

**Fluid Mechanics**  
**Prof. Subashisa Dutta**  
**Department of Civil Engineering**  
**Indian Institute of Technology - Guwahati**

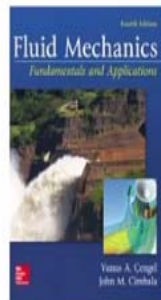
**Lecture – 17**  
**Fluid Kinematics**

Welcome you to this fluid mechanics class on to fluid kinematics, this is second lectures on fluid kinematics, this is what quite interesting lectures what today I will cover it with starting from the derivations of vorticity to; so the real time the vortex formations in Bay of Bengal which is super cyclones what is happening today. So, let us have a these lectures which started from very basics how the vorticity is there and how the fluid rotations we are talking about that.

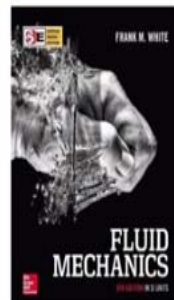
And where very larger scales is meso scale process of cyclones which can looks like a vortex process, a very real time near real time vortex formations what is available in Internet what I am going to show you, so this is quite interesting lectures for you to locate the vortex from smaller scale to the bigger scale.

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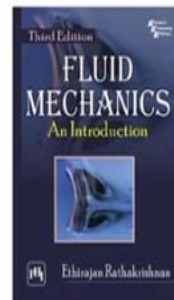
Reference Books for the Course



Yunus A. Cengel  
John M. Cimbala



Frank M. White



Eshwaraj Rathakrishnan

Today, I will cover again these 3 books and mostly I will talk about recently, we are talking about more Rathakrishnan's book and Cengel, Cimbala, so it is a good book on fluid kinematics.

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## Contents of Lecture

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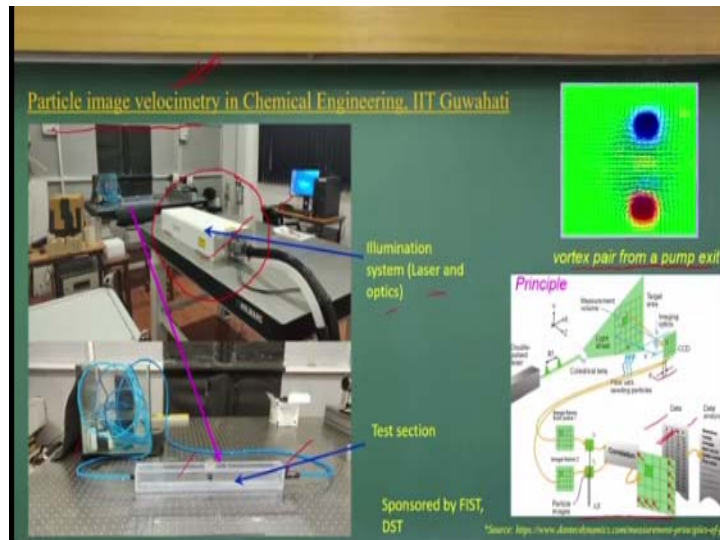
1. Particle image velocimetry in IIT Guwahati
2. Collapse of Water Column with Obstacle
3. Lagrangian and Eulerian Descriptions
4. Motion and Deformation of Fluid Element
5. Vorticity and Rotationality
5. Example problems on Velocity field application
7. Summary

Now, let us have the contents of the today lectures, I will very interestingly, I will show the experimental facilities what is there in Department of Chemical Engineering, IIT, Guwahati, particle image velocity meter which measures 3 dimensional velocity components, so when you measure the 3 dimensional velocity component, then you can understand how vortex formations happens, how the turbulence characteristics happens.

So, which is more important today's world to know it very, very micro scales the process to understanding as compared to macro scales or cross characteristic things what we did it in 20 years back or 30 years back, then again another examples from the; as a CFD solvers, the collapse of water columns with a obstacles that was very interesting will be surface flow that is what we will see it.

Then, we will go for, already I discuss about the Lagrangians or Euler descriptions, I will just touch upon that so, in these the flow patterns what we are getting it then, we will go for how the motion and deformation of the fluid elements, vorticity and rotational and some few examples best for the motion and deformations and followed by the summary.

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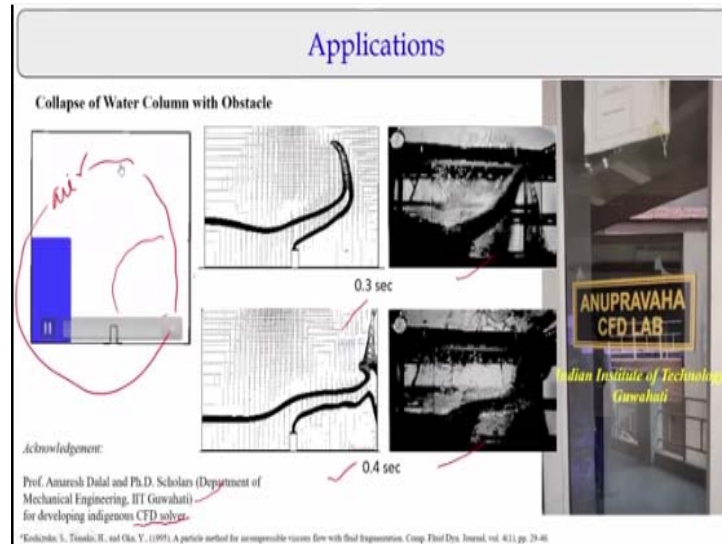
Now, if you look at the facilities what you have granted from the Department of Science and Technology come out of India, this facility is known as the particle image velocity materials, so where this is the facility we generate the laser beam, okay so and that laser beam passed through the test sections which has a 2 cameras to monitors how these the laser beams are changing it with an image processing, we can compute the 3 dimensional velocity fields.

