Fluid Mechanics

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Lecture - 14
Conservation of Momentum: Example Problems

Welcome all of you for this course on fluid mechanics. In the last class we discussed about conservation of mass and the momentum and its applications. To continue to that conservation of mass and momentum and its applications, today I will deliver lecture on this topic and also I will solve some example problems to illustrate it how we can use conservation of mass and momentum equations to solve real life problems.

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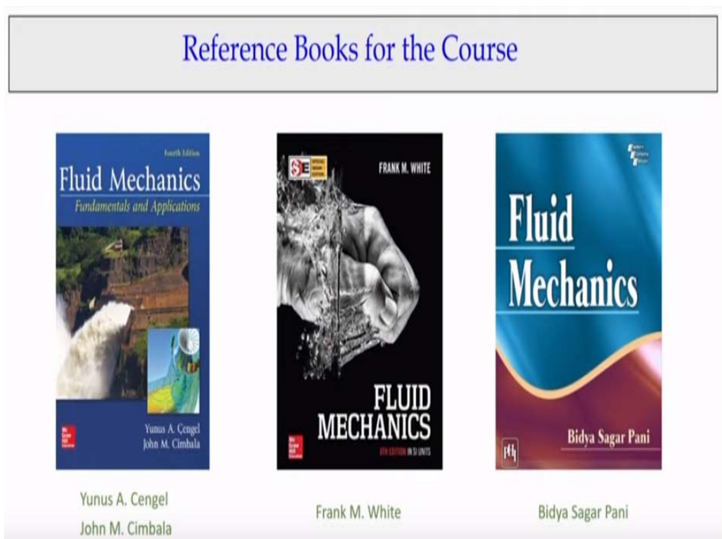

Again, I can talk about this that some of the examples the level of the fluid mechanics what I have been teaching it, its level of the fluid mechanics fundamental and applications, Cengel, Cimbala or close to the Fluid Mechanics by F.M. White.

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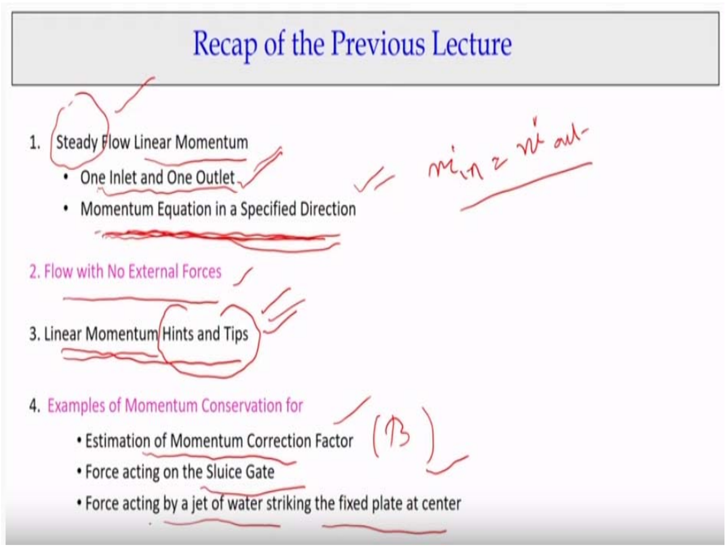

Let us come back to what we discussed in the previous class that we discussed that how we can approximate linear momentum equations from Reynolds transport theorems to a specific cases like one inlet, one outlet, which is very simplified problems, when you have a one inlet and one outlet. And second thing is that is you know it the momentum equation is vector equations.

$$m_{in} = m_{out}$$

But many of the case we can solve the problems in a specified directions or equating momentum equation in a specified direction. Then we can solve the problems. So we can align the specific axis rotation in such a way that using only one equation we can solve the problems. So these are the simplifications that the problems where we have one inlet and one outlet.

And the momentum equations we can apply for a specific directions. Most important qualities that we consider always is steady flow conditions. This is the approximations what we do it, the steady flow conditions where there is no change of the pressure or the velocity distributions with respect to the time. So it is a steady condition. We apply the steady linear momentum equations.

Again you simplified for the one inlet one outlet case, where the mass inflow will be equal to mass outflow. So this is very simplified case we will get it. Similar way, when I apply the momentum equation in specific directions that means, as you know it the momentum equations, we can write in three scalar components in x, y, z for Cartesian coordinate system.

So that way we can apply the momentum equations for the polar coordinate systems also. So if you look at that way, we can apply the momentum equations in a specific direction such a way that we can solve the problems. Flow with no external forces like a spacecraft, there is no external forces are there. In that case, what simplifications we do it.

Then we talk about when you apply this linear momentum equations, we should follow hints and the tips like we follow the free body diagrams in solid mechanics, whenever you apply the linear momentum equations, you have to draw control volume, the control surfaces. How to define this control volume and control surface whether it is a fixed control or movable control, how should be the control surface should be there.

That should be a major emphasis when you draw a control volume and the control surfaces and that hints we should consider it how these pressure distributions and velocity distributions we should assume it or we can have a proper assumption which is valid for that problems, that we should highlight it. In the last class we solved many problems like force acting on the Sluice gate, force acting a jet water striking the fixed plate at the centers.

Also we talk about the momentum corrections factors that means beta. How do you do you compute the beta value which is the momentum correction factors.

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Now let us come to today lectures what I will talk about that. We will discuss many things with a starting with a flow structures in hydraulic jump. Then again I will talk about or repeat the linear momentum hints and tips. That is what again I will repeat it. Again we will going to solve some of the previous GATE questions or example problems based on linear momentum and mass conservation equations.

Flow with no external forces, I will repeat that part. Then we will solve one examples, cases where no external forces is there specifically the spacecraft problems that what we will solve it and end of the day we will have a summary for these lectures which is more theoretical.

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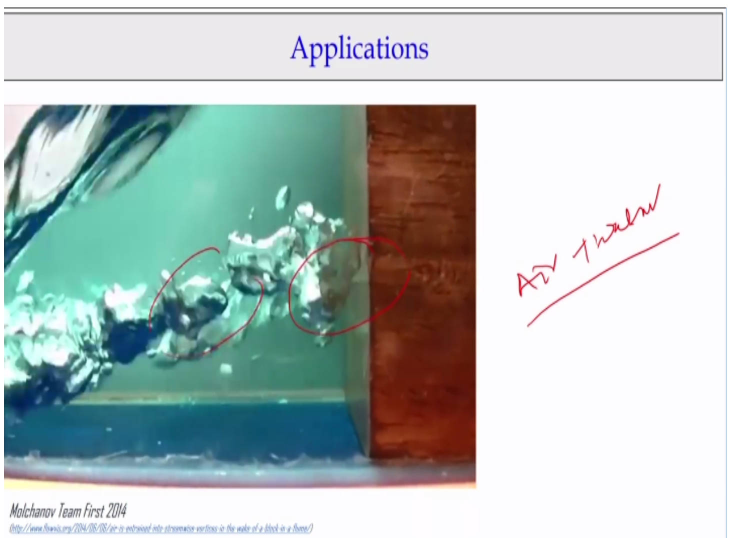

Now if you look, it is very interesting photographs what is there. It is available, you just type the flow visualizations. It is a great things are nowadays available if you just type the flow visualizations in any Google search engine you can see this type of flow visualization tools are available. If you look it what I am going to show from this photographs, it is clearly indicating that if I have flow jet and there is a mixing of air, there is mixing of air and the water okay, how the things are changing it.

If you look at that, the bubbles what is representing it here, those are all are the air bubbles. So and if this is the water jet, there is formations of water and air bubbles mixing with water. So interesting pictures are coming up and if you just visualize the flow, how it is very mixing of air and waters and that what change the flow systems.

So with this just a flow visualization tool let us come back to the our applications of linear momentum equations. Again, I am repeating this part of the tips of linear momentum applications, what are the tips are necessary to do that.

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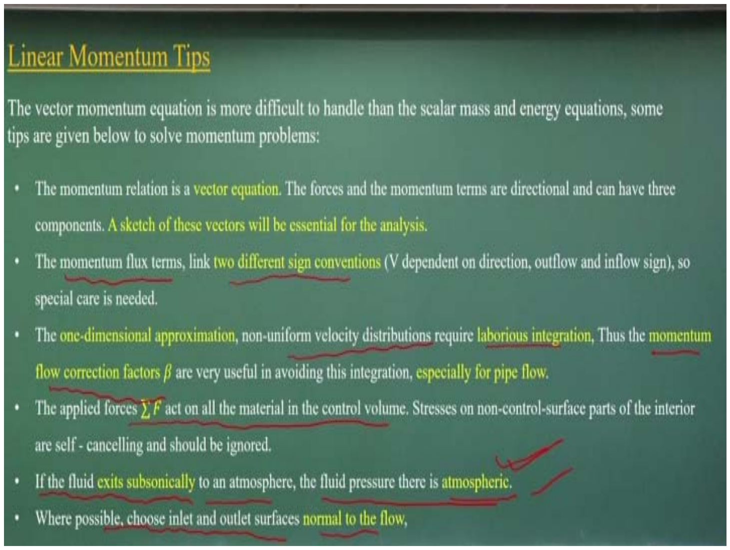

One thing you should remember it this momentum relationship what you get it is a vector equations. That means it has three scalar component, x component, y component and z component. We have the velocity vectors in the three component, V x, V y, V z. So similar way we have the force factors, which have three components. The momentum relationship what we have also the three components.

