Fluid Mechanics
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Lecture – 16
Fluid Kinematics

Welcome you to these lectures on fluid mechanics, so today I will cover the fluid kinematics, this subject what I like most. So, for today we will discuss about the fluid kinematics and very simple way I can tell that how we can describe the fluid flow patterns okay, so looking that I will give you very, very interesting examples today and you should visualize that how the fluid flows it; how complex fluid flows happens in the real fluid flow problems.

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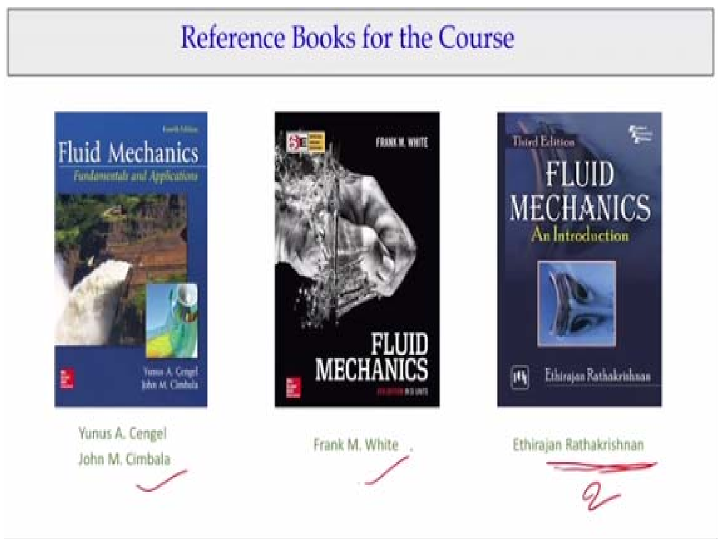

So, for today I will talk about that but let us say I have see these reference books, I am putting it to Professor's Rathakrishnan's books which is a fluid mechanics and introductions which is very concise book, okay and this book is very interesting books, it has a lot of component is there and very concise way it has been written it so, if are interested to read it conceptual wise, please follow these professor Rathakrishnan's books, the fluid mechanics and introductions.

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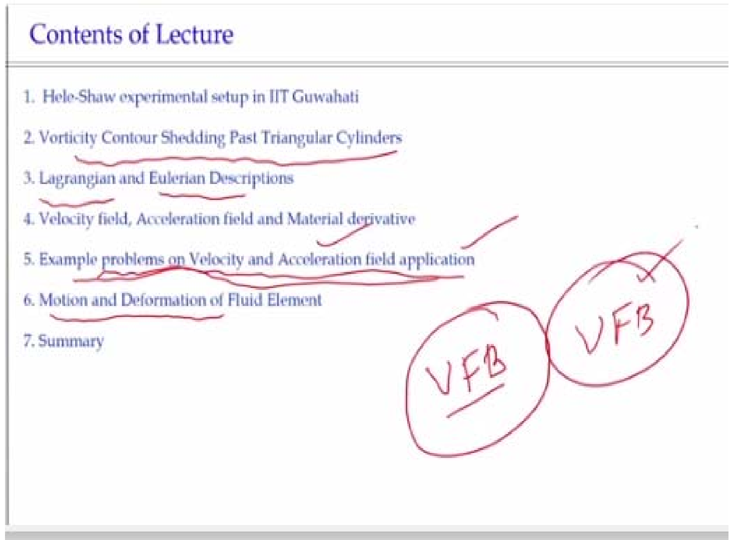

No doubt we have been following this 2 book of Cengel, Cimbala and F.M. White book, let us come back to the today lectures, I will start with the experiment setups which is called Hele-Shaw experiment setups to draw the streamlines, the streak lines and path lines, the flow fast in different type of objects, then I will talk about one example problems which CFD solvers which is the vorticity contour shedding past triangular cylinders.

Then we will describe about Lagrangian and Eulerian terms and as I said it earlier, when you talk about the fluid flow problems, we talked about 3 fields; the velocity field, pressure field and acceleration field but today beyond the velocity fields, we will talk about accelerations field also, material derivative and we talked about; we will have some example problems we can solve it for very simple way, we can solve the example problems which is had more led that (()) (03:02) question papers.

So, similar way we can have an example problems, where we can solve for the velocities and accelerations fields, then we will go for very interesting talk about the motion and the deformations of fluid flow. When you talk about the fluid kinematics, if you try to understand it in a solid mechanics, we describe the motions without considering the force component.

Or I can simplify way tell you that we try to describe the flow patterns, there is a force component is there but that force component we are not emphasized, so when you do that we call fluid kinematics that means, you are describing the fluid flow problems that means velocity field, pressure field, accelerations field, trace field and the tensors but we are not talking about the force components and all, which if you know the velocity field, if you know the accelerations field, we can always compute other components; the force component.

If you know the pressure field, you can find out the uplift force, the drag force on an object passing by different type of the flow phenomena, so the process what I am talking about that the fluid kinematics and the dynamics both are combined, complementing each others, they are not separate but for you to have an easy presentations, we tell that we consider the fluid flow problems where just we are talking about descriptions of these fields; the velocity field, pressure field, accelerations field and the density field.

So, we are talking about how does they vary, how do they vary that is what is our prime importance as you, I demonstrates many of the topic looking into that, then we will be talking about the motion and deformations of the fluid element and as I; you remember it I introduced you virtual fluid balls, I think today we will use very extensively this virtual fluid balls to describe the Lagrangians and Euler descriptions, describing the velocity field.

And try to understand very easily the flow patterns of pressure field, velocity field and accelerations field in a very complex problems using virtual fluid ball concept that is the my idea and that is what today, we will have very extensive explanations of the fluid kinematics using virtual fluid balls okay, virtual fluid balls concepts.

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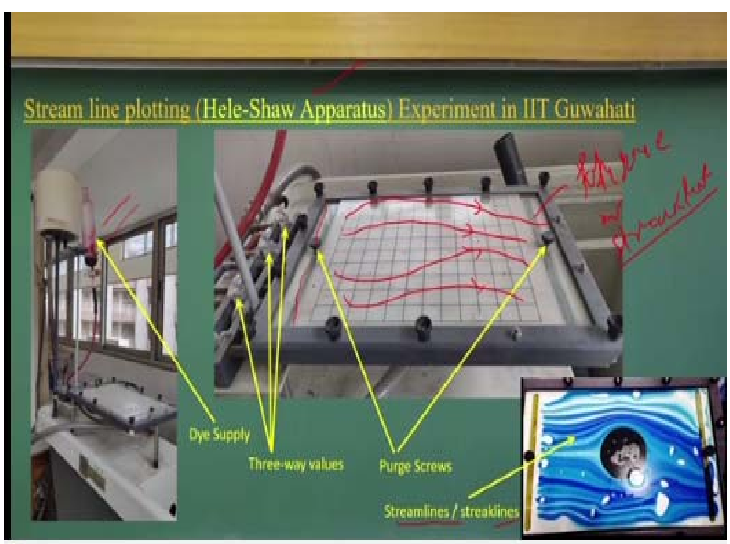

So, let us have a today lectures and let us start with the experimental setups which we call Hele-Shaw apparatus, okay where we can generate a constant flow and there are the patterns of the colour dye we can inject it, as we injected the colour dye from this; this colour dyes are follows the path lines or the streak lines, as we make the flow in steady, so the stream line and streaks line the when you look it, if I put the colour dye, so these what will be show me the streak lines or the path lines, if I tracing on that okay, path line or streak line.

Or since it is a steady flow, I can tell it that it is streamlines, so many of the times we are very much familiarize with the streamlines but when you look it, so what are these lines; is that the streamlines, different colours flow passing over this object, so we can see that how the stream line patterns are happening it, as we change the flow Reynolds numbers this pattern changes it, more we will have a discussions that how the flow past an object we have to describe it.

But this is the experiment we have use it to plot the stream lines using a colour dye and uniform plate flow okay, so that is a reasons we need graph paper, we trace these stream lines; straight the stream lines as we control the flow, we will have a different velocity, different the distance, you will have it the change of the flow patterns that what we do using Hele-Shaw apparatus setup which is there in IIT Guwahati.