I am not going more detail how we what is the basic principle of image; particle image velocity materials but there is instrument like a laser beams, then the test sections and the 2 cameras based on that with these principles, what we have given here to monitor the particles at different time frames, we can obtain the 3 dimensional the velocity fields, as you see it is very interesting photographs what we are coming from experimental data.

Because here you can get 3 dimensional velocity fields, if you look into that there are 2 vortex are forming just from a pump exit and the you can see these factors, you can see the vortex sheddings which is going down and 2 pair of vortex formations are there and so these quite interesting figures of vortex formations and the propagations of vortex pair that what we can go it in a very detailed way, if you have a facility like PIV or particle image velocimetry.

So, these very unique facilities that is what we have in a Department of Chemical Engineering IIT, Guwahati, so but what I am to tell it at present, we have the facilities to measure the 3 dimensional the velocity fields with very accurately, so that we can represent like this type of vortex forces what it happens it that can be possibility.

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Now, let us go to the one of very interesting again from the acknowledging see professor Dalal and students group in Department of Chemical Engineering, IIT, Guwahati who has developed indigenous CFD solvers, now if you look at this free surface, you can see that how interesting things is are happening here, you just see these collapse of water column with the obstacles.

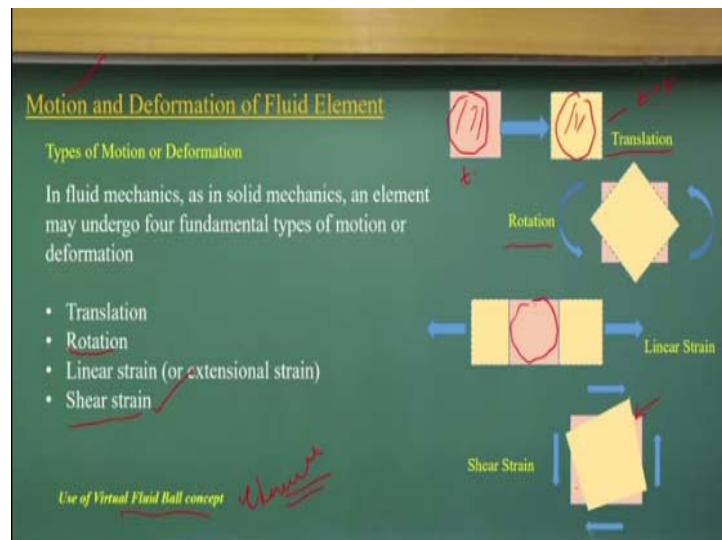
That means, you have water columns that maybe have a water tanks is collapsing it, then there is obstacles, how the flow patterns happens, if you look at these free surface is very interesting things what is happening it and this the CFD solver today's it is able to capture these so complex process if you look at how the complex processes are happening it, how the mixing of water and air is happening it.

So, if you look it at these are things, it is a really possible today's now it is not that difficult and this is what is showing with a comparison with the velocity fields and all with the experimental data sets. So, if you look at this way, there are very complex processes also we can get the streamline patterns, velocity pattern and the pressure patterns (( )) (06:59) surface variations patterns either using CFD solvers or having an experimental facility with particle image velocimetry.

So, if you have that you can get the solutions that so complex solutions also, we can get it and you can see how the pressure variability is there, how the velocity variability is there, how the density varies there and these are the problems where there is a mixing of the 2 domain problems, this is the water is there also, air is there so, how these 2 mixings are happening it

and that is what the complexity of the problems and which is (()) (07:44) say possible to do this type of the studies and which will give a very interesting thing.

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Let us come back to our basic fluid mechanics okay, so as in the last class I talked about 2 types of descriptions; one is Eulerian frame of descriptions, another is Lagrangian frame of description and that the descriptions we try to visualize as form of virtual fluid balls okay, so today I will talk about that I have the fluid element which is you know in any of fluid mechanics book.

They talk about the fluid element which is representing a certain space of the fluid particles which is much larger scale than the molecules levels or it is not that bigger scale to represent the flow process, so mostly I will talk about which is calling virtual fluid balls, okay. So, when you have a balls then it can have go through any type of motions and the differences like it can be translations like variations that you have a ball, it can be translated, it can go in a u direction or v direction.

Or it can have a displacement in a x direction, y direction, z direction and if the displacement is caused because of the velocity component, so that means if this the virtual ball, it is at the t time and this is a t plus delta t time, we will have more detail discussion how much it will travel it that depends upon the velocity at the time t and at after  $\Delta t$ , what could be the displacement which is simple things, the displacement is equal to velocity into  $\Delta t$ .

So, it is very easy concept that how much of displacement will be there, how much of translations will be there and the particle; the virtual fluid balls are the element; the fluid elements can go for a rotation that means, there is no translations motions that means, it is just go for a rotation, it is there, only it is goes for either clockwise it represents or the anti-clockwise it represents.

So, the fluid element can go for a rotation, so that means we can compute what could be this angular velocity, if it is going through a rotation; the clockwise rotations or anti clockwise rotations. So, if you try to understand that these rotations all depends upon the velocity fields, the velocity field the variations will cause the rotations of these fluid particles.

The velocity of the field also talk about how the translations motion will happen from point A to point B, as fluid particles are changing at the time  $t$  to  $t$  plus  $\Delta t$ , so all it depends upon the velocity fields, so that way we can have a translations or the rotations which is motions part, then we talk about this fluid element as in the solid, it could have the deformations, like it can have a linear deformation, like it can be elongated.

So, that means if you have given a just stress, okay or you compress it, so that means the elemental sides; the volume of the fluid element that may increase it or can decrease it that depending upon also the velocity distributions, how we have that is what will be trace the fluid elements or the compress the fluid element or if I have a virtual ball so, the basically the dimensions of balls will change.

