Unstructured Soil Mechanics

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Week – 03
Lecture - 07
Summary on Fundamental Principles and Constitutive Relationships

Hello everyone. Let us summarize whatever we have learnt so far. In the fundamental principles we have learnt that there are constituted relationships, state variables and material constants, for any given phenomena to define. To define any given phenomenon, we need to understand and distinguish what are the state variables, constitute relationships material constants. The governing equations in the continuum physics are basic principles represent fundamental physical laws which are independent of matter or independent of material.

So, they are same for any given material any given phenomenon such as conservation of mass, conservation of momentum conservation of energy etcetera. The dependent variables in these governing equations are the state variables. Because they represent the state of the system; however, we require additional equations to solve a given problem or to mathematically solve a given phenomenon. Such additional equations which are constitution dependent are called constituted relationships are depend on the material. Material dependent equations are called constitutive equations.

Constitutive equations therefore, are basically a relationship between 2 different state variables; such as a relationship between void ratio and effective stress, or a relationship between effective stress and shear stress. So, the equation constants in these state variables are called material constants. So, we required to understand the middle constant dependency on other state variables also. So, once we identify what are the different constitutive relationships for a given phenomenon and establish the relationships and identify the middle constants, we can understand the phenomenon.

So, this is the flowchart for the phenomenon prediction is like this.

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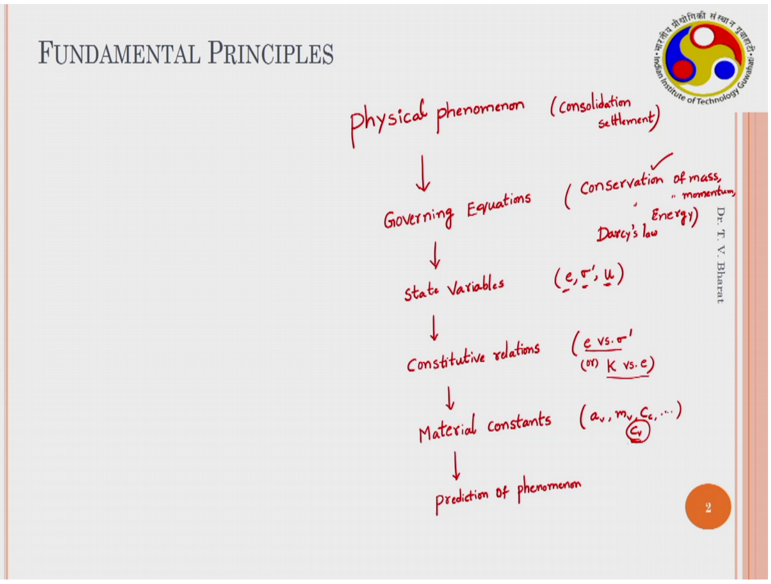

FUNDAMENTAL PRINCIPLES

Physical phenomenon (consolidation settlement)

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Governing Equations (Conservation of mass, " momentum, " Energy) Darcy's law

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State Variables $(\underline{e}, \underline{\sigma'}, \underline{u})$

↓

Constitutive relations $(\underline{e \text{ vs. } \sigma'}$ (or) $\underline{K \text{ vs. } e})$

↓

Material constants $(a_v, m_v, C_c, \ldots)$ $\boxed{C_v}$

↓

Prediction of phenomenon

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So, this is a physical phenomenon or observation. So, from this after observing the physical phenomenon we frame governing equations. For example, we can think of a consolidation settlement. This is slurry when a tailing points the mine tailings would settle with time. So, the phenomenon here is settlement of mine tailings with time. So, is the consolidation settlement? So, to predict that phenomenon we need to develop governing equations. So, such as we could utilize the basic physical laws like conservation of mass, conservation of momentum, or force equilibrium, and energy conservation.

So, here consolidation settlement. So, from this governing equations, we identify the state variables. May be the density or any other state variables we identify. So, from that so, in this particular case the state variables are maybe the void ratio is reducing. Because the volume is reducing so, the volume is void ratio is decreasing with time. So, therefore, void ratio is one state variable. The stress state would differ with depth may be the effective stress is varying with depth. So, you have another state variable like sigma hash.

