Fluid Mechanics for Civil and Mechanical Engineering
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Lecture No 12
Bernoulli Equation and its Applications

Welcome all of you for this very interesting lectures on Bernoulli Equation and its Applications. The last class we discuss about Bernoulli equation starting from its history of Bernoulli equation. Very briefly, I can say that because of the Bernoulli equation, having a simplification to the fluid flow problems, the industrial revolutions what it happened in Europe, the contribution of Bernoulli equation also helped a lot to design pipe flow, channel flow in Europe after this equations was suggested long back in 1752.

Now, today I will give a very simple way representation of this Bernoulli equation, how we can use for real fluid flow problems with some correction factors or we can use this Bernoulli equation as hydraulic gradient line, energy gradient line and we can apply these equations for a systems having pump and the turbine. So, basically today, I will talk more applications and how we can use the Bernoulli equation for real fluid flow problems. That is the basic concept what I will do and these are the reference books.

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And most of these materials partly we follow Cengel, Cimbala and F.M. White book.

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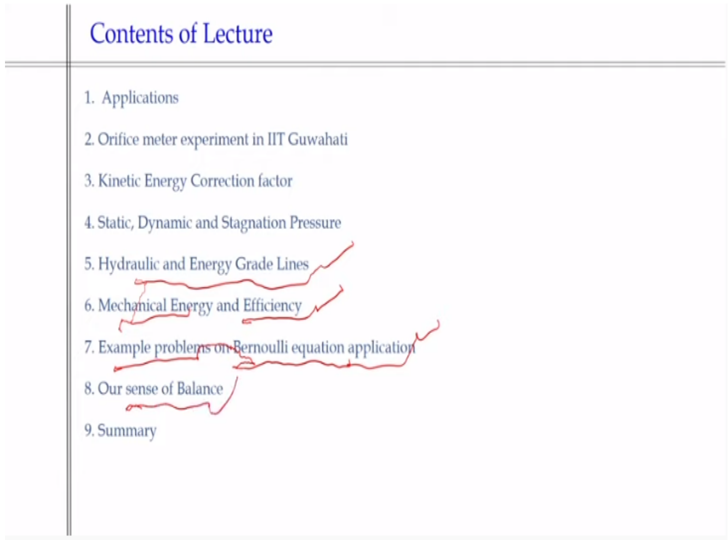

I will start with the applications, very interesting applications I will show to you, then I will go for orifice meter experiment in IIT, Guwahati, then I will talk about kinetic energy corrections factors. That means, for a non-uniform distribution of flow, when you apply this Bernoulli equations, we need to have a corrections factors if we are using average velocity. That correction factors for Bernoulli equations.

Fourth part, I will talk about how we can define the three different types of pressures; static, dynamic and stagnation pressures. Then will come hydraulic and energy gradient lines, that is the basic concept what we will talk about. Then I will talk about if we have a pipe flow systems, with a series of pump turbine systems, then how we apply it and how we can quantify different energy mechanical energy also the efficiency to the fluid flow problems.

Then, we will solve around four fluid flow problems, which are the gate and the engineering service problems will solve, which is part of the Bernoulli equations applications. Then concluding this lecture, we talk about, how our sense of balance is there. So okay, how it is works.

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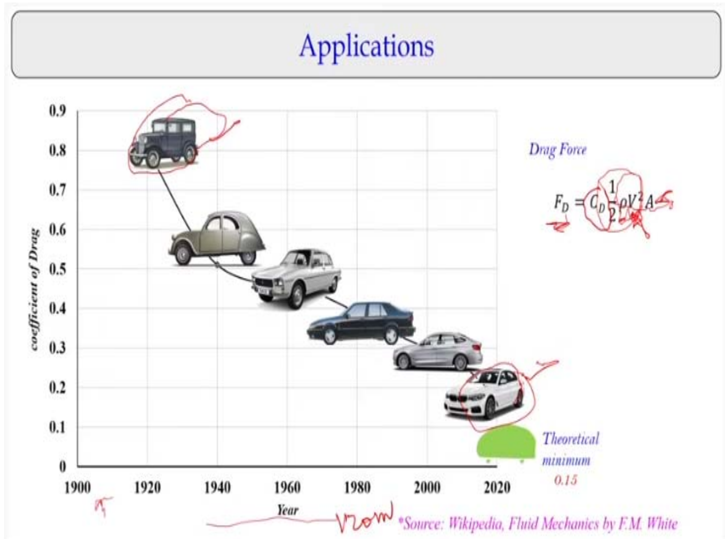

So, let us go back to very interesting examples, as you know it people say as of now, we have very fuel efficient car. That means the mostly car we talk about the drag force. The more the drag force, then you have to have more fuel to be spent it, fuel to burn it. So the basic idea is to, because V stands for the design velocities, area is a projected area, which is more or less constant. So, rho is the density of air. Only the C_D can be changed. That is what it happened the evolution of the C_D change from almost 120 years, okay.

Drag Force

$F_D = C_D \frac{1}{2} \rho V^2 A$

Starting from 1800s to 2020. So, the coefficient of drags, which earlier is close to the 0.8, that means the force is equal to 0.8 times of rho V square into the area that becomes reduced to the 0.15, theoretical minimum value, very close to theoretical value. And that because of you can see the shape of the cars, okay. And the shape of the cars are changing, the streamline, the drag force patterns are changing the flow around these cars when you are moving with a design velocity.

That is what it changes it, with a design velocity. The cars is moving with the V, which is the design velocity. On that, what is the drag force component. So that is what if you look it, we have achieved, within 100 years, the theoretical minimum value which is 0.15, very close to that. And you look the shape of the cars what today we have, which if you move with a design speed, then we will have a drag coefficient close to 0.15 and which is a strength of the fluid mechanics evolved the automobile sectors with a very efficient the fluid cars.

What is today is available and their drag ratio starts from the coefficient of drag C_D from point 0.8 to 0.15. That means as equivalent to 1/6. So, for example the 6 liters of the car if it is taking this one for one particular distance to travel it, now, we need just 1 liter, the same distance can be traveled because of the drag coefficient is almost 1/6. So, this is what is the evolutions happened is from the C_D equal to 0.8 to 0.15.

And that because of series of the experiments done for the automobile sectors to find out what could be the best shape of the cars. So that, the coefficient of drag should be minimum or should be coming to closer to theoretical minimum value of 0.15. So that is what it happened in the evolutions of what has happened. And that is the reasons we have a fuel efficient cars available nowadays and with a C_D value close to the theoretical minimum and this is what is possible because of the fluid mechanics as evolution with last 120 years.

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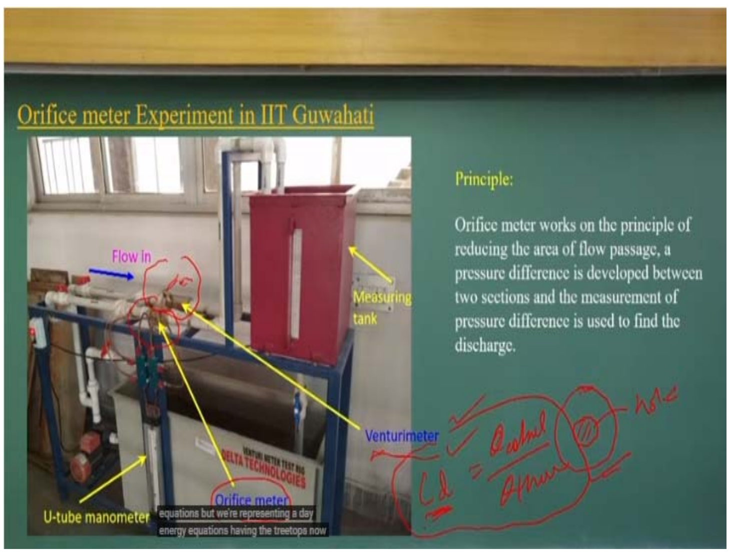

Now come back to the where simple experimental setups, we generally do the measuring the flow in a pipe, either in a venturimeter or the orifice meter. The orifice meter is a smalle device, that means it will be a circle and there is a hole inside this. This is called the orifice. In case of the venturimeter you have a diverging zone and converging zone. But in case of orifice, we do not have a diverging and converging zone, only you have a reductions of the flow area that is what is the orifice meter.

That means you have a simple plate with a hole with a constant, a particular diameters and that is what is fixed here. So, we can measure the pressure difference at two locations; one is at the fiber locations where the cross sections is really is less and another is the off stream locations. That pressure difference and if you apply simple Bernoulli equations and the mass conservation equation, you can find out what will be the discharge passing through venture or orifice meters.

But as I said it earlier, that whenever flow goes through these venturimeter or orifice meter, there will be energy losses. Because of the energy losses, they will be difference between theoretical discharge and actual discharge. Since the Bernoulli equations what we applied, we do not consider the energy loss components. We consider there is no energy losses, okay. No the frictional resistance is more or there is no vortex formations and all. But in the real fluid flow, there is energy losses.

Because of that, what do you will have, your theoretical discharge compute from the Bernoulli equations and the mass conservations will be more than actual discharge. So actual discharge, the difference between these, we introduce a coefficient of discharge, that means the CD.

$$c_d = \frac{Q_{actual}}{Q_{theoritical}}$$

Coefficient of discharge is C_D here, it is not a coefficient of drag, please do not have a conclusion between the two terms.

If you look at the device where we have a setup of venturimeters and orifice meters, the basically in a venturimeter we have a converging zones and diverging zones and in orifice we do not have a converging diverging zones, but we just incorporated a plate with holes, okay, with a internal hole of this. Because of that, it also create, the converging zones and the diverging zone.

And because of that, we can have a energy losses in a flow systems when the streamline are converging and the diverging, definitely there will be energy losses with this systems. As the energy losses will be there, the theoretical discharge computed using Bernoulli equations and the conservation equations will be higher than the actual discharge. That is the reason, we introduce the coefficient of the discharge, the C_D value, which is the ratio between actual discharge by theoretical discharge.

