 T^0$	• Viscosity	$M^1 L^{-1} T^{-1}$
• Volume	$M^0 L^3 T^0$	• Force	$M^1 L^1 T^{-2}$
• Velocity (speed)	$M^0 L^1 T^{-1}$	• Moment flux, Torque	$M^1 L^2 T^{-2}$
• Acceleration	$M^0 L^1 T^{-2}$	• Power	$M^1 L^2 T^{-3}$
• Volume flow (Discharge)	$M^0 L^3 T^{-1}$	• Work, Energy	$M^1 L^2 T^{-2}$
• Kinematic viscosity	$M^0 L^2 T^{-1}$	• Specific weight	$M^1 L^2 T^{-2}$
• Strain rate	$M^0 L^0 T^{-1}$	• Mass flux	$M^1 L^0 T^{-1}$
		• Surface tension	$M^1 L^0 T^{-2}$
		• Density	$M^1 L^{-3} T^0$

Now if you look at the fluid properties what we have some of the fluid properties if you know it is related to the dimensions okay that is the length area and the volume. So it is just a dimension and geometric dimensions so it has only unit in terms of the length okay. So that means length is  $L^1$   $L^2$  and area will be  $L^2$  the volume will be  $L^3$  so you can understand what is the velocity you know it length power unit time.

The acceleration similar way you can do it let me look at what is the distance or volumetric rate volume or unit time that is what we  $L^3$  divide by  $T$  That will be the volumetric then now we have come to the 2 different parts one is Kinematic viscosity as we discuss in newtons laws of viscosity is that it is independent of mass. So you can understand it has a dimensions of length and time.

Similar way we see a Strain rate also indifferent of mass and length. So if you look at these properties all are independent to mass. First I start from length area, volume then fluid properties like velocity, acceleration volumetric rate or the discharge kinematic viscosity and the strain rate. Now if you look at other part like pressures force per unit area similarly case force per unit area. So force you know it that will be the mass into acceleration the force will be mass into acceleration the pressure and stress will be the force per unit area.

That is what you can compute it. Similar way you can compute the viscosity dimensions if you know very basic equation of Newtons law of viscosity. Just Shear stress rate is proportional to Shear strain rate proportionality is the viscosity. So you can compute it what could be the dimensions of the viscosity. Similar way let us come to next levels with the momentum flux other torque that what we will have this unit the power, work and energy you can find out what will be the unit then I will be coming to the other 2 properties is specific weight and Mass flux.

Mass per unit time that is what is mass flux similar way the surface tension you can compute what will be the surface tension so that one is force per unit length. That is if we look at that force per unit length is really come into its surface tension. The density mass per unit volume so if you look it if you know very basic definitions of these fluid properties you can easily write these properties in terms of dimensions.

In terms of mass length and the time so it is very easy if are having any doubt over that you take a equations like you have a doubt about what could be the dimensions of viscosity then you will write newtons law of viscosities that is what  $\tau$  shear stress is equal to  $\mu dv/dy$  substitute the shear stress what is the unit  $dv/dy$  what is the unit then you will get the  $\mu$  because this is the dimensionless homogeneous equation.

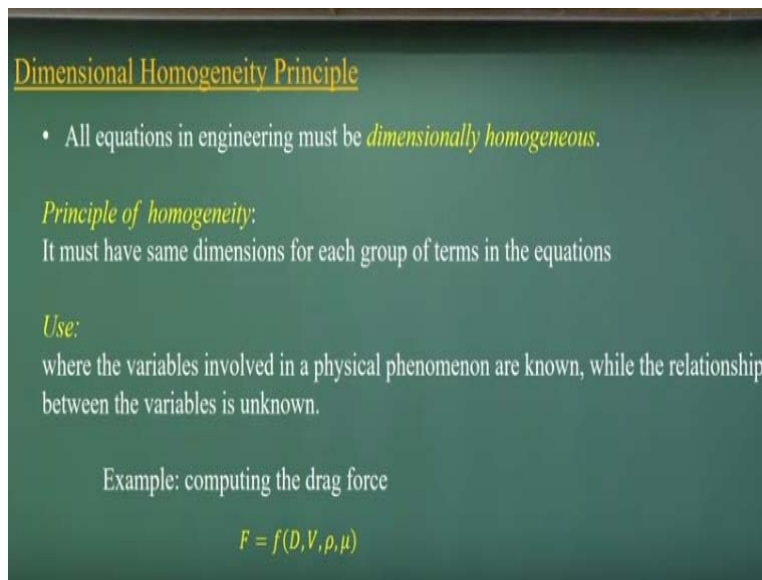
So what I am to talk about that if you are just forgot that what could be the dimensions of a particular fluid properties try to remember a dimensionless homogeneous equations with that properties are involved if you can get it that and substitute the dimensions of the properties like here this shear stress the velocity gradient and the  $\mu$  value then you can get it what will be the

dimensions of  $\mu$  and you will know what is the relationship between the kinematic viscosity and the viscosity or the dynamic viscosity.

So if you know the basic relationship between the fluid properties also we can compute what will be the dimensions of the fluid properties. So whenever you have the problems first you write the dimensions of the fluid properties and if you know the dimensions of fluid properties then you can easily find out what could be its unit okay in terms of kg per meter cube what is the unit of that what you can make it mass will be in kg length will be in meter or centimetres.

