Unsaturated Soil Mechanics
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Week - 02
Lecture - 06
Capillary Phenomenon in Unsaturated Soil-II

Hello everyone. We have discussed our Capillary rise in different capillary tubes with different radius.

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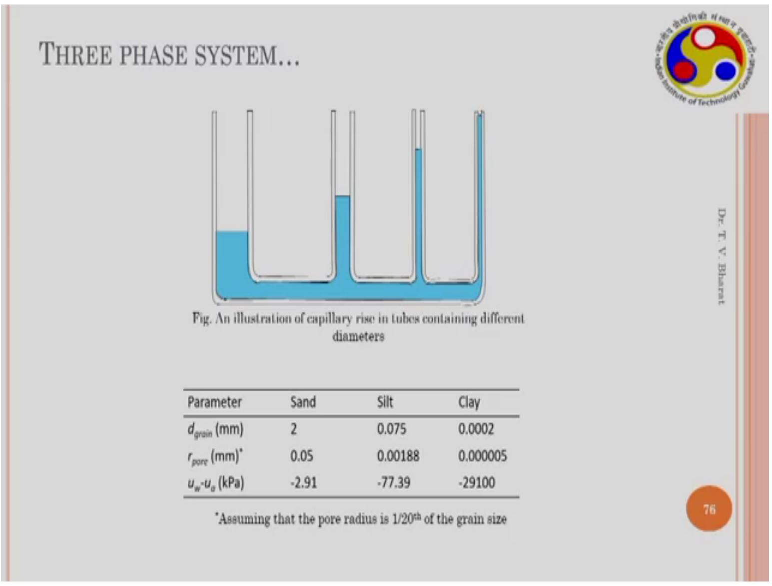

We have discussed the rise of capillary in capillary tubes of different radii. We have seen that if the if you have a capillary of diameter as small as the clay pore size, equivalent to the clay pore size, then the capillary raise would be too much, too high value as high as 2910, so nearly 3 kilometers.

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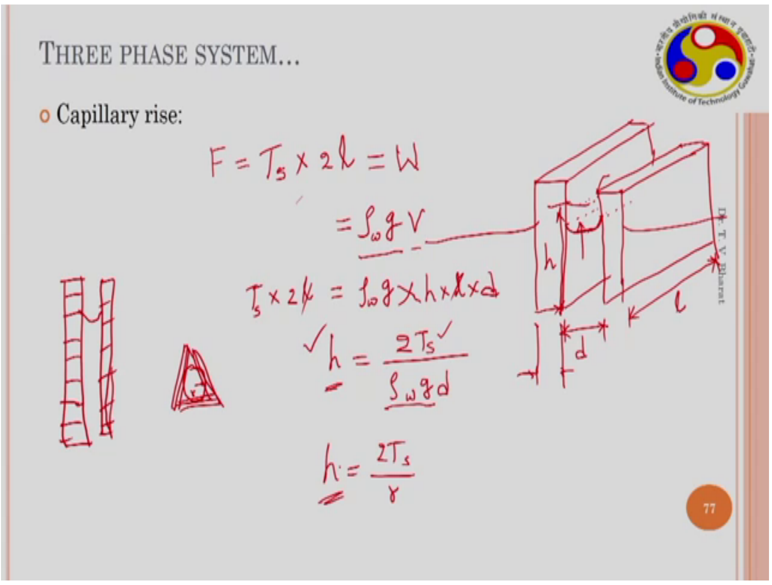

So, for clay particles usually they are plate like structures. If you assume that this is the plate, this is one plate and you have another plate. So when they are immersed; when they are immersed in water, so what is the rise of height one can find out? So, you would see that the water forms a capillary forms a curvature with the surface and it should rise up. So, what is a rise we can find out because we know the force due to surface tension? The force due to surface tension; force due to surface tension is T s times the total length.