So, we introduced the coefficient of discharge in this and many of the times we have to compute it, what is the coefficient of discharge. So now if you can try to understand it that many of the times we do not go inside in this fluid flow problems within the venturimeter or the orifice meter that how the flow streamlines, how the energy dissipater happens, that what we do not look it, that in depth. But in terms of these flow patterns, what is it the gross characteristics in terms of energy losses, in terms of reductions in a discharge.

That is what we introduce as a C_D coefficient, the coefficient of discharge. So, these are gross representations of change of the flow patterns, the pressure and velocity patterns. Because of that and flow structures. We need we do not look it very microscopically, how it is changing it, but as a gross representation, we just look it the coefficient of discharge as the relationship between the theoretical and the actual. There is a link between now experimental work and analytical farm work.

So, the Bernoulli equations is strength there that this is the equations we can easily incorporate the experimental relationship into these Bernoulli equations. So, that is the strength of the Bernoulli equations as compared to the other Euler forms or other Navier-Stokes equations form, where Bernoulli equations are very suitable, appropriate to incorporate any energy losses, energy to this a fluid flow systems or taking out from the fluid flow system.

All we can incorporate it in a Bernoulli equations looking as energy conservation equations, looking as a energy conservation equations. But we remember, this is a equations for along a streamlines, is a momentum equations, but we are representing as a energy equations, having the three talks.

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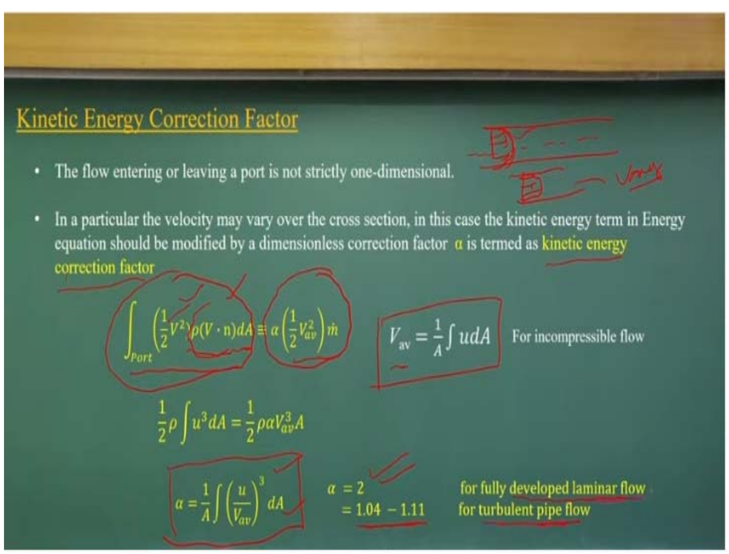

Now, let us come back that whenever you have a fluid flow problems, like fluid flow through a pipe. So definitely the velocity distributions is not uniform. The velocity distributions will be change it, from laminar to the turbulent. The turbulent will have logarithmic profiles and the laminar will be parabolic profiles. So velocity distribution changes, so what we do it that, it is very difficult for us to put it as the average velocity to compute the kinetic energy.

Because the flow within these pipes follow, having a parabolic flow velocity distribution. Now, we are looking it, if I consider the flow velocity distributions. Because of that velocity distribution, what is the total amount of kinetic energy? I can compute it with simple integrations, considering the velocity distributions. Then I will compute it, if I consider the average velocity, what will be the total kinetic energy. And since I am going to use average velocity, not the velocity distributions.

$$\int_{Port}\left(\frac{1}{2}V^2\right)\rho(V\cdot \mathrm{n})dA \equiv \alpha\left(\frac{1}{2}V_{av}^3\right)\dot{m}$$

$V_{\mathrm{av}} = \frac{1}{A}\int u dA$ For incompressible flow

I apply a correction factor for that, which is called kinetic energy correction factors. Let me repeat these things, what we are looking at that in a real fluid flow problem, always we have flow is not uniform, non-uniform distribution. that means it will have a either a parabolic distribution, logarithmic distributions. It depends upon the flow, through pipes, or open channels. The flow velocity distributions varies in a space.

So, when you apply the kinetic energy, we look at the total kinetic energy, which is we can integrate over that a small dA element with a. This is a mass flux, rho V dot n is a mass flux and this is a half rho V square is the mass flux into V square by 2 is a kinetic energy flux. That is what we will do the integrations over the total area dA and we have the kinetic energy. So, if you look it, the kinetic energy computations, using the V square, V average values will have the kinetic energy computed using average velocities.

Times of alpha, alpha is kinetic energy correction factors. The basically what it indicates that, if you look it, compute the kinetic energy best on average velocity, then what could be the correction factors for non-uniform distributions of velocity in the particular flow field condition. So, if I equating these things, then my alpha will comes like this part, it is very simple part.

$$\frac{1}{2}\rho \int u^3 dA = \frac{1}{2}\rho\alpha V_{av}^3 A$$

As you know, V average we compute, is average velocity, we can compute like this.

$$\alpha = \frac{1}{A}\int \left(\frac{u}{V_{av}}\right)^3 dA$$

$$\alpha = 2 \qquad \text{for fully developed laminar flow}$$

$$= 1.04 - 1.11 \qquad \text{for turbulent pipe flow}$$

So alpha can consider it V average u to the power 3 or cubic power dA and this integrations will give us the alpha. For fully developed laminar flow, this alpha value is about 2. Turbulent pipe flow, this is what varies from 1.04 to 1.1. That is what is the biggest problems many of the fluid mechanics book, they do not give the alpha value, they consider the alpha value is close to the 1 or indirectly they represent that flow is turbulent. The alpha value variations of 1.04 to 1.11, they do not consider it.

But that is not correct, any fluid flow problems, we should compute what is the alpha value and whenever we apply the Bernoulli equations, considering the velocity distributions, the kinetic energy, correction factors needs to be done it and that is the value which is varies for the turbulent pipe flow and for the laminar flow, which is equal to 2 value.

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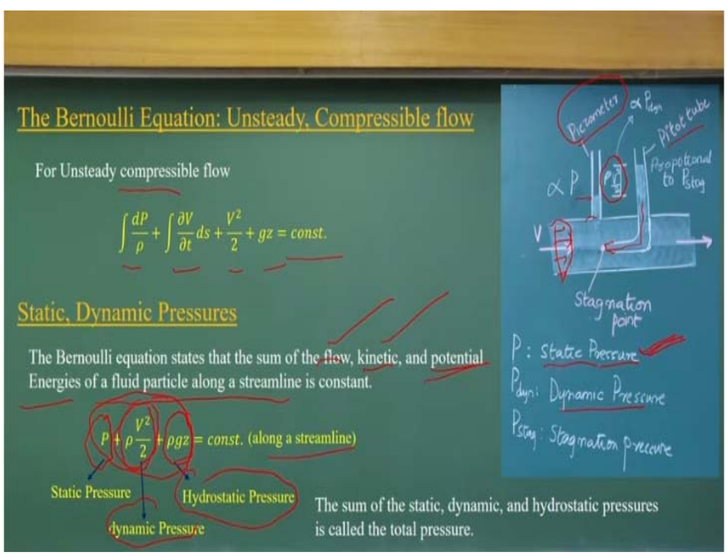

Now, let us come back to this Bernoulli equation what we have derived, that in case of unsteady compressible, will have the two integrals component.

$$\int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} + gz = const.$$

But in case we make it the steady and incompressible, then we get it these three forms as earlier I discuss it, this is summations of three energies, flow energy, which is because of the pressure at that locations.

How much work is done by that pressure, that is what will be the flow energy. You have a velocity field, which have a kinetic energy and the fluid particles or virtual fluid balls staying a particular height and from a **(()) (19:30)**, which gives potential energy. So, three energy along a streamline is a constant. The sum of these three energy flow, kinetics and the potential energy of a fluid particles along a streamline is constant. That is what we have written it.

$$P + \rho \frac{V^2}{2} + \rho gz = const.$$

Now, in terms of pressures, if you talk about, one is static pressure, that is the presses act on this fluid particles another is the dynamic pressure because of the velocity components. Please remember this $\rho \frac{V^2}{2}$we have used when you come the drag force, okay. And we have ρgz, which is the hydrostatic pressure. So, we have three pressure components, in terms of pressure if you look it. In terms of energy, we can define it is a flow energy, kinetic energy and potential energy of a fluid particles along a streamline is a constant.

Now, let me demonstrate it if I have pipe flow, flow is going through this, it has a velocity distribution. As you know it, the velocity near to wall is 0 and whatever the height of the fluid go, this is what will be represent is me the pressure. That is what will be represent me the pressure head, that is what the static pressures. So, basically this height will be the static pressure and that is what we measure through piezometers. But, if I insert a tube and put it inside this.

And as the flow is moving with velocity V and stuck over this, since it is a constant, it is a fixed point, the velocity has to be zero at that point. So, the stagnation point will be work on this. So, because of that, what will be happen it, the two energy components what we have the flow energy and the kinetic energy that what convert to as a head, as equivalent water head or the liquid head will get it that. So difference between these two will give us $\rho \frac{V^2}{2}$.

That is what is the kinetic energy head. The difference between this Pitot tube and the piezometer will give us the difference of these energy $\rho\frac{V^2}{2}$. But you can say that, what about the potential energy. These two energies are so high that, we do not consider the z difference what we have that may not have a that significance. That is the reason, we do not consider the potential energy part, but we can measure. When you have a pipe flow we can measure, using the piezometer you can measure the pressure.

Using the pitot tube, which **(()) (22:46)** can insert and have a sharp point to rise the fluid inside the tube. That is what measures both the components, the dynamic pressure as well as the static pressures. The dynamic pressure and the static pressure. Since we know the static pressure, you just compute what is will be the dynamic pressure. The stagnation pressure is a sum of static pressure and dynamic pressure.