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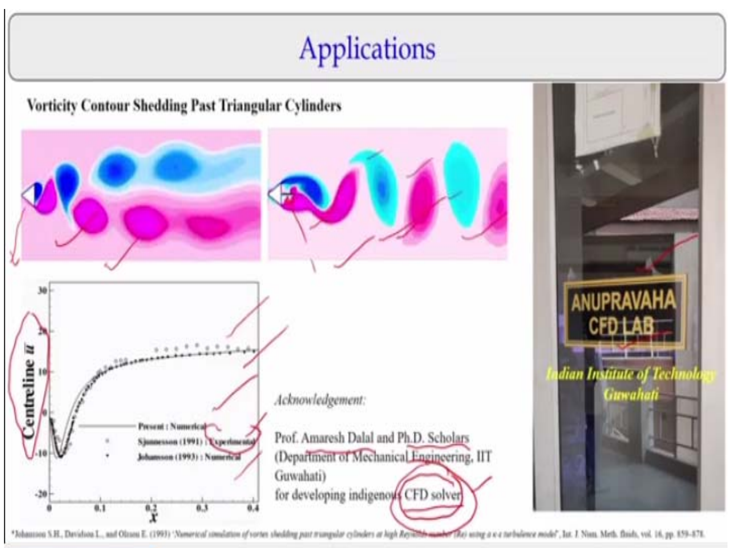

Now, if you look it is very interesting experiment numerically, it is done by professors Amaresh Dalal and the Ph.D. Scholar who develop the CFD solvers and they have very unique lab which called Anupravaha CFD lab, so I acknowledge there, these PPT to that okay, let us understand what is this PPT is, if you look in these colourful things. What it happens is that there is a triangular cylinders are there and there is a uniform flow and there just looking in to vorticity.

How the vorticity is going on, so you can see that the vortex what is generated from the often down, they behave differently and at different times, the shedding patterns are changing it, we will discuss about what is the vortex but you can understand it this type of vortex (()) (09:14), when you have a uniform flow passing through a triangular cylinders, okay the triangular cylinders you will have a this type of 2 dimensional pattern of the flow.

But if you just include a one small thread, if you look it that the flow pattern changes it, is that you see looking there is such a simple thread is there and the flow patterns which is very interesting flow patterns what is happening it that is what is changing it, the flow pattern of these and the flow patterns here, the streamline pattern, the streak line patterns, the path lines will be the differ and there are the vortex formations happening it.

And the series of the vortex the formations and moving downwards, is called vortex shedding or vortex contour sheddings what is happening it, okay and that is what is they have a compare with the experimental data and the numerical presentations what the varies of the central line, the average velocities and the x values and they are getting very close to what is the experimental region.

So, in a similar we can conduct any experiments okay, putting the triangular cylinders and we can look it how the vortex shedding patterns happens it, the same way we can do using this computational fluid dynamics solvers. So, if you look at today, we have a very sophisticated software develop in a IIT Guwahati, elsewhere also there are lot of solvers are there, no doubt we can visualize the flow.

That means, we can understand which are the stream lines, which are the streak and which are the path line, how the velocity varies it, how the pressure varies it, how the density variations, so those are things are today's it is a possible either through experiment or the CFD solver, it is a possible and you see these in very interesting experiments what the numerical experiment these are what the numerical experiments is done with 2 conditions with just having a tail end part here is nothing have a tail end and how the vortex settings are happening it.

So, if you can understand it when you have a kite flying in the sky, there is a vortex shedding just downstream of a kite also, so you can try to understand it or you can try to understand it if a bullet train; the train of more than 300 kilometre per hour is moving with a start speed, we have a triangular shape, it can have a this type of vortex sheddings, so this process what we are representing is that you have what I am giving example is that let you have the not the triangular cylinders which is moving may be very fast of 300 kilometre per hours.

And it can have a vortex shedding like this, it can have a vortex shedding like this, so try to understand these simple problems what it happens for real engineers problems that we try to understand it okay.

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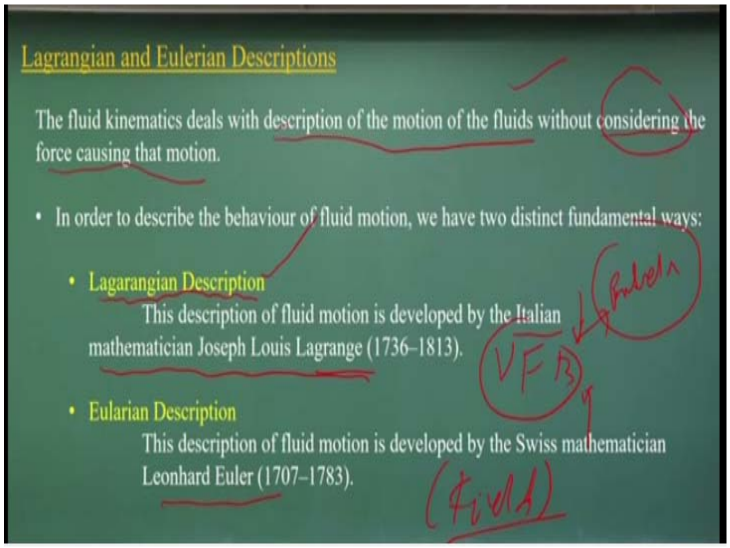

Now, let us come back to that as given in the textbook that we describe the fluid flow things okay, descriptions of motion of the fluid flow without considering the force causing that motions, I just differ on this statement, there is a force components which causes the flow motions but here we are not considering, we are not empathised, I think this considering what sentence is not appropriate.

But we can say it is, we are not emphasized what is the force system, more we are oriented to know it what is the descriptions of the motion of the fluid as I demonstrated earlier, the flow fast in triangular cylinders, so we are bothering about how the descriptions of the fluid flows okay, so that is a reason we will talk about now virtual fluid balls, okay that is the way the fluid motions are describing 2 fundamental ways.

One is Lagrangian descriptions; it as a long ago okay, 1736, 1813's, the Italian mathematician describes Lagrange describe these motions okay that is what called the Lagrangians, okay. Similar way you have the description of fluid motions which the field concept and that is the

Lagrangian particle concept what we call the virtual fluid ball concept which is I can say that intermediate of that, that is what the way I try to represent you that virtual fluid balls are intermediate between these which not the particles or the concept or not the field concept, it is intermediate.

So that student can visualize the fluid flow and once you visualize the fluid flow, as you see that you can solve many things and or you can use that knowledge to interpret it, the very, very complex fluid flow problems solutions obtained from CFD solvers, as I demonstrated earlier very, very complex fluid flow problems we can get a solution from the CFD solver but the questions comes is; are they correct, so that is the reasons we need to have a basic fundamental knowledge on the fluid mechanics.

And the concept of the virtual fluid balls used intermediate of these 2 descriptions; Lagrangian, Euler descriptions, one is a field okay which we described earlier, the velocity field and pressure field and the density field or we talked about the particles level, okay fluid particles that means, a group of the molecules we are talking about. I think similar way, we are talking about virtual fluid balls is intermediate of particle levels and the field level, it is a conceptually the virtual ball.

So that we can understand at the field level as well as we can understand at the particle level that is the my concept and we are intermediate between these Lagrangians and Eulerians concept now, if you look at that what is a Lagrangians and the Eulerian concepts okay.

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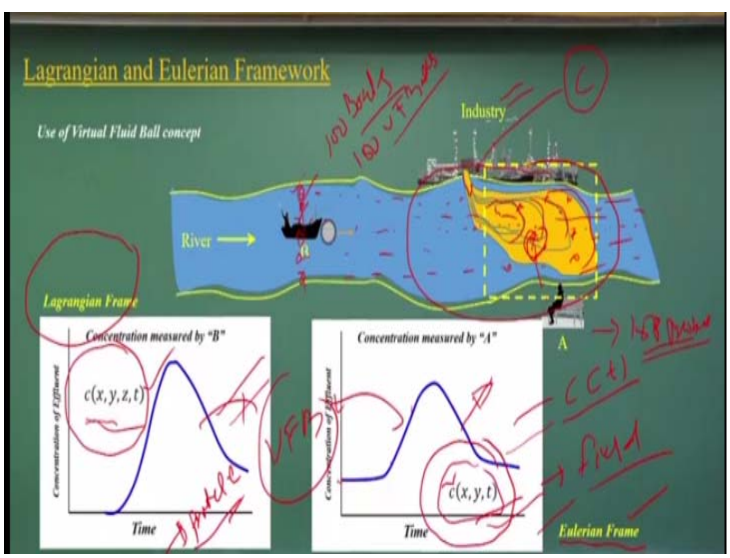

If we look at a person is sitting here, okay, he has a group here and there is an industry okay, it is polluting the waters and that is the pollutions are released it here is having the concentration C, can have any pollutions, okay, may be dissolved oxygen or whatever be okay, there is a concentrations of pollutions are happening it and the pollutions loop is moving like this, like this.