See, if I have a virtual ball concept that means, what I am talking about that the dimensions of the ball can change it okay, if I have a give a linear strain okay, positive strain it can expand it, if I give the compressing it, then it will be reduced the size, the similar way we can have a shear strain, we can have a the shear strain to the fluid particles like, you can have a shear strains to the particle.

So, there is a difference between this rotation and the shear strains okay, you can see the graphical how they are different okay, we will discuss more. So, basically if you look at that the fluid can go through 2 type of motions; translations and rotations also, it can have a deformations of linear strains and the shear strain.

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**Motion and Deformation of Fluid Element**

**Translation**

The rate of translation vector is described mathematically as the velocity vector.

In Cartesian coordinates

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

The fluid element has moved in the positive horizontal (x) direction; thus u is positive, while v (and w) are zero.

Fundamental types of fluid element motion or deformation: translation

*Use of Virtual Fluid Ball concept*

Now, come to the translations which is very easy concept okay, which is the velocity factor which is responsible for shifting a fluid particle from A location to B locations, it depends upon the velocity components like in this case, you have a small u, v and w is a scalar velocity component in x, y, z directions respectively.

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

So, that means at the  $\Delta t$  time, after the  $\Delta t$  times, these particles which is there it can move it at a displacement of u into  $\Delta t$ , v into  $\Delta t$ , w into  $\Delta t$ .

So, that way it will move in these 3 directions to get a new positions factors, so it is a simply translations from A point to the B point with y the velocity vector components, okay that means this particles has the velocity vector component u, v, w and at the t time, t plus delta t time, it will move it with a in the x direction would be  $u\Delta t$  and  $v\Delta t$  and  $w\Delta t$  from these origins locations.

So, it is a very simple thing as you know it in solid mechanics okay, if a particles are moving with a u, v, w what could be the display positions vectors at  $t + \Delta t$  times, so it is very easy we can have this ones, okay but many of the times we see the 1 dimensional components, other components you may consider it is a 0.

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**Motion and Deformation of Fluid Element**

**Rate of Rotation (Angular Velocity)**

At a point is defined as the average rotation rate of two initially perpendicular lines that intersect at that point

Rate of rotation of fluid element about point P

$$\omega = \frac{d}{dt} \left( \frac{\alpha_a + \alpha_b}{2} \right) = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

Here, the average rotation angle is thus  $(\alpha_a + \alpha_b)/2$ , and the rate of rotation or angular velocity in the xy-plane is equal to the time derivative of this average rotation angle.

The rate of rotation vector is equal to the angular velocity vector and is expressed in Cartesian coordinates as

$$\vec{\omega} = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \mathbf{i} + \frac{1}{2} \left( \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \right) \mathbf{j} + \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \mathbf{k}$$

Now, coming to the rate of rotations, which is called angular velocity okay, what could be the angular velocity of the fluid point. See if I take it this is the fluid element okay, it will have a this the velocity variations that means, if this is the u velocity is here at this point, you will have  $u + \frac{\partial u}{\partial x} \Delta x$ , as the x direction is changes. If this is the v velocity here at this point we will have a  $v + \frac{\partial v}{\partial y} \Delta y$ .

What I am telling it if I have a u and v, it is a 2 dimensional element where we have consider it is easy to understand it and it is the z direction which is a perpendicular to this surface, so that way it will have a rotations along the z directions; along the z directions perpendicular to z directions, it will have a rotation in on xy plane. So, if is I have a, u and v velocity at this point, I will have a velocity at this point will be  $u + \frac{\partial u}{\partial x} \Delta x/2$ .

So, similar way at this point I will have the velocity, the v;  $\frac{\partial v}{\partial y} \Delta y$  but this will be the u velocity, so similar way I can write the what could be the velocity at this point. So, now if you look it when you take a fluid element in the 4 corner points, we have the velocity variation because of this velocity component okay, because of this velocity component and this velocity component, the fluid particles is going to rotate it, is it correct?

The velocity component of the v at these locations that what will we try to rotate it, the velocity component v, u the gradient the relative difference of velocity component will be move it, so that way what we can have that how we can compute the relative velocity difference between



these point that what multiplying with the dt will give a distance and then we can get it what will be the angular rotations, very simple way.

$$\omega = \frac{d}{dt} \left( \frac{\alpha_a + \alpha_b}{2} \right) = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

If you want to have very detailed derivations writing from the Taylor series, find out in the distance travels that computing's all these component of the velocity at each points, you can follow of any of my wave lectures which is there in NPTEL's, so but let us not I spend more time on that, let me take you very concept wise that the angular rotations, the rate of rotation of the fluid element about the point P will be the  $(\alpha_a + \alpha_b)/2$  which we can easily write in terms of the partial derivative of B and the u component, the scalar component velocity.

It can be easily written it, if you follow the Taylor series and with these things okay, this is what that means this is the angle of rotations is given in a functions of in terms of the scalar; the partial derivatives of the scalar component of P and u in a x and y directions respectively but if you want to make it 3 dimensional like if you look at this fluid particles okay, I just computed for the z directions okay.

Similar way, I can find out what could be the rotations in the x directions and the y directions; that what will come it the similar like this, so this is what the rate of rotations factors, it has x component rotations, the y component rotations and z component, these are complete rotations factors is in terms of the change of the velocity gradient in x, y, z directions.

So that; so please try to have a simple relationship to try to know it whenever you are getting the compute the rotations in the y; z plane you need to have a scalar gradient component in x and y.

$$\vec{\omega} = \frac{1}{2} \left( \frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial z} \right) \vec{i} + \frac{1}{2} \left( \frac{\partial u}{\partial z} - \frac{\partial \omega}{\partial x} \right) \vec{j} + \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \vec{k}$$

Similar way, if you want to have these, computing these rotations in the ith plane that means you are looking at the scalar gradient A of z and the y plane, so those things you can just re-remember it or it is not necessarily you try to simplify it as get a fluid elements and you just think it.