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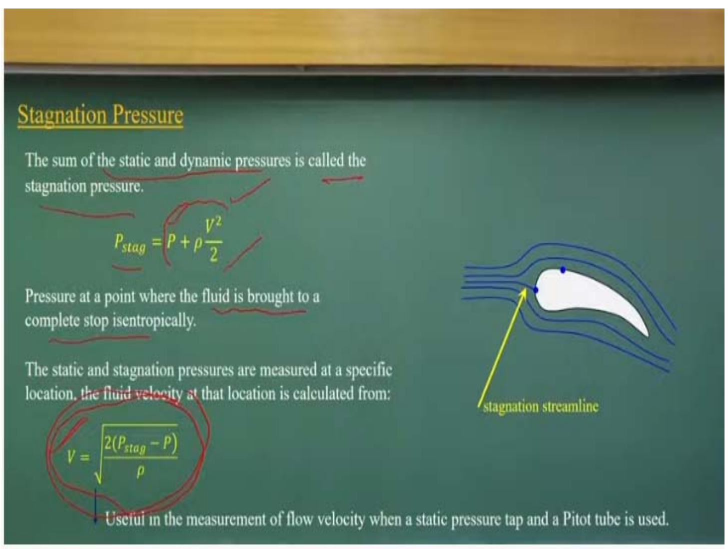
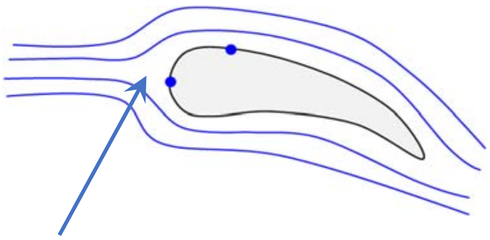


Now, if you look it that same concept what we are talking about. As I say that the sum of the static and dynamic pressure is called the stagnation pressure. That whenever you have an object, which is at the stationary conditions, it is at rest conditions. At that point, will have the, the pressure will be the, because the velocity at that point of the fluid flow have to the zero. So we get is the stagnation pressure, we will have a two component.

$$P_{stag} = P + \rho\frac{V^2}{2}$$



stagnation streamline

The static pressure component and the dynamic pressure component. So that is what we get it, because pressure at point where fluid is brought to a completed stop, then we will have a stagnation pressure will be two components, one is a static pressure component another is dynamic pressure component and those equations we can rearrange it to compute it what will be the velocity. I think this is what is used in even in modern era, in any aircraft wings, you can see there are pressure sensors and based on these pressure sensors, we can easily compute it, what will be the air speed.

$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

So to measure the air speed in any modern aircraft, still also use the same concept of the measuring the pressure and as you measure the pressures at this stagnation point and you know the atmospheric pressures, then we can easily find out what could be the airspeed is coming that. This is the equations used for this to measure the flow velocity when static pressure staff at pitot tube is used.

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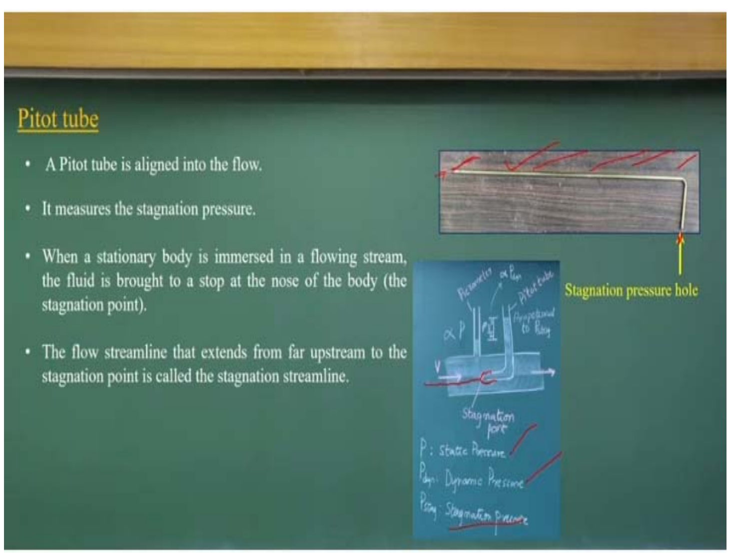

This is the same concept of the pitot tubes as I explained to you, the basically it measures the stagnation pressures, which have both the components and if you have the piezometer, then you can differentiate and find the dynamic pressures and once you know the dynamic pressure, you can compute what could be the velocities. The flow streamline that extend from upstream to the stagnation point is called stagnation streamline, the basically this streamline, which is just come it hit over the mouth.

Just pinpoint of this hole, this is what the pitot tube, this is not a big instrument, this is a small tube inclined here. So, there is a pressure hole, at this point. This is the pressure holes to measure it, this is the points where the stagnation pressure is develop it and that is what will be the two components, static pressure and dynamic pressure and using these with the help of the manometer we can measure the pressure.

And once you know the pressures, you can compute it dynamic pressures and you can compute the velocity. It is a just a applications of Bernoilli's equation.

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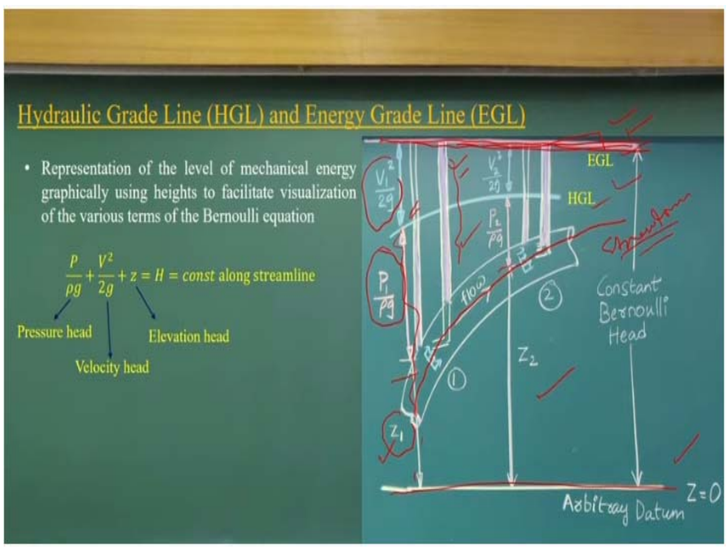

Now, another interesting applications of Bernoulli equations is that energy gradient line and the hydraulic gradient line. This is very great simplifications of the fluid flow problems. Like you may have fluid flow problems with pipe arrangement, the dock arrangement and all. Any flow, as I say that, it can have the flow distributions. But as total energy, along a streamline, if you can draw a streamlines and you want to quantify how these energies are changing it, okay,

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = const \text{ along streamline}$$

That itself as you know it, fluid flows from higher energy to the lower energy, not the higher elevation to lower elevation. Please do not have that things. The fluid flows from higher energy to the lower energy. As it flows from any pipe flow, the channel flow, any doc flow, okay, always there will be a flow which starts from the higher energy to lower energy. Whenever flow from higher energy to lower energy, definitely there is an energy loss is happens.

And these energy losses if you can quantify experimentally, that is what I will discuss when I discuss the pipe flow, how very interestingly this energy losses in the pipe are quantified and then you apply the Bernoulli's equation for designing total pipe network. Same way if you know it in a different type of open channel flow, how much of energy losses is happening, you apply this Bernoulli's equation. Also, you can design the open channel flow.

The basic concept here is called that we should always gauge energy gradient line, hydraulic gradient line. So, any flow patterns, we have to draw the energy gradient line and hydraulic gradient line. What is energy gradient line? Now, we are representing the Bernoulli's equation again in a different forms, it is the same things, okay. Same energy we talk about in terms of head, in terms of meter, okay. That means as equivalent if I have any liquid will be there, how much the lift will be there because of the pressure.

Because of the flow energy, because of the kinetic energy, because of the potential energy. That is the reason, we define it as a head. The pressure head, the velocity head, and the elevation head. It is nothing else, they are all three energy, the flow energy, kinetic energy, and the potential energy, constant along a streamline. And this energy gradient line and the hydraulic lines, what the difference between that. Energy gradient lines consider all these points.

That means, you have Z_1, this is a flow energy, this is the kinetic energy. If you do not have the kinetic energy or the velocity head, then we can have the piezometer lines. If you consider the velocity head, then we have energy gradient. So, always we should have a flow from higher energy to lower energy, okay. This gauge is not that appropriate, that should be the gradient of energy gradient lines. Otherwise, it is not possible to have a flow, okay.

If having a flow, if there is flat energy gradient line, there will be no flow. There is no flow when you have a constant energy gradient, there is no slope of energy gradient line. So, definitely there will be scope of energy gradient line, okay. And that what is drive the flow. So, you have energy gradient line and you can have a pitot tube and the piezometer to measure the hydraulic gradient and the energy gradient, and the energy gradient line and then we can sketch it as energy gradient line to get the hydraulic gradient line.

And many of the times, we can consider a **(()) (30:50)** from where you can compute the Z_1 Z_2, the elevations head of the, that means we define a streamline, then we are just drawing the energy gradient lines for this streamline, for this streamline we are drawing the energy gradient line, we are drawing the hydraulic gradient lines. The hydraulic gradient lines is not, it does not consider velocity head, it just consider pressure head and the elevations head.

Just like you just put a piezometer, then you get it what is the pressure head, also the elevation head is know to us that the what will come it. As soon as you put the piezometers whatever the point to will come it, that what will define us hydraulic gradient line, if I put a pitot tube, the line what will by the top surface will be the energy gradient line. So, whenever we design the pipe flow or channel flow and all, we need to draw energy gradient line and the hydraulic gradient line.

