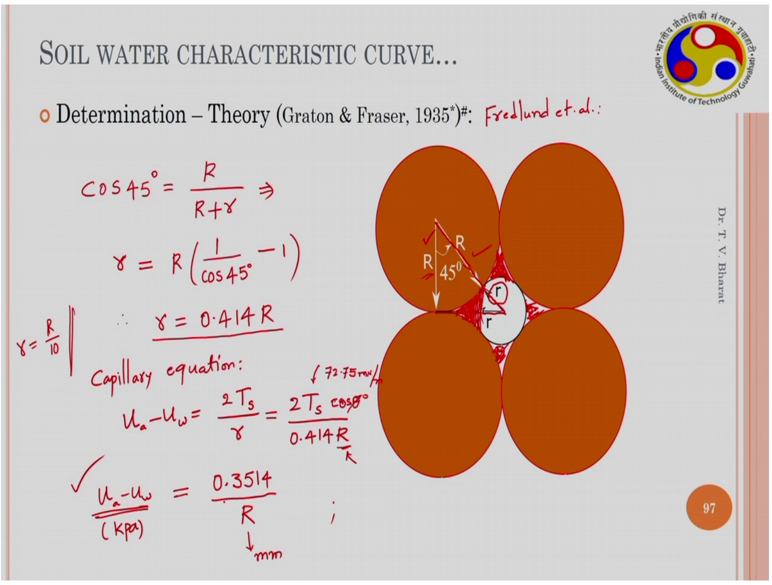
Unsaturated Soil Mechanics
Dr. T.V. Bharat
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Week - 03
Lecture - 09
Concept of Water Retention and Soil Water Characteristics-II

Hello everyone, so far we have seen that the soil water characteristic curve is a fundamental constitutive relationship for unsaturated soils, this relationship is very important for understanding the flow behavior through unsaturated soils and shear strength of unsaturated soils. Before going into the determination of the state variable such as suction and water content independently and relating them to establish the constitutive relationship. Let us try to understand theoretically how can we get some information on how this soil water characteristic curve changes when the pore structure or pore geometry changes or when the pore size changes.

(Refer Slide Time: 01:11)



Simple theory as we have earlier developed using Young Laplace equation or Toroidal approximation or Capillary rise equations could be used to estimate the suction in the soil, such simpler relationship also exists from Graton and Fraser the such model is given in Fredlund, Rahardjo and Fredlund text book on Soil Mechanics for Unsaturated Soils Wiley publications, New York. Now, if you consider assume that a simple cubic pattern exists where soil 4 soil grains are placed in this particular manner in a loosely compacted fill. So, then the radius of the then also assume that the soil is uniform soil, so that all particles are having the same diameters or radius.

So, if the radius of the particle is capital R and this is a pore which is also assumed to be of spherical then you ignore this portions which are in between this circular pore and the particles we ignore this pore space then. So, we can assume that this is a pore space governed by pore space exist due to the arrangement of these 4 particles in simple cubic packing. So, if so then the radius of the particle is capital R and this is straight line so this forms a triangle so therefore this side is capital R plus small r, small r is the radius of the smaller radius of the pore. So, this side is capital R plus small r and this side is capital R and this side anyways we do not require here.

So therefore, if you utilize we know these 2 sides we know this side and this a side. So, if you utilize the cosine rule if you assume if you use cosine rule cos 45 is equals to capital R divided by capital R plus small r. So, from this you can obtain the relation between this pore radius to the radius of the grain soil grain, this is capital R times 1 over cos 45 minus 1, therefore r is simply 0.414 capital R. If you recollect our earlier assumption that as I assumed that for simplicity small r radius of pore is related to capital R by in a manner that r is equals to one-twentieth or one-tenth of capital R we have used 0.1 times capital R for understanding the capillaries height.