If the persons here is putting a probe here, measuring that concentrations okay, then we call Eulerian frames that means, you have a fixed locations at that fixed locations, you are measuring how the concentrations were rigid that means this part, so initially there will be a some concentrations but as the probe is coming it, concentrations will increases as flow the concentration will go down.

So, this C is a functions of t, the concentrations which is were rigid at the functions of t, so we are not targeting about the fluid particles, we have the fixed locations. At the fixed locations whatever the fluid particles are touching about that those fluid particles are concentrations we are measuring using this probe and we are plotting it.

If we are doing that, then we call Euler frame okay, that is what is Eulerian frames that means your position is fixed, you are getting the velocity field, the pressure field, the concentrations field at that point locations, if that person's will going to measure the concentrations at different points, then he can develop a concentrations variability map or in a x, y and the t time; the space as well as the time that is what will happen it.

$$c(x, y, t)$$

The portion what is the A which is Eulerian frame reference, if he can use these consider not the single probe, if he can put let be 100 probes okay and measure the concentrations and develop the variability of the c with respect to positions and the time, then we will have these function, other one is that Lagrangian frame, what do you mean by that? Look at this particle B, okay, there is a person okay, with a boat, he is just traveling along the rivers, measuring the concentrations.

$$c(x, y, z, t)$$

As he is travels it, that means is a particles are moving it okay, this fluid particles are moving it or virtual fluid balls are moving it that is what we have concept, that the virtual fluid balls are moving it, as the flight; the virtual fluid balls is moving it as soon as it reaches here, the concentrations increases and as it is go through these, then concentration decreases so, you will have these functions.

So, if you have a let you have a not a single boatman, you may have a 100 boats are there okay or you have the 100 virtual fluid balls, then you can visualize that how the C varies with respect to positions and time, since we are describing this concentrations field based on 2 ways of measurement; one is using a particles, as particles moving it or another is using the probe.

We have a fixed locations, we are not bothering about which particles are coming, which fluid particles are hitting over that but with that locations with a time, we are plotting it how the C varies with respect to time, so this is A. Eulerian frames descriptions of the concentrations, this is what Lagrangian frame of descriptions motions where we track about the series of the fluid particles or the virtual ball.

How their concentrations are changed and based on that we have define it that how the flow patterns are happening, so either in a Lagrangian frame or Euler frameworks, there is no problems, Euler framework is a very easy because you are not looking at particles levels but today what I as I introduce you to the virtual fluid balls, we can have the earlier they used to tell it we cannot follow the Lagrangian frameworks.

Because to describe this flow field, the concentrations flow field, we may need 1000 numbers of the particles to describe it which is too difficult to do it earlier but now, today it is possible because you have a computational facilities you can track it and you can visualize the fluid flow, so that is the reasons I have introduced you about virtual fluid ball which is just intermediate of these.

And you just think it you have the balls; 1000 balls, I am rolling it these 1000 balls here, as the balls moves it, for each balls how these concentrations are changes it and that concentrations if I plot it, I will get the C variability in terms of space and the time or if I have a 1000 probes putting it in a 1000 locations measuring the concentrations also, I will get a C field, so this is the field descriptions, this is of particle description.

As in your textbook say that the Lagrangian flows is not necessary, I do believe that could be long back but now having so big, the best computational facility with us today, we can think it intermediate the virtual fluid ball concept where you can think it 2 ways how to understand in a Lagrangian frame as well as the Euler frames, there is no superset of Eulerian frame it is the expect to Lagrangian's frame.

Let us discuss both the frames are equal levels because today, what the complex fluid flow problems we are solving it, the field approach also fails its many of the times, we should also look at that Lagrangian frameworks, we said easy to understand it as compared to the field approach.

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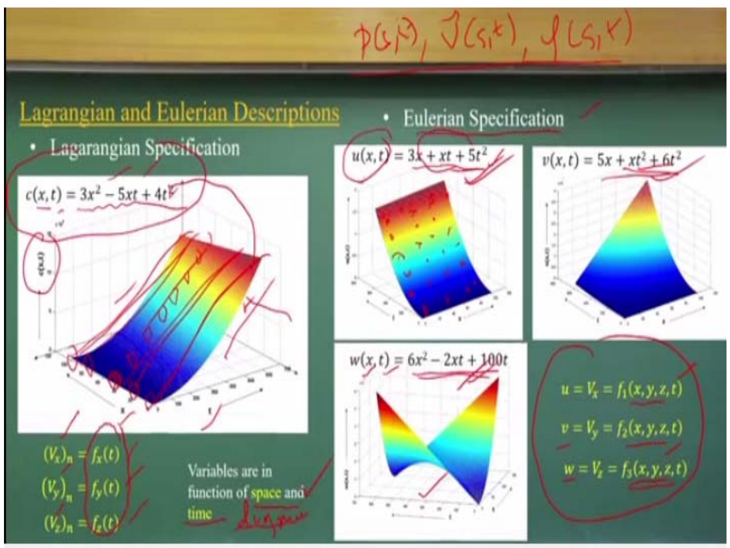

So, let look it, we do talk about this Lagrangian Euler descriptions okay, the most interesting things we should talk about the what I told you that I have the pressure okay, which varies with the positions and time, I have the velocity factors which is having varying with the positions and time and I have the densities which varies its positions at time, either Eulerian frames or the things but if you look it there is a space coordinate, there is a time coordinate.

Any variables; the velocity, the pressure, the accelerations all the component we should look it in a space time diagrams okay, I think many of the fluid mechanics books they do not tell it but I do encouraging you just think it the time is another dimensions as it x, y, z dimensions. As we have the space dimensions x, y, z for a Cartesian coordinate systems similarly, we have the time dimensions, always you plot the x, y, z and the t.

Then, you try to understand the process okay, we do not have a 4 dimensions representations but we can always think it that we can present the things in a space and the time diagrams, please try to understand I can describe this process of pressure field, velocity field, density field with in a space and time diagrams, okay that means there will be x and y coordinates, z coordinate and the time coordinate that is the 4 dimensional component, okay.

So, you try to understand that, that way we can define the flow and if you know it how does it varied this velocity, the pressure, the density, accelerations, then it is easy to visualize that so, always we should have a 3 dimensional prospective view of how does sorry; how do these fields vary and if you can understand that, then we can solve many of the problems that is the reasons I typically, I am putting these graphs of 3 dimensional graphs to you.

Let we just told it in the last slides that we measure the concentrations which varies with positions and the time okay, here this is position only the x coordinates along the x direction okay, so which can define is a function of x and t, the plot will be like this, just look it, okay. This is a function of x and t, the plot between x and the t and the c is varies like this, so this is the space time diagrams, this is what space time diagrams and plotting the concentrations.

Space time diagrams, where I am plotting the concentration, if I put it a particular virtual fluid balls as a boat is moving it, it has the velocity. As the fluid virtual fluid balls are moving with a different time, I will have the velocity in x, y, z directions I can write it; it is a functions of the time, is it correct? In a Lagrangian description or the virtual fluid ball concept, if I were rolling balls or the particles, I describe this the particles velocity component of Vx, Vy, Vz as a functions of a time.

$$(V_x)_n = f_x(t)$$
$$(V_y)_n = f_y(t)$$
$$(V_z)_n = f_z(t)$$

Because we are tracking the velocities of the particles so, if I have a n number of particles or the virtual balls, so I can get this f and those concentrations can satisfy these equations which will be the solutions what we are talking about, this is what the our solutions nothing else, so that way we can define the Lagrangian specifications imaging the space and time diagrams with the velocity components, the pressure components, the functions of time.

So, we are talking about the virtual fluid ball concepts in space and the time diagrams, how does they vary it and if I know that variations of the virtual fluid balls, which is will be functions of the time, may have the different functions with the time and that functions if I know it, I can know it how the C varies with respect to the space and the time, to demonstrate you, we have consider a one dimensional space in space coordinate systems and the time, okay that is easy to describe it.

But you can have a 4 dimensional figures and 5th dimensional figures which we do not have to visualize you but that is the reasons we have only the space coordinates in only one directions. Let me look at this Eulerian specification; Eulerian specifications what do we do it; we measured the velocity using the proofs and number of proofs in different points at the different locations.