And the time will be in second, hour, day we can have a different timeframe. So it is a very easy task to make it a dimensions of the fluid mechanics properties.

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Dimensional Homogeneity Principle

- All equations in engineering must be *dimensionally homogeneous*.

*Principle of homogeneity:*  
It must have same dimensions for each group of terms in the equations

*Use:*  
where the variables involved in a physical phenomenon are known, while the relationship between the variables is unknown.

Example: computing the drag force

$$F = f(D, V, \rho, \mu)$$

Let us go for what is the principal of homogeneity all the equations not all the equations so most of the questions in engineering the most of the equations of engineering dimensionally homogenous not all that is the let me have a repeat these things that means what it indicates as that the dimensions of the equations will be the same okay the left side of dimensions LHS should have a dimensions of right hand side.

Then the equations are dimensionally homogeneous so that means what do we have to look at that for any physical political properties is and all so somewhere it follows this dimensional

homogeneous concept. Those are concept we have used it for designing the experiment for example if I compute a drag force like we have the experiment set up what I showed it that I have a cylinders I am making a velocity  $V$  and I try to know it what will be the drag force.

Okay if it is that kind of conditions if that is the condition if is the velocity  $V$  these the diameters are the cylinders and the drag force  $F_D$ .

$$F = f(D, V, \rho, \mu)$$

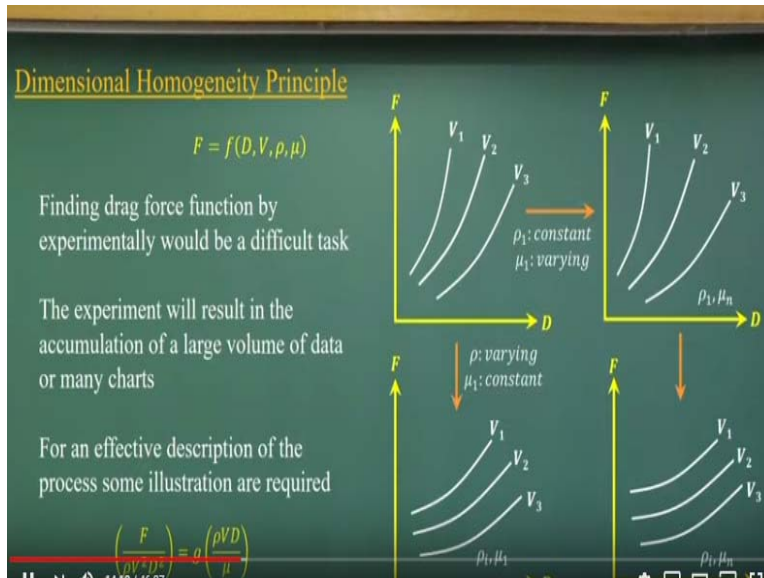
I am looking it so other 2 properties also matters it  $\rho$  and the  $\mu$  is the viscosity. So it depends upon the viscosity that means it depends upon whether you have an air or the water so it density varies the viscosity varies. So the drag force what is happening is it is a function of  $D$  is the diameter of the cylinder  $V$  is the velocity of the flow.

And the  $\rho$  and the  $\mu$  is the fluid properties related to density and the dynamic viscosities the viscous force components it depends upon the like you can know it the drag force court in oil will be the different compared to drag force in air. So that with the  $\mu$  will take care of what type of drag force will be there  $D$  also depends if a bigger diameter or smaller diameter  $D$  also matters.

So the drag force is a function which we do not know it okay  $D$  is the functions of the diameters  $V$  velocity of the flow and the density and dynamic viscosity if we start now I have to design the experiment to finding out this  $F$  component how do we do it there is 2 way to do.

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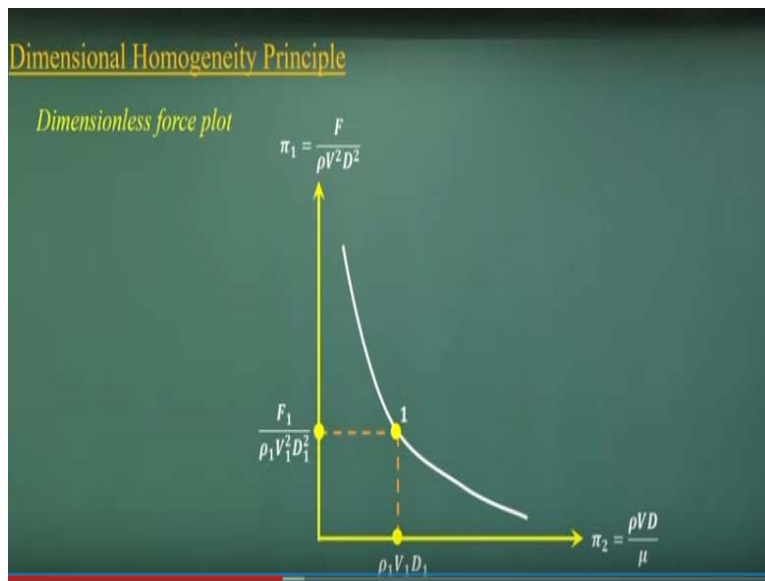
First is that I know it what I can do it since  $F$  is a function we get 4 independent variables I will do a 4 type of experiments that means when I am conducting an experiment between changing the  $D$  and different  $V_1, V_2, V_3$ . I have a making a constants of  $\rho$  and  $\mu$ . Similar way I will change the other parameters keeping other parameters constants. See if I am trying to do that because how to try to get a functions on that I will give it 2 parameters fixed and I will run for a series of experiment.