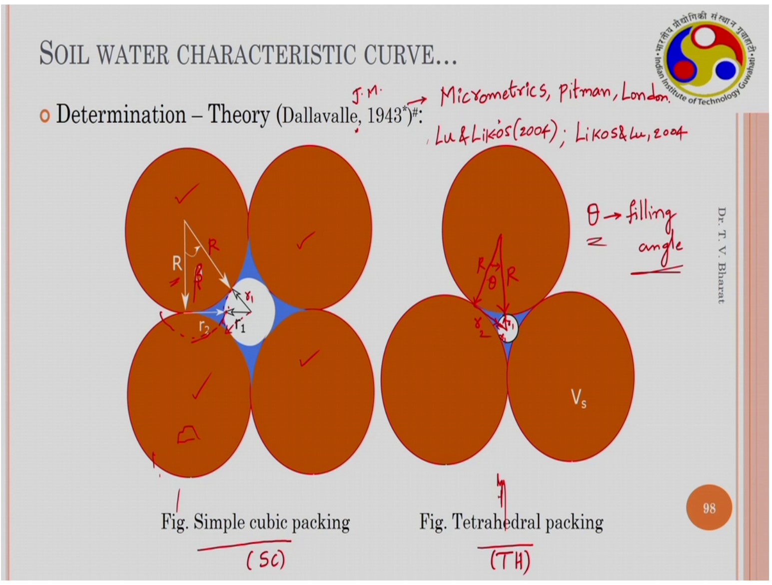
So, similarly here from simple cubic packing if you assume that this is a pore then you get this relationship that is small r is equals 0.414 R approximately half of it. So, when you use the capillary equation which we used earlier, if you assume should that the capillary equation is valid that will be valid when this pore space is ignored. So, you can assume that the pore is a circular one, so then your capillary equation that is u minus u w is equals to 2 T s by small r that is valid that is equals to 2 T s by 0.414 times capital R. So, now everything is in known quantities Ts surface tension is known at any given temperature we can substitute the value and capital R is the size of the pore.

So, if you can simplify this by substituting Ts as 72.75 milli Newton per meter at standard temperature and pore diameter, if you substitute in terms of mm then to get u a minus u w in kilo Pascal, you may have to multiply with 10 power minus 3 then if you simplify this you get u a minus u w is equals to 0.3514 divided by capital R where R should be in mm. So, this simple expression could be used the radius of the grain or soil solids and the suction here we assumed that the cos theta r cos theta there is a cos theta term is absent because theta is assumed to be 0 that means it is perfectly waiting. So such a simple expression could be derived, however the issue is that with such expressions we cannot derive the relation between u a minus u w and water content how much water is present.

For example some water is filled the pore is here we are assuming that the pore is empty, then the total suction that could be developed at this particular instance is this much. But however, in many a situations you said that there is only some portion of water that is present, for example this whole thing is filled with water. So, then in that particular case similarly here and here then you would see that as the water content increases the angle here are the meniscus curvature changes this may become flat at one given point of time entire water is filled in.

So, when the water content changes u a minus u w changes how these changes could not be understood using such expressions. This is useful to estimate what could be the capillary height or capillary raise in a simple situation like this, but it does not give insights into solid water characteristic or relationship.

(Refer Slide Time: 07:33)



There is a good approximation that was done way back in 1943 by Dallavalle which is Dallavalle is a Dallavalle 1943 is a book on Micrometrics Pitman London publication and this is Dallavalle J.M 1943, this particular procedure is mentioned in Lu & Likos text book this particular theory is. The theory behind this the concept this Dallavalles model is discussed in Lu and Likos text book, which is given in references initial references and also one of the publications by the same authors Likos and Lu in the same year which is which is in American society of civil engineering journal, the journal of geotechnical and geoenvironmental engineering.