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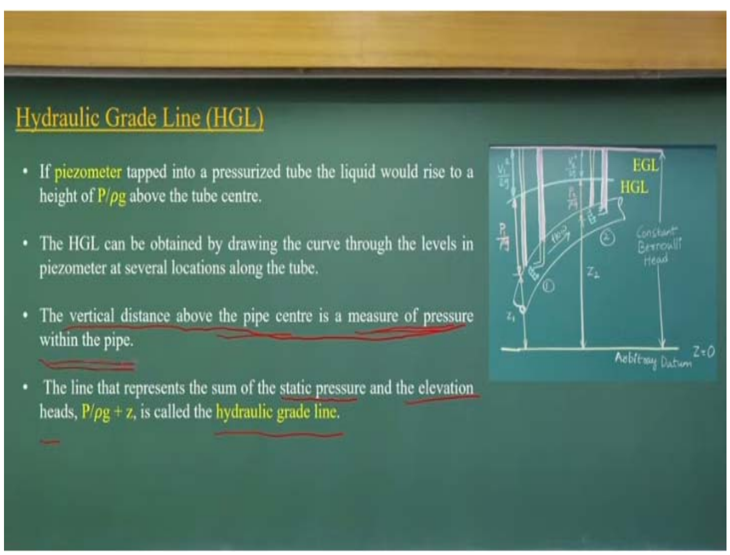

That is what I have explaining it that, so the basically piezometers and the hydraulic gradient line what I have said it and the hydraulic gradient line has some of the static pressures and the elevations. And the vertical distance above the pipe center is measure of the pressure within the pipe, okay. Within the pipe, what is the distance above the pipe centers, is a measure of the pipe pressure within this.

$P/\rho g + z$, is called the hydraulic grade line

What is talking about this sentence is that even if you are putting it here, what is the pressure we are getting it that what is the pressures within the pipe, it is measure it, more or less the same value, the elevations will not be that difference.

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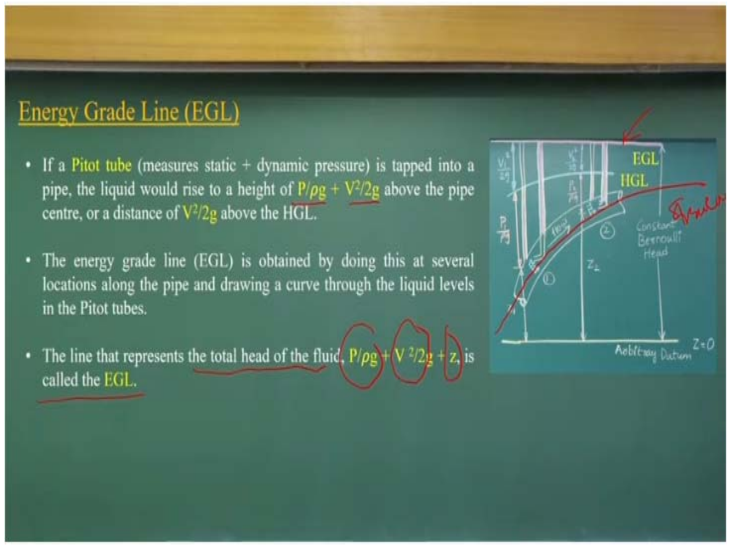

This is what I explain it, if a pitot tube is tapped in the pipe, the liquid could rise in height, which will have the static head and the dynamic head above the pipe and that will be above of dynamic pressure of the $V^2/2g$. The velocity head part and that is what we can draw it, represents a total head using energy gradient line, which represents the total head of the fluid flow energy, kinetic energy and the potential energy, whenever we draw a streamline, we can compute the energy gradient line, hydraulic gradient line.

So, again I continue to say that energy gradient line should have some slope, so there is will be the flow. The flow will be from higher energy level to the lower energy level. The hydraulic gradient may have the difference, okay that is not a big problems, but the flow goes from high energy level to low energy level. The line that represents the total head of the fluid, $P/\rho g + V^2/2g + z$, is called the EGL

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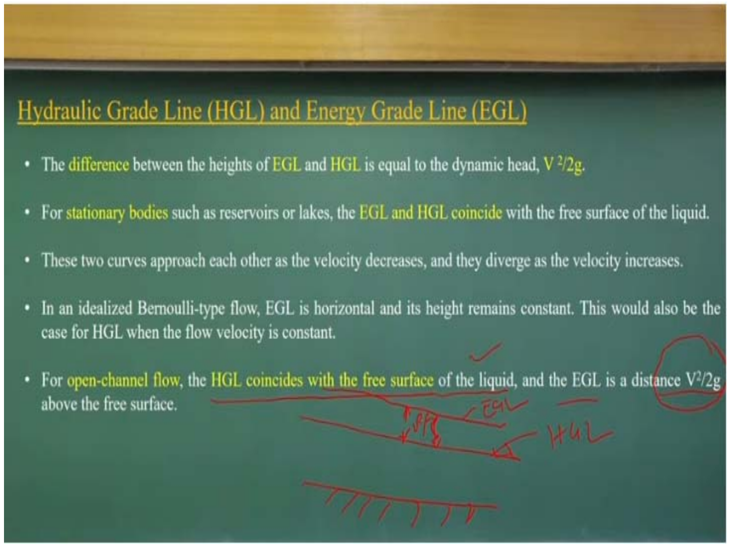

And okay, we are not discussing about open channel flow heads, but in case of open channel flow, the hydraulic gradient lines coincides with the free surface of the liquid okay. Because there is no pressure head. So, whatever the water surface free surface, that what will be hydraulic gradient line and the energy gradient lines will have a included the velocity head above the free surface. That means, if you consider a open channel flow, this is the free surface.

This free surface will be representing us hydraulic gradient line and $V^2/2g$ of this, adding this velocity head, will get energy gradient line, okay, in case of open channel flow because there is no pressure head. But in case of the pipes, we can have a piezometer to measure it, what could be the hydraulic gradient lines. Energy gradient line to measure it we need to have a pitot tube to compute what will be these things.

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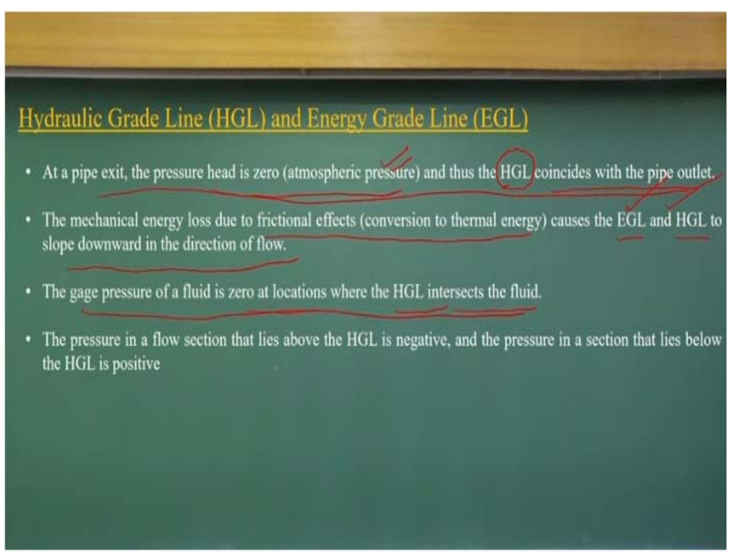

Now very basic things we should understand it, whenever a pipe exit, that means flow is going out. The pressure head becomes atmospheric pressure, no doubt about that. And that is the reason, the hydraulic gradient lines coincidence with the pipe outlet. So, when the pressure head becomes zero, that is what we said it, in case of open channel flow, when there is no pressure, is pressure is atmospheric pressure, that the surface becomes hydraulic gradient line.

Exactly same way, when you exiting, a pipe is exiting the flow to the atmosphere, that means your hydraulic gradient line should coincidence with the pipe outlet. Mechanical energy losses due to the frictional effects, which is converting from thermal energy, the causes the energy gradient line, hydraulic gradient line, to a slope downwards in the directions of the flow, that is what I try to explaining to you. That there will be energy losses whenever you have the flow systems, okay.

So, there will be a slow downward in the direction of flow of energy gradient line and also the hydraulic gradient lines. But energy gradient lines should have this, but sometimes maybe hydraulic gradient may not have the slope downward in direction of the flow. The gauge pressure of the is zero at the locations when the hydraulic gradient line intersect the fluids. The same things I think we are repeating it whatever we have said it in the first point.

This is the same point what we are talking about here, that it happens it, if there is a gauge pressure, okay. Pressures in a flow sections lies above the hydraulic gradient line is negative. Pressure in a section when the lies below the hydraulic gradient line is positive and the negative that is the difference what we get it. Pressure in a flow section that lies above the hydraulic gradient line is negative and pressure in a section that lies below the hydraulic gradient line will be the positive value.

So whenever will solve the pipe flow problems, will show it how the pressure it changes negative to positive directions in respect to the hydraulic gradient line.

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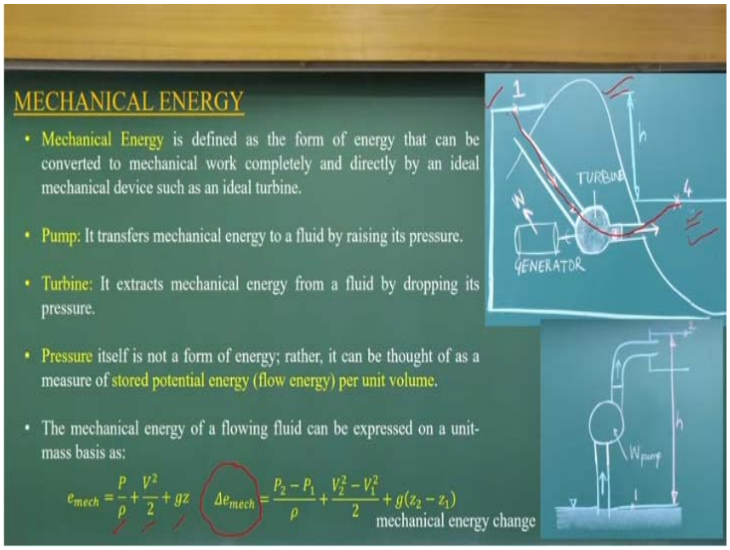

Now, let us consider is the problems when we generally use as a engineer having a pump and turbine systems. As you know it, the pump is transfer from mechanical energy to fluid by rising pressures, okay. Transfer the mechanical energy to a fluid. So, fluid gains the energy because of the pumping systems by rising its pressures, that is the. Similar way, the turbine does opposite things, that it takes out or extract the mechanical energy from the fluid by dropping it pressure.