Let be a 10 experiment I will do it to get a curve if that is the conditions that means for the 3 independent variables I will do it 10 into 10 into 10 into 10 that is what will come out to be the 1000 experiment. So if it is a 1000 experiment then it is too expensive for us but what we have to look at that? Is there any some sort of dimensions relationship is there between this independent variables and the dependent variable if they have the dimensional relation and that relationship you can get it we need not to do this 1000 experiments.

We can do less number of experiment that means you can do a 10 experiment if you do a dimensional analysis to design these experiments you just do a 10 experiment to complete this process what we have to do it we make a non-dimensional curve. That  $\frac{F}{\rho V^2 D^2}$  if you just substitute the dimensions you will see that it does not have a dimension of this  $g \left( \frac{\rho V D}{\mu} \right)$  also have a does not have a dimensions.

$$\left(\frac{F}{\rho V^2 D^2}\right) = g\left(\frac{\rho V D}{\mu}\right)$$

You just substitute it and try to find out  $g\left(\frac{\rho V D}{\mu}\right)$  which is very well known equations which we call the Reynolds equations. So okay Reynolds numbers okay we will discuss more about that in pipe flow and so if we look at that what I developed now I instead of F functions we are looking at G functions which is a really sensitive tool non-dimensional group. One is left side and the right side this would have the same and I will be looking at different functions if it that did the conditions  
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I should conduct a relationship of experiment of different Reynolds numbers and can get it from the experiment but we need the force and non dimensionless as a  $F_1$  functions and void heat wave functions getting these. So only 10 experiment are enough for this study as compared to conducting the 1000 experiment. So the non dimensionless experiments making us that we can make a dimensionless force plot.

And we can conduct the experiment with the different Reynolds numbers and plot because once you do with different Reynolds experiment you will get the course you know the velocity you move the diameters of the cylinders then you can compute it what will be the non dimensionless force component here and if you get this curve then it is easy to find out at different Reynolds numbers what will in the force.

So one is that these dimensionless analysis is saves us though doing a large number of experiments second is that when you put a dimensionless force it easy to interpret to the data as compared to the doing the individual basis. So the dimension analysis plot and this analysis help us to save the large number of experiments as well as this it help us to interpret it data better way as compared to do individuals.

$$\pi_1 = \frac{F}{\rho V^2 D^2}$$

$$\pi_2 = \frac{\rho V D}{\mu}$$

Almost all the time do any experimentalists they start designing this experiment using dimensional analysis. It is not only the fluid mechanics economic models socio economic models you will talk about any experiment what you conduct it we always do a dimensional analysis to find out the how many experiments we should conduct.

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**Buckingham's  $\pi$ -theorem**

The number of independent dimensionless groups

Used to describe a phenomenon

Which involve  $n$  variables =  $n - r$  numbers ( $r$ , no of basic dimensions needed)

**Example:**

Drag force on a sphere in a fluid stream

Variables:  $F, V, D, \mu, \text{ and } \rho$  ( $n = 5$ )

Basic dimensions:  $M, L, \text{ and } T$  ( $r = 3$ )

} =  $5 - 3 = 2$  number independent dimensionless groups

Mow let us commit how to do that is Buckingham's  $\pi$  –theorem it is a very simple  $\pi$  –theorem concept is that so we will have a number of independent dimensionless groups so how many number of independent dimensionless groups I look at it if there is a n dependent variables okay we know it basic dimensions are 3 mass length and the time so that means we can group it a non

dimensionless number will be the  $n-r$  please remember as you will know it the dimensions will be 3 but not always be 3 okay?

That is again I highlighted right I need to have a judgment for that if you know that the particular defendant variables have it its having these many of dependent variables if you can visualize that what are the force component? What about the variables are going to play for that components then you will do a non-dimensional analysis to find out the real relationship between dependent and independent variable?

Now let us come back to that same problems if a drag force on a sphere okay let us see easy now okay same concept in a fluid streams the same problems okay. You will have a sphere here and you have the flow is coming here like a cricket balls okay? You are throwing the cricket balls okay and you would try to find out that what could we the drag force okay what could be the drag force is acting on that.

What will be the drag force acting in that so if that is the conditions I can visualize that it has a dependency of velocity the dimensions of this the diameter of the sphere the fluid properties like  $\mu$  and the  $\rho$ . So the total number of dependent and independent variables are 5 basic dimensions are 3. So we can make 2 number of independent dimensionless groups is correct its very easy things that we have to find out the defendant variables and the indifferent variables count that and we know the basic dimensions are the 3 mass length and the time and we just subtract it and find out that we can have a 2 number of independent dimensional group.