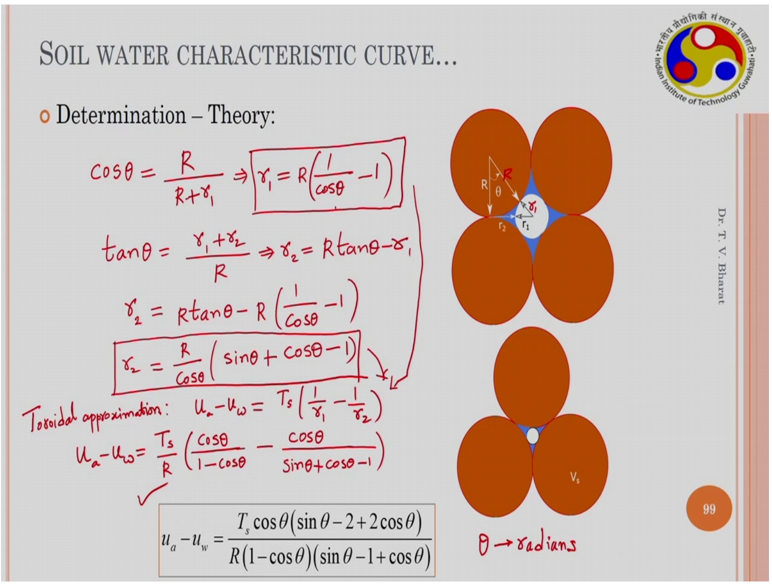
So, in that publication also you could find the entire derivation and the concepts given by Dallavalle. Here you can assume 2 different types of arrangements in soil particles one could be simple cubic packing which is similar to what we assumed previously and another one is a tetrahedral packing SC packing or TH packing. In simple cubic packing you have 4 particles which are arranged in this manner this represents soil in a loosest possible state and this represents soil in a densest possible state when you have uniform soil exist. So, now you have a water lens between these particles, so that is indicated with the blue colour hatching, that is indicated with blue colour particles are in this colour these are particles these are 4 particles and so the water lens is in blue colour.

So, again the radius of the particle is assumed to be capital R and the curvature of the lens is r 1 and inside. So, this in this plane this r 1 is in this plane. So, this is in this plane r 1 and r 2 is in the other plane r 2 will be in other direction. So, in the inside here other direction what we have seen earlier in a toroidal approximation this is similar to that, so r 2 is an autoplane direction. So, now if you assume this is r 2 and this is r 1 then this side is r capital R plus small r 1 and this is r 2 plus r 1 and this is capital R.

So, from similar to this even in TH packing this would be seen this is capital R and this is theta, here theta is not the contact angle theta is filling angle. Filling angle indicates what is the presence of water what is the volume of water that is present in the pore space, when the filling angle increases the amount of water with in the pore system increases their curvature of the meniscus gets reduced.

So, this theta is not same as the contact angle which is discussed earlier, I should have used a different notation for this probably beta or something, but as I have used theta everywhere in all the figures so I keep it as it is. So, then remember that theta is a filling angle. So, now similarly here for the TH tetrahedral packing this is the capital R and this is theta and similarly here this is r 1 and this is this is again same r 1 and this is r 2. So, the triangle same it forms the same triangle and TH packing as well as in simple cubic packing, so therefore when you derive the expression for suction it would be same for both the packing's.

(Refer Slide Time: 12:08)



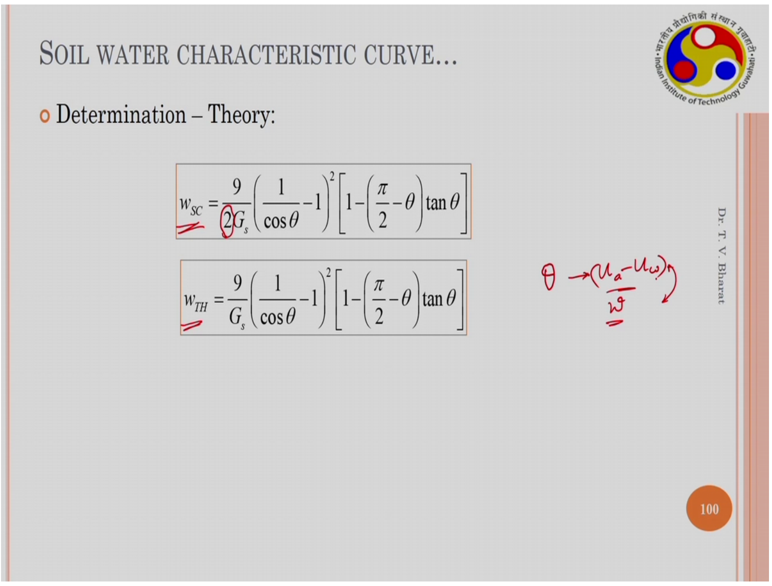
So, from the triangle here this particular case if you take if you consider cosine rule similar to the earlier one, cos theta is capital R by this is capital R this is r 1, so therefore capital R plus r 1. So, therefore, r 1 can be written in terms of capital R as r times 1 by cos theta minus 1.