So, total length if you consider length of the particle as small l. So, the water can interact with the surfaces at two interfaces along l plus l 2 l. So, this is the total force due to surface tension. So, which should balanced by the weight of the column when it rises, so the weight of the column should be equals to the density of water times g times the volume. So, this should be in kilo Newton's kilo Newton per meter cube. This whole thing is kilo Newton per meter cube into meter cube should be kilo Newton. So, that is a weight.

So, this should be density of water can be found formed and volume is if the rise of height is say h and the distance between the particles is small d. So, then it should be volume is high times high times l times d. This is T s into this is T s times 2 l. So, when you simplify you get an expression for h h equals to 2 T s by rho w into g times d.

So, this expression one can use for estimating the capillary rise. Here the surface tension we can find out depending on the temperature. At different temperatures, we know the surface tension the rho w density of water is also known at a given temperature g is 9.81. We can substitute r together we can substitute the unit weight of water which is a 10 kilo Newton per meter cube at a standard temperature and d is the space.

Generally the particle size or particle thickness would be the clay particle thickness would be as small as 1 nanometer to little coarser ones maybe ten 10 meters distance between the particles may be as high as maybe 100,2000 nanometers are when slightly more. So, if you substitute you can approximately calculate what could be the rise of height. So, this could be in meters. So, for then we need to assume that the particles have stacked up, each particle stacks up like this and our particle stacks up like this and which forms which forms a cavity or which forms a small pore which forms a pore and then water should rise up. This is the most unlikely rather than that if you assume that the particles are stacked like this.

These are clay particles which are stacked and it forms a tac-toy kind of thing, then this pore size could be approximated as a spherical and we can use the earlier expression to find out the earlier expression that is a height is equals to 2 T s by simply r. You can use, we can assume that the radius of pore size could be related to pore, related to the particle size or pore size could be estimated from different means. And if you substitute, you can approximately estimate what is the capillary raise. So, there are various way researchers have estimated this by relating the pore radius to d 10 different particle d 10 etcetera.

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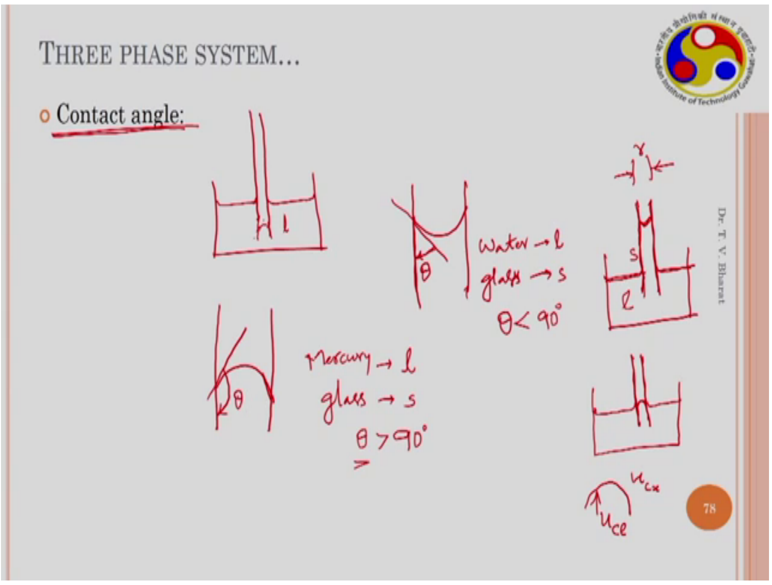

We have seen that if you insert a capillary tube in water in a beaker of water. A capillary tube made up of a glass, glass tube. We have seen that there is a raise of water, water raises and forms meniscus like this. We have discussed earlier that this raise is, because there is an adhesive force between the water molecules and the glass surface, because of these adhesive forces, water tries to creep up creep and then, it tries to occupy the maximum possible surface with a thin film.

So, in turn as there are cohesive forces between the molecules, the water molecules get pulled up and it forms a meniscus like this; this is what we have seen. So therefore, apart from the radius of the capillary tube apart from the radius of the capillary tube, the adhesive force between the water molecules and the solid surface wall that also plays an important role in governing the capillary rise. Such adhesive forces are quantified using contact angles, such adhesive forces are quantified using a contact angles. If the adhesive force between liquid and solid our capillary surface are weaker than the cohesive force between the individual water molecules with a drawn from the surface of the wall.