But many of the time we reserve it into the scalar components like writing the momentum occasions in x directions, y directions or the z directions, we do that. Second thing what I am to highlight is that momentum flux terms. The most of the times when you compute the momentum flux terms, we should look it what is the relationship, what is the angle between velocity vectors and the normal vector to the control surface.

If the scalar product of these two vectors is positive sign, that means it will have a positive flux is coming into the control volume. If it is a negative sign, it indicates it is going out from the control volume. So that is the scalar product of the velocity vectors and the normal vectors to the control surface. That what to consider. Always you scale the velocity vectors and the normal vectors then try to find out what is the angle between them.

Most of the times what you make it these two surface should have a such a way that the theta the angle between these two vectors should be zero or the pi, okay 180 degree. So it will be easy for us to do vector product. So that way we get either a positive or the negative sign convections. And most of the times if you look it the flow are not uniform velocity distributions. It is non uniform velocity distributions.

Because of that, we should have the momentum flow correction factor beta because of the velocity distributions is not uniform. But many of the time we simplified it and say that the beta is equal to the 1. We consider it that the flow distribution is uniform and we do the or we solve the problems.

But if you try to understand it sometimes we can use the beta value, the momentum flow correction factors when it will be much larger than beta equal to 1. We can use that for especially for the pipe flow. When beta is close to 1, it may help us indirectly not to consider the velocity distributions. But when beta is more than the 1 we should consider the beta values to find out approximated how much the momentum clocks going through these control surface.

Similar way when you apply this control volume concept, within the control volumes we do not talk about that as it can be considered is a trace field and that can self canceling each others. The mostly we would consider the control volume gross characteristics. Within the control volume we do not consider it how the flow variations are there, the pressure variations and velocity variations or the density variations we do not consider that inside the control volume.

And this is what very good it is not atmospheric is a quite valid conditions when you have the fluid exits to an atmospheric and the fluid flow is subsonic. Then we can assume it the fluid pressures at that locations is atmospheric pressures. These assumptions is quite valid. So many of the times we use whenever the flow jets are going to the atmospheres and we anticipated is that the flow is subsonic.

That means the flow Mach number is less than 1. In that case, we can have the fluid pressures we can assume it or it is quite valid the atmospheric pressures. And the to appropriate inlet outlet surface, generally we take it flow normal to that. The otherwise we can solve the problems using the scalar product of the velocity and the normal vectors. It is possible to do it, but it will be laborious to do the integration part or it takes time to solve the problems.

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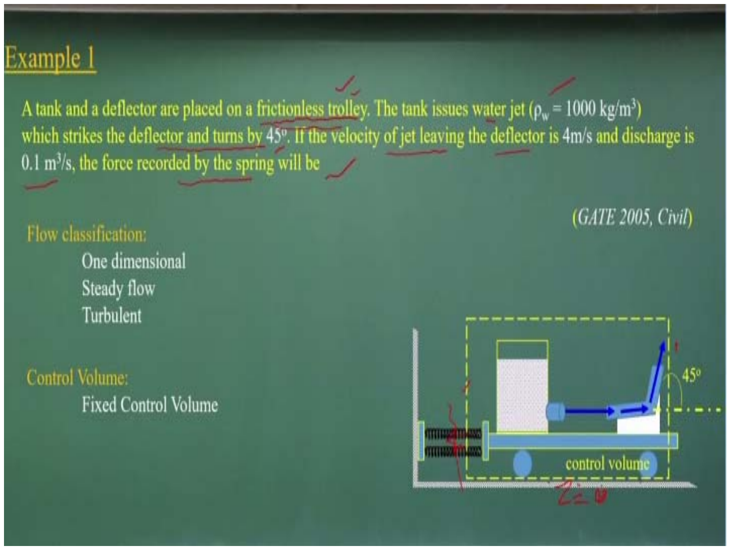

Now, let us come it to solve example one which is GATE 2005 civil engineering problems. The problem is that there is a tank or deflectors are placed in a frictionless trolley. Okay, there is no frictions components, okay. The water issues the water jet which is have a the density of the water is 1000 kg per meter cube. It strike to a deflectors, turn 45 degree.

[A tank and a deflector are placed on a frictionless trolley. The tank issues water jet (ρ_w = 1000 kg/m^3)
which strikes the deflector and turns by 45°. If the velocity of jet leaving the deflector is 4m/s and discharge is 0.1 m^3/s, the force recorded by the spring will be]

If velocity of jet leaving the deflector is 4 meter per second and discharge is equal 0.1 meter cube per second, what could be the force record by the spring, that is the problem. So if you look at that that is what we have sketched it here. There is a tank which is having the waters and getting jet of the waters from this tank and there is a frictionless trolley that means there is no frictional force acting on this.

The problem is quite simplified for us. And there is a deflector which is making a 45 degrees to turn these things. We need to compute it, how much force is going to act on the spring. That is the problem. It is very easy problem, we can solve it. But let us follow very systematic approach what I discussed in the last class, we will go step by step. First step is that to classify the problems.

Flow classification:

One dimensional

Steady flow

Turbulent

These problems is one dimensional flow and steady flow because the discharge what is coming out from this jet can consider is a steady. Flow will be the turbulent and consider a fixed control volume okay. That is what the control volumes I have consider it. So the force what is acting here that is a component here. Other the locations if you look at this water jet is going out from this.

Other locations we can take it the pressure equal to the atmospheric pressures. And here the shear stress is equal to zero because frictionless trolley. So there is no friction force is acting on this. So it will becomes zero. So there is no force component is here due to the friction. So water jet that component will come it. That is what the water jet is coming out from this. So we have chosen the control volume, fixed control volume. Our object is to compute what is the force recorded by the spring.

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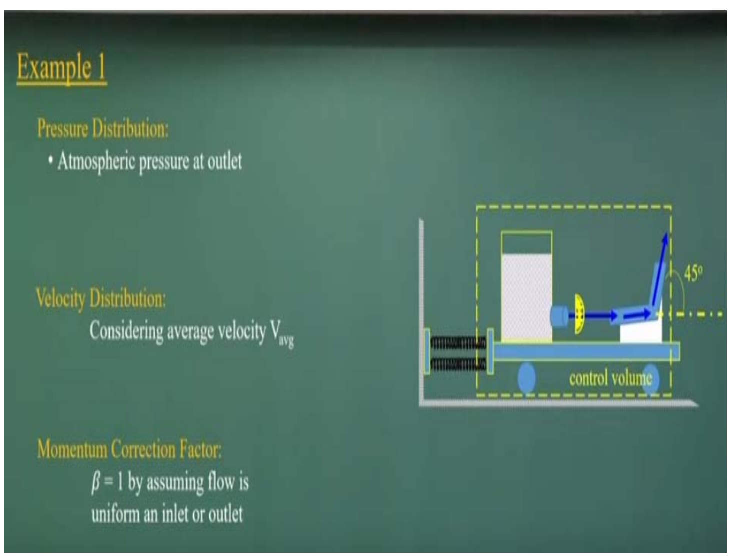

So the pressure distributions at the outlet will be the atmospheric pressure. Here also we have consider the velocity distributions again average velocity. No doubt there will be a velocity distributions will come it here. After deflectors also velocity distribution will come it, but we here assume it is uniform velocity and the beta will be the one value for this.

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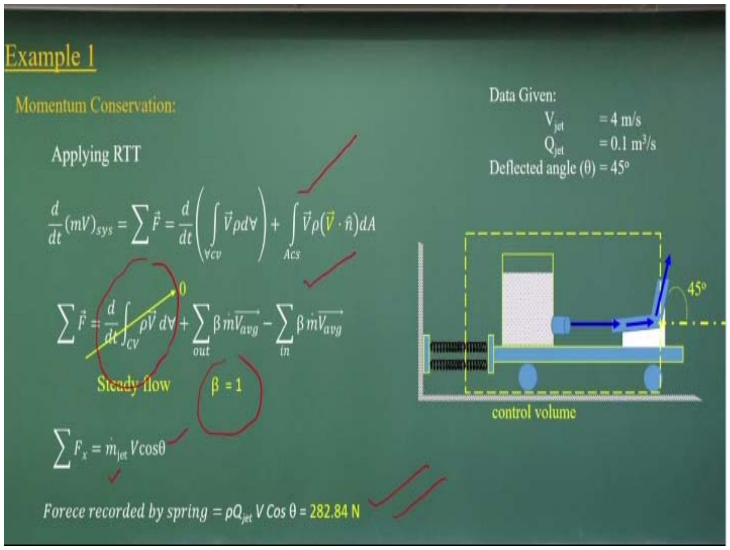

Now let us apply the directly the momentum conservation equations here. If I apply this momentum conservation equation this some of the forces should equal to the rate of the change of momentum flux within the control volume is equal to the net outflux of momentum flux passing through these control surface.