But if you come to the virtual ball concept, the same concept I can tell it there will be virtual ball concept. What I am talking about now that not the fluid elements, it is a virtual fluid balls that means, there will be a series of balls will be there, since there is a velocity gradients are there, so these balls will try to move these directions, these balls have try to move to these directions.

Because of that there will be start a rotation, the direction can change it, they can have a clockwise direction so basically, if you think it there are the balls are there as they have the different the velocity gradients are there, so they will move different displacement, as a jointly, we can see there is a rotation, what is happening it, as a rotation what is happening in a z plane as well as we can compute for x and y directions.

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**Motion and Deformation of Fluid Element**

**Linear strain rate**  
 Defined as the rate of increase in length per unit length.

Mathematically, the linear strain rate of a fluid element depends on the initial orientation or direction of the line segment upon which we measure the linear strain.

Increase in length of line segment PQ into line segment P'Q', which yields a positive linear strain rate.

If the flow is incompressible, the net volume of the fluid element must remain constant.

*Use of Virtual Fluid Ball concept*

Linear strain rate in some arbitrary direction  $X_\alpha$  is defined as the rate of increase in length per unit length in that direction

Now, coming to the very simple thing is called linear strain rate that means, what is the strain rate; rate of increase the length per unit length, this is very simple definition, okay. So, rate of increase in length for unit length that means, let you have a the initially, you have a the fluid element which is starting from P to Q okay, at the point P you have the velocity  $u_\alpha$  but at the Q, we should have a based on the Taylor series, you can have this velocity.

$$u_\alpha + \frac{\partial u_\alpha}{\partial x_\alpha} dx_\alpha$$

At the dt times, these P point will be moved to P', the Q point will move to Q', the distance between these 2 will be the different, so that is the reasons they will be linear strain, distance travelled by this P and Q will be the different as the velocities are the different at the P and Q,

if the velocity difference between the P Q not there, then you will may not have a linear strain, will not have a linear strain.

Since, there is a velocity variation, the fluid element which connected as a virtual fluid balls from P to Q; Q moves more as compared to the P, so you have a stretching of fluid virtual fluid balls, stretching of the virtual fluid balls that means, they will stretch it, there will be the same dimensions but there will be stretching it, so they are will be elongated as the width will be reduces, the length will be change it.

So, you have this part, so increase in length of PQ like the segment yields a positive linear thing, when a flow is incompressible, the net volume of fluid element was to remain constant. If the fluid is incompressible that means, the density does not change within the fluid flow domain. It means that the net volume of the fluid must be constant now, if you look at these problems that so, you can find out this linear strain rate is very easily that linear strain rate length you know it divided by this per unit; increasing length you know it divided by this the original length will get it from that what will be the linear strain rate.

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**Motion and Deformation of Fluid Element**

**Volumetric strain rate**

- The rate of increase of volume of a fluid element per unit volume is called its volumetric strain rate or bulk strain rate.
- This kinematic property is defined as positive when the volume increases.

Volumetric strain rate in Cartesian coordinates:

$$\frac{1}{v} \frac{Dv}{Dt} = \frac{1}{v} \frac{dv}{dt} = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

For incompressible flow, volumetric strain rate is "0"

$$\nabla \cdot \vec{V} = 0$$

So, we can compute it what would be that is why we can easily compute it, what it will be linear strain rate which is the gradient of the linear strain rate in the x directions, the y directions and the z direction. If I am looking for a poly metric strain rate that means, will be sum of these strain rates what we are looking at. Let me have a very simple examples how you have either; like you have this type of flow the conduits are you have.

When a fluid particle is coming like this or virtual ball is coming it, it has the velocity variation, as soon as its comes here, it has to be bigger size, so it have a is to occupy this space if you would have this, so these fluid particles from this point to A to B when passing through, it has gone through shear strain deformations. So, if you look at this fluid flow problems, which is very easy, things that there is a smaller diameter pipe, it is a connected to a bigger diameter pipe.

When a fluid elements is coming from this point as goes to this point has to be increased, there will be a linear strain rate what is going to happen in this case. So, because of that there will be reductions of fluid, so if we look at this way whenever the fluid particles passing through a smaller to bigger dimension or bigger to smaller dimension, it goes through a strain rate formations.

Volumetric strain rate in Cartesian coordinates:

$$\frac{1}{v} \frac{Dv}{Dt} = \frac{1}{v} \frac{dv}{dt} = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Now, if you look at; if I look at a volumetric strain rate is a sum of the shear strain rate in the x, y and z directions, as you know it for incompressible flow, the volumetric strain rates becomes 0 that means, what is this; is a dot product of delta and v is equal to the 0, so this becomes a conservations equations; the mass conservation equation for us, if you consider the flow is incompressive.

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**Motion and Deformation of Fluid Element**

**Shear strain rate**

- Shear strain rate is a more difficult deformation rate to describe and understand.
- Shear strain rate at a point is defined as half of the rate of decrease of the angle between two initially perpendicular lines that intersect at the point.

Shear strain rate, initially perpendicular lines in the x- and y-directions:

$$\epsilon_{xy} = -\frac{1}{2} \frac{d\alpha}{dt} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

*Use of Virtual Fluid Ball concept*

Now, let us go for the shear strain rate, what is this? The shear strain rate is a half rate of decrease of angle between 2 initially perpendicular lines that intersect at the point, what it is saying that if you look at these figures that I have a figure like this, the fluid element at the  $t_1$  time at the fluid element at the  $t_2$  time. At the  $t_2$  time, this line is moving  $\alpha$  and  $\alpha_{a-b}$  angles.

So, we are looking at half of that angles, the decrease of angle between 2 that means, because of shear strain rate, you can identify the angle and how this angle are changing it okay, on average how much of angle it is a changing it with a 2 perpendicular lines, okay, you can understand it if I have the shear stress is acting it, the strain will be the angular rotations.