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## Buckingham's $\pi$ -theorem

### Example 1

Develop the independent dimensionless group using Buckingham's  $\pi$ -theorem

from  $F = f(L, V, \rho, \mu)$

Independent dimensionless groups:

Variables:  $L, V, \rho, \mu,$  and  $F$  ( $n = 5$ )  
Basic Dimensions:  $M, L$  and  $T$  ( $r = 3$ )  
Repeating Variables:  $L, V$  and  $\rho$  ( $j = 3$ )

} =  $n - j = 5 - 3 = 2$  number independent dimensionless groups from pi theorem

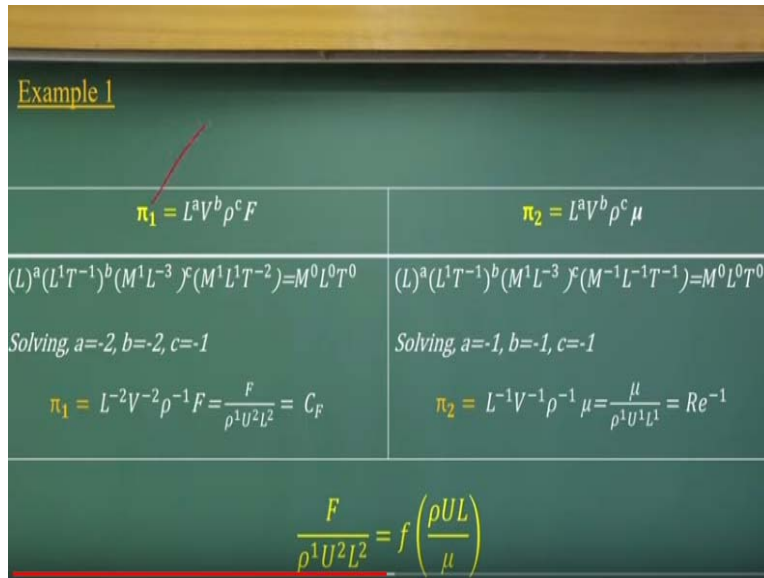
Dimensions of each variable:

$F$	$L$	$V$	$\rho$	$\mu$
$M^1 L^1 T^{-2}$	$L^1$	$L^1 T^{-1}$	$M^1 L^{-3}$	$M^1 L^{-1} T^{-1}$

Now I have to develop the dimensional groups of that so if you look at that way first what I have to do it I have to write the dimensions of each force, Mass into excellence the length the velocity density and dynamic viscosity. If you do not remember that dimensionless viscosity the dimensionless please remember newton's law of viscosities okay? Anyone you have to remember it okay so we put do not remember Newton's law of dynamic viscosity please remember what newton's laws of viscosity.

If you remember it, you can originally compute it what could be the mu dimensions that is what always you can prove any of the exam that is I want to give you that in order to remember all the dimensions of the fluid properties but you should know the Basic equations some of the basic equations you can easily find out what could be the dimensions of that depending on independent variable.

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Now to look at that this is called a  $\pi$ -theorem it is a very easy concept that though please do not have a conclusion with the  $\pi$  means 3.14 it is a non-dimensional form of  $p$  is okay. So if it is that I can have a 3 variables okay that is the length velocity and the  $\pi$  and the force these are called the repeating variables length velocity and viscosity I will put the repeating variables and with the  $F$ .

Similar way the length velocity  $\rho$  will be the repeating variability and then I substituting the  $\mu$  okay I do not know what power  $x$  point of length, velocity and the  $\rho$  will make it a non-dimensional form of  $\pi_1$  or I do not know what is the power experiment could be there to make a non-dimensional form of length, velocity density and the dynamic viscosity to make a  $\pi_2$  will be non-dimensional.

$$\pi_1 = L^a V^b \rho^c F$$

$$(L)^a (L^1 T^{-1})^b (M^1 L^{-3})^c (M^1 L^1 T^{-2}) = M^0 L^0 T^0$$

Solving,  $a=-2, b=-2, c=-1$

$$\pi_1 = L^{-2} V^{-2} \rho^{-1} F = \frac{F}{\rho^1 U^2 L^2} = C_F$$

Then I will take  $\pi_2$  to this which is a reciprocal of Reynolds number okay that is equal to the Reynolds number. That means one of the  $\pi_1$  is the functions of a  $\pi_2$  that is what you have written it only we are just the reciprocal of Reynolds numbers you make it here another Reynolds number format. So this is what our non-dimensional form this is what is our non-dimensional form.

$$\pi_2 = L^a V^b \rho^c \mu$$

$$(L)^a (L^1 T^{-1})^b (M^1 L^{-3})^c (M^{-1} L^{-1} T^{-1}) = M^0 L^0 T^0$$

Solving,  $a=-1, b=-1, c=-1$

$$\pi_2 = L^{-1} V^{-1} \rho^{-1} \mu = \frac{\mu}{\rho^1 U^1 L^1} = Re^{-1}$$

$$\frac{F}{\rho^1 U^2 L^2} = f\left(\frac{\rho U L}{\mu}\right)$$

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Dimensionless groups in fluid dynamics

The following variables may be essential for the complete description of most of the fluid flow phenomena.

- Pressure
- Length
- Viscosity
- Surface tension
- Speed of sound
- Acceleration due to gravity
- Density
- Velocity

p  
L  
μ  
σ  
α  
g  
ρ  
V

Three basic dimensions are required to describe above eight variables, so  $(8-3) = 5$  independent dimensionless groups can be formed with these variables.

So similar way let go for what are the variable in general in fluid flow we will get it. As I told it earlier that when you talk about fluid flow problems the mostly we talk about the velocity and the pressure field okay that is the basic things velocity and the pressure field. Then when you talk about that you know it there would be gravity force which will really taking care by acceleration due to gravity.