And pour water pressure also changes with time. So, therefore, it could be another state variable. So, once you identify we can develop constitute relationships that provide interdependency of different state variables. So, this could be a relation between void ratio and effective stress. So, this is a void ratio versus sigma dash. Or it could be the hydraulic conductivity also changes with depth. So, hydraulic conductivity is a material constant, but it could change with void ratio. So, you may require another relationship such as this.

So, once you will have the constitutive relationships, we have we identify the material constants. The material constants could be a v or m v or could be your c c etcetera. So, once we have this again or could be c v also. So, these are the material constants. Once we identify these constants, now we can predict the phenomenon. Phenomenon is settlement versus time. If the phenomenon does not, then we compare this phenomenon with the physical phenomenon. If it does not match, there will be more constitution relationships required.

Therefore, so, by providing those constitutive relationships we try to match the prediction try to get the good prediction of the phenomenon. So, for example, the one dimensional consolidation can be terzaghi, which uses only conservation of mass, and which uses Darcy's law also; which combines these 2, and it provides a governing equation, which creates the governing equation the state variables that it considers is pour water pressure, sigma dash and e dash e. So, the constitutive relationships are y dash versus sigma dash, which are assumed to be constant at any given change in the effective stress.

And it uses k versus e relationship. A k is assumed to be constant, for a given increment of the loading, k is assumed to be constant. And which drives the coefficient of consolidation. In from the phenomenon we in fact, predict the c v value. And we match the governing equations are we match the development model with the physical phenomena observation to determine the c v. This coefficient of consolidation is used in the field for estimation of consolidation settlements the rate of consolidation settlements.

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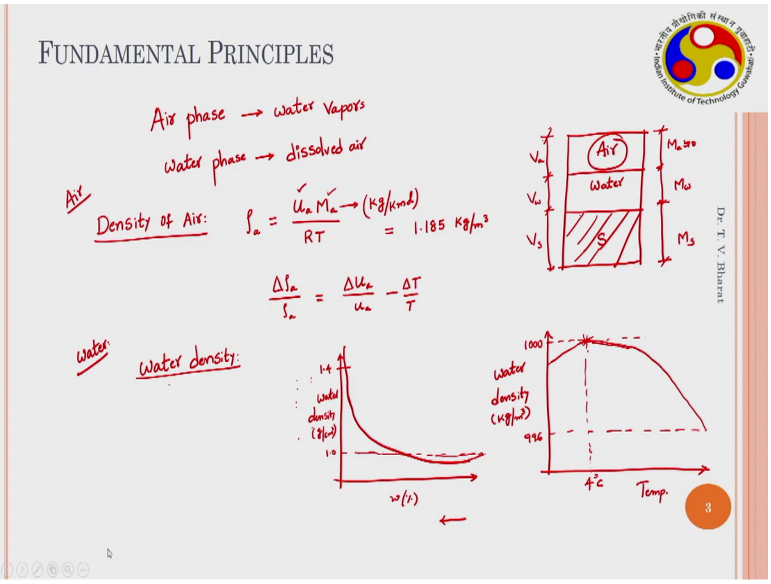

FUNDAMENTAL PRINCIPLES

Air phase → water Vapors

water phase → dissolved air

Air

Density of Air: $\rho_a = \frac{u_a M_a}{RT}$ $M_a \rightarrow$ (Kg/Kmol) $= 1.185$ Kg/m^3

$$\frac{\Delta \rho_a}{\rho_a} = \frac{\Delta u_a}{u_a} - \frac{\Delta T}{T}$$

water

Water density:

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Similarly, in unsaturated soils also we need to identify what are different state variables so that we can develop governing equations and it can develop constitute relationships for understanding the phenomena. In unsaturated soil mechanics we have seen that we have 3 phase system; where we have water air water and solids. So, this is solids, and you have air and water. So, you have volume ratios generally consisted on this side. So, volume fractions are considered generally in this side, volume of air, volume of water, and volume of solids. And this is mass of air which is 0.

And mass of water and mass of solids. So, the air phase generally defined as the poor space which is not occupied by water. So, there is called air phase, and water phase is the poor space which is not occupied by a air; however, the air phase contains the air phase also contains water vapor, air phase contains water vapors. And water phase contains dissolved air. As a solute you have dissolved air. And the phases are no means pure; the water can exist in the air phase as water vapor. And air can exist in water as dissolved air depending on different conditions, like pressure temperature etcetera.