Data Given:

V_{jet} = 4 m/s

Q_{jet} = 0.1 m^3/s

Deflected angle (θ) = 45^o

Momentum Conservation:

Applying RTT

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall + \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

Steady flow, $\frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall = 0$

$$\sum Fx = \dot{m}\text{jet}\, V\cos\theta$$

$Forece\ recorded\ by\ spring = \rho Q_{jet}\ V\ Cos\ \theta$ = 282.84 N

You can look it the cos theta component will come it as the x directions and force recorded by spring if I compute what will be the mass, ρ into Q. Q object which known to us, V is known to us, θ is known to us, we can compute what will be the force acting in terms of Newton.

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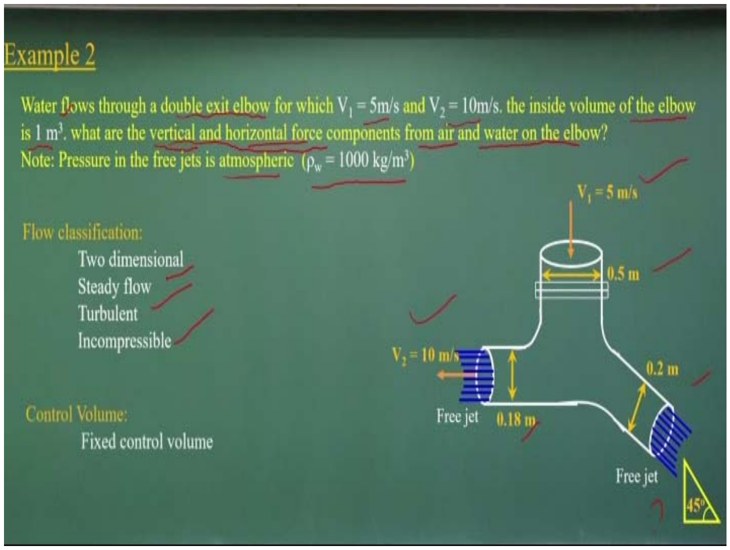

Now let us go to the second example 2 where we have the water flows through a double exit elbow. Okay this is elbow type of concept, where we have a V 1 velocity and V 2 velocity. One is 5 meter per second another is 10 meter per second. Inside the volume of the elbow is 1 meter cube, okay the volume of this. What are the vertical and horizontal force component of from air and water on this elbow if pressure in the free jet is atmospheric and the unit weight of the water will be 1000 kg per meter cube.

[Water flows through a double exit elbow for which V_1 = 5m/s and V_2 = 10m/s. the inside volume of the elbow is 1 m^3. what are the vertical and horizontal force components from air and water on the elbow?

Note: Pressure in the free jets is atmospheric ($\rho_w = 1000\ kg/m^3$)]

Now if you look it the sketch of the problems, there are the free jet in the two part. And there is a velocity here, the velocity here, but the velocity is unknown here. We do not know the velocity of this part. There is a diameter of this elbow part. The first we have to apply the mass conservation equations. Then we will apply linear momentum equations to compute what is the vertical, horizontal force components.

Flow classification:

Two dimensional

Steady flow

Turbulent

Incompressible

Is very simple things that here we will apply mass conservations equations. Then we will apply two momentum equations, one for the vertical directions and other is horizontal directions to get what is the vertical force component, what is the horizontal force component. First let us define the problem. Problem is two dimensional.

As you seen it that here the flows are coming, there is not change of the flow volume within this, this. So we can consider is a steady problem, turbulent, incompressible. That what we can consider as a flow classification. We can consider is a fixed control volume like this. This is what will be the control volumes. We have the surface like this. We have a surface like this.

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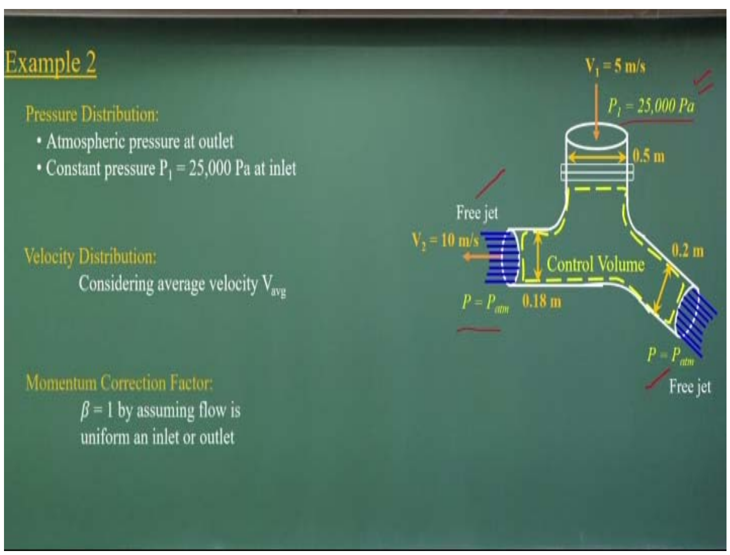

So if we have this fixed control volumes, first we will find out that the pressures. The pressures at the inlet is given is 25,000 Pascal. At the outlet is pressure is atmospheric pressure. That what is there. So you have the free jet at these two locations where pressure is the atmospheric pressures. At these two point you have a pressure equal to atmospheric pressure, but at this point I have the pressure which is 25,000 Pascal, Newton per millimeter square.

So if you have that and we use a concept of average velocities. We neglect the velocity distributions. We use the beta equal to the 1 as for the inlet and the outlet.

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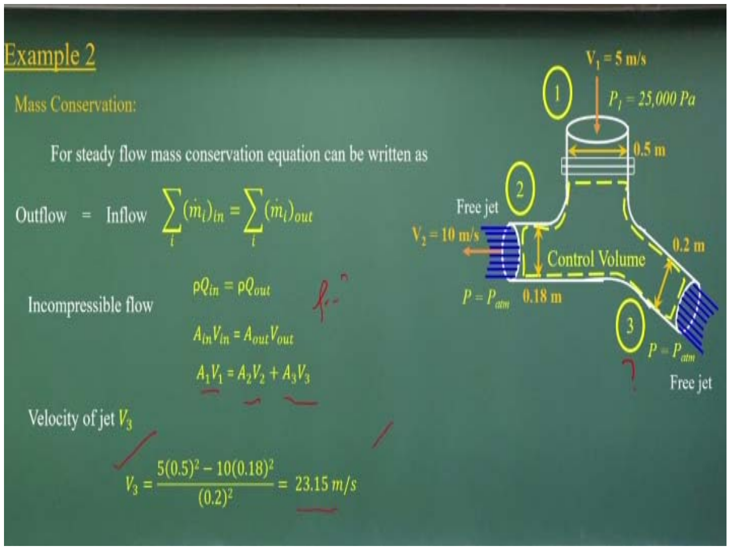

Then we will apply mass conservation equations that let us apply mass conservation equations to compute it what could be the velocity at the section 3. So we are applying the mass conservation equations for these control volumes that to compute what could be the velocity at this point. So if I apply this mass conservation equation which are very simple form is rate of mass influx into the control volume is equal to rate of the mass outflux from this control volume.

For steady flow mass conservation equation can be written as

Outflow = Inflow

$$\sum_i (\dot{m}_i)_{in} = \sum_i (\dot{m}_i)_{out}$$

$\rho Q_{in} = \rho Q_{out}$

$A_{in}V_{in} = A_{out}V_{out}$

$A_1V_1 = A_2V_2 + A_3V_3$

Velocity of jet V_3

$$V_3 = \frac{5(0.5)2 - 10(0.18)2}{(0.2)2} = 23.15\ m/s$$

Simple substituting the area and the velocity of V_1, V_2 and you can compute it the velocity of z passing through the section 3 will be 23.15 m/s.

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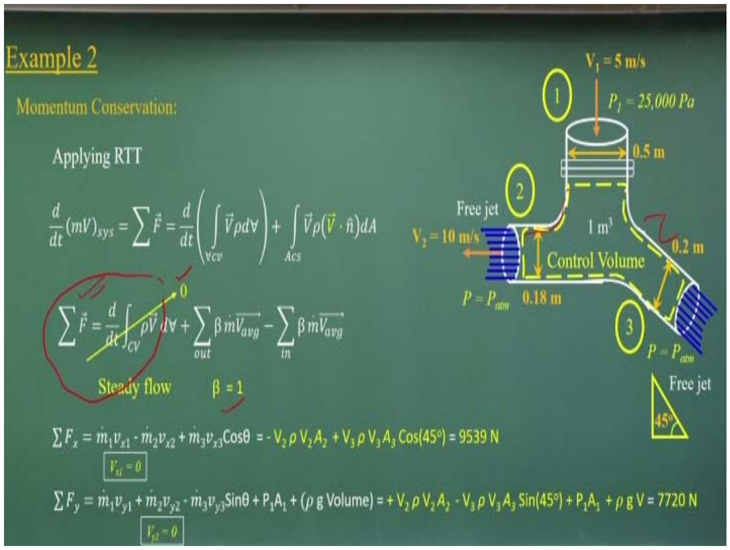

So if I apply this momentum conservation equations for this control volume, when I apply this momentum conservation equations, the first things what I writing the control volume equations, Reynolds transport theorems and then I am simplifying it. So again it is a steady problem, okay. And I look it the momentum flux in and out assuming the beta is equal to the 1. And our objective is now to compute the force components.