The length dimensions will be there then comes out to be mass functions will be the density will be within the mass the viscosity will define the flow resistance. So 2 is what is very important is called the surface tensions when you have an interface of 2 liquids 2 fluids you will have a surface tension. Same way as we discussed many of the times the speed of sound also matters for us to know it flow numbers or incompressibility of the flow or compressibility of the slow.

So all to tell if we look it any fluid flow problems okay can have all the dimensions or some of this that means pressures, length, viscosity, surface tension speed of the sound, acceleration due to gravity then Density and the velocity these are the prime variables okay. See if you look at how many are there? 8 variables 3 basic variables. So that all these 8 variables can we make a 5 independent dimensional groups that is the concept of the fluid mechanics okay.

So any most of the fluid flow problems okay. Now you do in a very extensive different way but most of the fluid flow problems we can define with these 8 variables that means we can group them into a 5 independent dimensional groups so what I did.

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Dimensionless groups in fluid dynamics

Some of the dimensionless groups formed from previous mentioned variables of fluid phenomena are:

- Reynolds number
 
$$R_e = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho VL}{\mu}$$
- Froude number
 
$$F_r = \frac{\text{Inertia force}}{\text{Gravity force}} = \frac{V}{\sqrt{Lg}}$$

(GATE 2013 CE)

Now i can see the show you figures I do believe it these figures will talk many things what I may not express with these so-called limited times time-space. If you look at the things what it happens that when we have fluid flows in different force acts and the different force dominate at different conditions and when you do the fluid flow problems, we should point out which one is dominated.

And you would try to solve for the dominated case so that is the way we look in the fluid flow problems to solve to find out or to judge which is the forces are dominating it like for examples when you have a laminar turbulent proof like this case okay you could see the laminar turbulent what i discussed in the first class okay by showing the what is exciting and all. So these are what?



These 2 forces are dominated here one is inertia forces the momentum blocks and another easy viscous component.

These 2 forces are dominant and those 2 forces the ratio is Reynolds number. You can compute this

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho VL}{\mu}$$

I am just leaving you to just do that exercise otherwise we will discuss when your assignment mode okay you can compute it that the inertia force by viscous force is a Reynolds numbers. That is what we can do it using virtual fluid concept also. So let us have that its a ratio between a fluid flow problems where inertia forces versus viscous force that one the  $\frac{\rho VL}{\mu}$  is defined as the Reynolds numbers.

So often you use the Reynolds numbers because though mostly we consider the inertia force by viscous force dominancy else you can see the vortex on the hills okay that is what we discuss a lot how the vortex formation happens in and very smoke just smoke cloud from mouth or you can have the formation of vortex settings all it depends upon what Reynolds numbers of the flow we have.

In a similar way if you go for the next numbers it is called Froude number it depends on the fluid flow where your gravity force also have a dominance as compared to the viscous force. Like for examples it will talk about this spill way here the viscous force are not that dominant the gravity force works dominancy levels. So we can have a ratio between inertia force by gravity force that is what is flow Froude numbers that is what we define it that is what is defined by this as non-dimensional.

So if you look it this is the velocity length into g of square root also the unit dimensions of the velocity. So it becomes non-dimensional so how to get its this component I think in this basic lectures I cannot complete that wave celerity concept here but I will tell it in higher classes if you go for postgraduates and all or the open channel flow definitely you know it how this  $\sqrt{Lg}$  comes a gravity force components.


The same process its happens wave cloud pattern also it happens n wave pattern generated by a small fluid you can look at this because its very interesting figures you can see this all sort of the flow dominancy. The top lines are the flow dominancy of inertia force that is what it drive the fluid versus the viscous force in this case the inertia force and the gravity force. I just repeat you please remember these 2 numbers because this is a very common numbers are called Reynolds numbers and Froude problem number.

$$Fr = \frac{\text{Inertia force}}{\text{Gravity force}} = \frac{V}{\sqrt{Lg}}$$

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Dimensionless groups in fluid dynamics

- Weber number


$$W = \frac{\text{Inertia force}}{\text{Surface tension}} = \frac{\rho V^2 L}{\sigma}$$


Bubble forming

- Euler (cavitation) number

$$E_u = \frac{\text{Pressure force}}{\text{Inertia force}} = \frac{-\Delta p}{\rho V^2}$$


Cavitation forming in turbine



While bullet traveling in water

Now if you look at other 2 numbers we not many times we use it but nowadays the level of the fluid mechanics problems what we are solving it we look at these formulas. One is called where the surface tension dominancy is there. For examples you can look at this bubble formation all of this because of the survey tensions. So in this case if I proposition on this bubble formation then I should look it the Weber number is represents as a ratio between inertia force by surface tension.