So, solids are the generally the range of solids varies from very fine grained soils such as clays silts and organic matter to coarse grained soils like sand and graphite. So, the air phase when you consider, the air phase has certain state variables, such as density of air, which is important state variable which governs to flow through unsaturated soils. So, density of air is an instrumented from this particular equation. Here u a is the air pressure, molar mass of air, R is gas constant T is temperature. So, molar mass of a substance is a mass of substance divided by the amount of substance, which is expressed in kg per kilo mole or gram per mole.

And u a is the mass at pressure which is expressed in kilopascal. R is a gas constant which is 8.314 joule per Kelvin mole. Temperature is in Kelvin. And the density of air we have estimated at standard temperature of 25 degrees, and 101.325 kilopascal pressure. This value is 1.185 kg per meter cube. The density of air is sensitive to the pressure and temperatures. So, variation of density can be represented by the mathematic equation mathematical equation as this. So, the density of air increases with increase in a air pressure, but decreases with increase in the temperature. As with increase in the elevation from the mean sea level the pressure decreases.

The atmospheric pressure decreases therefore, the density of air decreases. This was explained simple illustration where if you fill the water bottle with air at higher elevation, close it with air tight and bring it to lower elevations or bring it down from the hillside. And when you bring it down because the air pressure increases in the surroundings, this can crush the water bottle, this was explained during this discussion, this was explained earlier. So, the temperature increases the density decreases, this is this could be understood from sea breezes we see on the shores.

Because the land quickly gets heated up, but the air above the sea relatively is cooler compared to the air above the land. So, therefore, the air above the land density decreases therefore, it gets uplifted, this is filled the space is filled by the air which is coming from the sea. So, you get a cool breeze. So, this is how the air circulates. Coming to the water, this is air, coming to the water. If you look at the water density, water density has direct influence on the physical and mechanical behavior of unsaturated soils. The water density varies with temperature in this manner.

As the temperature decrease the volume of water decreases. So, therefore, the density increases, and richest one critical value and beyond that the volume increases our density decreases. So, this maximum density which is achieved are lowest to volume which is achieved is at 4 degree Celsius. And which is thousand kg per meter cube. These are on 9 9 6. Sorry, increase in the volume below 4 degrees is called anomalous expansion of water. Because of this the ice floats on water. Ice which is the lesser dense is less conductive to temperatures; because of this the aquatic life survives in the cold countries. Because once the ice forms, and it floats at the surface ice is less denser compared to waters colder than this compared to waters.

Because of this phenomenon anomalous expansion, and when it floats it will act as a thermal insulation are partially thermal insulation and it does not allow heat to transfer. So, that beneath the ice the temperatures are maintained at higher values. Because of this the aquatic life survives. If whole thing freezes continuously then aquatic life cannot survive. Apart from dependency of water density and temperature, the water density significantly changes due to physical chemical effects. We have seen that the clay particles are collide, they are plate like particles which has negative charge on the surface due to isomorphous substitution.

Therefore, there are positive ions acquired from the environment during the formation or during chemical weathering process. And they are present at the surface. So now, when the water is available the water hydrates all these cations, and also hydrates internal lattice and also external surface. When this water absorbs, the water is available is a thin film around individually clay platelet.

This water content can be as high as 15 percentage at edge rate state for one molar at rich clays, this was explained earlier. So, therefore, water as it got absorbed on the surface of the clay, which is tightly held on the surface in absorbed form. The density of water is different from the free state.

So, if we can draw density of water or water density in gram per centimeter cube, and the gram metric water content on x axis. It is observed that, the water density can increase to very high values. This value 1.4 and this is 1. So, as the water content decreases when we are going in this direction water content decreases the density of water increases; because the water which was held in the absorbed state has very high densities. The densities can even reach to 1.8 gram per centimeter cube. Form on more right rich clays, this was recent observations given by Luke in 2018.