We have the consider the control volume inside this control volume, you can think that there will be a frictional resistance from the surface of the 1. But that part we are neglecting it as compared to the force component what is acting it. So this way we can neglect the frictional force which is there near the wall, that part we are neglecting it to compute it what will be the force component.

Applying RTT

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall + \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

Steady flow, $\frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall = 0$, $\beta = 1$

$\sum Fx = \dot{m}1vx1 - \dot{m}2vx2 + \dot{m}3vx3\text{Cos}\theta = - V_2\, \rho\, V_2 A_2 + V_3\, \rho\, V_3 A_3 \text{Cos}(45^o) = 9539\text{N}$

$V_{x1} = 0$

$\sum Fy = \dot{m}1vy1 + \dot{m}2vy2 - \dot{m}3vy3\text{Sin}\theta + P_1A_1 + (\rho \text{ g Volume}) = + V_2\, \rho\, V_2 A_2 - V_3\, \rho\, V_3 A_3 \text{ Sin}(45^o) + P_1A_1 + \rho \text{ g V} = 7720 \text{ N}$

$V_{y2} = 0$

So if I apply this part, then F x will come it to the 9539 Newton. Similar way the force component acting on this y direction that one will be the 7720 Newton.

The approximations and all the velocity component, zero component if you look it and we have resolved the force, momentum flux component as a cos theta and sin theta. We have also considered the force due to the pressures at this point, which is and also we consider the weight of the fluid here. If you look at this, the weight of the fluid is considered in the y direction, that components are there.

The force due to the pressure one also we have consider it and these are all momentum flux component that what total what is come in these parts. So there is a force F x direction and F y direction. We noted that the force due to the pressure at this point we have computed which is can consider as a gauge pressure, it is not the absolute pressure.

And also we have considered the weight component of rho g into the volume. Combining the all the terms we got it the force acting the y direction will be 7720 Newton.

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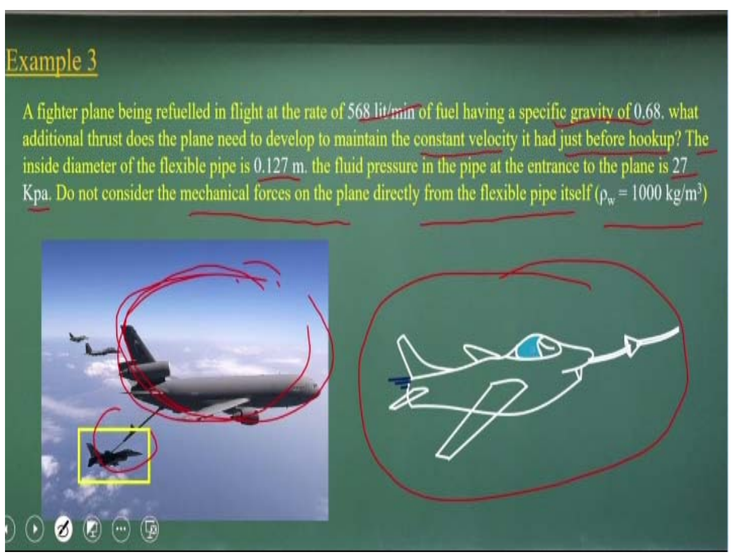

Let us take another example. A fighter plane being refueled in a flight at the rate of 568 liter per minute of fuel having specific gravity of 0.68. What additional thrust does the plane need to develop or maintain constant velocity it had just before hookup. The inside diameter of the flexible pipe is 0.127 meters. The fluid pressure in the pipe at the entrance to the plane is 27 kilo Pascal.

[A fighter plane being refuelled in flight at the rate of 568 lit/min of fuel having a specific gravity of 0.68. what additional thrust does the plane need to develop to maintain the constant velocity it had just before hookup? The inside diameter of the flexible pipe is 0.127 m. the fluid pressure in the pipe at the entrance to the plane is 27 Kpa. Do not consider the mechanical forces on the plane directly from the flexible pipe itself (ρ_w = 1000 kg/m^3)]

And if you neglect the mechanical forces on the plane directly to the flexible pipe and you consider the unit weight of water will be 1000 kg per meter cube. The problem what graphically it is shown it in this figure, this is the main aircraft and this is the what the fuelling aircraft okay which is fuelling this the aircraft from this sides. That is what new technologies what is develop it.

In air itself we can refuel the aircraft from this part. So if it is that is the conditions, what could be the additional thrust does the plane need to develop to maintain the constant velocity between these two. They should move it the same velocity. They should move it with the same velocity so that the refueling can be done. That is the problem. Now we try to look it what is the additional thrust is necessary.

The problem is now simplified in this part that there is aircraft like this and it is hooking it, the fuel hooking to this one.

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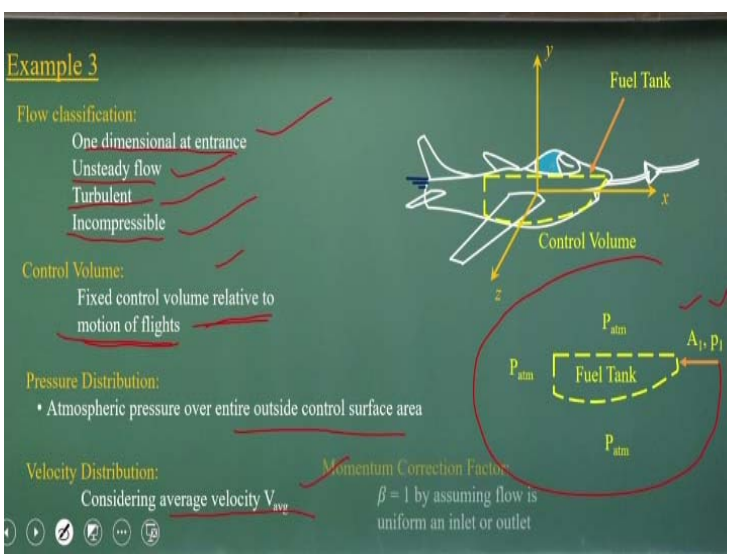
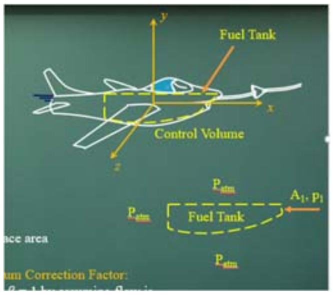


And if I define the fuel tank and this coordinate of x, y, z the problems can be considered is one dimensional. Fluid is unsteady flow, but we can simplify it make it the steady flow; turbulent, incompressible. So these three conditions will be there; one dimensional, we are not considering the velocity distributions and all. The unsteady, turbulent and incompressible flow. That is what the classifications.



Flow classification:

One dimensional at entrance

Unsteady flow

Turbulent

Incompressible

Here it is movable control volume, but since we have the two flights are moving it with respect to the refueling aircraft we can consider is a fixed control volume, relative motions to the flight okay. So that the basic assumptions of these type of problems is a fixed control volume which is related to the motion of the flights. If that is the conditions, the pressure distributions enter outside the control surface.

We can consider is atmospheric pressure what will be there like this the problems now, everywhere you have atmospheric pressure and the pressure, the fuel what is injecting it, it is P 1 pressure having area A 1. The everywhere the control surface can be considered is P equal to the atmosphere pressure. The velocity distributions again we can take is average velocity distributions V average value, beta equal to the 1.

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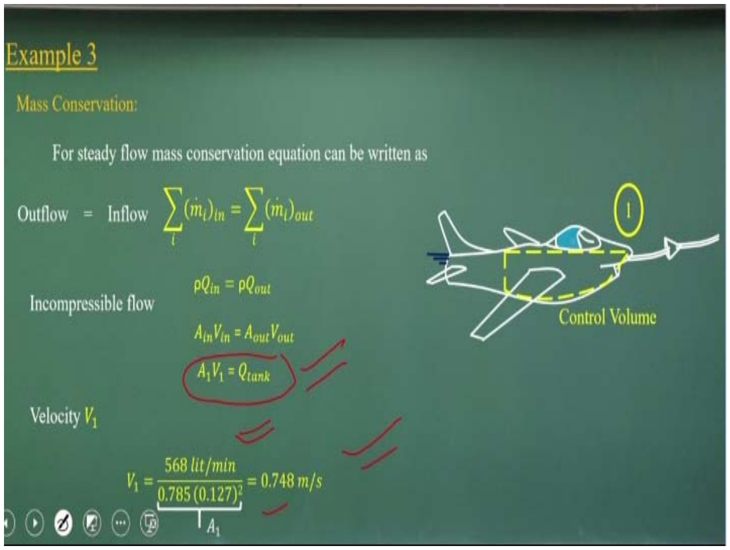

Now let us apply the mass conservation equation where is inflow minus outflow and this case will be,

Outflow = Inflow

$$\sum_i (\dot{m}_i)_{in} = \sum_i (\dot{m}_i)_{out}$$

Incompressible flow

$\rho Q_{in} = \rho Q_{out}$

$A_{in}V_{in} = A_{out}V_{out}$

$A_1V_1 = Q_{tank}$

$$V_1 = \frac{568\ lit/min}{0.785\ (0.127)2} = 0.748\ m/s$$

This is what the discharge is given to us, the volumetric discharge is given to us divide by the area will get it the velocity, that the velocity part what we have.