So, this is what the **(()) (38:27)** turbine. So, when you have a flow systems and a pipe network of your campus, you can see there is a series of pumps are there to increase the energy in the flow systems, the mechanical energy in the flow systems. And the turbines are there wherever you have a hydropower projects. So you need to have a turbine, they extract the energy from the flow systems by dropping its pressures. So increasing the pressure and dropping the pressure.

Pressure itself is not a form of energy as we all say that, it is just a flow energy or storage potential energy per unit volume that is the pressures. So, if I look it this, again coming back to the Bernoulli equations, I have the three terms which will be define us as an energy term, okay. And if I have a two points, I am differentiating between these two energy, the difference part will show me the net energy difference will come it. That means, for example, I have a these systems.

There is a dam, the reservoir, water is going through a turbine and generators, coming out at the three, then it is related to the tail end reservoir. So, we have upstream reservoir, I have a tail end reservoir, there is a connections between the pipes and in between, I have the turbines and the generators. If you look it, if I take a flow streamlines, assuming the flow is comes like this and coming to this point, okay. So, I can find out what is the mechanical energy difference between these two part, just differentiating this energy at the one end flow level locations.

$$e_{mech} = \frac{P}{\rho} + \frac{V^2}{2} + gz$$

$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

Similar way, two and three locations we can find out. And this is a very simple pumping arrangement you can know it that we use a pumping to enhance the mechanical energy in the fluid flow systems by rising its pressures and because of that we have a lifting the waters in a water head tank as you know it.

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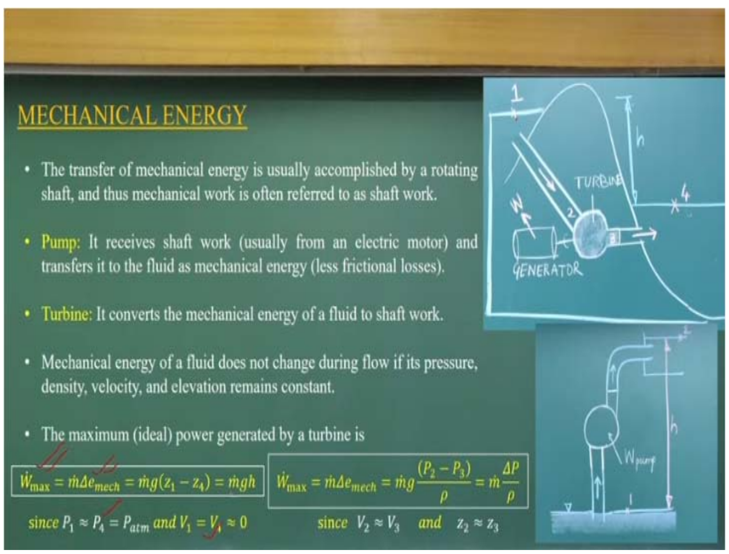

This mechanical energy what we get it and if I multiply with a mass rate, I will get the work done part, okay. How much of work done we are getting it or how much power we are getting it, maximum ideal power generated by turbines will be m dash and the energy part, mass flux part which we will be talk about work per unit time is the power. So, basically, we are looking for the power part, which will be, since this energy per unit mass, so we have multiplied with the mass rate to find out what will be the power will be there.

For example, if I take it the same problems, I need to know it what is the maximum ideal power generated by this turbine. Assuming it that all are 100% efficient systems that maximum power generated from these one and two four points. As you know it, the pressure at these two points will be atmospheric pressures and both are the same. The velocity at one and four, the velocity becomes zero, only this elevations difference, the potential difference what will be get it, that what is maximum ideal power generate by the turbine, okay.

$$\dot{W}_{\max} = \dot{m}\Delta e_{mech} = \dot{m}g(z_1 - z_4) = \dot{m}gh$$
$$since\ P_1 \approx P_4 = P_{atm}\ and\ V_1 = V_4 \approx 0$$

$$\dot{W}_{\max} = \dot{m}\Delta e_{mech} = \dot{m}g\frac{(P_2 - P_3)}{\rho} = \dot{m}\frac{\Delta P}{\rho}$$
$$\text{since}\quad V_2 \approx V_3 \quad and \quad z_2 \approx z_3$$

But if I consider the two and three, this before turbine and after turbine. At that locations, I have a $V_2 \approx V_3 \quad and \quad z_2 \approx z_3$. Substitute this value, I will get it the power generated by the turbine will have a mass flux and ΔP, the pressure difference between P_2 and P_3 by the ρ value. So, these are very simple calculations, you can do it.

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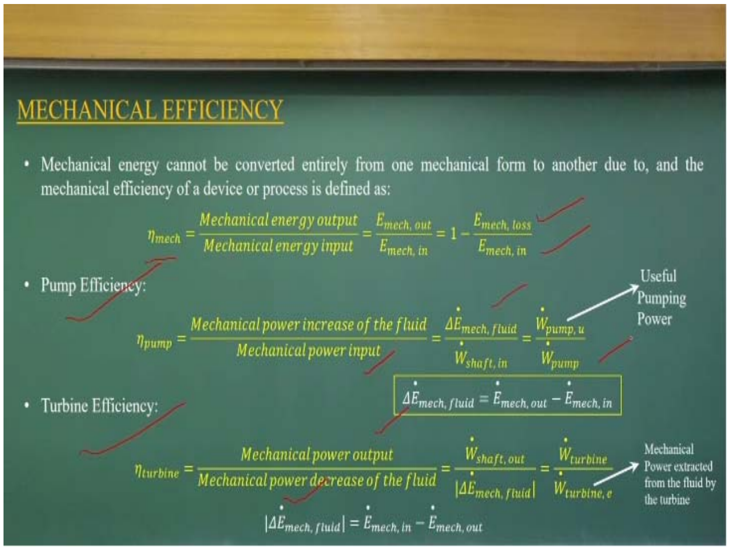

Now, whenever we have a system, there is energy losses. We cannot convert total mechanical energy entirely from mechanical pump or pump. There will be some heat energy losses, the sound energy losses, will have the energy losses from that. Because of that, there is an efficiency looped to that.

$$\eta_{mech} = \frac{Mechanical\ energy\ output}{Mechanical\ energy\ input} = \frac{E_{mech,\ out}}{E_{mech,\ in}} = 1 - \frac{E_{mech,\ loss}}{E_{mech,\ in}}$$

If efficiency equal to 1, that means whatever input that what we get the output. Like we are quantifying what is the mechanical efficiency of the systems. If mechanical energy input and output, the ratio, that is what is efficiency of that.

Or in terms of loss, we can also quantify it, like this, very simple. Similar way, pump efficiency and the turbine efficiency, input and output.

$$\eta_{pump} = \frac{Mechanical\ power\ increase\ of\ the\ fluid}{Mechanical\ power\ input} = \frac{\Delta\dot{E}_{mech,\,fluid}}{\dot{W}_{shaft,\,in}} = \frac{\dot{W}_{pump,\,u}}{\dot{W}_{pump}}$$

$$\Delta\dot{E}_{mech,\,fluid} = \dot{E}_{mech,\,out} - \dot{E}_{mech,\,in}$$

The ratio is between output to input, that is what is the efficiency of the pump or the turbine. The mechanical power increases of the fluid, by mechanical power input, turbine mechanical power output by mechanical power decreased by of the fluid. That is what will gives the turbine, the efficiency of the turbine systems.

$$\eta_{turbine} = \frac{Mechanical\ power\ output}{Mechanical\ power\ decrease\ of\ the\ fluid} = \frac{\dot{W}_{shaft,\,out}}{|\Delta\dot{E}_{mech,\,fluid}|} = \frac{\dot{W}_{turbine}}{\dot{W}_{turbine,\,e}}$$

$$|\Delta\dot{E}_{mech,\,fluid}| = \dot{E}_{mech,\,in} - \dot{E}_{mech,\,out}$$

And these are the basic powers, in terms of powers also we have defined them as a efficiency components.