As we have done it that and measure the velocities and if we take functions with the x and t diagram and that what is defined as the velocity field which varied with respect to x and t. So, if I have a the velocities having a 3 scalar component of u, v, w's and is varies with respect to x and the t, we can define their different functions, you can see it how complex things are happening it as you are going; boating from these functions to these function and these function.

$$u = V_x = f_1(x, y, z, t)$$
$$v = V_y = f_2(x, y, z, t)$$
$$w = V_z = f_3(x, y, z, t)$$

So you; if you just look at the shape of these functions is quite interesting, so that way you can build a very, very complex functions (()) (29:20) so but in this Eulerian frames, since we define this velocity; the u velocity component, the scalar velocity in the x direction is a functions of positions and time and the v is the positions and time and w will be the positions and time.

So, we define as a space functions and the time functions of u, v, w and as I demonstrated you just showing these 3 functions how the shape changes are the colour changes okay, so you can imagine it that if you have a 2 different variables, it changes the; your shape changes or your surface changes, so any complex fluid flow problems we can define through these functions.

Because as with the simple descriptions of this what is we need okay, that is the reasons I just encourage you as the students, please use the MATLAB code or Mathematica code which is available, use these different functions and try to look into functional behaviour, then visualise that, if you visualize that really you will appreciate the fluid mechanics and many of the mechanics if you talk about or the graphics, I think the tools today what is available to you as the MATLAB visualizers, Mathematica visualizers, please use that to just visualize a complex functions, how does it looks when it varies in x and t.

T, again I will tell it is a another variables, so with a 2 space, the variability is like this, if you talk about there will be variability in 3 dimensional, 4th dimensional and 5th dimensional and you can imagine it what completes shape of the figures we can get it, so I just encouraging you the students is please use the MATLAB visualizers, the mathematical visualizer, try different type of functions of these 2 variables, 3 variables, plot it, visualize it.

I think if you do that, you will appreciate the fluid mechanics, you will be engineers which is that perfectly know what type of shape it is okay that is the strength of an engineer's, it is not like what is there in the textbook but how do you visualize the equations or visualize the flow probe, fluid flow problems, thermal problems, you talk about any problems, all we can use the functions to define the field.

That is the reasons I encourage it please use MATLAB visualizers, the mathematical visualizers, use different type of functions with x and t, x and y, xyz, 3 variable, 4 variable plot it, see how does how do they really vary it and what completes the nature of the curve surface we get it; 2 dimensional, 3 dimensional surface that what my point to tell you.

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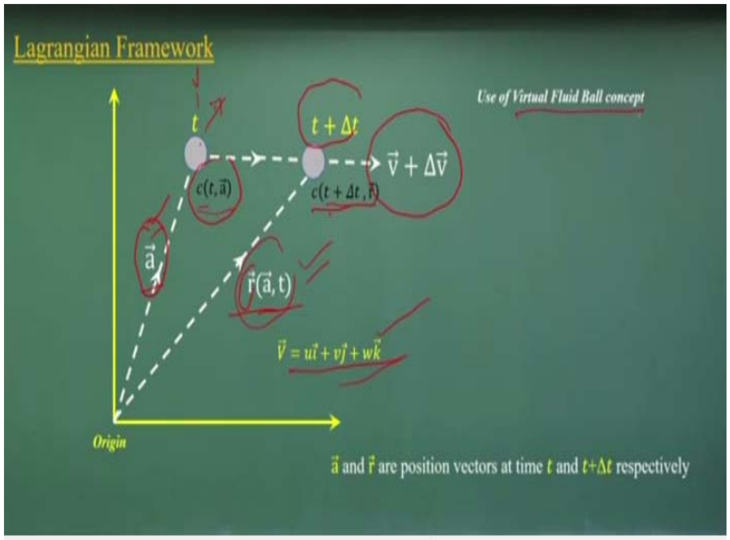

Now, coming back to the lagrangian frameworks okay, which is nothing else, which is the virtual fluid ball concept, so that means at a time t, I have a positions vectors where I know this the C which is varies with the t and this the a positions factors and the $t + \Delta t$ time, there is a change of; let me I consider the velocity v is here at this point this velocity changes, $\vec{v} + \Delta\vec{v}$ at the time equal to t+Δt, the concentration changes it.

It has a functions with the positions vector of $\vec{r}(\vec{a},\ t)$, the r is a positions vector is a depending upon accelerations and the time factors, so if you look in that way if I have the fluid particles, I can describe as a particles motions at time t and $t + \Delta t$, the concentrations of that the in that fluid particles may be dissolved oxygen, maybe BOD, whatever the component of the concentrations is that that what is vary from t, time to $t + \Delta t$.

So, this is what the position vectors, the particle movements at the $t + \Delta t$, we are describing to these concentrations varies from c and how these also depends upon the positions factors at the a, at the initial position vector and the $t + \Delta t$, we have a position vector and you have a the velocity vectors like this which is having the scalar components, so if you try to understand is that I have a, n number of virtual fluid balls are there.

And I on to track it, so the each fluid balls are having the velocity and as at the time t, it is having the v velocity at $t + \Delta t$, it will have a $\vec{v} + \Delta\vec{v}$, there is a change of the positions, there is the change of the velocity that means, there is a accelerations, there is a force components. So, if I were tracking of n number of virtual fluid balls that means, I am describing the velocity field, I am describing this the pressure field, I am describing the accelerations field, I am describe the density field that is and these things how do they vary with the positions factors that what we are illustrating it.

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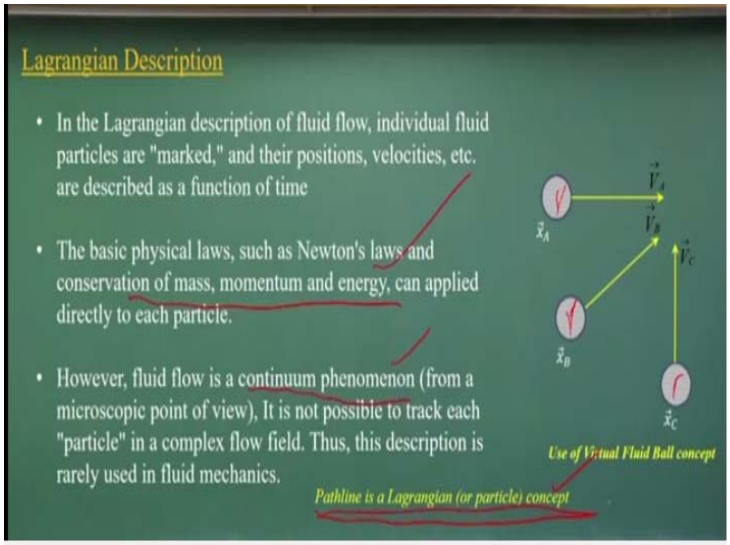

So, that what the Lagrangian framework concept with a virtual fluid balls now, if you talk about that as is told in many books that when you describe the Lagrangian, where the individual fluid particles will marked it positions, their velocity and these same things, I think I discuss in very beginning is that when you have a n balls should we consider and this would have a different velocity and their directions, okay Va and Vb is the velocity factors can have a different directions, can have a different magnitudes.

And can have move any directions, okay but here so that way you have a positions, their positions are changing it, the velocity is changing it, the pressure field is also changing it, that what we can find out to define this the flow descriptions that how the flow varies it, how the velocity varies, how pressure variations are there basically, when you do these in a Lagrangian framework, it is a very easy that mass conservations, and the particle levels, momentum conservations energy we can apply it very easily as compared to the Eulerian framework.

Because we know it, we track of the fluid particles but again I will tell that fluid flow is a continuum phenomenon, no doubt about that, that is what we discuss lot. What my idea to tell you that today because we have a so sophisticated fluid mechanics tools are available, these virtual fluid balls we can used to pressurize the flow because lot of the solutions are available from the CFD solvers, there is a high sophisticated instruments are nowadays available to measure the 3 dimensional velocity, pressure field.

But we need to now interpret it that data, that data we can interpreted into strengthen our flow visualization, how to standard that flow visualization, if I have a understanding how the virtual fluid balls moves it, if I understand that I really I can have a confidence to analyse complex fluid flow problems solutions what we get it from CFD solver, from advanced fluid mechanics labs having 3 dimensional velocity pressure measurements and all.

So, what I am to debate it here, it is okay in your textbook, it is written it the descriptions is not rarely used but I can say in present era we should use the virtual fluid ball concept to understand for. What is the virtual fluid ball? Again, if you talk about in terms of path line, streamliner, streak lines, it is a path line, we have a particular concept; the Lagrangian concept we already discuss about the path line, it is nothing else, it is the path line, okay.