That is the component what we are getting but you know there is a cavitation surface when they present at certain levels then what are the fluid forms which we discussed in the BC classes what we taught it in a lecture 1 lecture 2. Okay so what did happen in that case? We try to look at how

much pressure drops which generate us the cavitation process. Pressure drops the change in the pressure if the more the change in the pressure there is a likelihood to have a cavitation force divide by this inertia forces is defined by the Euler numbers or cavitation.

$$W = \frac{\text{Inertia force}}{\text{Surface tension}} = \frac{\rho V^2 L}{\sigma}$$

We remember it in these Euler numbers inertia force is in the below okay as compared to the other component. So we have a 4 numbers with us the Reynolds numbers, Froude numbers, Weber numbers and Euler number these cavitation which happens when your turbine moving it we can see that there is a chance to have the cavitation formation near the turbine okay. So but there are the conditions where the both the things are dominance that dominates like a surface tension and the cavitation which is happens it is very interesting stuff.

$$Eu = \frac{\text{Pressure force}}{\text{Inertia force}} = \frac{-\Delta p}{\rho V^2}$$

Like that if a bullet traveling in waters it surface tensions as well as the pressure drops creates the cavitation process. So we can see this the bubble formations and all these formations are also collapsing of the bubbles as the bullet travels it which because the inertia force is the dominance is there. Surface tension dominance is there also there is a dominance of the pressure force drops which generates the cavitation process.

That is the way, there are the water vapour formation are there and that workforce are collapsing it is that process if you look at the very interesting process what is happening it just firing a bullet in water that means to the process are happening in say dominance with respect to the inertia forces we have to do dominance f that.

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## Dimensionless groups in fluid dynamics

- Mach number

$$M = \frac{\text{Flow speed}}{\text{Sound speed}} = \frac{V}{a}$$



Boom flight



Mach number: 0.785



Mach number: 3.35

- Drag coefficient

$$C_d = \frac{\text{Drag force}}{\text{Dynamic force}} = \frac{D}{\frac{1}{2} \rho V^2 A}$$



Similarly, Prandtl number, Rossby number, Specific-heat ratio, Roughness ratio, Rayleigh number, Pressure Coef etc.,

$$M = \frac{\text{Flow speed}}{\text{Sound speed}} = \frac{V}{a}$$

Now you can look in there another 2 components which is a Mach number flow speed by the sound speed okay this numbers becomes 1 when the flow speed is equal to the sound of the speed okay? If you look at the Mach number of Boeing 737flight max will be 0.785 which is the subsonic flow but if you talk about the fighter jet Sr 1 black bird the Mach number is 3.35 that means the pure velocity its much larger than the speed of this jet fighter jet is larger than the speed of the sound.

You can understand it but where but in this is not the case in case of your domestic flight but the fighter flight knows that. So you need a different technology and all the things if you are really interested just google it and get it what the appropriate for the fighter jet and what is appropriate for the Boeing 737. So these are the 2 different Mach numbers and the low pattern changes it the compressibility changes it in both the cases the compression matters because when Mach number more than 0.3 we talk about the compressibility.

So both the cases we have the low compressibility so that way we can know it that with respect to Mach numbers the flow speed and the sound speeds okay with aerodynamics and more advanced level you will know it. So you can move it but is the range of style and in other words you know it as we discuss it the drag coefficient which is,

$$C_D = \frac{\text{Drag force}}{\text{Dynamic force}} = \frac{D}{\frac{1}{2}\rho V^2 A}$$

and just substitute the dimensions and you can find out is a force component when you have a  $\rho V^2 A$ .

Why does the half is there please you think it so if you have a drag force we say earlier that today technology has developed and all we do the modelling at the full scale levels the prototype levels if you at these things and with a smoke or a jet exactly streamlined patterns how it happened at different velocity and the drag force component? So this is what its designed for speed to moving more than 200 kilometre per hours.

That is the reason even if there is a cyclone we cannot have a rolling but what happened to the bus? It has not designed for the 200 kilometre per hours. So that is the reason so you can see them rolling of the bus in the cyclone situations but that will not happen in case of well-designed car what do you have because these are all designed for high speeds. The design velocity itself is close to 200 kilometre per hour.

So we will not have a rolling conditions what we had seen it bus is ruling conditions when you have it the cyclones that is what we mean out there because it is a well-tested at the prototype levels it is not in the model level it is a prototype levels and to find out what is the drag force mesh under drag force it has designed all the components with the width should be there where the lightweight will be there.

Whether how many passengers are sitting inside or not sitting inside all the condition they tested it and to find out the best the safe which even an extreme design conditions of the speed of relatively 200 kilometre per hours. Still this will be the not going to be ruling conditions what we saw the ruling of a bus in cyclone Fani.

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## Buckingham's $\pi$ -theorem

### Example 2

Develop the independent dimensionless group using Buckingham's  $\pi$ -theorem

$$\text{from } Q = f\left(R, \mu, \frac{dp}{dx}\right)$$

Independent dimensionless groups:

Variables:	$R, \mu, \frac{dp}{dx}$ and $Q$ ( $n = 4$ )	}	= $n - j = 4 - 3 = 1$ number independent dimensionless groups from pi theorem
Basic Dimensions:	$M, L$ and $T$ ( $r = 3$ )		
Repeating Variables:	$R, \mu$ and $\frac{dp}{dx}$ ( $j = 3$ )		

Dimensions of each variable:

$Q$	$R$	$\mu$	$\frac{dp}{dx}$
$L^3 T^{-1}$	$L$	$M L^{-1} T^{-1}$	$M L^{-2} T^{-2}$

So for let us coming to the closure to this before that let us have it 2 or 3 examples which are very easy things I will not repeat much things that let us have a find it an independent of dimensional group using there is a Q which is a volumetric discharge is a function of r radius dynamic viscosity and the gradient of the pressure okay.

$$Q = f\left(R, \mu, \frac{dp}{dx}\right)$$

We have n 4, j we know the 3 mass ,length and time because all these things as soon as you write the dimensional discharge R,  $\mu$ , and dp/dx.