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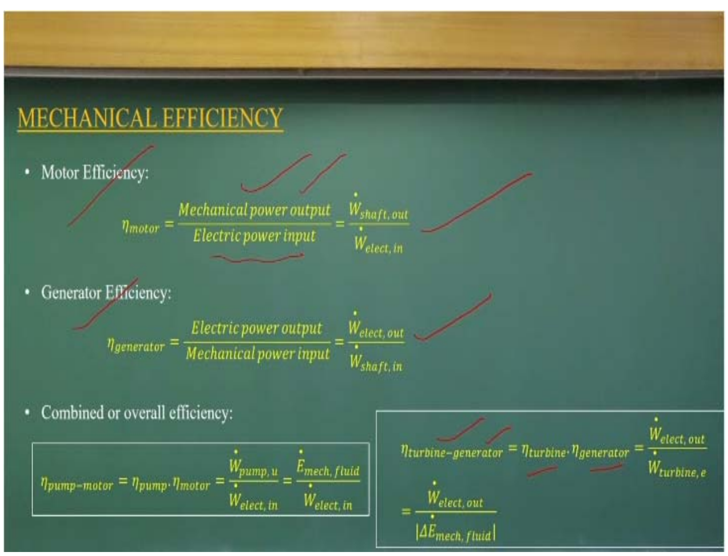

Similar way, you can have a motor efficiency and the generator efficiency. Electrical power input for the motors, what is the mechanical power output. Motor generates from electric to the mechanical power. That is the reasons you have a input is electrical power, output is mechanical power. That ratio we get it.

$$\eta_{motor} = \frac{Mechanical\ power\ output}{Electric\ power\ input} = \frac{\dot{W}_{shaft,\,out}}{\dot{W}_{elect,\,in}}$$

Similar way, generators we have the mechanical power to electrical, but when you have a combined systems, pump on motors, then you have a efficiency of the both the systems you have to input.

$$\eta_{generator} = \frac{Electric\ power\ output}{Mechanical\ power\ input} = \frac{\dot{W}_{elect,\,out}}{\dot{W}_{shaft,\,in}}$$

Pump and the motors, that is what we will define it. Similar way, if you have a turbine and generator, you have efficiency up to the things which are independent, you need to know the combined efficiency, that means you have to have a multiplication factor for that and that what is given here.

$$\eta_{pump-motor} = \eta_{pump} \cdot \eta_{motor} = \frac{\dot{W}_{pump,\,u}}{\dot{W}_{elect,\,in}} = \frac{\dot{E}_{mech,\,fluid}}{\dot{W}_{elect,\,in}}$$

$$\eta_{turbine-generator} = \eta_{turbine} \cdot \eta_{generator} = \frac{\dot{W}_{elect,\,out}}{\dot{W}_{turbine,\,e}} = \frac{\dot{W}_{elect,\,out}}{|\Delta \dot{E}_{mech,\,fluid}|}$$

Now let us come it to the example problems, okay, which are GATE and engineering service problems.

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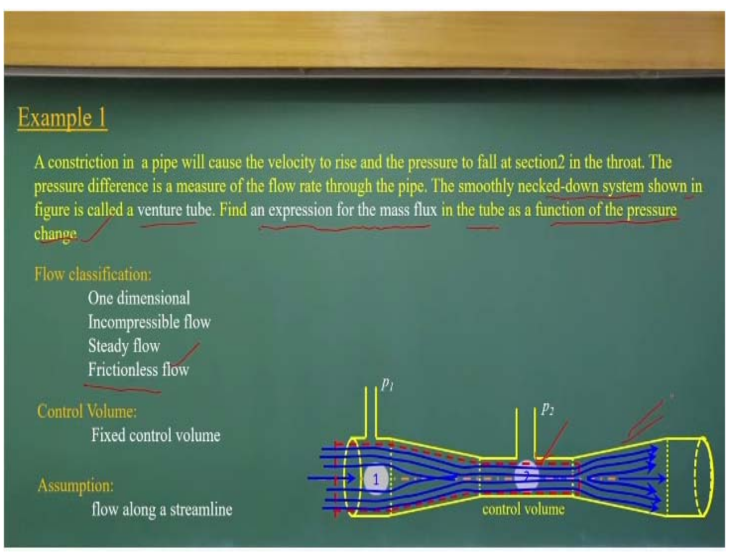

These are very easy problems, here we will apply mass conservations and Bernoulli equations, that is all. We are not going to do much here. Mass conservations and the Bernoulli equation what will be use it. But I always encourage you gauge the control volumes as well as draw streamlines. After the drawing the streamlines, the apply the Bernoulli equation for the two points, okay. That to be looked it this part and always highlight it, what are the assumptions behind that.

[A constriction in a pipe will cause the velocity to rise and the pressure to fall at section2 in the throat. The pressure difference is a measure of the flow rate through the pipe. The smoothly necked-down system shown in figure is called a venture tube. Find an expression for the mass flux in the tube as a function of the pressure change]

What are the assumptions you have put it, that should be highlighted before solving any fluid flow problems. Now, look at this example one, which is a constrict in a pipe will cause the velocity to rise it definitely. Okay, there are, just like a venturimeter, here there is a converging zones and the diverging zone, okay. And because it is converging zones, the velocities to rises, pressure to be fall it. That is what we know it very beginning of Bernoulli equations.

Flow classification:

One dimensional

Incompressible flow Steady flow

Frictionless flow

The pressure difference is a measured of flow rate through the pipe. The smooth neck down system shown in the figure is called venturi tube. Find an expression for the mass flux in the tube as a function of the pressure and the pressure change, which is very simple things. Will just highlight it how to use mass conservation equation and Bernoulli equations. Now, let us gauge it, what are the flow classifications. Here we can assume it one dimensional flow because incompressible, steady and frictionless.

That is what is our strength is, because we are directly applying the Bernoulli equations as there is no energy loss component, we are not including that. Fixed control volumes and we need to apply this Bernoulli equations along a streamline. The streamlines could be like this. There will be converging zones and the diverging zones, streamlines will be the converging and the diverging. Or you put the virtual fluid balls you can have the track of the flow path.

The path lines of the virtual fluid balls also could be like this, okay. They will converge it, more or less parallel in these regions. So, there will be no curvatures of streamlines in this regions. Then they will be the divergence on the streamlines, okay. So that the flow structures what we get it in venturi tube.

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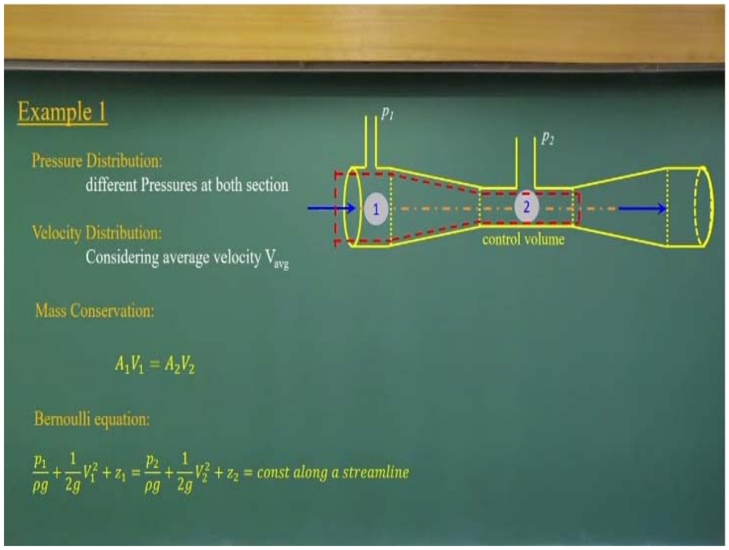

And we need to compute it in terms of average velocities. So, we are not considering is turbulent or laminar velocity things. We are not putting any kinetic energy correction factors, which supposed to be done it, but in this case we have not done it. We just apply the mass conservations and the Bernoulli equations assuming that there is a uniform flow distribution, it does not happen like that. We use the average velocity here.

$$A_1V_1 = A_2V_2$$

$$\frac{p_1}{\rho g}+\frac{1}{2g}V_1^2+z_1=\frac{p_2}{\rho g}+\frac{1}{2g}V_2^2+z_2 = const\ along\ a\ streamline$$

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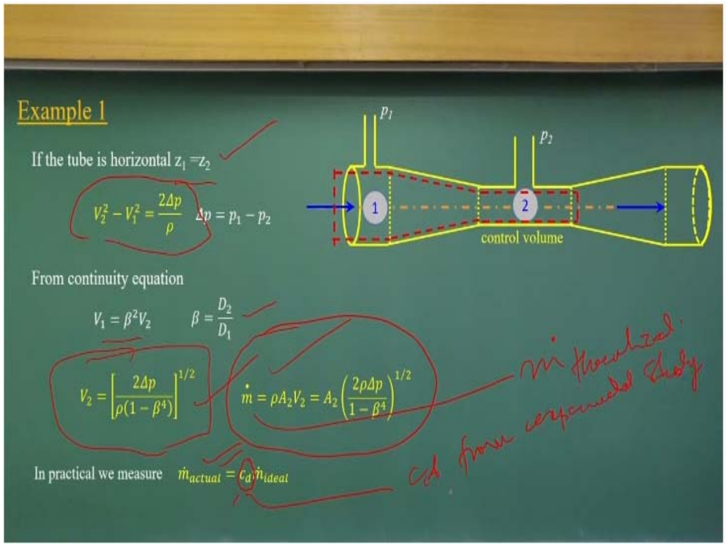

And applying this, as the tube is horizontal, z1 = z2, you can write in terms of velocity difference in terms of pressure and you apply the continuity equations to get it V2 in terms of pressure difference and the beta which the ratio between the d2 and d1, representing the diameters and the sections 2 and the diameter at the sections 1, respectively. That is what we will get it and finally, the mass flux will be rho AV, rho AV. So, area at the sections 2 you know it, V2 you know it, then we can compute it what will be the mass flux.

If the tube is horizontal $z_1 = z_2$

$$V_2^2 - V_1^2 = \frac{2\Delta p}{\rho}$$

$$\Delta p = p_1 - p_2$$

From continuity equation

$$V_1 = \beta^2 V_2$$

$$\beta = \frac{D_2}{D_1}$$

$$V_2 = \left[\frac{2\Delta p}{\rho(1-\beta^4)}\right]^{1/2}$$

$$\dot{m} = \rho A_2 V_2 = A_2 \left(\frac{2\rho\Delta p}{1-\beta^4}\right)^{1/2}$$

In practical we measure

$$\dot{m}_{actual} = c_d \dot{m}_{ideal}$$

But please remember, as we have not considered energy losses, we need to incorporate a Cd value, which is the coefficient of discharge to these computations to find out what will be the actual value. This is what is m dot theoretical value. And this Cd comes from experimental study. The Cd comes from the experimental study.