It is a very simple thing, it is a path line, we are describing the path line of the Lagrangian concept as I saw you the Hele-Shaw experiments; it is a, these are the path lines we just; individual fluid particles we just dye it, give a colour to that and we track over that how it is going on that is what is the path line, so the Lagrangian concept is the path line that is what we follow it.

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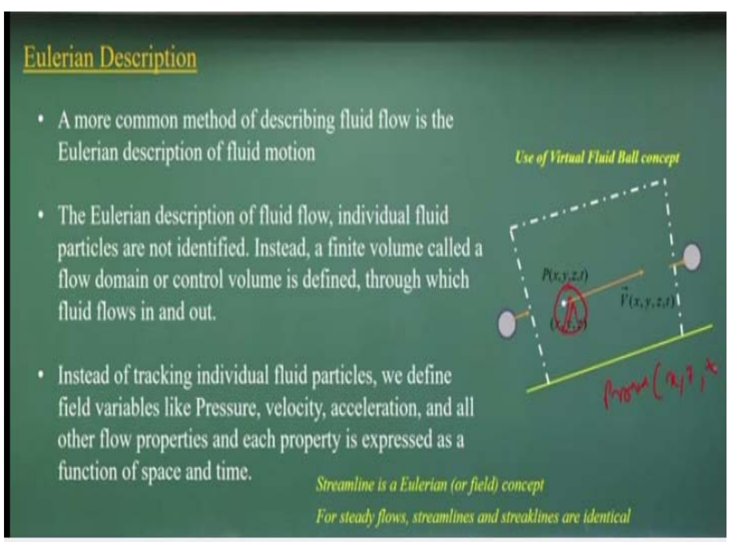

But in case of Eulerian concept as I said that we have the proof, okay and we that proof has positions P(x, y, z), where it is in terms of 3 dimensional.

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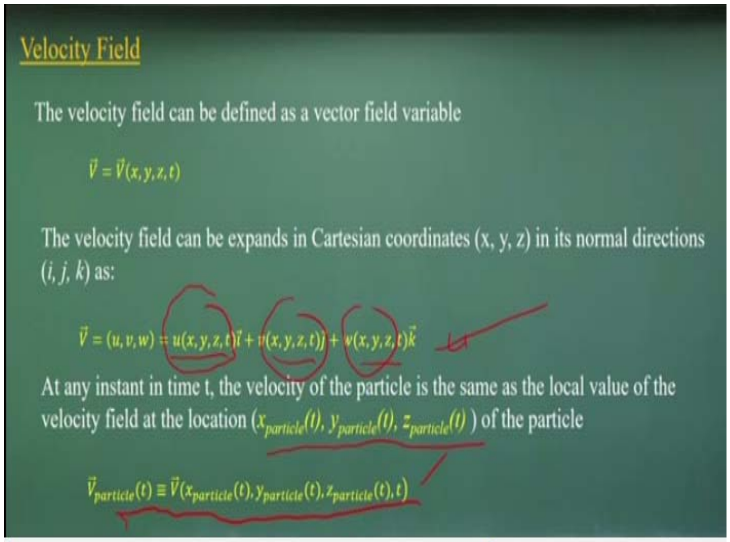

And then you have the time we measure, like in case of the velocity so, we define in terms of a scalar field of u, v, w,

$\vec{V} = \vec{V}(x, y, z, t)$, which varies with the positions and the time, these are the velocities scalar component in u directions, the velocity scalar component in v directions means, y direction and z directions that what having a functions with the positions and the time, this is what the velocity field what we have described it, either you can get it from CFD solvers or you can get it from very advanced fluid mechanics with a measurement facilities, okay.

$$\vec{V} = (u, v, w) = u(x, y, z, t)\vec{i} + v(x, y, z, t)\vec{j} + w(x, y, z, t)\vec{k}$$

At any instant in time t, the velocity of the particle is the same as the local value of the velocity field at the location ($x_{particle}(t)$, $y_{particle}(t)$, $z_{particle}(t)$) of the particle

$$\vec{V}_{particle}(t) \equiv \vec{V}(x_{particle}(t), y_{particle}(t), z_{particle}(t), t)$$

But if you take it, you take virtual fluid balls or the particles that means, you track upon that the particles that is what we described it, is the velocity is a particle velocity at the time how it is varies.

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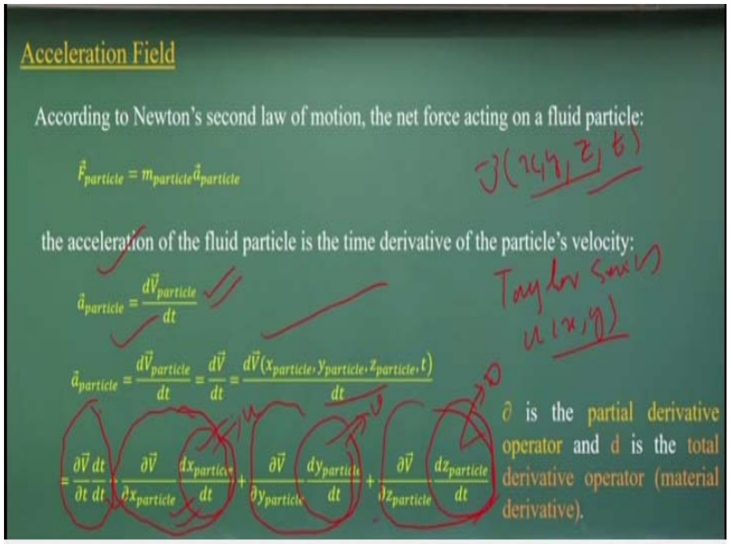

So, similar way if I am looking at Newton's second law also see that force is equal to mass into acceleration, force is vector component, acceleration is the vector component, okay and both are the parallels okay, so force and the vector, at the particle levels like in solid mechanics, the force we can put is mass into acceleration.

$$\vec{F}_{particle} = m_{particle}\vec{a}_{particle}$$

Now, let us look it at the particle levels, if I had to find out what is the acceleration of the fluid particles?

Is nothing else is a time derivative of velocity of the particles, as you know it from class 10, 11th and 12th I am just doing this time derivative of the velocity; particle velocities with respect to time and that is what represents the accelerations, at the particles levels you will have these. Now, if you look at this the velocities has a variability in a positions and the time because of that though when you define a; derivative with respect to time you will have a local component okay, you will have a with a x particle directions, y particle directions and z particle direction.

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt}$$

This is nothing else if you are considering is a 2 variables like I just discussed you the Taylor series, if you remember it defining for the 2 variables in this case, I have a Taylor series of 4 variables the x, y, z and the t. If you expand it and take it only these 4 terms, you will get this component nothing else, we are near to the same Taylor series concept what we discussed in a single variables, independent variables, how the Taylor series expansion, the 2 independent variables how the Taylor series expansion.

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt} = \frac{d\vec{V}}{dt} = \frac{d\vec{V}(x_{particle}, y_{particle}, z_{particle}, t)}{dt}$$

$$= \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x_{particle}}\frac{dx_{particle}}{dt} + \frac{\partial \vec{V}}{\partial y_{particle}}\frac{dy_{particle}}{dt} + \frac{\partial \vec{V}}{\partial z_{particle}}\frac{dz_{particle}}{dt}$$

If you do that, you will have a the same concept what is there and by the dt, you will get these component, so mathematically we are not things you just try to understand it that the same Taylor series we have applied it but the v is a having the independent the variable 3 in a space and the time that is a reason you have the length d resistance, so if we look at this way, the $\frac{dx_{particle}}{dt}$, what is that?

The velocity in the u directions, $\frac{dy_{particle}}{dt}$, what is that; the velocities in v directions, $\frac{dz_{particle}}{dt}$, the particles; the change in the z direction without or times will be the; which is the definitions of the velocity.