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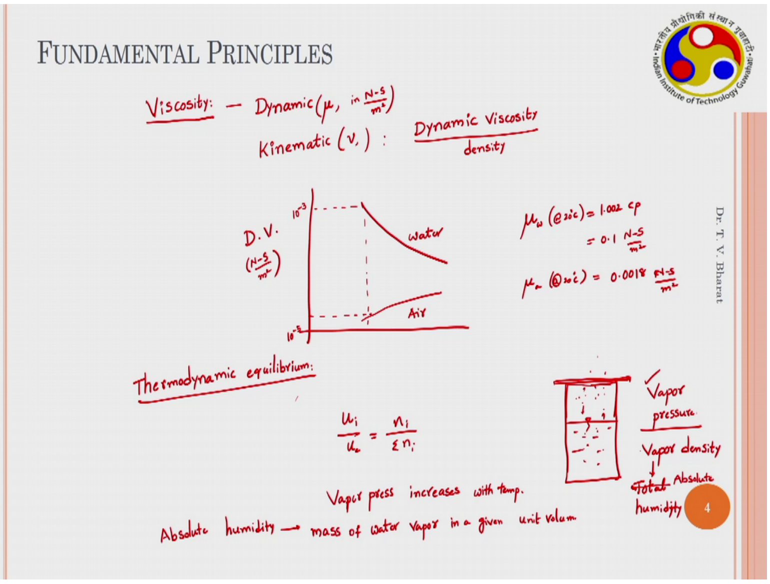

FUNDAMENTAL PRINCIPLES

Viscosity: – Dynamic (μ, in $\frac{N\text{-}s}{m^2}$)

Kinematic (ν) : $\frac{\text{Dynamic Viscosity}}{\text{density}}$

μ_w (@ 20°c) = 1.002 cp = 0.1 $\frac{N\text{-}s}{m^2}$

μ_a (@ 20°c) = 0.0018 $\frac{N\text{-}s}{m^2}$

Thermodynamic equilibrium:

$\frac{u_i}{u_a} = \frac{n_i}{\Sigma n_i}$

✓ Vapor pressure

Vapor density

↓

~~Total~~ Absolute humidity

Vapor press increases with temp.

Absolute humidity → mass of water vapor in a given unit volum.

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Other than the density of water and air there is a viscosity of air and water. Viscosity of fluid is defined as the ability of fluid to deform under shear stresses. So, the hydraulic conductivity of water and air depend on or get influenced by the viscosity of air and water respectively. The viscosity influences the hydraulic conductivity of both the air and water. So, therefore, the determination of viscosity of air and water are important. Dynamic viscosity which is represented with mu has units of has units of Newton second per meter square or centipoise this is express the resistance to fluid flow.

Another one is kinematic viscosity, which is exposed to the nu, which is the ratio of dynamic viscosity to the density, which is used in analyzing the Reynolds number. So, if we see the dynamic viscosity of water and air, and which viscosity for the water the dynamic viscosity decreases, but for air viscosity increases. But for air the viscosity increases. So, this is for water and this is for air. So, this value at one particular temperature, it is values around 10 to the power minus 3 in Newton second per meter square. And at the same temperature, this value may be very small this is how the ranges are.

So, mu of water at 20 degrees is 1.002 centipoise or 0.1 Newton second per meter square, and mu of air at the same temperature is 0.0018 Newton second per meter square. Air viscosity increases with temperature. As the temperature increases, the air molecules get additional energy and they collide each other therefore, the viscosity increases. On the other hand, the water viscosity decreases with increase in the temperature, because the increase in the temperature makes the water molecules to overcome the intermolecular forces.

So, this decreases the dynamic viscosity of water, understood the thermodynamic equilibrium between different phases. We have understood from a given beaker or a taken one glass of water. For example, if we take a glass of water. So, this is water. In this you have, water molecules, these water molecules which are available at the surface would leave the surface and go to the air phase, because the intermolecular force at the surface is not strong enough to hold them back into the liquid. So, they go into the air phase.

For example, of this is closed by a lead, the water molecules which going to the vapor are in the air phase would eventually have to come back by hitting this plate and again go into the water phase. After equilibrium, the number of molecules which are leaving from the water phase would be equal to the number of molecules which are coming in to the water phase. So, at this equilibrium for example, this is coming out and this is going up, this is leaving the surface. So now, the molecules which are actually coming and coming to the water phase are called condensation. And molecules which are leaving the surface are called vaporizing.