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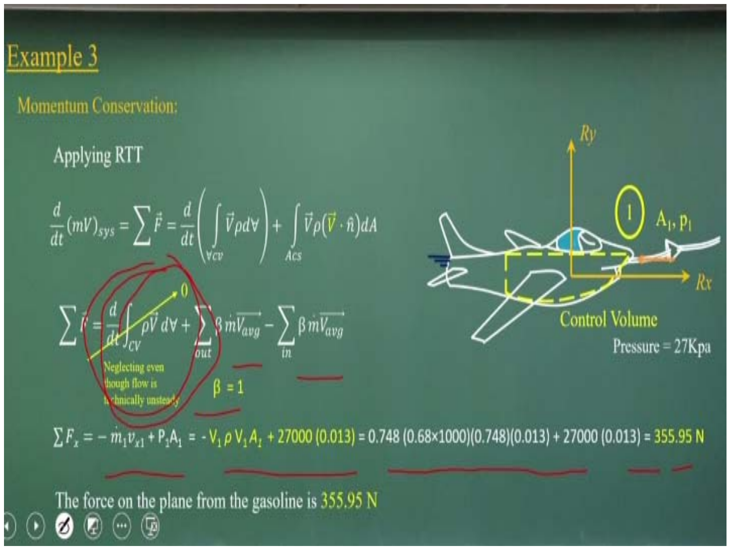

Now we have to find out what is the force acting in R_x and the R_y directions, we have the pressure component here. Applying this Reynolds transport theorems and the simplifying the tops. Here, this is the main concept what we have consider it that there is a change of the momentum flux within the control volume. But the right of the change of the momentum flux within a control volume, is not that significant order as compared with the momentum thrust what is coming from the fuel.

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall + \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

Neglecting even though flow is technically unsteady, $\frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall = 0$

$\beta = 1$

$$\sum F_x = -\dot{m}_1 v_{x1} + P_1 A_1$$

$$= -V_1\rho V_1 A_1 + 27000\,(0.013)$$

$$= 0.748\,(0.68 \times 1000)(0.748)(0.013) + 27000\,(0.013)$$

$$= 355.95\, N$$

So this component will be there, but it is not that significant. Since it is not significant we make it to zero and make it the problems in a steady nature. Then we have a two momentum flux components, one is influx another is outflux, beta is equal to the 1. So if I apply it, I will have a simply the momentum flux in x direction, the pressure into the area and that what I will get it and in terms of value, I will get it 355 approximately Newton.

That what the force will be acting on the x direction. Now take it another examples which are again simple examples what we are talking about, but is the control volume which is the moving conditions. That means moving control volume problems we will going to solve it.

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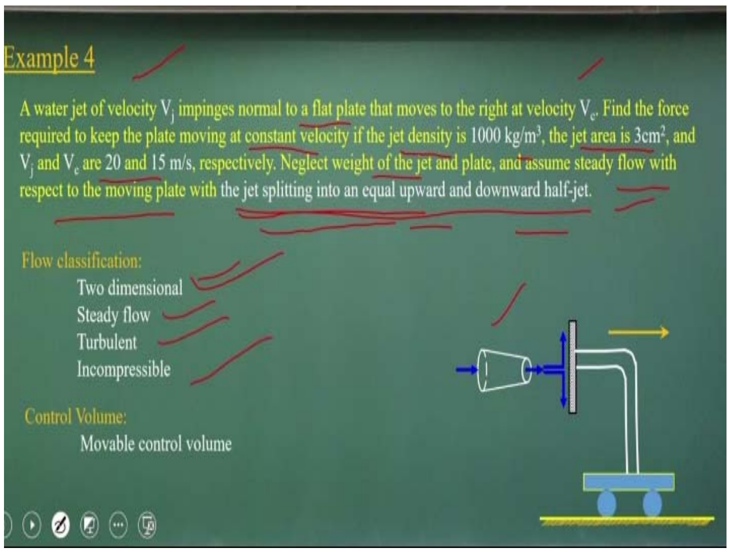

Like let we will have a water jet, the if you look it this is the water jet, is impinging this normal to a flat plate and moves to the right to the velocity V c, okay this that means, this flat forms is moving with a velocity V c. Find the force required to keep the plate moving at the constant velocity if jet density is this, the water density is this. Jet area is given and V j the velocity of water jet and the V c the velocity of the frame is respectively 20 and 15 meter per second.

[A water jet of velocity V_j impinges normal to a flat plate that moves to the right at velocity V_c. Find the force required to keep the plate moving at constant velocity if the jet density is 1000 kg/m^3, the jet area is 3cm^2, and V_j and V_c are 20 and 15 m/s, respectively. Neglect weight of the jet and plate, and assume steady flow with respect to the moving plate with the jet splitting into an equal upward and downward half-jet.]

Then neglect the weight of the jet and the plate, assume the steady flow with respect to moving plate. Jet is splitting into two equal upward downward half-jet. So these are the assumptions, quite valid for a jet flow problems is quite valid. So as the flow is impinging and making into two directions, vertical directions. One is jet is coming down, another is up.

Flow classification:

Two dimensional

Steady flow

Turbulent

Incompressible

You will have a two dimensional flow problems becomes a steady the turbulent and incompressible flow. Movable control volume concept only the things what we have here to know it that we will apply this movable control volumes to know it the pressure distribution part.

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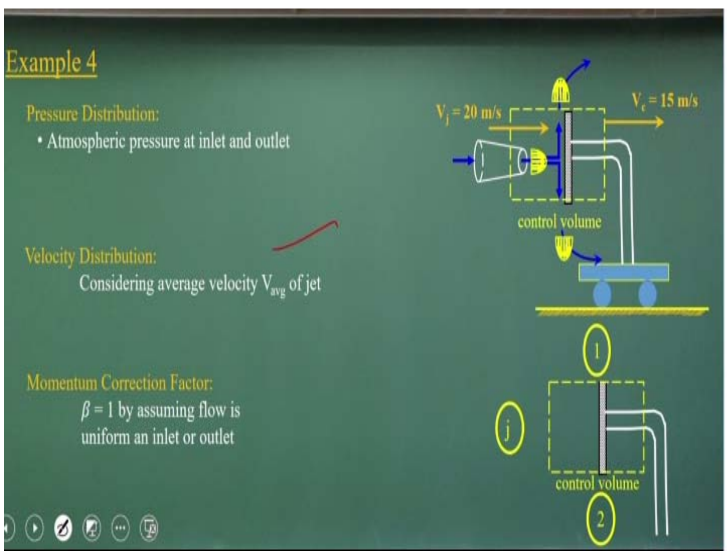

The pressure distributions in atmospheric pressure distributions will be the atmospheric pressure distribution the we will consider the average velocity conditions and the beta equal to the 1. Okay that the great simplifications we used to do it.

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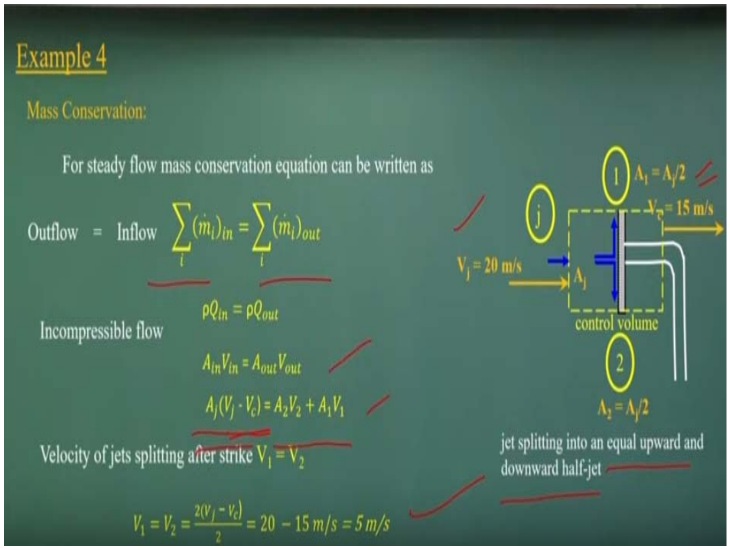
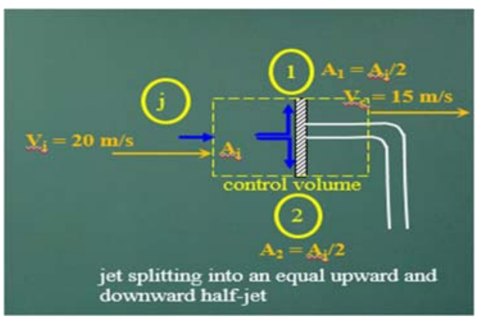


And if this is the V 1 to this cross section V z is water is coming into that and it is splitting into two part is equally part. So this is velocity of jet, this is what the moving control volume is moving with 15 meter per second. Jet is splitting into equally upward half-jet, the half of what goes to upward direction, half of water goes to the downward direction. If is that let us first apply the mass conservation equations.