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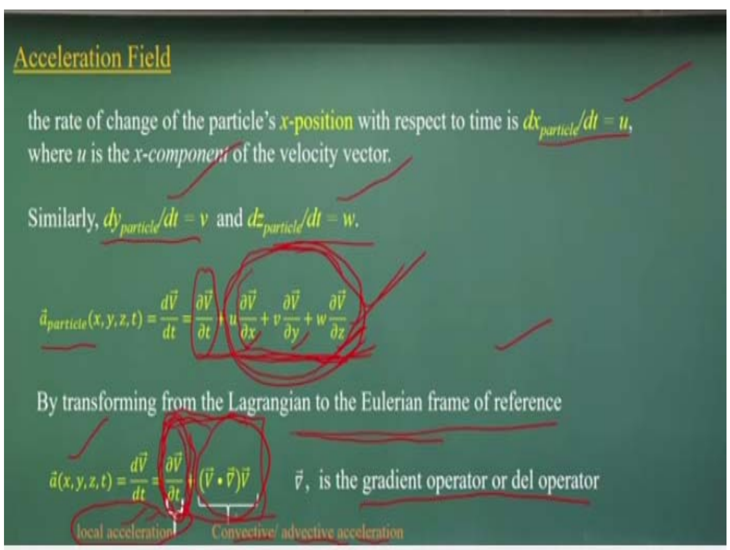

So, if I put it that I will get it the accelerations fields as I explaining it that, I will have a particle, the change of the position the particles in the x direction with respect to time will give you the velocity component in that directions, so this is for x component, so y component, z component, so the finally this accelerations will have this form, accelerations will have this form.

the rate of change of the particle's *x*-position with respect to time is $dx_{particle}/dt = u$,

where *u* is the *x-component* of the velocity vector.

Similarly, $dy_{particle}/dt = v$ and $dz_{particle}/dt = w$.

$$\vec{a}_{particle}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial t} + v\frac{\partial \vec{V}}{\partial t} + w\frac{\partial \vec{V}}{\partial t}$$

So, by transferring this what do we do it actually, when you talk about either particles of large number particles or you talk about the probes, both are the same in the both frame of reference, you will get it as the same things whether you particle tracking, the probing, you will have the acceleration speed like the same at the particles and the Euler frame of reference, you are getting it these accelerations will have a 2 component.

This component and this component, if you look at expanding forms, this is what vector calibration, it is okay, it is called del operators, okay between the V and delta that is what the del; the dot product of 2 vectors okay that is what is represented this ones, if you remember it the dot product of 2 vectors, okay. So, if you look at that what is there; these the velocity is changing with respect to time and it is given is the partial derivative.

$$\vec{a}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \bullet \vec{\nabla})\vec{V}$$

$\vec{\nabla} =$ is the gradient operator or del operator

Convective/ advective acceleration

$$(\vec{V} \bullet \vec{\nabla})\vec{V}$$

local acceleration

$$\frac{\partial \vec{V}}{\partial t}$$

That means, with respect to time, how much of variations and when you are considering that you do not consider the variability in the x and y, z that what is called the local accelerations but this component if you look it, it is called convective or advective accelertaion that because the particles are having different velocity gradient okay, since they have a different velocity gradient, so let us look at this component of acceleration and particles or the Eulerian and Lagrangian frames, they will be the same.

If you look at that this is what is called the local accelerations, the velocity changes as with respect to the time only, so that means at the probe equations because of only the velocity changes at that locations will give us the local accelerations. Velocity is also changes because of change of the velocity field that is the velocity variances x gradient, y gradient, z gradient and the u, v, w component.

These components change of the velocities is called convective or advective accelerations component, so this is 2 components; one is local acceleration component and other is convective acceleration. In vector (()) (45:37), we can very simple way represented the local accelerations and the convective acceleration component.

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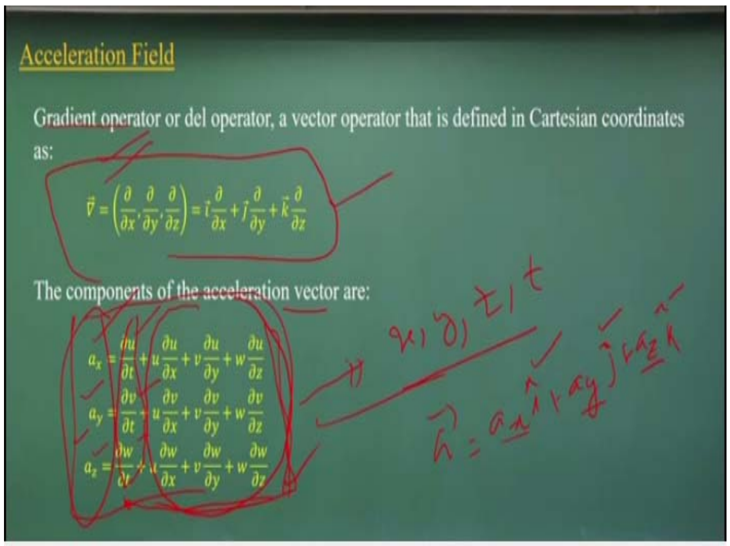

Now, if you look at these same acceleration fields, if I looking it as a delta operators or the gradient operator, you will know it we can define it as i, j functions like this

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

and the accelerations component; the scalar component of accelerations in the x direction, y and z direction can have it like this. So, what we have done it we just put it the accelerations,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

So, this is the convective acceleration term and this is what we have local acceleration term. In a Cartesian coordinate systems, where we have a x, y, z and the t and these are showing these accelerations which will be

$$\vec{a} = a_x i + a_y j + a_z k$$

so these are the acceleration scalar components in i, j and k direction respectively. So, we can define it the accelerations component that means, this equation shows that if I know the velocity components and the velocity partial derivative components, then we can know it what is the acceleration component.

So, we can know accelerations variability in the space and the time; space and the time, if I know this velocity variability, the partial derivative of velocity with respect to the space x, y, z, also the time variability of this velocity with respect to time, if I know this component, I can compare this scalar component using this equation, you can easily remember this equations which is just a replace of u, v, w, it is nothing else representing the a_x, a_y, a_z component.

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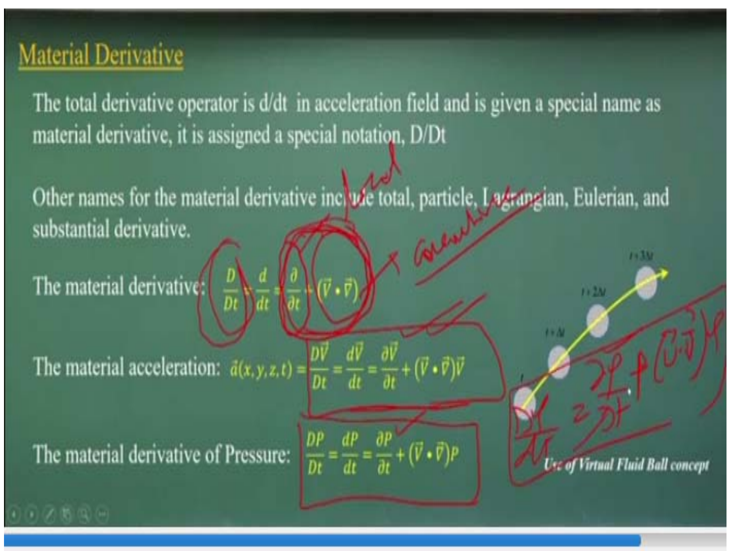

But many of the times people talk about material derivative, it is nothing else, it is that the change it is a special name of material derivative means, when you talk about a particles along the particles, you compute the derivative with respect to time is a total derivative or the material derivative or the particle derivatives is nothing else, we already proved it that these derivatives we can define as these functions that is what we have defined it, okay.

The material derivative:

$$\frac{D}{Dt} = \frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{V} \bullet \vec{\nabla})$$

So, that means at the particles levels how it is varies with the time; it has a 2 component and the Eulerian frames; one is the local component and other is convective component, those the gradient we have represented like this, is a particle derivatives that means, it is a the derivative of a particles are moving it, material derivative, particle derivative all are the same, the derivative as we are getting of the velocity field and pressure field as I see that if I have a series of the boats okay, I measure how does it changes it.

That what is my total derivative compound, okay but if I am looking it in Eulerian frames which will be this local derivative and convective terms; local derivative; local and convective term this because of velocity gradients in a space domain that is what we represent the accelerations. The similar way if I talk the particles, it has some pressures, how these particles derivative changes with the time, I can define like this.

The material acceleration:

$$\vec{a}(x, y, z, t) = \frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \bullet \vec{\nabla}\right)\vec{V}$$

If my particles having the density as I earlier say that particles can have the density, my virtual fluid balls can have a different density, if I define that I can define with a material derivative of the density,

$$\frac{D\rho}{Dt} = \frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \left(\vec{V} \bullet \vec{\nabla}\right)\rho$$

so if you look at that either is a density, the pressure or the velocity all the things we can define in terms of particle derivative forms or the material derivative form.