So, therefore, the number of molecules which are hitting the surroundings or surface, this plate the pressure which is exerted by these molecules, this vapor molecules on the lead surface is called vapor pressure. Are the number of molecules which are available in the space also can be defined based on the density? So, number of vapor molecules are the mass of this molecules per unit volume can be defined as vapor density. So, this vapor density is nothing but the total humidity. And the relative humidity is defined based on the vapor pressures.

So, based on the partial pressure of all gas components which are proportional to the molar fraction of each component; this is what we have defined, u i by u a is equal to n i by sigma n i. So, partial pressure of each component is relate to the molar fraction of those components. Or it could be expressed in terms of volume fraction also, because the ideal gas volume is known that is 32.414 liter per mole. So, this vapor pressure increases with increase in temperature because the temperature is increased. More energy is given to the molecules of water, and they can overcome the intermolecular forces.

And then they try to escape from the surface. Therefore, more number of water molecules exists in the gas phase, gas phase therefore, more pressure is exerted. So, therefore, the vapor pressure or vapor density increases with increasing the temperature. As we defined the total humidity is total absolute or absolute humidity sorry, this is absolute. So, this is absolute. So, the absolute humidity is mass of water vapor in a given unit volume is called absolute humidity.

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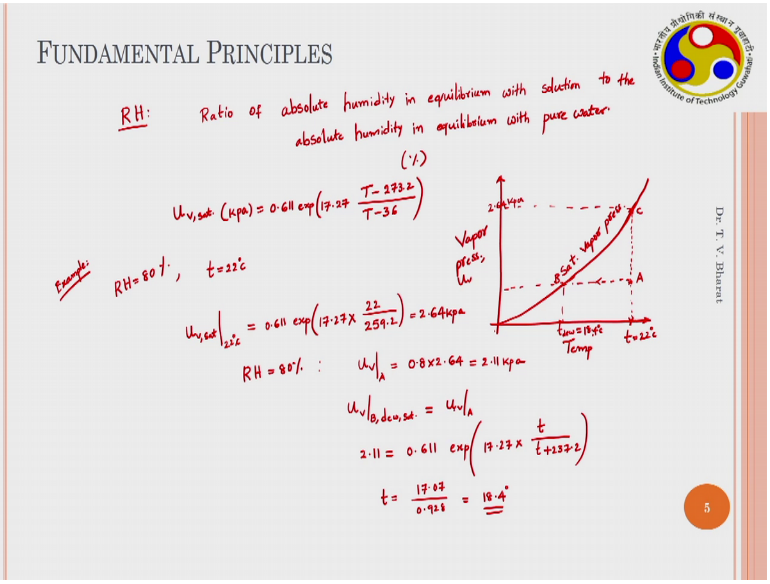

FUNDAMENTAL PRINCIPLES

RH: Ratio of absolute humidity in equilibrium with solution to the absolute humidity in equilibrium with pure water. (%)

$u_{v,sat.}\,(kpa) = 0.611 \exp\left(17.27\,\frac{T-273.2}{T-36}\right)$

Example: $RH = 80\%$, $t = 22^\circ c$

$u_{v,sat}\big|_{22^\circ c} = 0.611 \exp\left(17.27 \times \frac{22}{259.2}\right) = 2.64\,kpa$

$RH = 80\%$: $u_v\big|_A = 0.8 \times 2.64 = 2.11\,kpa$

$u_v\big|_{B,dew,sat.} = u_v\big|_A$

$2.11 = 0.611 \exp\left(17.27 \times \frac{t}{t+237.2}\right)$

$t = \frac{17.07}{0.928} = \underline{\underline{18.4^\circ}}$

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Other hand, the relative humidity R H is defined with respect to the partial pressure of vapor.

So, this is the ratio of absolute humidity in equilibrium with some solution to the absolute humidity in equilibrium with pure water; which is expressed in percentage. So, if you take pure water, and the density of vapor which is present on the above this water pure water which defines the absolute humidity, and if you take salt water or any other solution, the humidity or relative density above this salt solution. And the ratio of these 2 the absolute humidity above the salt solution in equilibrium and the ratio of absolute humidity of the vapors above the salt solution and the vapor density above the pure water, and this ratio is called relative humidity.