So, this retraction makes the surface to curve because when if this is in this you are immersing a capillary in this you are immersing a capillary, the forces between the adhesive force between the water molecules and the glass surface or the surface of the solid are weaker than the cohesive force between the individual water molecules. Then water has water with the draws from the surface and it retracts and which forms a concave surface. So, it forms a convex surface like this. So, it forms a concave in the liquid phase. So, when the concave surface forms, so the pressure within the concave side would be higher. We have seen that the pressure in the concave side would be higher than the pressure on the convex side.

So, therefore, the liquid surface depresses in the capillary tube in order to compensate the increase in the pressure. So, if you take a capillary I will redraw it; if I take a capillary, this is the surface of the fluid in the container and when water retracts, it forms a meniscus and this meniscus concave in the liquid phase means that the pressure in the liquid is higher. So, when the pressure in the liquid is higher to balance this. So, there should be a depression, the water should start collapsing; the water the capillary would start depressing or collapsing within this capillary tube.

So, at equilibrium you would see that at equilibrium you would see that there is a depression of this fluid within the capillary tube you would see. So, best example is a mercury in a glass tube mercury. You use mercury as liquid and use a glass tube then it will depress. This could be better quantified using the contact angle. If I magnify this capillary tube and this is the meniscus and the interaction that takes place with the solid and this is a contact angle. Essentially the contact angle is a measure of angle between line tangent to the gas and liquid interface. The gas and liquid interface is this and line defined by liquid and solid boundary and is measured within liquid. So, this is contact angle theta.

So, similarly here the contact angle would be this is in water; this is water as liquid and glass tube as solid. So, the theta is less than 90. In case if you take mercury if I magnify this, this is the capillary and this is the surface water surface what air water interface. So, here the contact angle would be this. So, here mercury as liquid mercury used as liquid and the glass tube, there is a solid surface and theta would be more than 90 degrees; theta would be more than 90 degrees. So therefore, liquid surface interacts with the solid with some angle known as contact angle theta. So, the nature of the curvature meniscus whether it is a concave or convex indicates the fluid is wetting or not or repelling fluid; so the fluid kind of ripples from the surface in this particular case. So therefore, it concave it convexes up like this or concave in the liquid phase.

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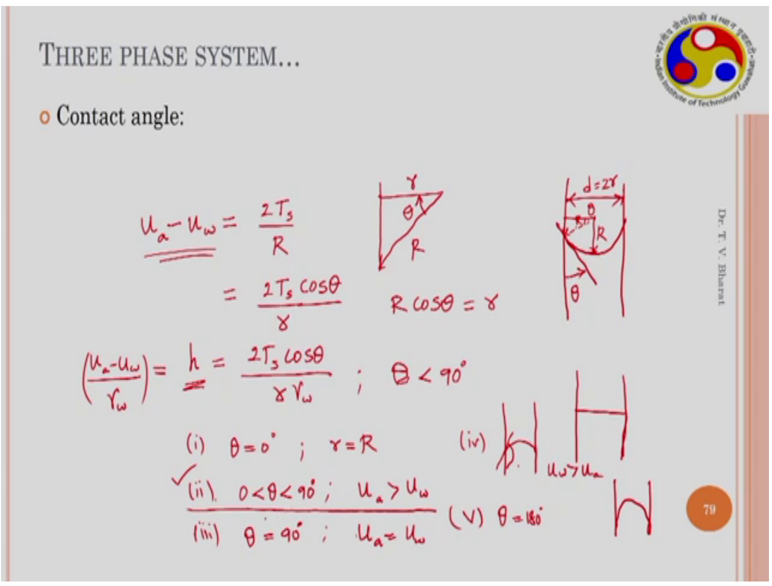

So, at equilibrium let us consider the capillary tube again and you have the meniscus and there is a meniscus and you have a contact angle theta. The radius of the curvature is capital R and this will be same here this is the capital R the radius of the capillary tube, if you sorry the radius of the capillary tube is R, then this is d is equals 2 r.