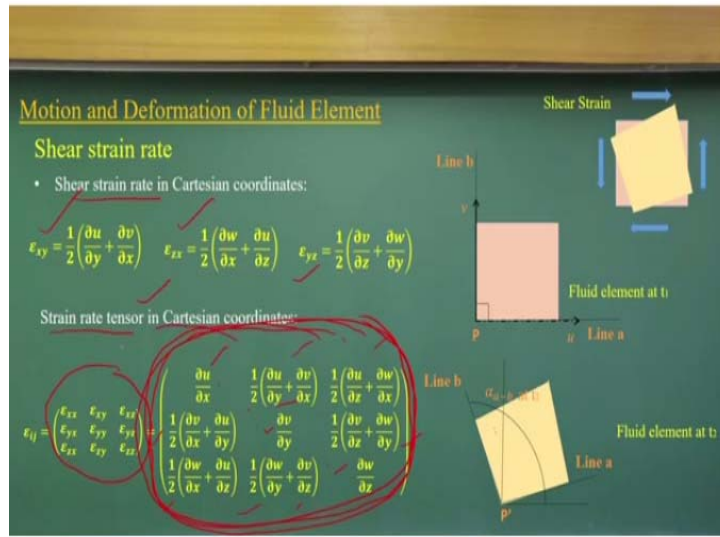
$$\epsilon_{xy} = -\frac{1}{2} \frac{d}{dt} \alpha_{a-b} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

And since is the fluid element we will have to have a 2 axis and with that 2 axis formations we are looking at that what is the angle rate of the change of decrease of angle between 2 initially perpendicular line that what intersected that, so again you can find out what will be the shear strain rate with the half of this change in the  $\alpha_{a-b}$ , okay that angle between these 2 change that is what again you can estimate it.

And you will find out the linear shear strain rate will be the functions of u and v, the scalar velocity components, the partial gradient component what you will get this function like this. So, you can easily understand it that if I have the fluid part; balls also can find out how much of rotations are happening it and because of if in case of angular rotations, both the axis are rotating in the same rate.

But in this case, the rotation rate are the difference because of that you can see that deformations, the fluid element deformations.

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So, as of now what we have done it, we can get a shear strain rate in a Cartesian coordinate for different plane, xy plane, zx plane and yx plane and finally, we can write the shear strain rate tensor which will be linear strain formations and the formations of shear strain rate. So, it has a 9 component of stress strain rate tensors okay, so you can find out the strain rate tensor functions which would have a 9 components.

Shear strain rate in Cartesian coordinates:

$$\epsilon_{xy} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\epsilon_{zx} = \frac{1}{2} \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)$$

$$\epsilon_{yz} = \frac{1}{2} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)$$

Strain rate tensor in Cartesian coordinates:

$$\epsilon_{ij} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) & \frac{\partial v}{\partial y} & \frac{1}{2} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \frac{1}{2} \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) & \frac{1}{2} \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) & \frac{\partial w}{\partial z} \end{pmatrix}$$

So, if you look at that this strain rate tensor is a functions of partial derivative of the u, v, w scalar velocity components, there could be positive, there could be linear part but they are all depends upon the velocity variations in x and yz of planes, so how would they velocity variation and that velocity variation causing as the shear strain tensors. So, as you know from the fluid mechanics or the solid mechanics, stress strain; stress is a proportional to the shear strain rate.

So, we can easily establish the stress and the shear strain rate tensor functions very easily, if you know the shear strain rate tensors but in these 20 hours' lectures, I am not going more details about these functions because it is limited to this way to just describe you how the shear strain rate functions are there.

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**Vorticity and Rotationality**

**Vorticity vector:**

- It is a measure of rotation of fluid particle
- Mathematically defined as the curl of the vector  $\vec{V}$
- It is equal to twice the angular velocity of a fluid particle

$$\zeta = \vec{\nabla} \times \vec{V} = \text{curl}(\vec{V})$$

**Rate of Rotation (Angular Velocity) vector:**

- Rate of rotation vector is equal to half of the vorticity vector

$$\vec{\omega} = \frac{1}{2} \vec{\nabla} \times \vec{V} = \frac{1}{2} \text{curl}(\vec{V}) = \frac{\zeta}{2}$$

*Boundary layer flow at a curved surface (viscous effects are important)*

*Irrotational fluid particles*

*Rotational Boundary layer region*

*Rotating fluid particles*

*Boundary layer flow at a curved surface (viscous effects are important)*

\*Source: Image is reproduced from Cengel and Cimbala Fluid mechanics textbook.

So, let us look at another component is called the vorticity or the rotationality, when you talk about a vorticity vectors, the basically we are representing as a vorticity vectors which actually the measures of rotations of a fluid particle, which if you can look at the rotations of fluid particles can be twice of the angular velocity, so it is the curl of V okay;

$$\zeta = \vec{\nabla} \times \vec{V} = \text{curl}(\vec{V})$$

and half of the curl of the V is as you know it is angular rotations, you just substitute the v functions and do the cross products, you will get the; divide by 2, you will get the rate of rotations.

$$\vec{\omega} = \frac{1}{2} \vec{\nabla} \times \vec{V} = \frac{1}{2} \text{curl}(\vec{V}) = \frac{\zeta}{2}$$

Or indirectly that it has the a relationship with a vorticity with the rotations because this is what again a representations, vorticity vector, if I know the cross product of the  $\nabla$  and V, which is 0, then we can say that the vorticity is 0 that means, there is no rotations, there is no angular rotations like for example, if you look at very interesting photographs or figures okay which is modified from Cengels and Cimbala's books.

I think you please remember these figures okay which is very interesting figures for a fluid mechanic student point of view that when you have this a flow passing over the plate, you can anticipate it that you will have a viscous effect zone which is called boundary layer formations, there is a zone where there is large gradient of velocity vectors, the velocity will start from 0 to a large gradient will be there.