So, similarly to get the relation between r 2 and capital R you can consider tan rule, tan theta is equals to r 1 plus r 2 divided by capital R so therefore r 2 is equals to capital R tan theta minus r 1. So, if you simplify this r 2 in terms of only capital R so you can substitute the value of r 1 from this expression. So, then this is the capital R into 1 by cos theta minus 1, so which when you simplify this further this results in capital R by cos theta into sin theta minus r cos theta. So, r when it is taken out cos theta sin theta plus cos theta and minus 1 so this is r 2. For this particular case earlier we have derived toroidal approximation for estimating the pressure drop across the meniscus when you have non spherical curvature which is based on young Laplace equation.

So, which is u a minus u w is equals to Ts times 1 by r 1 minus one by r 2. So, if we substitute r 1 and r 2 into this equation, so it results in u a minus u w is equals to Ts by r into cos theta by 1 minus cos theta minus cos theta by sin theta plus cos theta minus 1.

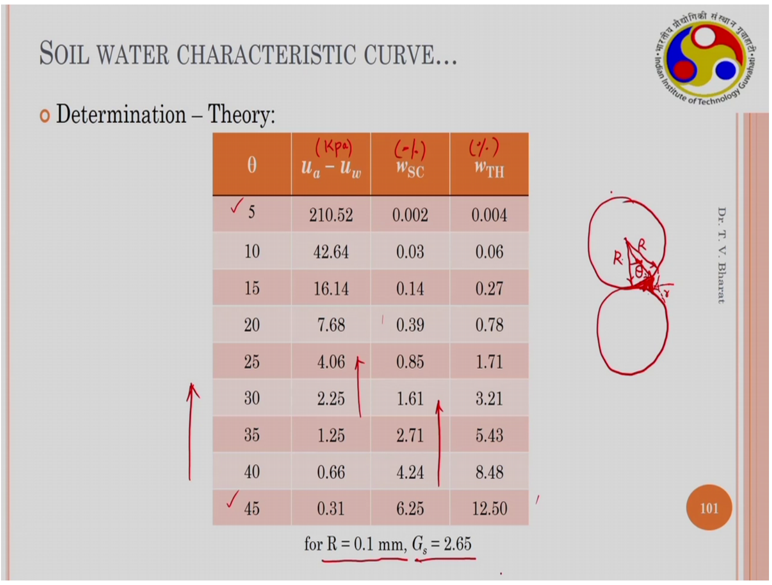
So, if you simplify this you will get this equation u a minus u w is equals to Ts by r into, if you take cos theta also in outside then sin theta minus 2 plus 2 cos theta divided by 1 minus cos theta into sin theta minus 1 plus cos theta, here theta should be substituted in radians. So, this a useful expression to estimate the suction in the meniscus or suction in soil due to this curvature r 1 and r 2.

(Refer Slide Time: 15:50)



Similarly, using the geometry the water content is estimated for SC packing and tetrahedral packing and that full derivation can be seen in the sighted paper or the book. So, the only difference between these 2 expressions is that the 2 is present here in the denominator in sc packing and 2 is absent here. So, the water content expression is this, so for the same theta filling angle you can derive you can estimate the value of u a minus u w and you can also estimate the value of water content, so that you can relate these 2. So, at the same water content what is the u a minus u w you can obtain.