The material derivative of Pressure

$$\frac{DP}{Dt} = \frac{dP}{dt} = \frac{\partial P}{\partial t} + \left(\vec{V} \bullet \vec{\nabla}\right)P$$

Some people tell different names but all are the same, we are talking about in terms of Lagrangian framework to link it with the Eulerian frameworks which is having a local and convective terms, so that is the a breaching between these derivative component between the Eulerian frameworks and the Lagrangian frameworks. The virtual fluid ball concepts and that is what is link it as I said it.

So, as this fluid having this velocity field, we can compute the accelerations field, we can compute this material derivative of the pressures, we can compute the material derivative of density with the same format, we can compute and we can solve the problems in this way.

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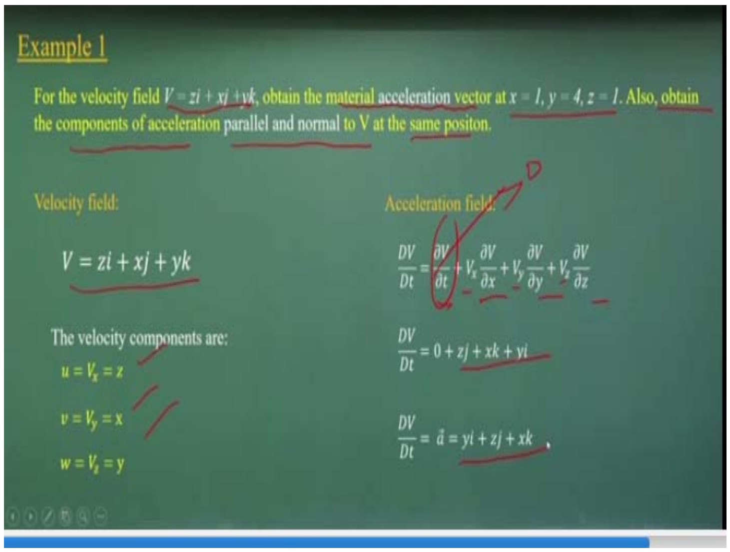

Now, let us solve this very few 2 examples problems; one is that the velocities field is given to us to compute the material accelerations factors okay, as we discuss it and the positions has given it, after that you obtain the component of acceleration is parallel normal to this V, okay so, let us solve the first problem is that, that means I know the velocity field, I know this u, v, w components, okay.

[For the velocity field $V = zi + xj + yk$, obtain the material acceleration vector at $x = 1, y = 4, z = 1$. Also, obtain the components of acceleration parallel and normal to V at the same positon]

Velocity field:

$$V = zi + xj + yk$$

The velocity components are:

$$u = V_x = z$$

$$v = V_y = \text{x}$$

$$w = V_z = \text{y}$$

Acceleration field:

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + V_x \frac{\partial V}{\partial x} + V_y \frac{\partial V}{\partial y} + V_z \frac{\partial V}{\partial z}$$

$$\frac{DV}{Dt} = 0 + zj + xk + yi$$

$$\frac{DV}{Dt} = \vec{a} = yi + zj + xk$$

I will do this partial derivative, substitute the Vx, Vy, Vz and the time derivative, there is no time components okay, so there is these components becomes 0, velocity does not have any time component that is become 0, so we just substitute it and you will get these accelerations as this one.

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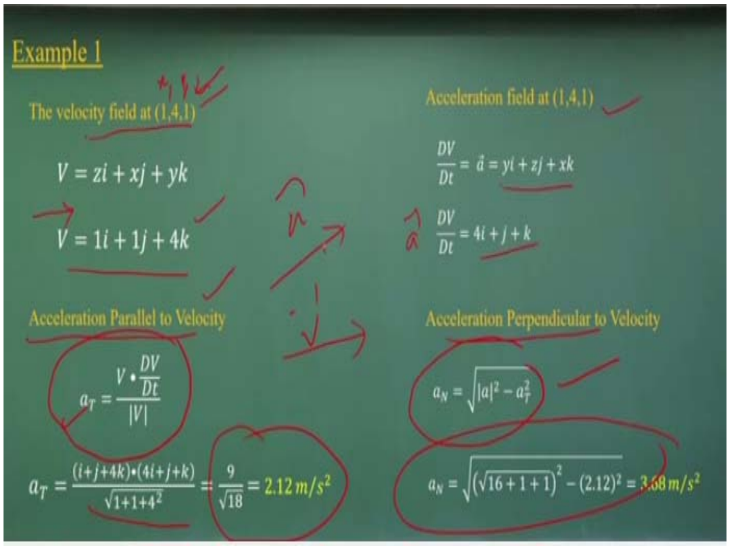

Then you substitute the; at the velocity field this 1, 4, 1, this x position, y and the z we just substitute it to get this velocity factors, the similar way we get this acceleration as we substitute the value. Now, what is telling that you find out the component of accelerations which is parallel to the velocity that means, we should do the dot product by magnitudes okay, you just try to understand the few vectors relationship, how to compute the component which is parallel to the velocity field.

The velocity field at (1,4,1)

$$V = zi + xj + yk$$
$$V = 1i + 1j + 4k$$

Acceleration Parallel to Velocity

$$a_T = \frac{V \bullet \frac{DV}{Dt}}{|V|}$$

$a_T = \frac{(i+j+4k)\bullet(4i+j+k)}{\sqrt{1+1+4^2}} = \frac{9}{\sqrt{18}} = 2.12\, m/s^2$

You will have this component and the acceleration perpendicular to these ones, I think you follow the any vector calculus book, you can find out how to; if I have a 2 vector velocity and acceleration vectors how to find out the accelerations which is a parallel to the velocity and what is the acceleration component parallel to this velocity okay, there is a 2 vectors now okay, one is acceleration factor and other is velocity vector.

Acceleration field at (1,4,1)

$$\frac{DV}{Dt} = \vec{a} = yi + zj + xk$$

$$\frac{DV}{Dt} = 4i + j + k$$

Acceleration Perpendicular to Velocity

$$a_N = \sqrt{|a|^2 - a_T^2}$$

$$a_N = \sqrt{\left(\sqrt{16+1+1}\right)^2 - (2.12)^2} = 3.68\, m/s^2$$

Next questions asking is that what is accelerations parallel to the velocity that the components what will get it here and what is that acceleration component; it is perpendicular to this that is what you just follow vector algebra, okay.

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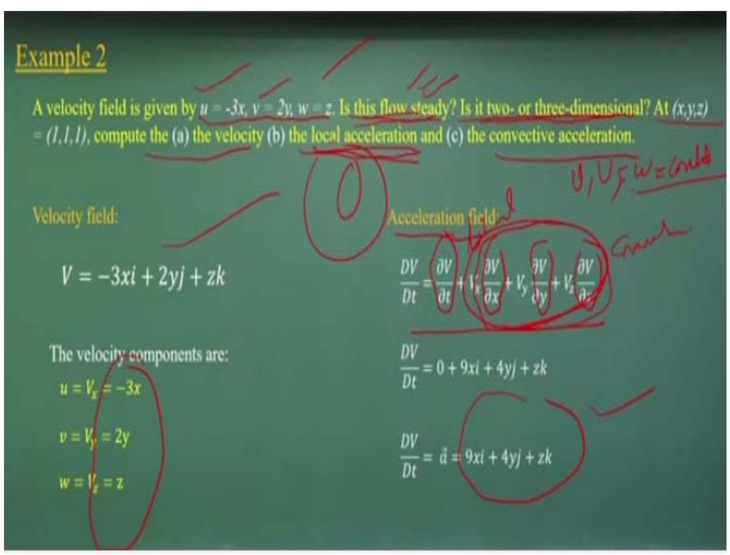

[A velocity field is given by *u = -3x, v = 2y, w = z*. Is this flow steady? Is it two- or three-dimensional? At *(x,y,z) = (1,1,1)*, compute the (a) the velocity (b) the local acceleration and (c) the convective acceleration.]

Now, talk about another very simple interesting problem that the velocity field is given to us okay, v, w, small v, u all it is given to us, is this flow steady, you can look it, there is no time components, so we can say this flow is steady, it is very straight forward. Is it a 2 dimensional, 3 dimensional; you just look it, there is a x component, y component, z component, so it is a 3 dimensional in terms of the positions.

Now, let x, y, z compute the velocity, which is very easy, local accelerations convective acceleration, so this is very easy problems as compared to previous ones, so you know the velocity field, you know these u, v, w components, you know these accelerations which this is part is local accelerations, this part is convective acceleration, you as we say these flow is steady without doing any calculations, we can say that local acceleration is 0.
Velocity field:

$$V = -3xi + 2yj + zk$$

The velocity components are:

$$u = V_x = -3x$$

$$v = V_y = 2\text{y}$$

$$w = V_z = \text{z}$$

Because there is no time component, if this flow is steady, the local acceleration becomes 0, we need not to do any calculations, with some common sense we can say that when you have the steady flow, local accelerations becomes 0 because your velocity field does not have a time component, it is independent of time. If it is that, there will be no local acceleration component, there will be convective accelerations component will be there.