Variables:  $R, \mu, \frac{dp}{dx}$  and  $Q$  ( $n = 4$ )

Basic Dimensions:  $M, L$  and  $T$  ( $r = 3$ )

Repeating Variables:  $R, \mu$  and  $\frac{dp}{dx}$  ( $j = 3$ )

Remember this is a dp/dx that means find the pressure, dimension divide by the length dimensions you will get a dp/dx decision. So once you know this time instance as you know that it will form a 1 independent dimensional group.

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### Example 2

$$\pi_1 = R^a \mu^b \frac{dp}{dx}^c Q$$

$$(L)^a (M^1 L^{-1} T^{-1})^b (M^1 L^{-2} T^{-2})^c (L^3 T^{-1}) = M^0 L^0 T^0$$

Solving,  $a=-4$ ,  $b=1$   $c=-1$

$$\pi_1 = R^{-4} \mu^1 \frac{dp}{dx}^{-1} Q = \frac{Q \mu^1}{R^{-4} \frac{dp}{dx}} = \text{Constant}$$

$$\frac{Q \mu^1}{R^{-4} \frac{dp}{dx}} = \text{Constant}$$

We will follow the same concept I am not going to repeat it here again you put a substitutes of power x point of abc substitute this dimensional values and equate with non-dimensional value you will get abc value and finally we will get a the relationship between the 4 which will make a non-dimensional group it is a very example simple things but first thing is that we should remember the dimensions of the fluid variables.

$$\pi_1 = R^a \mu^b \frac{dp}{dx}^c Q$$

$$(L)^a (M^1 L^{-1} T^{-1})^b (M^1 L^{-2} T^{-2})^c (L^3 T^{-1}) = M^0 L^0 T^0$$

Solving,  $a=-4$ ,  $b=1$   $c=-1$

$$\pi_1 = R^{-4} \mu^1 \frac{dp}{dx}^{-1} Q = \frac{Q \mu^1}{R^{-4} \frac{dp}{dx}} = \text{Constant}$$

$$\frac{Q \mu^1}{R^{-4} \frac{dp}{dx}} = \text{Constant}$$

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## Buckingham's $\pi$ -theorem

### Example 3

Develop the independent dimensionless group using Buckingham's  $\pi$ -theorem from  $\delta = f(P, L, I, E)$

Independent dimensionless groups:

Variables:  $P, L, I, E$  and  $\delta$  ( $n = 5$ )  
Basic Dimensions:  $M, L$  and  $T$  ( $r = 3$ )  
Repeating Variables:  $L$  and  $E$  ( $j = 2$ )

} =  $n - j = 5 - 2 = 3$  number independent dimensionless groups from pi theorem

Dimensions of each variable:

$\delta$	$P$	$L$	$I$	$E$
$L^1$	$M^1 L^1 T^{-2}$	$L^1$	$L^1$	$M^1 L^{-1} T^{-2}$

And this is another example 3 is where what the basic differences here is that we have a dimensional groups we have to make it with a conditions this is not Froude problems we have a variable which is E value is there and you have a P you have length you have a deflection something like you have a deflection. So you have a length dimensions you have pressure the dimensions you have length I and you have an even.

In this case what you have to look at that when you have this type of 2 conditions where you have a having this mass so we cannot make a 3 variable combine it to make a non-dimensional only the 2 variable is considered to make a non-dimensional problem. So our instead of 3 we have used 2 because 3 variable we cannot combine it to make a non-dimensional problem. So because only these 2 variables here having a dimensions where the mass is there. So in that case you will consider  $j =$  or  $r=2$  so you make a 3 independent dimensional groups from a pi theorem.

Variables:  $P, L, I, E$  and  $\delta$  ( $n = 5$ )

Basic Dimensions:  $M, L$  and  $T$  ( $r = 3$ )

Repeating Variables:  $L$  and  $E$  ( $j = 2$ )

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Example 3		
$\pi_1 = L^a E^b I$	$\pi_2 = L^a E^b P$	$\pi_3 = L^a E^b \delta$
$(L)^a (M^1 L^{-1} T^{-2})^b (L^4) = M^0 L^0 T^0$	$(L)^a (M^1 L^{-1} T^{-2})^b (M^1 L^1 T^{-2}) = M^0 L^0 T^0$	$(L)^a (M^1 L^{-1} T^{-2})^b (L^1) = M^0 L^0 T^0$
Solving, $a=-4, b=0$	Solving, $a=-2, b=-1$	Solving, $a=-1, b=0$
$\pi_1 = L^{-4} E^0 I^1 = \frac{I}{L^4}$	$\pi_2 = L^{-2} E^{-1} P^1 \mu = \frac{P}{E^1 L^2}$	$\pi_3 = L^{-1} E^0 \delta^1 = \frac{\delta}{L}$
$\frac{\delta}{L} = f\left(\frac{P}{E L^2}, \frac{I}{L^4}\right)$		