The mass influx is equal to mass outflux. If that is the conditions you have a these values, okay. Please remember it we consider here the relative velocity component along the and it has the V_1, V_2 velocity which is going out where the velocity components, we do not have any velocity components in the y direction. But is only moving in the x direction, that is the reason.

Outflow = Inflow

$$\sum_i (\dot{m}_i)_{in} = \sum_i (\dot{m}_i)_{out}$$

$\rho Q_{in} = \rho Q_{out}$

$A_{in}V_{in} = A_{out}V_{out}$

$A_j(V_j - V_c) = A_2V_2 + A_1V_1$

Velocity of jets splitting after strike $V_1 = V_2$

$V_1 = V_2 = \frac{2(V_j - V_c)}{2} = 20 - 15\ m/s = 5\ m/s$

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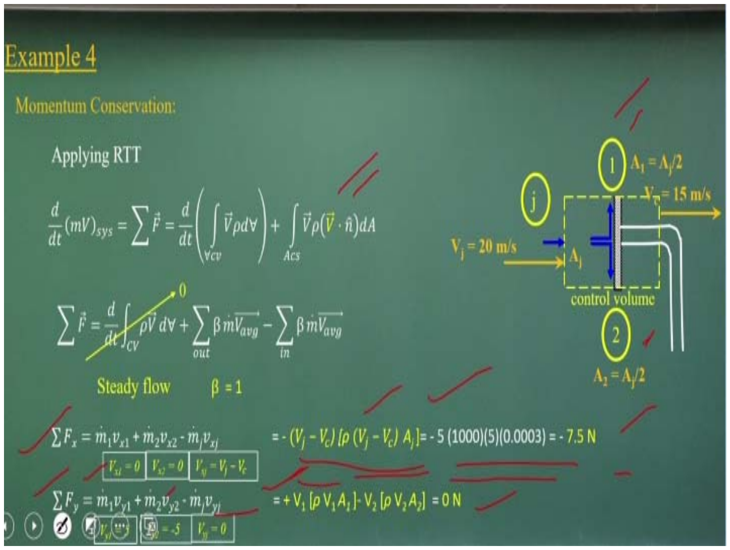

Now I am applying momentum conservation equation. We remember it I need to apply the relative velocity component here as the part the considerations here and beta equal to the 1 and I apply the relative velocity component. That what you can look it how the relative velocity components are used for the mass flux and the momentum flux component and that what is substitute to get it what is the force component. So the problem is similar nature.

Applying RTT

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall + \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

Only what we are doing it, we compute the relative velocity. The V xz which is V j – V c. V j is the velocity of jet minus V c is the velocity of the platforms moving with V c. That in the x direction. The flow what is going in the direction of 1 and 2 as you know it does not have any component in the x direction, we can make it zero.

So only the force we have because of the impinging of jet what is working with a relative velocity component that what we have substitute to finally we got it what is the amount of force acting because of these things. Similar way, the y direction also we can substitute it and apply it. As it expected it when you have a the same amount of water goes in the 1 and 2 the same velocities, they will be cancelled each others.

$$\sum Fx = \dot{m}_1 v_{x1} + \dot{m}_2 v_{x2} - \dot{m}_j v_{xj} = (V_j \text{ -} V_c)[\rho\ (V_j \text{ -} V_c)\ \ Aj\ ]$$

$$= -\ 5\ (1000)(5)(0.0003)\ =\ -\ 7.5\ N$$

So the net force will be act is zero, the force component will be the positive and the negative. The net momentum flux will be the zero. That the force component will come it.

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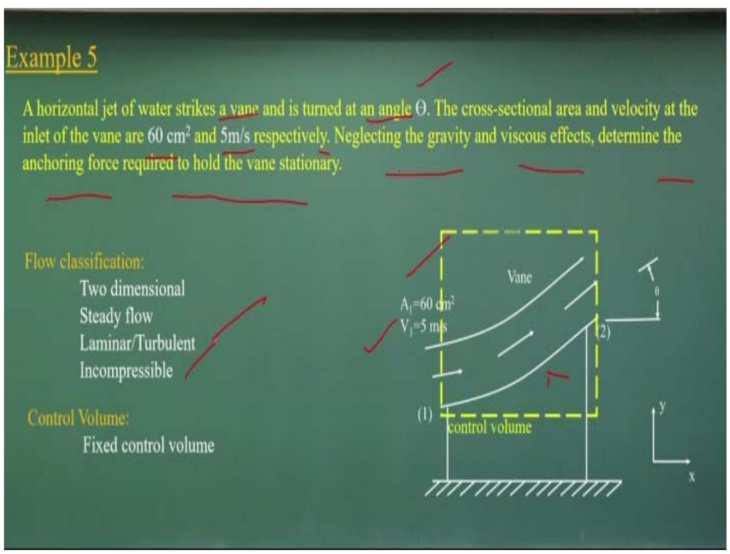

Now let us come to the example fifth which is again a simple problem where a horizontal jet of water strikes a vane, this is the vane, okay? Is turned the angle to theta degree. Cross-sectional area and velocity at the inlet of the vane is 60 centimeters, 5 meter per second respectively neglecting the gravity and viscous effect, okay?

[A horizontal jet of water strikes a vane and is turned at an angle ϴ. The cross-sectional area and velocity at the inlet of the vane are 60 cm^2 and 5m/s respectively. Neglecting the gravity and viscous effects, determine the anchoring force required to hold the vane stationary.]

Determine this anchoring force required to hold this vane at the rest or the stationary. This is very simple problems okay. There is flow jet is coming. We are making angle theta which is coming out from this jet. The area is given, velocity is given and we need to compute it what amount of force is required to anchor this vane so that the vane will be at the rest conditions or the stationary will be there.

Flow classification:

Two dimensional

Steady flow

Laminar/Turbulent

Incompressible

Flow is two dimensional, steady. We do not know whether flow is a laminar or turbulent or incompressible flow. Fixed control volume. This is what the control volumes.

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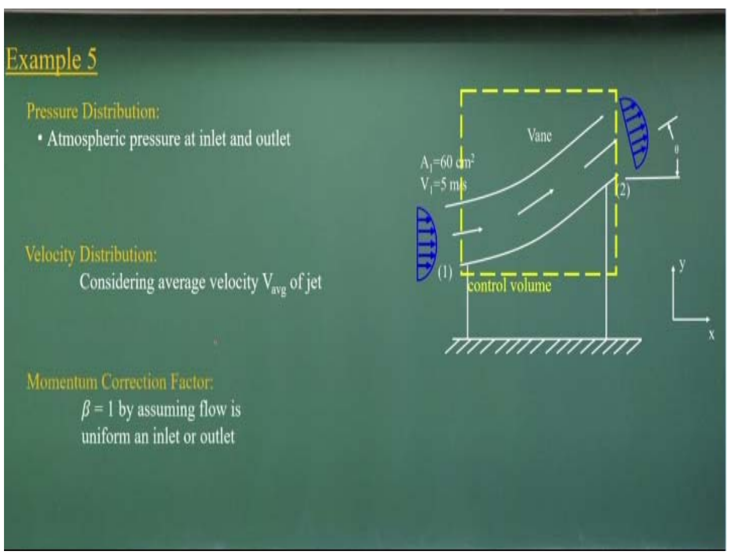

And the jet is there is atmospheric pressure and the velocity distribution again we can consider is a uniform. That is what not a conditions when you have the free jet in it going through the vane V but we can assume it the velocity distribution is uniform.

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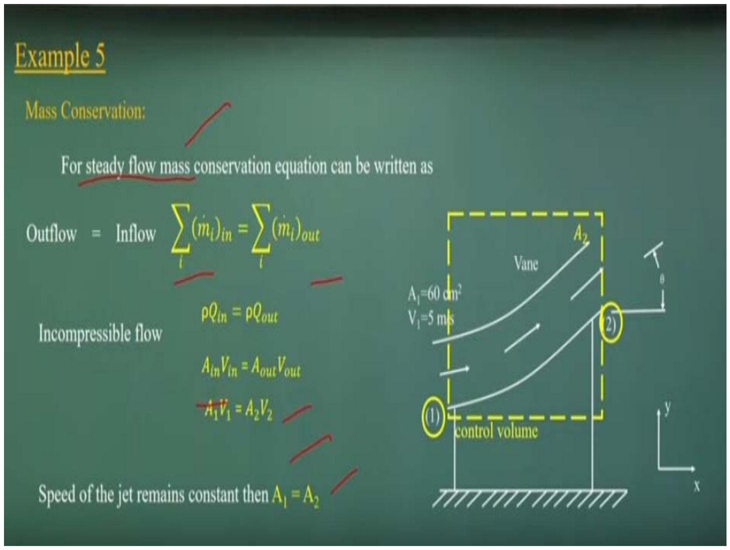

And if that is there we are computing the first mass conservation which is very easy things here being a steady flow we will have a mass influx is equal to this is a very standard things as we discussed earlier and that is what we will show it since the speed of the jet remains constant, the area of flow A 1 = A 2 following the continuity equations as the flow is incompressible.