So, this relative humidity decreases as the salt concentration increases. And it also depends on the type of salt which is present in a given solution. And also it depends on temperature. The saturated vapor pressure is defined to understand what are the general ranges of relative humidity because even though you have pure water near by the relative humidity cannot be 100 percent. The relative humidity decreases due to the presence of some solute or presence of some salts in the solution are due to these factors the relative humidity decreases and to understand the variation the relative humidity.

And formation of dew points etcetera, we can we require the variation in saturated vapor pressures with temperature. So, this is vapor pressure and this is temperature. So, this is how the saturated vapor pressure varies. This increases exponentially nearly. So, the saturated vapor pressure, this can be expressed as an equation in terms of kilopascal. This is called Tetens equation 0.611 times exponential of 17.27 T minus 273.2 divide by T minus 36. So, this is Tetens equation and we have other equations also. We have I have discussed other questions also for fitting this saturated vapor pressure.

Generally, the soil or in the atmosphere, the vapor pressure may be somewhere here. The temperature of the atmosphere is this much, and the vapor pressure is vapor pressure is represented by point A here. Point A here, now we can reach the saturated vapor pressure in 2 ways: one by decreasing the temperature. So, we follow this curve, and we hit this one we get the saturated vapor pressure. Or if you increase the vapor pressure by keeping the temperature same and we also reach the saturated vapor pressure line. So, more often we see that in atmosphere suddenly when the temperature drops, from here to here, that is the formation of dew or condensation that takes place.

Let us solve one simple problem to understand the dew formation. Which is you have a soil sample. So, in atmosphere the currently the relative humidity is 80 percent. When the humidity is 80 percent and the temperature is 22 degree Celsius. And the question is what is dew point temperature when the vapor density remains constant for the dew point to form are to hit this saturated vapor pressure line what is the temperature required. So, this information is sufficient to estimate dew point temperature. First we can estimate what is the saturated vapor pressure at this particular temperature. This temperature is 22 degrees Celsius, and this is dew point temperature which is not known.

So, we can estimate the saturated vapor pressure corresponding to point A. So, this is B, saturated vapor pressure at A, or at 22 degrees is equals to 0.611 exponential of 17.27 into multiplied by 22 divide by 259.2, which is equal to 2.64 kilopascal. So, we estimated what is point C? At point C the vapor pressure is 2.64 kilopascal; however, the relative humidity is known this is 80 present. Therefore, we can calculate to u v at A. U v at A is 80 percent of 2.64, which is 2.11 kilopascal.

So, therefore, we know the vapor pressure at point A. If this is known, when the temperature is decreased the vapor pressure remain same, but this vapor pressure on the vapor pressure at the dew point. So, the saturated vapor pressure at point B and vapor pressure at A both are same, because it maintains the same vapor pressure, but the temperature only decreases. So, therefore, we can estimate we can say that the vapor pressure at B that is dew point that is saturated is equals to vapor pressure at A.

So, we have the saturated vapor pressure where the at the dew point. So, that is 2.11 which can be equated to 0.611 exponential of 17.27 times. So, this temperature is not known. So, this is a T plus 237.2 in terms of degree Celsius if you write. And when we solve this, we can get T equals to finally, 17.07 by say 0.928 is equal to 18.4 degrees. So, this is 22 and this is 18.4 degrees. If nearly 4 degrees drop in temperature takes place, then dew forms. We see that the small droplets drop.

You can feel the water droplets. So, this is dew formation. And this concept is very, very important in future understanding dew point potentiometer principle, which is developed for estimation of total section of the soil. Which are the important state variables for unsaturated soil mechanics, and in the dew point potentiometer? So, sample is taken, but the samples initial humidity is not known. Only temperature is known then the sample is placed in the dew point potentiometer. Temperature is reduced gradually so that they can find out so that the dew point can be found out using some physics principle.

Once the dew point is found or dew point temperature is found, the existing relative humidity can be found. This is a inverse analysis of this particular problem same problem. So, these concepts are very, very important for understanding the principles of several operators used for estimation of state variables of unsaturated soils.

Thank you.