So, here this is a normal to this line, this tangent; here this line is perpendicular to this tangent and if I draw a line, if I draw a radius, this is again tangentt perpendicular to the wall say. Therefore, this angle again should be theta. So, essentially if I again magnify this, this is capital R and this is the wall and this is small r, this angle again should be theta.

So, R cos theta would be equals to small r. So, earlier we have derived a Laplace equation. According to Laplace equation, the pressure drop across a across of air water interface u a minus u w is equals to 2 T s by R r is the radius of curvature. So, if we substitute r from here, this would be r into cos theta. This is the same expression again we have derived for raise of capillary tube. So, here u minus u w that could be written as h times gamma w, then this would be 2 T s cos theta by r gamma w, because in order to maintain the hydrostatic equilibrium, the induced pressure u a minus u w should be balanced by the raise of capillary.

So, therefore, h is equals to u a minus u w divided by gamma w. Here h is positive, if theta is h is positive theta is less than 90 degrees; theta h is negative if theta is more than 90 degrees so; that means, there is a depression this can be directly understood what is the capillary rise or capillary depression that takes place if contact angle is changing.

So, the contact angle is very useful to quantify whether there is a depression that is taking place or there is a capillary rise that is taking place. There are several possibilities for the contact angle. The pressure drop across the interface in a capillary tube for any contact angle could be estimated using this particular expression and the possibilities here are probably one theta is equals to 0; contact angle is 0 degrees means perfectly wetting; it is a perfectly wetting. So, in that particular case the small r is equals to capital R because cos theta is equals to 1.

So, second possibility is that the contact angle vary between 0 and 90 degrees this is a partially wetting. So, partially wetting surface this is a typical value we see in soils. So, in this particular case the u a is u a minus u w is positive. So, u a is more than u w. So, the third possibility is that theta is equals to 90 degrees. If there is 90 degrees, so then it is a perfectly flat surface.

So, r equals to infinity or u a minus u w is a u a is a equals to u w. So, you have a perfectly flat surface. So, here you have flat surface like this. So, when you have a very large capillary, then you will have u a is equals to u w because cos theta is 0; so you got u a minus u a is equals to u w or u a minus u w is equals to 0.

So, when you have a theta between 90 and 180 that is a capillary depression that takes place, here this is the angle. So this one is a, this one causes capital depression; the pore water pressure the u w would be more than the atmospheric pressure.

So, there is a typical case in mercury and some of the argon of flips soils would exhibit this kind of a behavior. And if theta is perfectly one eighty degrees, so it is a perfectly repellent material. So, these four five possibilities could be there could be considered for our soils. This is a more relevant theta varies generally between 0 and 90 degrees Celsius.

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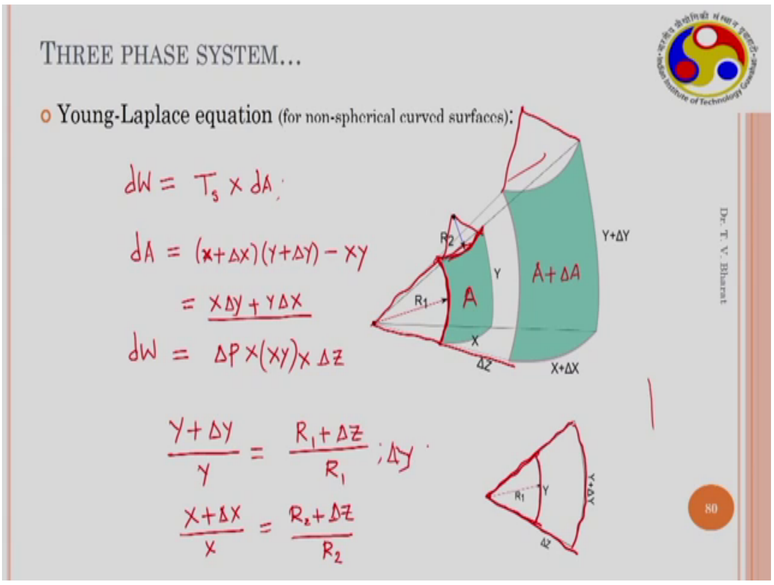