Outflow = Inflow

$$\sum_i (\dot{m}_i)_{in} = \sum_i (\dot{m}_i)_{out}$$

Incompressible flow

$\rho Q_{in} = \rho Q_{out}$

$A_{in}V_{in} = A_{out}V_{out}$

$A_1V_1 = A_2V_2$

Speed of the jet remains constant then $A_1 = A_2$

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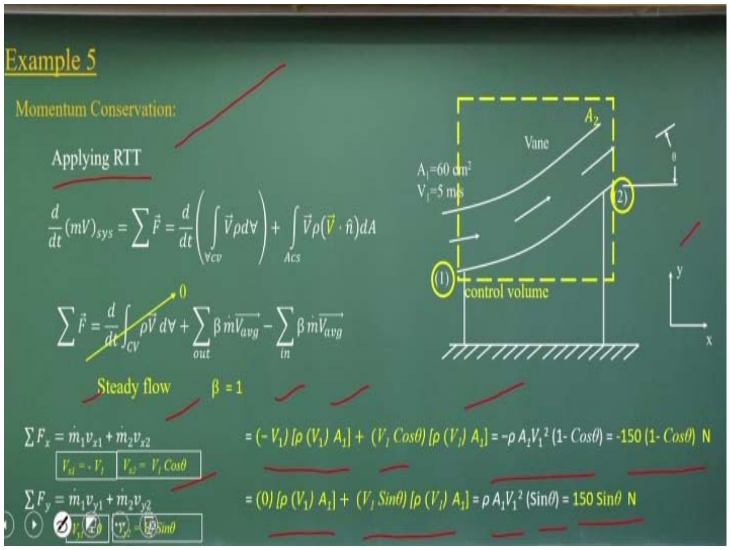

Now need to apply the momentum conservation equation, apply the Reynolds transport theorems to compute what will be the force. First again steady flow conditions which is standard assumptions we have been doing it for simplifying the problems for the problems here what we have considered it and β equal to 1.

Applying RTT

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall + \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

$$\frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall \text{ tends to } 0$$

$\sum Fx = \dot{m}_1 v_{x1} + \dot{m}_2 v_{x2} = (-V_1)\,[\rho\,(V_1)\, A_1] + \,(V_1\, Cos\theta)\,[\rho\,(V_1)\, A_1] = -\rho\, A_1 V_1^{\,2}\,(1-Cos\theta) = -150\,(1-Cos\theta)\;N$

$V_{x1} = -\,V_1$

$V_{x2} = V_1\, Cos\theta$

$\sum Fy = \dot{m}_1 v_{y1} + \dot{m}_2 v_{y2} = (0)\,[\rho\,(V_1)\, A_1] + \,(V_1\, Sin\theta)\,[\rho\,(V_1)\, A_1] = \rho\, A_1 V_1^{\,2}\,(Sin\theta) = 150\, Sin\theta\;N$

$V_{y1} = 0$

$V_{y2} = V_1\, Sin\theta$

The velocity component x direction and the mass flux that what we will equate it, it will be this value. Similar way if I am to compute the force acting in this y direction, I will have mass flux coming into momentum flux in the x y directions will be the zero. Influx is not there, it is coming imposing on the x direction.

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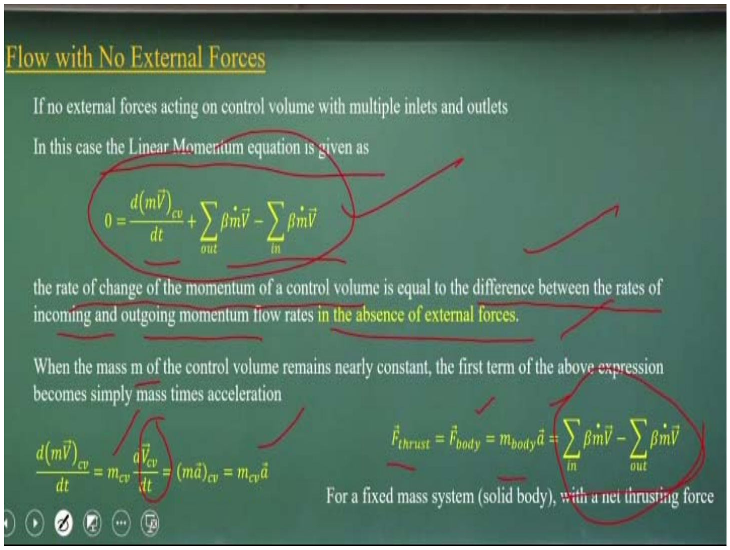

Now let us come it before ending this lecture, let us have a one examples very interesting examples of decelerating the spacecraft before landing on the earth. This is what very interesting problems. That means when you have a spacecraft there is no external force acting on that. And if you have that there is a multiple inlet and outlet. You can write the linear momentum equations sum of the force in the systems is equal to zero.

$$0 = \frac{d(m\vec{V})_{cv}}{dt} + \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$$

$$\frac{d(m\vec{V})_{cv}}{dt} = m_{cv}\frac{d\vec{V}_{cv}}{dt} = (m\vec{a})_{cv} = m_{cv}\vec{a}$$

You will have a these two point. The rate of the change of the momentum flux within the control volume is equal to net outflux of momentum flux passing through this control surface. That is what is written here. Rate of change of momentum, within a control volume is equal to the difference between the rates of incoming, outgoing momentum flux rate in absence of external force.

This is what the text, this is what mathematical explanations for this. So if you assume it is a mass of the control volume is m which remains constant. Then we can write this part is mass of control volume dV by dt. That is acceleration. So mass of control volume into the acceleration. The thrust of the force will going to act it is force acting on the body, the mass body into the acceleration.

$$\vec{F}_{thrust} = \vec{F}_{body} = m_{body}\vec{a} = \sum_{in} \beta \dot{m}\vec{V} - \sum_{out} \beta \dot{m}\vec{V}$$

That what net outflux of the momentum flux which is going out from this. This is very interesting problems. We will consider it when there is no external force acting on a control volume even if you have a multiple inlet and outlet.

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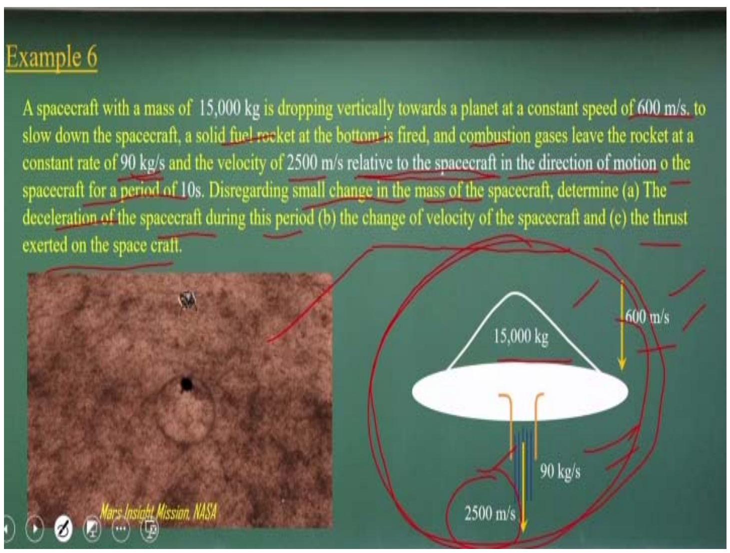

Let us see this, the figure what is showing is that when a spacecraft are landing on the surface, it decelerate it. It decelerate the terminal velocities and then it land on the earth or the any planets. So how do you do decelerate it? That is what we do it to decelerate that. It is the problems what you have consider it a spacecraft with a mass of 15000 kg dropping vertically towards a planet at a constant speed of 600 meter per second.

To slow down to spacecraft a solid fuel rocket at the bottom is fired. Combustion of gases leave the rocket at a constant rate of the speed of 90 kg per second. This is the what the mass rate is going up. The velocity will be 2500 meter per second relative to the spacecraft. Please remember this is what the main point, relative to spacecraft in the directions of the motions.

Then if that is what for the spacecraft for a period of the 10 second, this disregarding or neglecting the small change of the mass in the aircraft determine the deceleration of the spacecraft during this period, change of the velocity of the spacecraft and what is the thrust exerted on the spacecraft. So if you look it this is the spacecraft. This is what the Mars inside mission, how the diagrams what is from the NASA that is what we have illustrated.

Looking at these problem text whatever we discuss it the problems like this there is a spacecraft having 15,000 kg weight, the speed is 600 meter per second, which is injected a mass 90 kg per second, mass flow rate. And it has a relative velocity which 2500 meter per second. What is the amount of force acting due to that? What is the decelerating because of these firing of the gases from this rocket. And what is the change of the velocity that is what we will solve it.