(Refer Slide Time: 16:39)

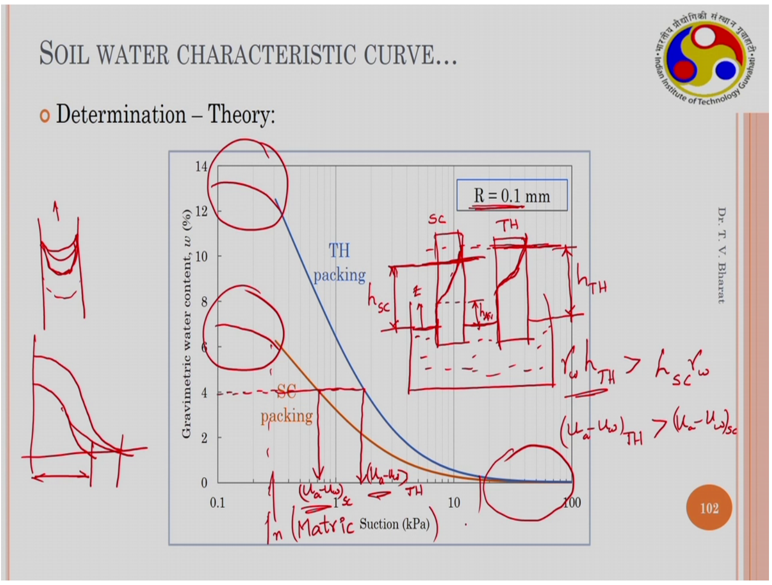


So, the calculation or the values for different values of filling angle are given here when the filling angle is changed from 5 degrees to 45 degrees and when you convert to radians and then substitute into the expressions given earlier, you get the u a minus u w estimated in kilo Pascal and here it is in percentage given here.

So, for a given values of R equals to 0.1 mm and G s is 2.65 specific gravity is 2.65 and the radius of the grain is 0.1 mm. So, here important observation is that, as the filling angle increases the suction value decreases, but the water content increases. So, this can be understood from this if we draw 2 grains only and this is the water that is present in between the grains this capital R and again this is capital R and this is theta when the theta value reduces the water content decreases because when theta is here then only water content is up to here.

So, then water content becomes when theta value decreases, then water content is only up to here earlier this was up to here. So, the water content decreases with the change in decrease in theta. So, whether decrease in theta the water content decreases from 6.25 to nearly 0 when theta is 5. But as the water content decreases the value of R changes the radius of curvature changes and because of that the U a value is increasing U a value is increasing nearly from 0 at 45 degrees to 210 kilo Pascal at 5 degrees. So, this is qualitatively correct this is qualitatively correct. So, let us. So, again for TH packing we will analyze by plotting it.

(Refer Slide Time: 19:04)



So, when you plot it for sc packing and TH packing for the same R value R is equals to 0.1 mm suction on x axis is a matrix suction matrix suction on x axis on log scale is on log scale and gravimetric water content on y y axis. So, the change in the curve curvature the change in the soil water characteristic curve can be seen for SC packing and TH packing.

This indicates that for the same value of water content say 4 percentage of water content the suction value in the sc packing is smaller than the suction value in the TH packing. So, this is u a minus u w in TH packing. So, this is u a minus u w in sc packing for the same water content this give this could this can be understood when you have a water reservoir in which you have 2 soil columns immersed 2 soil columns. So, this is water. So, this is a soil column with sc packing this is soil column with TH packing. So, this is loosely compacted and this is densely compacted soil. So, then you would see that there is a water content water increases into the soil in both the soils. So, you would observe that the water content variation with height. So, this is z you would see that the water content will be same remains same up to their entry value and after that it starts decreasing and decreases like this.