Since, there is a large gradient of velocity variation is there as I said it earlier that velocity vector change from one point to other point, so the fluid particles will not go straight line, they will start rotating it, so that way this figure is retreating that when the fluid particles entered here, there are large velocity gradients are there, the turbulence is there, so the boundary layer formations are the zone where viscous effect dominates okay.

Those regions you will see the vorticity would be there or you just have the cross product of the  $\nabla$  and the  $V$ , will show the vorticity and the graphically, you can see the face of this one's okay, it is just a vorticity okay, it is a rotation, the fluid particles will go under the rotations but just the particles which is much above this okay, where there is no vortex, you can see the face of this one's okay, they are just same, there is no rotation.

So, then we tell it irrotational fluid, this is the outside of the boundary layers, you can see that fluid particles are moving it or the virtual fluid balls are moving it without any rotations but within the boundary layer formations, the small regions near to a surface, you will see there is a change of the velocity gradients; the drastic change of the velocity gradient and those the regions; a thin region is called boundary layer.

There; because there is large variations of velocity factors induces the fluid particles to be start rotating it, once you start rotating it so, once particles start rotating, other one will start rotating so, all the particles will start rotating it and that what it is having formations of Eddies formations and all which we will discuss in the later on in a pipe flow chapters.

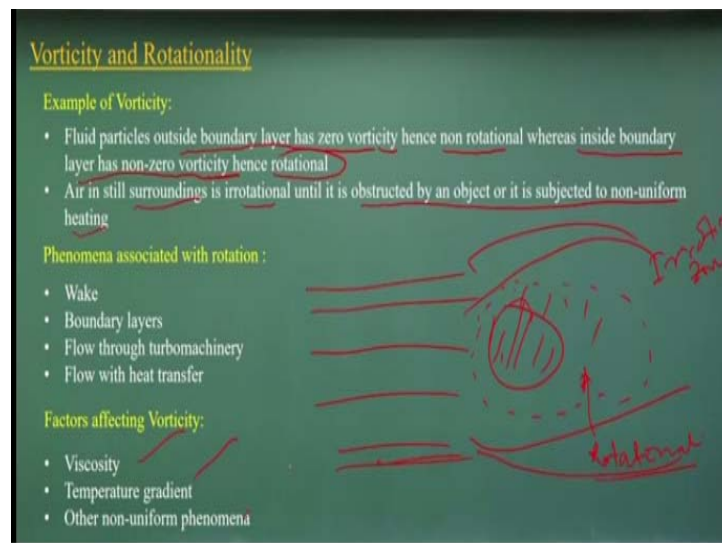
But you try to understand it like these figures, you please try to visualize these figures and see there are will be the regions where the rotations will be very dominate and there is a regions you will not have a rotations that much of a dominate, it can consider is irrotational zones, where the fluid particles will not go for any rotational, they will have the translations, there



may have the linear shear strain formations, may have the linear strain formations but they will not have a rotational activity.

So, that what we measure in terms of vorticity, so please do not have a very confusions between the vorticity and angular vector because vorticity is easy to define is a cross product between the delta and the V where is when you talk about angular rotations , we have half of that so, it is very easy the people who are not looking the angular rotations, they are looking it in terms of how the vorticity is playing it or vortex formations happening it, they talk about at the vorticity level not at the rotations level.

**(Refer Slide Time: 35:25)**



These are very examples as I said it that when the fluid particles outside the boundary layer has a zero vorticity non-rotational zone but inside the boundary layers, it will have non zero vorticity, so flow is a rotational, air is still surrounding the irrotational but it is obstructed by object or these things, so air flow wherever is that if there is no obstructions, so it will be the irrotational.

But when you are start putting a obstructions like a any cylinder curl type of subject, so you will see that very close to these regions, will have a lot of the rotational activities but the outside these are irrotational zone, so you try to understand it how is the formations and this is the formations which is called the boundary layers and (()) (36:26) formations, it happens for flow through the turbo machinery, the wake formations and all and which causes viscosity temperature gradient and the non-uniform phenomena.

**(Refer Slide Time: 36:37)**

**Vorticity and Rotationality**

Vorticity vector in Cartesian coordinates:

$$\zeta = \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \vec{i} + \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \vec{j} + \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \vec{k}$$

Two dimensional flow in Cartesian coordinates:

- z component of velocity (w) = 0
- u and v are independent of z
- Vorticity vector points either in z or -z direction

$$\zeta = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \vec{k}$$

Vorticity vector in Polar/ cylindrical Coordinates:

$$\zeta = \left( \frac{1}{r} \frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z} \right) \vec{e}_r + \left( \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \right) \vec{e}_\theta + \frac{1}{r} \left( \frac{\partial(ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right) \vec{e}_z$$

Two dimensional flow in Polar/ cylindrical Coordinates:

$$\zeta = \frac{1}{r} \left( \frac{\partial(ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right) \vec{e}_z$$

Now, in terms of vorticity vector coordinates if I put it in an i, j, k, I will have this component, if I am looking a 2 dimensional flow, the flow does not have the z component okay and u and v are independent to z, then I have only one components, so many of the flow you know it we can define in terms of 2 dimensional flow, it is easy to visualize, easy to understand it okay, as compared to the 3 dimensional flow.

Vorticity vector in Cartesian coordinates:

$$\zeta = \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \vec{i} + \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \vec{j} + \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \vec{k}$$

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So, that is what we can see the vorticities can define in terms of the scalar velocity gradient component, partial gradient component in x and y directions and the k indicating the directions where the vorticity directions is the z directions. So, vorticity vectors also can retain in terms of polar and cylindrical coordinate systems for 3 dimensional and also the 2 dimensional's.