So, what do we get it is a convective acceleration that is what we solve it and get these acceleration similarly, if you look at if velocity does not vary, these are all this gradient, if velocity does not vary in x, y, and z direction, my convective acceleration should be 0, so you just imagined it what is the velocity field is given to you, as I told earlier is that try to visualize the fluid flow by drawing these 3 dimensional figures in Matlab mathematica, any you have a lot of resources nowadays to have a 3 dimensional plots okay.

Even if in a Microsoft Excel, we can have a 3 dimensional plots, so making 3 dimensional plot and try to understand how do they varied, as I told you that when you compute the accelerations field, if the problems is lacking is the local acceleration is there or not, first look at whether the flow is steady or unsteady, if it is steady it independent to the time, there is no time component in u, v, w, we can say the steady flow.

Acceleration field:

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + V_x \frac{\partial V}{\partial x} + V_y \frac{\partial V}{\partial y} + V_z \frac{\partial V}{\partial z}$$

$$\frac{DV}{Dt} = 0 + 9xi + 4yj + zk$$

$$\frac{DV}{Dt} = \vec{a} = 9xi + 4yj + zk$$

And we can say that local acceleration should be 0, only it is a convective acceleration, similar way if my u, v, w all are constant, they do not vary with the time, all are constant, the convective acceleration becomes 0. So, you can visualize this thing, the problems of the fluid mechanics are too easy but you try to understand the functions how do they vary.

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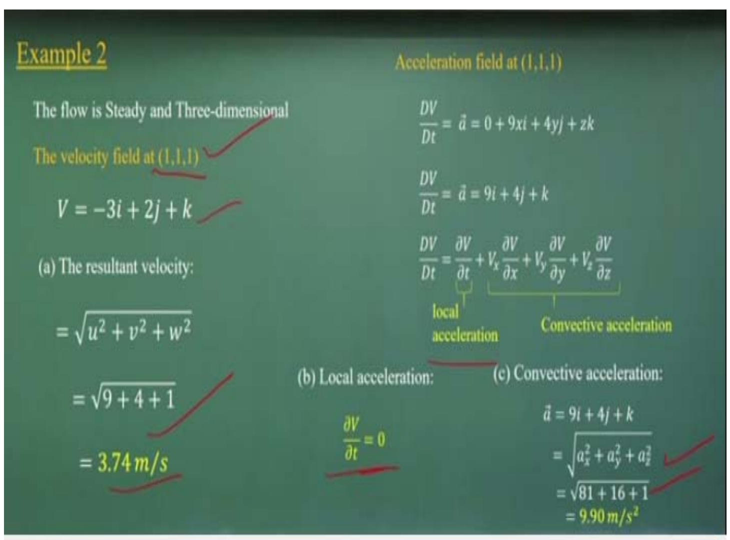

If you understand that we can use these simple mathematics to solve these things now, to compute the velocity field and acceleration field, we just substitute the velocity; at the positions 1 and 1, what should be that and what should be the resultant velocities as you know from vector calculations similar way, these local accelerations components also and convective accelerations component we can compute it.

The flow is Steady and Three-dimensional

The velocity field at (1,1,1)

$$V = -3i + 2j + k$$

(a) The resultant velocity:

$$= \sqrt{u^2 + v^2 + w^2}$$

$$= \sqrt{9 + 4 + 1}$$

$$= 3.74\, m/s$$

Acceleration field at (1,1,1)

$$\frac{DV}{Dt} = \vec{a} = 0 + 9xi + 4yj + zk$$

$$\frac{DV}{Dt} = \vec{a} = 9i + 4j + k$$

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + V_x \frac{\partial V}{\partial x} + V_y \frac{\partial V}{\partial y} + V_z \frac{\partial V}{\partial z}$$

b) Local acceleration

$$\frac{\partial V}{\partial t} = 0$$

(c) Convective acceleration:

$$\vec{a} = 9i + 4j + k$$

$$= \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$= \sqrt{81 + 16 + 1}$$

$$= 9.90\, m/s^2$$

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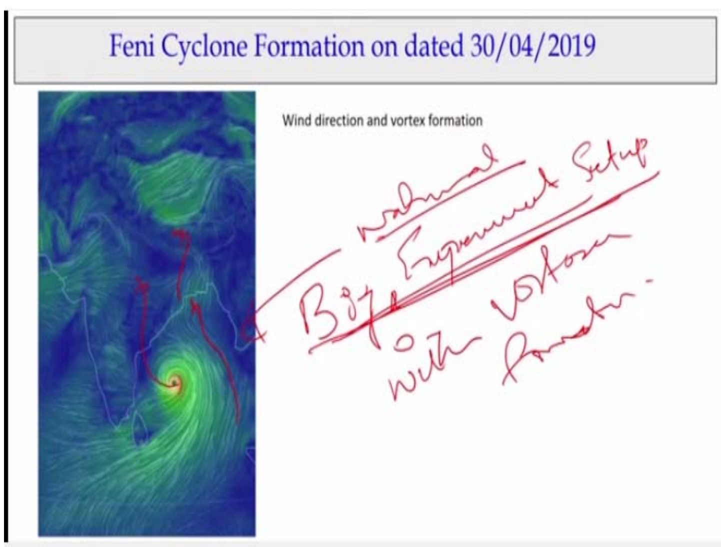

So, we can have a references this, with this let us conclude today lectures showing you very interesting photographs of today photographs which is available in Internet, today is 30th April

2019, okay you can see there was cyclone formations happened in Bay of Bengal wind directions and vortex formation, is it not very interesting; we talked about a small Hele-Shaw experiment set up, it is a big experiment set up, big, big is it correct; very big experiment set up of with vortex formation.

And these cyclones we are predicting it, we are tracking it what could be the cycle, you see these figures, you try to understand how complex flow patterns happens during the cyclone formations, how the vortex pendants, how the wind directions are changing it, so we have a big experiment, natural experiment setups is a cyclone formation, okay not this small experimental setup but the Hele-Shaw experiment setup with a colour dye experiment.

But todays we are fortunate enough to see the process at different scale, smaller scales to much, much bigger scale, with this let me finish these lectures with just showing you the summaries as I told it earlier.

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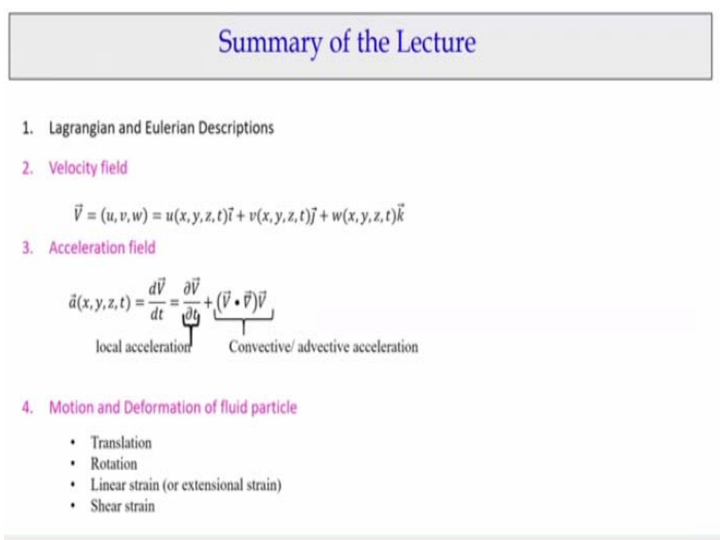

And today lecture, I just spend a lot of time you to just understand what is the difference between Lagrangian and the Eulerian descriptions and we talk about the velocity field and acceleration field.

1. Velocity field

$$\vec{V} = (u, v, w) = u(x, y, z, t)\vec{i} + v(x, y, z, t)\vec{j} + w(x, y, z, t)\vec{k}$$

2. Acceleration field

$$\vec{a}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \bullet \vec{\nabla})\vec{V}$$

3. Motion and Deformation of fluid particle

    Translation

    Rotation

    Linear strain (or extensional strain)

    Shear strain

The next class we talk about more about the motion and deformations of fluid particles, vortex, vorticities, all we will discuss using virtual fluid balls, thank you a lot for this.