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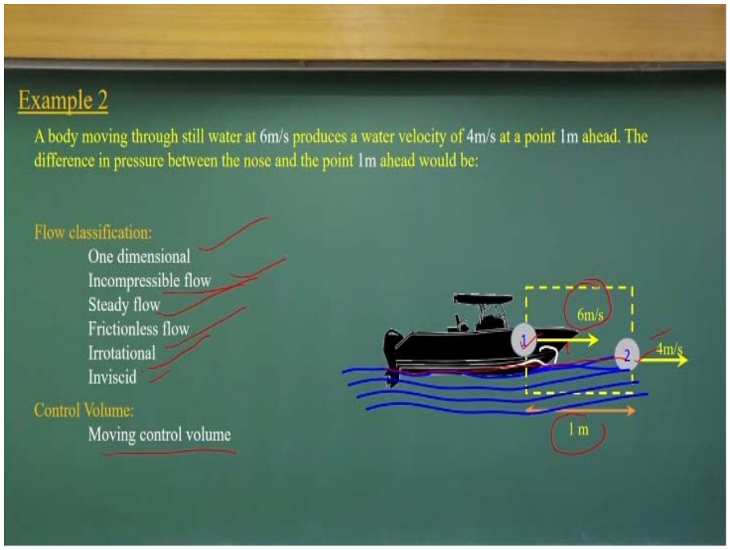

Let us have another examples very, interesting examples that a body is moving with a still waters of 6 meter per second produces water velocity 4 meter per second at 1 meter ahead. That means there is a distance between these two points is 1 meters, this is what 6 meter per second, the flow is, there is a reservoir type of conditions where the still waters are there and the velocity at this point is 4 meter per seconds.

[A body moving through still water at 6m/s produces a water velocity of 4m/s at a point 1m ahead. The difference in pressure between the nose and the point 1m ahead would be:]

Flow classification:

One dimensional

Incompressible flow   Steady flow

Frictionless flow

Irrotational

Inviscid

Now, we have to compute it, what could be the pressure difference between the nose and the point 1 meter ahead could be. So, we can assume, draw the streamlines, which is passing through this point 1 and point 2, incompressible flow, steady, frictionless, irrotational, inviscid, okay. It is a moving control volume, but we are not looking it as a moving control volume, we are just looking the pressure difference, we are not looking at the mass fluxes here.

So, we will just apply the Bernoulli equation between one and two, because they are at the same horizontal levels.

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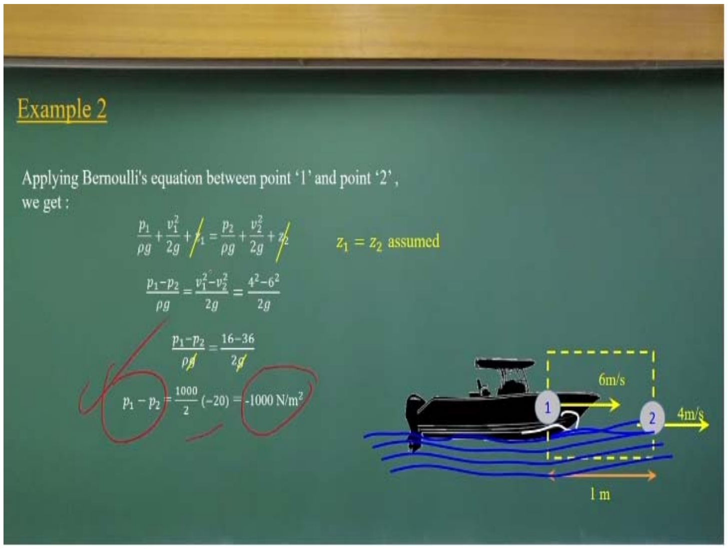

The z1 = z2 only will have the pressure difference will be equate with the pressure head is equate with the velocity head. And by substituting the pressures and the velocity, we will get it what will be pressure difference. It is very simple problems, but only you have to visualize the problems, how it looks like this. We have to sketch for that.

$z_1 = z_2$ assumed

Applying Bernoulli's equation between point '1' and point '2' , we get :

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{p_1 - p_2}{\rho g} = \frac{v_1^2 - v_2^2}{2g} = \frac{4^2 - 6^2}{2g}$$

$$\frac{p_1 - p_2}{\rho g} = \frac{16 - 36}{2g}$$

$p_1 - p_2 = \frac{1000}{2}(-20) = -1000\ N/\mathrm{m}^2$

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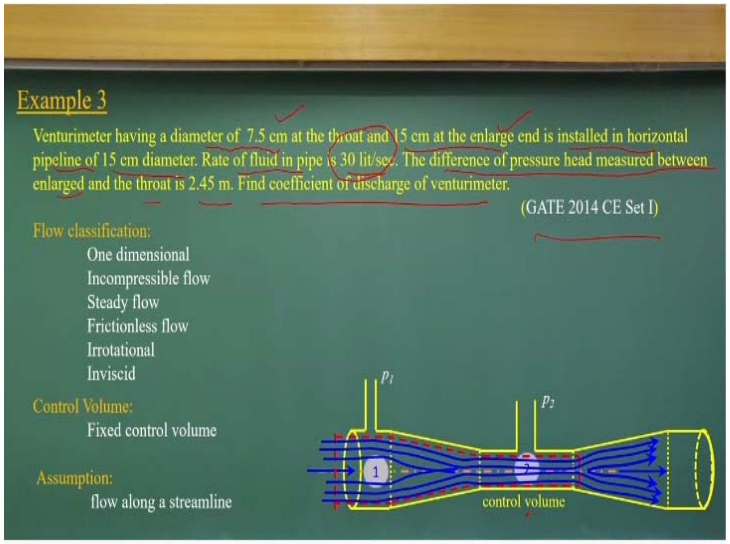

Third problems, which is very easy. The similar way as we discuss for the venturimeter problems in example one, where in this, this is the GATE 2014 questions, where we have a venturimeter having a diameter 7.5 centimeters at the throat levels and 15 centimeter at enlarge is installed in a horizontal pipelines of 15 centimeter dia. Rate of the fluid pipe flow is 30 liters per second. The difference of presser head measure between the enlarged and the throat locations is 2.45 meters. Find the coefficient of discharge of venturimeters.

[Venturimeter having a diameter of 7.5 cm at the throat and 15 cm at the enlarge end is installed in horizontal pipeline of 15 cm diameter. Rate of fluid in pipe is 30 lit/sec. The difference of pressure head measured between enlarged and the throat is 2.45 m. Find coefficient of discharge of venturimeter.]

Flow classification:

One dimensional

Incompressible flow Steady flow

Frictionless flow

Irrotational

Inviscid

If you look at this problems, we already discussed in example one, the same problems only, the numerical values are given for us to solve it. So, I will not take much time for this, only you can see the same streamline you have to draw it and you apply mass conservation equations and the Bernoulli equations to solve it to find out because here, the actual discharge is given to us, we have to compute theoretical discharge. Once you know this theoretical discharge, you can compute the coefficient of discharge.

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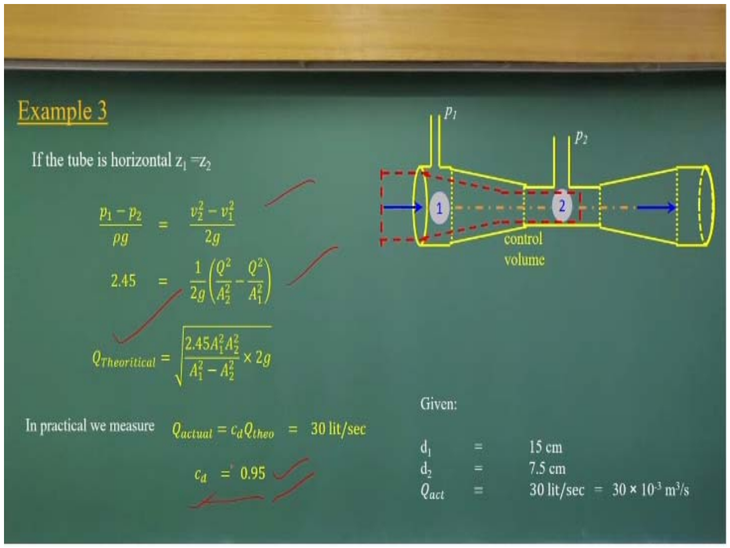

$$A_1 V_1 = A_2 V_2 = Q\text{theoritical}$$

$$\frac{p_1}{\rho g} + \frac{1}{2g} V_1^2 + z_1 = \frac{p_2}{\rho g} + \frac{1}{2g} V_2^2 + z_2 = const\ along\ a\ streamline$$

Given:

d_1 = 15 cm

d_2 = 7.5 cm

Q_{act} = 30 lit/sec = 30×10^{-3} m^3/s

If the tube is horizontal $z_1 = z_2$

$$\frac{p_1 - p_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$$

$$2.45 = \frac{1}{2g}\left(\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}\right)$$

$$Q_{Theoritical} = \sqrt{\frac{2.45 A_1^2 A_2^2}{A_1^2 - A_2^2} \times 2g}$$

In practical we measure

$$Q_{actual} = c_d Q_{theo} = 30\ \text{lit/sec}$$

$$c_d = 0.95$$

So, this is a very simple things. Again, we are putting the mass conservations equations and the Bernoulli equations, which is a energy conservation equations and by just putting the substituting these will get a theoretical discharge and we will get the actual discharge you know it, so you will get the Cd value, which comes out with 0.95.