Vorticity vector in Polar/ cylindrical Coordinates:

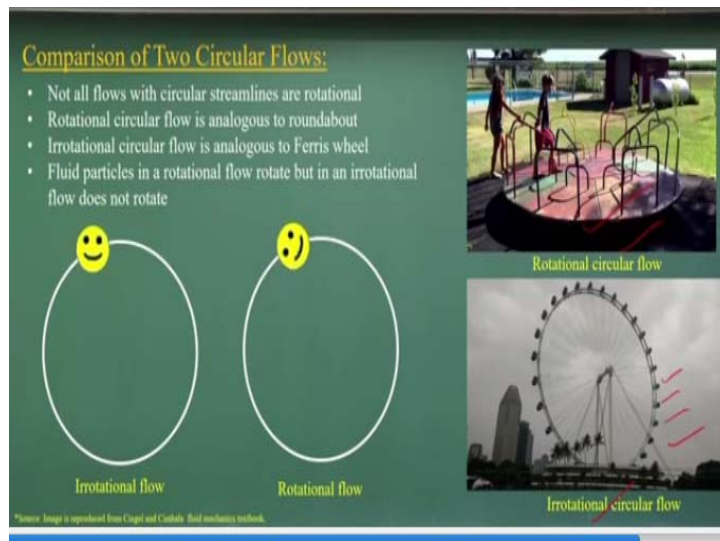
$$\zeta = \left( \frac{1}{r} \frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z} \right) \vec{e}_r + \left( \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \right) \vec{e}_\theta + \frac{1}{r} \left( \frac{\partial(ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right) \vec{e}_z$$

Two dimensional flow in Polar/ cylindrical Coordinates:

$$\zeta = \frac{1}{r} \left( \frac{\partial(ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right) \vec{e}_z$$

So, those are interested in high level of fluid mechanics, please look at the similar way.

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Now if you look at these figures, you can understand it which is irrotational, which is a rotational okay and the figures which is there in here you can look it which is a rotational, which is a irrotational okay, because of that you enjoy this the wheel okay because of its irrotational circular flow, so you just sit it, you enjoy it because it is irrotational but this is the rotational circular flow.

So, that is what is happens it and these figures what is there very interesting figures drawn by my students and you can really visualize these figures and I do believe it if you visualize these figures will never forget what is difference between irrotational and rotational flow. So, with this I can start solving the 2 example problems and concludes today's lectures.

**(Refer Slide Time: 39:08)**

**Example 1**

A two-dimensional flow field is specified by  $V = 3yi + 3xj$ . State whether the flow is steady, irrotational, and check whether the given field is feasible. Find the stream function and determine the volume flow rate passing between streamlines through the points (1,3) and (3,3).

Velocity field:

$$V = 3yi + 3xj$$

The velocity components are:

$$u = V_x = 3y$$

$$v = V_y = 3x$$

The flow is Steady

$$\frac{\partial V}{\partial t} = 0$$

The flow is irrotational

$$\zeta_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} = 3 - 3 = 0$$

For the field to be feasible one, it must satisfy the continuity equation

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

So, if you look at the first point that I am looking a there is a 2 dimensional velocity field state whether the flow is steady, irrotational and check whether the given field is feasible that means, whether the mass conservations happens or not okay, the or the flow is incompressible and mass conservation principles will rotate. Then, we are talking about the stream functions and determining the volume flow rate passing through between the stream line passing through these 2 points okay.

[A two-dimensional flow field is specified by  $V = 3yi + 3xj$ . State whether the flow is steady, irrotational, and check whether the given field is feasible. Find the stream function and determine the volume flow rate passing between streamlines through the points (1,3) and (3,3).]

Even if I have not discussed about the stream function but I will just introduce you what is the stream functions and based on that, you can always think how to get it the stream functions variability. Now, the flow is steady because you can easily locate this u and v component does not have any time component, so the steady flow; flow is irrotational, you can find out the velocity gradients and find out the flow is irrotational.

Velocity field:

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For the field to be feasible one, it must satisfy the continuity equation

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

And from the continuity equations or the linear volumetric strain rate functions, you can find substitute these values, you will see that this is equal to 0, that means the flow field is possible one.

**(Refer Slide Time: 40:28)**

**Example 1**

The stream function  $\psi(x,y)$ , is in the differential form

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = -V_y dx + V_x dy$$

$$= -3x dx + 3y dy$$

$$\psi = \frac{3}{2} y^2 - \frac{3}{2} x^2 + C$$

The discharge between streamlines through (1,3) and (3,3) is

$$q = q_{\psi(3,3)} - q_{\psi(1,3)} = \psi_{(3,3)} - \psi_{(1,3)}$$

$$q = \frac{3}{2} (3^2 - 3^2) - \frac{3}{2} (3^2 - 1^2)$$

$$q = -12 \text{ units flowing from right to left}$$

Now, coming back to the stream functions what we define it okay, if you remember it, if these are my stream functions, which varies in x and y, I am talking about the steady stream lines okay, x and y directions as you know it, the stream functions change in the x directions okay, should be equal to  $-V_y$  and stream function changing in y directions should be equal to the  $V_x$ ,

The stream function  $\psi(x,y)$ , is in the differential form

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$$\psi = \frac{3}{2} y^2 - \frac{3}{2} x^2 + C$$

This what you can physically interpret it without having this thing that the tangential component of the velocity factor that what is represent as these functions.

So, considering this the definitions of the stream functions, you can establish this is what the functions behaviour relationship between the stream functions gradient and all, so if as these functions is 2 independent variables, we can define like this substituting this value, we get a stream functions. As you know the stream functions, you can find out the discharge between them which is the difference on that and substitute it here you will get a 12 units.

The discharge between streamlines through (1,3) and (3,3) is

$$q = q_{\psi(3,3)} - q_{\psi(1,3)} = \psi_{(3,3)} - \psi_{(1,3)}$$

$q$ , volume flow rate is

$$q = \frac{3}{2}(3^2 - 3^2) - \frac{3}{2}(3^2 - 1^2)$$

$$q = -12 \text{ units flowing from right to left}$$

Please I can encourage you to read more on the stream functions and all similar type of questions comes in gate or engineering service.