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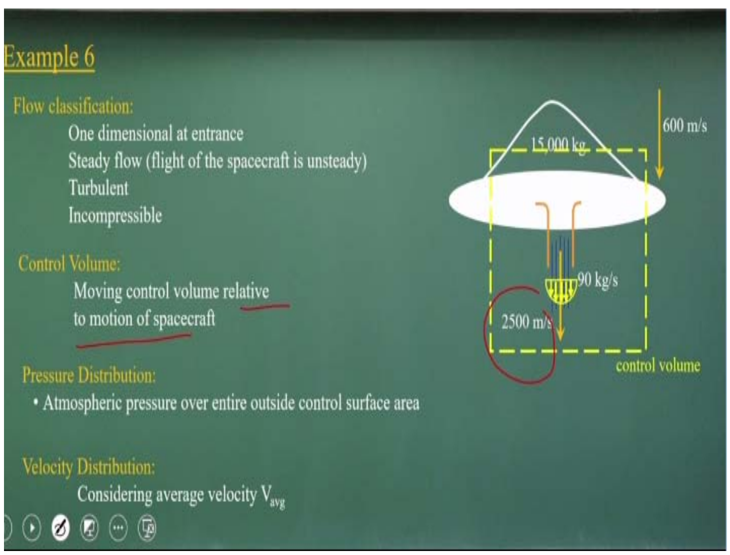

Now the problem is one dimensional because only one jet is there. We can use as a steady problems, turbulent, incompressible. This is a moving control volume, relative to motion of the space aircraft. Here we are talking about the relative velocities. We are not talking about the absolute velocity of jet and if it is that we consider the control volumes and try to find out what will be the force is acting because of this firing of the gases from this space aircraft.

Flow classification:

One dimensional at entrance

Steady flow (flight of the spacecraft is unsteady)

Turbulent

Incompressible

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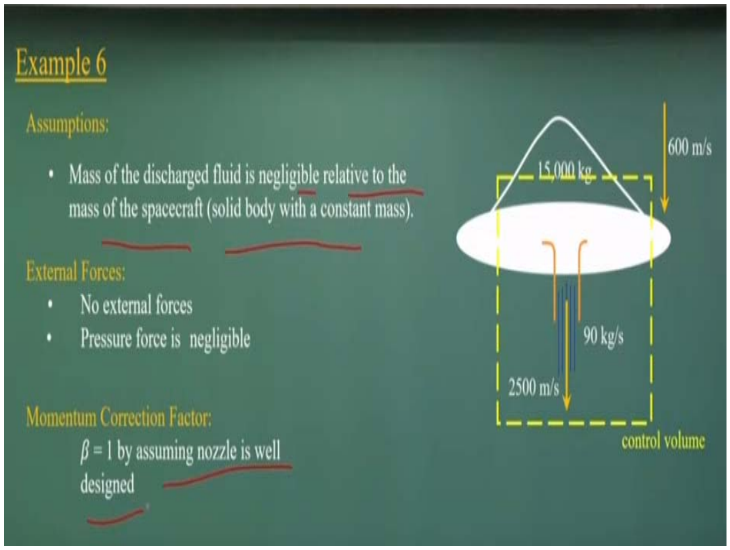

Now if I have a another assumption as you said, the mass of the discharged fluid is negligible, is relative to the mass of the aircraft. This will be the solid bodies moving with a constant mass, okay. No external forces. The pressure forces can be negligible. Beta is equal to the 1, assuming the nozzle is well designed, okay. So because a well designed nozzle have a close to the uniform velocity distribution. So beta will be 1.

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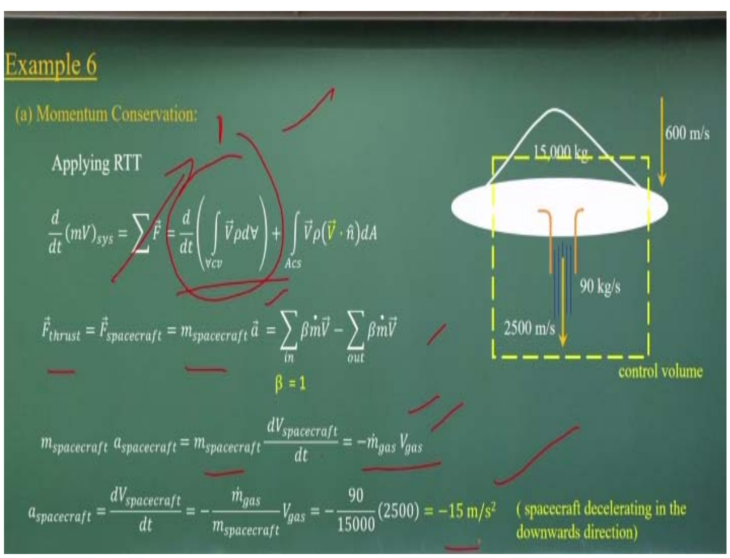

And let us apply the Reynolds transport theorems, okay. In this case, as we have considered earlier, okay. So there is this force component is zero. Only the change of the rate of the momentum flux within the control volume that what will force thrust, that will be the mass of aircraft and the accelerations. That what will be net outflux of momentum flux what is going out from this. There is no inflow.

Applying RTT

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\vec{F}_{thrust} = \vec{F}_{spacecraft} = m_{spacecraft}\,\vec{a} = \sum_{in} \beta \dot{m}\vec{V} - \sum_{out} \beta \dot{m}\vec{V}$$

$$\beta = 1$$

$$m_{spacecraft}\, a_{spacecraft} = m_{spacecraft}\frac{dV_{spacecraft}}{dt} = -\dot{m}_{gas}\, V_{gas}$$

$$a_{spacecraft} = \frac{dV_{spacecraft}}{dt} = -\frac{\dot{m}_{gas}}{m_{spacecraft}}V_{gas} = -\frac{90}{15000}(2500) = -15\ \text{m/s2}$$

So you will have a because of gas injections you have a momentum flux what is m dot V gas. That what is the will be the spacecraft mass into the acceleration. This way we can compute it the acceleration will be -15 per second square. So this is the what the accelerations of the space aircraft in the negative directions. That what is decelerating in downward direction.

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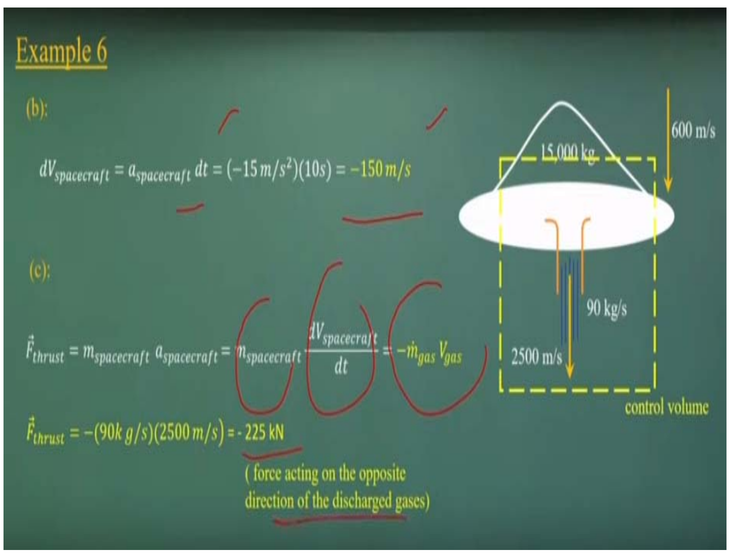

Second part of the problem is that compute the change of the velocity.

$$dV_{spacecraft} = a_{spacecraft}\,dt = (-15\,m/s^2)(10s) = -150\,m/s$$

And what will be the force acting on that which is very easy to compute and the momentum flux or mass of aircraft and the acceleration we can compute what is the thrust is acting on these,

$$\vec{F}_{thrust} = m_{spacecraft}\, a_{spacecraft} = m_{spacecraft} \frac{dV_{spacecraft}}{dt} = -\dot{m}_{gas}\, V_{gas}$$

$\vec{F}_{thrust} = -(90k\, g/s)(2500\, m/s)$ = - 225 kN

this is what will come it to 225 kilo Newton force acting opposite directions of discharged gases, that is what happens it.

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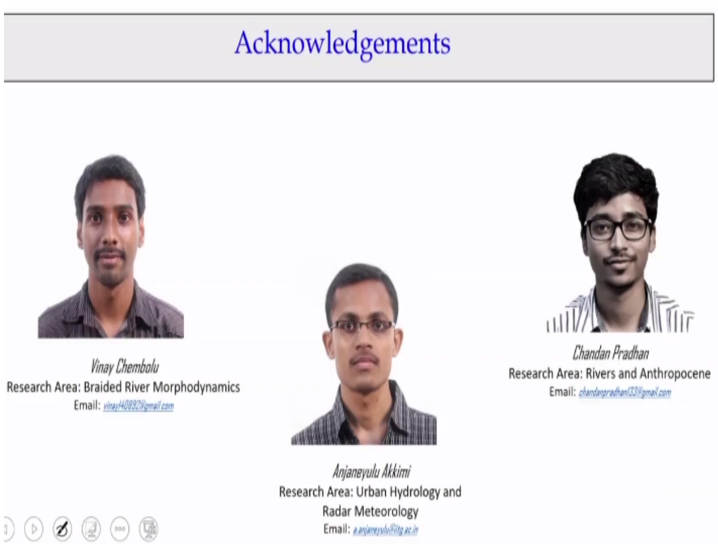

And let me acknowledge before ending this course is that my 3 PhD students they have been working hard, drawing the figures, illustrating the figures, bringing lot of examples, discussing lot before presenting you. So I do acknowledge their efforts and not to improve the knowledge of the fluid mechanics but teach the fluid mechanics in simpler form. That is what is possible because of them. I do again acknowledge their help. Okay, with this let us conclude today lectures. Thank you lot.