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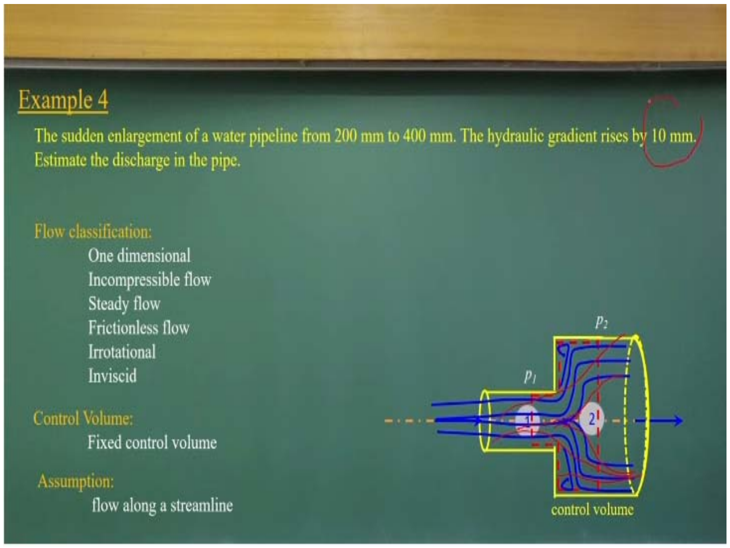

Another problems, we are very interesting problem, here there is sudden enlargement of pipeline from 200 millimeter to 400 millimeters. The hydraulic gradient raises by 10 mm okay. It is just given, their hydraulic gradient lines has increased by 10 mm. Then, we can estimate what will be the discharge in the pipe. So, again we will apply same mass conservation equation and the energy conservation equations around the streamlines. You can see that mostly the streamlines will have a divergent.

[The sudden enlargement of a water pipeline from 200 mm to 400 mm. The hydraulic gradient rises by 10 mm. Estimate the discharge in the pipe.]

Flow classification:

One dimensional

Incompressible flow   Steady flow

Frictionless flow

Irrotational

Inviscid

And there will be energy distributions on these systems, but let us, we do not know how much of energy distribution, but we know it from data, there is the hydraulic gradient rise by the 10 mm.

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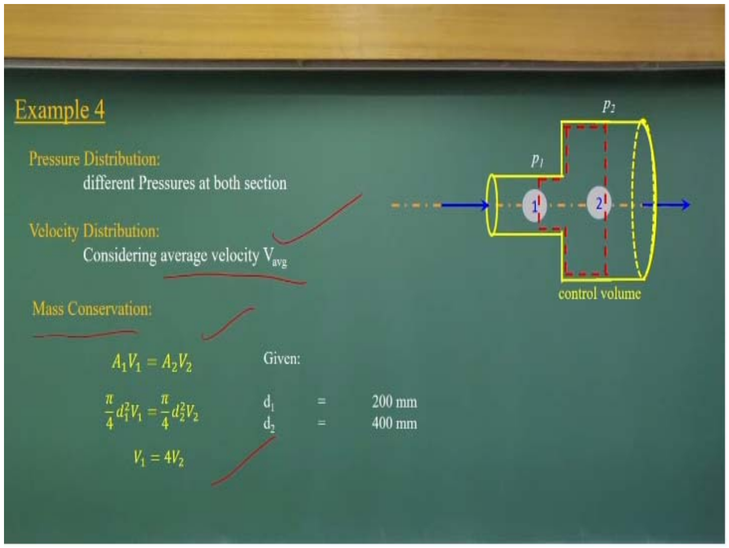

So, you have a mass conservations equations and considering the average velocity not considering kinetic energy correction factors, so you can know it. This mass conservations equations as you know it, $A_1V_1 = A_2V_2$

$$\frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2$$

Given:

d_1 = 200 mm

d_2 = 400 mm

$$V_1 = 4V_2$$

substituting this diameter we can get it the relationship between V1 and the V2.

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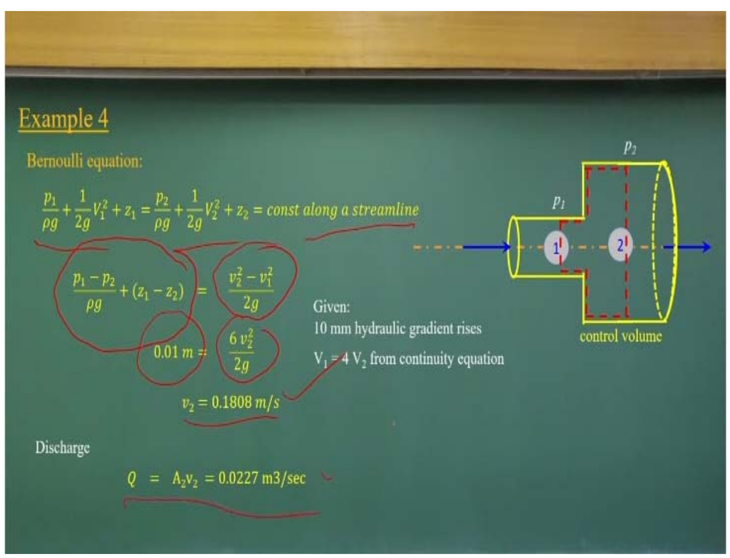

And then if I substitutes the Bernoulli equations, okay, along this constant lines. So basically if you look it, these two head, the pressure head and the z1 z2 is not given to us, but that is what will be reflect in terms of piezometric head.

$$\frac{p_1}{\rho g} + \frac{1}{2g}V_1^2 + z_1 = \frac{p_2}{\rho g} + \frac{1}{2g}V_2^2 + z_2 = const\ along\ a\ streamline$$

$$\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) \quad = \quad \frac{v_2^2 - v_1^2}{2g}$$

That is what is given the difference between the piezometric head is 0.01 meters. So we are substituting directly on that. We know this, the velocity and their relationship, we can compute the velocity. And once you know the velocity, we can compute the discharge. Given:

10 mm hydraulic gradient rises

$$0.01\ m = \quad \frac{6\ v_2^2}{2g}$$

$$v2 = 0.1808\ m/s$$

$$Q \quad = \quad \text{A2v2} \ = 0.0227\ \text{m3/sec}$$

So this way, it is very easy problems, only you have to look it, it has given the hydraulic gradient rise, that means it is representing us the pressure head and the elevations head. These two are we are including it when you put the two piezometers, what will be the raise, because of that, that is what we have of 10 mm hydraulic gradient raise and that is what we have substitutes.

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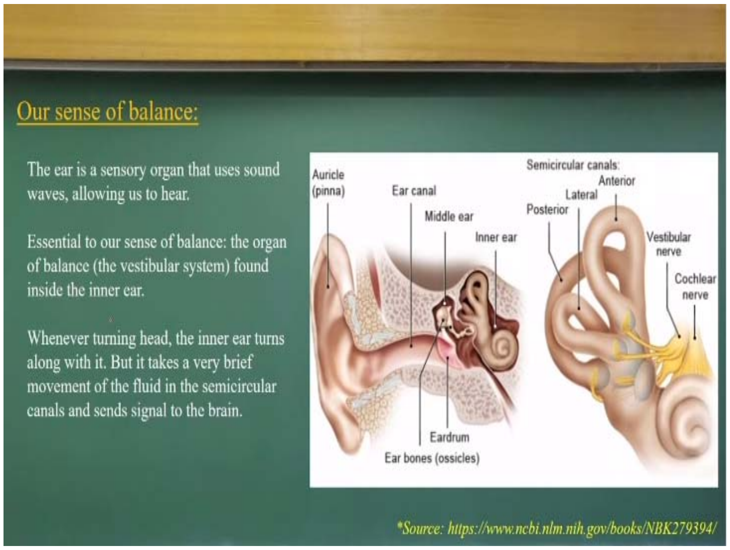

Let me conclude this lectures with the sense of balance of a organ systems. If you know it, many of, we know it this ears is for the acoustic sound waves. That is the reasons we can hear it. And whatever I am speaking you can hear because of the ear, which is sensory organ to uses the sound waves allow us to hear it. But this ear also have a composition organ of balance inside the ear, which gives up the sense of balance.

Like, if I tilt it, I can get it from my brain, I am tilting it, okay. That is what the sense of balance. I'm not going to talk much details, but I am just telling you that the nature has given so beautiful arrangement of our body structures and the brain, which we cannot replicate it and this is what it happens all in the fluid flow sensorial organ systems. So, that way, we look it very smaller components what we teach in the fluid mechanics, but the human body is the structure still give us lot of lessons to be learnt about the fluid, about the sensing organs and all.

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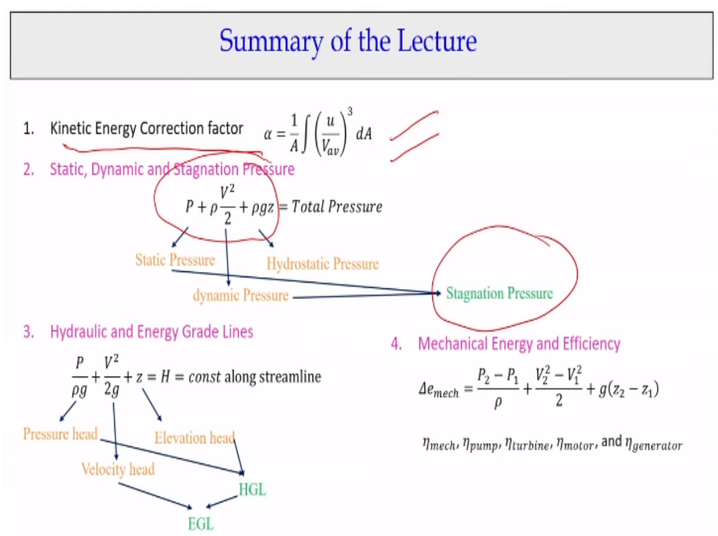

And that is the same things and this times we talk about the kinetic energy corrections factors, how we need to be compute when you have the velocity distributions is uniform. And we also seen Bernoulli equations, we can represent it in terms of the pressures, static pressures, dynamic pressures and hydrostatic pressures and two of the components that can represent us the stagnation pressures.

$$\alpha = \frac{1}{A}\int \left(\frac{u}{V_{av}}\right)^3 dA$$

$$P + \rho\frac{V^2}{2} + \rho g z = Total\ Pressure$$

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = const \text{ along streamline}$$

$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

And we also discussed about the hydraulic gradient line and the energy gradient lines and the mechanical energy and the efficiency. With this, let us thank you all for this presence here.