**(Refer Slide Time: 42:04)**

**Example 2**

A flow field is given by  $u = y^2, v = -xy, w = 0$ , value of  $z$  component of the angular velocity at the point (0,-1,1) is

(GATE 2018, Civil)

The velocity components are:

$$u = V_x = y^2 \quad v = V_y = -xy \quad w = V_z = 0$$

Rate of Rotation (Angular Velocity):

$$\omega = \frac{d}{dt} \left( \frac{\alpha_x + \alpha_y}{2} \right) = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega = \frac{1}{2} (-y - 2y)$$

$$\omega = -\frac{3}{2}(y)$$

$$\omega = \frac{3}{2} \text{ at } (0, -1, 1)$$

[A flow field is given by  $u = y^2, v = -xy, w = 0$ , value of  $z$  component of the angular velocity at the point (0,-1,1) is]

Now, coming to second example is a too easy examples as compared to the first one which is a gate 2018 civil engineering questions here,  $u, v, w$  components are given it okay, compute

the z component of angular velocity at the point, so the point is also given, the x, y, z coordinate is given it, you have to find out the z component of angular velocity that is again the sketch of these figures which is giving alpha and alpha B, you can find u, v, w.

$$u = V_x = y^2$$

$$v = V_y = -xy$$

$$w = V_z = 0$$

Then, you know this relationship between these things and you just substitute it, you will get it which value is comes out to be 3/2, so the basically you try to understand it, if you find out the scalar components differentiate these scalar; partial differentiations you would do it with respect to y, x or y, then you just substitute it, then you can get what will be the angular rotations in the z directions which is here.

Rate of Rotation (Angular Velocity):

$$\omega = \frac{d}{dt} \left( \frac{\alpha_a + \alpha_b}{2} \right) = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

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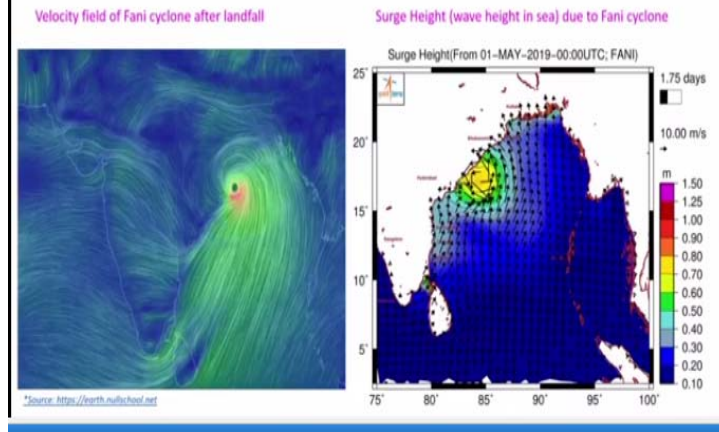
$$\omega = -\frac{3}{2}(y)$$

$$\omega = \frac{3}{2} @ (0, -1, 1)$$

And after that you substitute the coordinate x, y and z at that point, then you will get what will be that.

**(Refer Slide Time: 43:15)**

## Fani Cyclone landfall on dated 03/05/2019



So, with this let us start these figures if you can look it and as its; as I said it is very real time, today is 3rd May 2019 and today is the cyclone is passing through the state of Odisha and you can see these big vortices how it is moving it and how the velocity vectors, the vortex is moving it and how it is causing extreme rainfall events and how it has make it almost the coastal wealth is stands still no flights are running, no trains are running it.

So, these are all it is a possible now because of the high level of fluid mechanics software's we have and which is running at different levels to predict the wind velocities, predict the vorticities, predict all these processing can really see it and also predict the surge height all it is possible now because of our in-depth knowledge of the fluid mechanics, the huge of computational fluid dynamics.

And that is the reason, it is easy to for us to predict a flood or predict the cyclones at the accuracy of is just were one hours difference of accuracy, we can predict at what time it will come it, where it will come it, all these are possible because our knowledge of fluid mechanics and because of our capability to simulate the very complex fluid flow problems in a weak area like if we look at what could be the large area at the global scale also, it is a possible to predict it.

And that is what today's we are able to predict the cyclones even if an accuracy of less than half an hour's, so we can exactly tell it at what time it is going to come it way, so it is all possible and how will be the wind velocity and what will be the wind directions all we can get the path of the cyclone tracks and all these are all possible because of our knowledge of fluid mechanics



and our knowledge how to solve this complex fluid flow problems using computer (CFD) (45:37) or CFD solver.

**(Refer Slide Time: 45:40)**

### Summary of the Lecture

**1. Motion and Deformation of fluid particle**

- Translation  

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$
- Rotation  

$$\omega = \frac{d}{dt} \left( \frac{\alpha_a + \alpha_b}{2} \right) = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$
- Linear strain (or extensional strain)  
 For incompressible flow, volumetric strain rate is "0"  $\nabla \cdot \vec{V} = 0$
- Shear strain  

$$\varepsilon_{xy} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

**2. Vorticity and Rotationality**

$$\zeta = \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \vec{i} + \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \vec{j} + \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \vec{k}$$

- Rotational flow: Roundabout playing bench is example for circular rotational flow
- Irrotational flow: Ferris wheel is example for circular irrotational flow

### 1. Motion and Deformation of fluid particle

#### Translation

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

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- Rotational flow: Roundabout playing bench is example for circular rotational flow
- Irrotational flow: Ferris wheel is example for circular irrotational flow

With this, let me conclude it, these are all the summary which is listing all the equations, so just have a really it is a good lecture to look at this vorticity, it is a smaller scale to the bigger

scale with the cyclonic patterns, with this let us have a; give you a thankful for this lecture,  
thank you.