Slowly we are inching towards understanding important state variables in unsaturated soil mechanics. So, one of the important state variable in unsaturated soil mechanics is this quantity u a minus u w. So, this is the pressure drop across the water interface that is what we have understood and this one we call suction very soon. We introduce as suction this quantity very soon. And once we understand this suction concept, then we bring in important constitutive relationships relationship in unsaturated soil mechanics such as soil water characteristic grow.

So, before understanding let us try to understand the equation for the pressure drop in case if you have a non spherical curved surfaces. So, this is one particular case where the diagram shows there is a curved surface a small area delta A, a small area sorry a small area small surface with the area A. A curved surface which has two different curvatures; one is R 1, another one is R 2. Here this is R 1 and this is R 2 two different curvatures you have.

So, you can imagine this as a balloon which is a one small part of it is considered and which has the curvatures in two different directions R 1 and R 2. And when it is blown giving extra energy or extra pressure, this would assume this by changing its area from A to A plus delta A. So, in this particular case we can estimate what is a pressure drop across the interface by analyzing what is the work required work done for expanding this one this area to this let us try to understand. So, based on the work needed to increase the interface area by infinitesimal amount delta A so, the work required work done is d w is equals to T s, surface tension times the change in area.

So, surface tension has units of Newton meter or milli-Newton per meter and this has units of meter square. So, therefore, the multiplication of these two would result in work; it is the joule or Newton meter. So, you can estimate what is a small area. So, small area or change in area is nothing but x plus x plus delta x sorry I am using capital X x plus delta X times Y plus delta Y minus X times Y. So, this is the change in area from this segment to this segment.

So, this is nothing but X delta Y plus Y delta X plus X Y that gets cancelled and you will get plus delta X delta Y that is too small those second order term, you can ignore. Then this is because this is the infinitesimally small area that is increasing due to small work that is done due to the work that is done.

So, this is the change in area. So, this causes a pressure change across the interface that pressure changes again d w is equals to the pressure change. You can estimate that is that is d P times X times Y; so that is area. So, pressure times area there is a force times the movement, this is a small move this has element has move from here to here while expanding. So, this is a delta z work done for force work done for moving this delta Z increment delta Z then this is work. So, work done is equals to delta P times X Y is the area times the delta Z is a distance between these two segments. So, here caution these are X Y and this is into.

So, here this should be this can be simplified, if this segment is considered as considered like this. So, this is the ones triangle; this is formed with this and so this is the bigger triangle. So, from the property of triangle this arc length is equals to R times the theta. From that triangle property of the similarity of the triangles, one can obtain the Y plus delta Y divided by either Y is equals to R 1 plus delta Z divided by R 1. Similarly you can consider other triangle from this. This is one this is another triangle R 2. And similarly you can also time X plus delta X by X is equals to R 2plus delta Z by R 2.