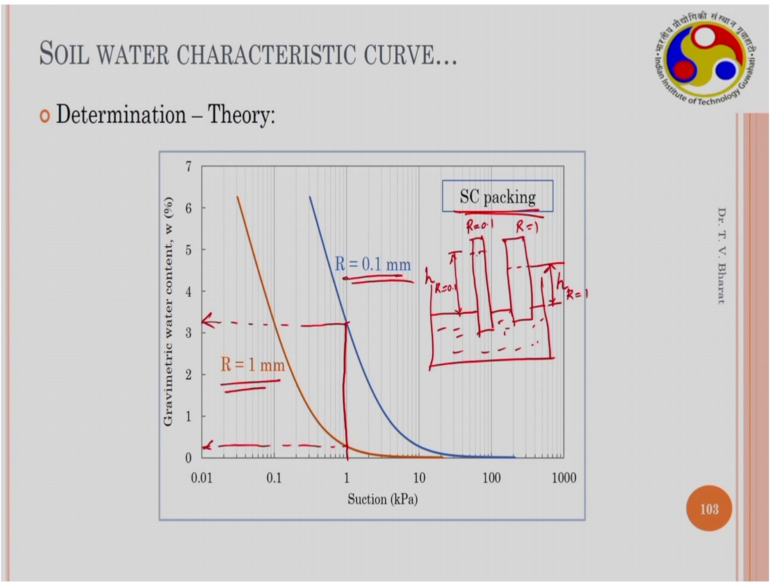
And in this particular case because is densely packed; that means, TH packing the radius of the pore is smaller than the pore radius in sc packing if you observe if you go back in these 2 cases the pore diameter in TH packing is much smaller than the pore diameter in the sc packing. So, therefore, the densely packed 1 the TH packing will have smaller pore radius even though the size of the particles same particles are same uniformly uniform soil with the size is same, but; however, because this is a densely packed the pore radius is smaller. So, because the pore radius smaller you will have this is a air entry for this particular case this is h a a v a e v. So, in this particular case, the water content would be same up to higher depth and after that it decreases. So, at the same water content if you want to consider at some depth say here. So, when it depletes or the water content here would be same as the water content somewhere here.

So, here the depth is smaller r at the air entry value if you assume that the water where water content where it decreases or something are same water content if you want to start compare for example, here and then here, soils will may have the same water content, but; however, this is depth is higher this depth is higher than this particular depth. So, this is h sc packing, this is h TH packing. So, h for TH parking is larger than h for sc packing if you multiply with gamma w on both sides this is nothing, but u a minus u w for TH packing and u a minus u w for sc for sc packing. So, this is what we have observed here u a minus u w u a minus u w for TH packing is higher than u a minus u w for sc packing at the same water content. So, this is qualitatively perfect; this would explain the physics of capillary action or capillary mechanism within the soil mass.

However this theory could be applied for uniform soils and here if you observe that the suction value below this value you would not be able to get. So, you would expect that air enter value you cannot obtain here from this particular case, you will not be able to get the air enter values because the surface effects are not considered. So, the changes in the contact angle the change in the contact angle as a suction increases is not considered. Because we have seen earlier that when you take a column initially this may be the contact angle, but as the suction increases we have seen that the contact angles changes it reaches a proceeding angle after that it starts decreasing. So, such concepts are not considered here. So, therefore, the initial state of the soil water characteristics the initial state of the soil water characteristic curve at very small suctions is not considered here.

Moreover here if you see if both are merging together here, because again the surface characteristics or surface forces would play a role in the residual portion. Here the residual portion in the residual region residual water content region or residual portion of the curve, again the surface characteristics are not taken into account. So, therefore, both are merging together otherwise, if you plot for 2 different soils one is densely compacted and other one is a loosely compacted. So, there would be some difference where the water content decreases, but here the such a restriction is not made and both are merging at higher suctions. Otherwise it explains qualitatively very well in between these 2 regions because a capillary effects are dominant in this particular zone.

(Refer Slide Time: 25:43)



Again if you consider the soil water characteristic curves obtained from the theory Dallavalles theory for same packing sc packing, but; however, for 2 different radius of the particles. So, then again if you see again if you compare at the same suction value, the fine grained soil will have more water content compare to the coarser soil. So, for the same suction R for the same water content you will have lower suction in largest grain size particles and larger value of the suction in the smaller grain size particles. This is again could be understood from the same mechanism where when the when you have 2 columns one with the fine grained soil and another one was with the coarse grained soil of same diameter sorry coarse grained soil immersed in now water reservoirs. So, this columns are of the same diameter.

So, then soil with smaller grain says contains more water or higher rise higher capillary rise compared to the larger diameter soil particles. So, the h capillary rise in this is R equals to 0.1 and this is R equals to 1. So, the rise of capillary in R equal to 1 would be smaller compared to R equals to 0.1. So, this is a quite obvious and that could be corrected replicated using this particular theory. So, qualitatively this theory could be used to understand especially for coarse grained soils and uniform soils where the approximate characteristics of soil water retention could be obtained.

Thank you.