That is what the difference now you can look at these tables we look at the  $\pi_1 \pi_2 \pi_3$  we take 2 repeating variables okay and 3 non-repeating variable substitutes all these panels solving it getting the values  $E^b$  finally we get a relationship is that. So if you are good in a solid mechanics you can interpret it what they are okay it is so very easy to interpret it this part because you can know it what they are okay it is not very difficult.

$$\pi_1 = L^a E^b I$$

$$(L)^a (M^1 L^{-1} T^{-2})^b (L^4) = M^0 L^0 T^0$$

Solving,  $a=-4, b=0$

$$\pi_1 = L^{-4} E^0 I^1 = \frac{I}{L^4}$$

$$\pi_2 = L^a E^b P$$

$$(L)^a (M^1 L^{-1} T^{-2})^b (M^1 L^1 T^{-2}) = M^0 L^0 T^0$$

Solving,  $a=-2, b=-1$

$$\pi_2 = L^{-2} E^{-1} P^1 \mu = \frac{P}{E^1 L^2}$$

$$\pi_3 = L^a E^b \delta$$

$$(L)^a (M^1 L^{-1} T^{-2})^b (L^1) = M^0 L^0 T^0$$

Solving,  $a=-1, b=0$

$$\pi_3 = L^{-1} E^0 \delta^1 = \frac{\delta}{L}$$


$$\frac{\delta}{L} = f\left(\frac{P}{EL^2}, \frac{I}{L^4}\right)$$


So this is moment of inertia okay that you can interpret it what these are the deflections values and all so I am trying to do is that the dimensional analysis would not do for fluid mechanics any experiment we do these dimensional analysis to know the relationship between dependant variables and the independent variable in terms of dimensionless formats.

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### Wright Brothers Flight

- The Wright Brothers developed world's first successful airplanes like **Wright Flyer** and **Wright Flyer III**.





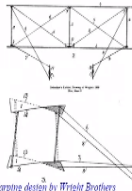


**Wilbur Wright**  
1867–1912



**Orville Wright**  
1871–1948

- The Wright Brothers closely followed bird's wings for balance and control and successfully developed "wing warping".

- Wilbur flew their first plane for 59 seconds at a height of 852 feet in 1903.

Sources:  
[http://www.nasa.gov/images/content/100000main\\_1903flyer.jpg](http://www.nasa.gov/images/content/100000main_1903flyer.jpg)  
[http://www.nasa.gov/images/content/100000main\\_1903flyer.jpg](http://www.nasa.gov/images/content/100000main_1903flyer.jpg)  
[http://www.nasa.gov/images/content/100000main\\_1903flyer.jpg](http://www.nasa.gov/images/content/100000main_1903flyer.jpg)  
[http://www.nasa.gov/images/content/100000main\\_1903flyer.jpg](http://www.nasa.gov/images/content/100000main_1903flyer.jpg)

Wing Warping design by Wright Brothers

So with this let us have a the closing this lectures before this I think today what do we have advantage to fly from one place to other place by aircraft? It is possible because of these 2 really adventurous people in the world in early 19th century they tried to do it something just looking the nature they had an if you are really interested to know this you just put it the Wright brothers google it and you can know it what way they struggled their life how many failures they faced it.

Before get a succeed to have it what they have succeeded their first plane they succeeded with a 59 seconds in the air at the height of 852 feet okay and because of their success okay now we are having the aircrafts and all because people the human being felt it was easy for now to fly it but what they did it how did they success?

These are not that well educate on this but they are adventurous spirit a spirit they wish to look they had a series of the failures they left their country came to other country and in that period I

will tell it that the Europe universities was were looking to develop something like this top professors are in Germany, Europe all these countries people were trying to look it. But nobody has this type of adventurous what they have that.

So what do you like to tell to these when you do the experiment you do accept the failures? But all the failures sometimes make a success to it. That is what these people proved it and all these failures will give us a opportunity to think why do we have the failures. One thing what they did it just they follow the birds wing for balance and the control which is necessary to fly. Just looking at this bird.

If you look at that it is called wing warping that is what they have designed here which nobody in the university professor could do that thing and that what is the spirit adventurous spirit makes the world in different forms. The basic knowledge because you try you feel it you learn it and that learning spirit makes the things which is something what nobody has invented So with great salute to these 2 brothers and someone who has a very so with this we conclude this lectures and you know it what is the dimension.

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### Summary of the Lecture

1. **Dimensions of Fluid Mechanics Properties**
  - Viscosity  $M^1L^{-1}T^{-1}$
  - Pressure, stress  $M^1L^{-1}T^{-2}$
  - Surface tension  $M^1L^0T^{-2}$
  - Force  $M^1L^1T^{-2}$
2. **Dimensional Homogeneity Principle**
  - All equations in the engineering must be dimensionally homogeneous
3. **Buckingham's Pi Theorem and Examples**
4. **Dimensional Groups in Fluid Mechanics**
  - Reynold's Number, Froude Number, Weber Number, Euler Number and Mach Number

- Viscosity  $M^1L^{-1}T^{-1}$
- Pressure, stress  $M^1L^{-1}T^{-2}$
- Surface tension  $M^1L^0T^{-2}$
- Force  $M^1L^1T^{-2}$

What is the dimension homogeneity fluid flow and what is the dimensional groups in the fluid mechanics will repeat these things in the next class with tis let us I think you for your attention for this?