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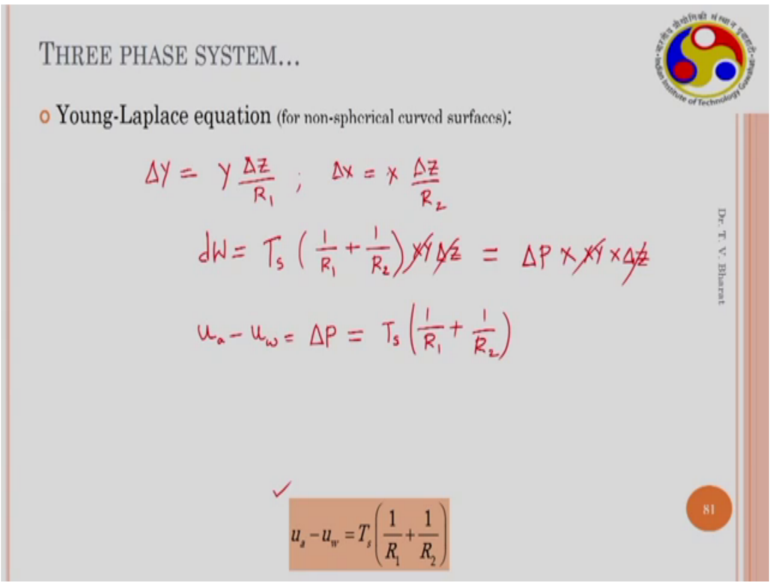

So, this is one plus delta Y by Y is equals to 1 plus delta Z by R 1. So therefore, delta Y is so, the delta Y delta Y; therefore the delta Y is equals to the Y times delta Z divided by R 1 and delta X is equals to X times delta Z by R 2. So, when you substitute this in the area and write the work done, then this is equals to the T s times this is a T s times T s times 1 over R 1 plus 1 over R 2 and X Y terms you can bring them out and delta Z term you have.

And when you equate this to the work done due to the pressure drop, this is a delta P times X Y times delta Z. So, then delta Z X Y get cancelled. So therefore, the change in pressure or pressure drop across the interface is equals to T s times 1 by R 1 plus 1 by R 2.

So, this is the expression here delta P is u; this is the delta P is here the delta P is u a minus u w. So, there is a pressure drop across the interface. So, this is the expression you get, there is a angle Laplace equation. So, this Laplace equation could be an angle of Laplace equation could be applied for understanding the soil behavior.

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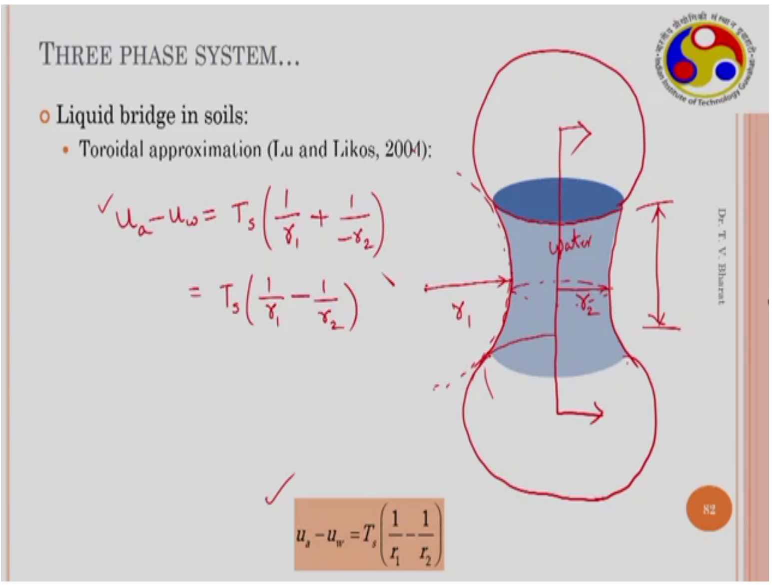

So, when you have two soil grains which are located; so one in here, so other one is here. So, this is water. So, there is a liquid bridge that forms there is a meniscus that forms that there is like this. So, with a curvature, the radius of curvature R 1 with radius of curvature small r 1, there is a interface that forms. So, this radius of curvature changes in this direction.

So, if you consider a plane; along this plane this appears as if it is a horse saddle. The radius that changes in along this plane; so, that is along this one is r 2. So, it seems in this particular case the r 2 is convex and r 1 is concave. So, in that particular case the liquid bridge like the pressure drop across a liquid bridge, in this particular case would be u a minus u w is equals to T s times 1 by r 1 which is equal to capital r 1 in the Young's Laplace equation plus 1 by minus r 2. Earlier in Young's Laplace equation both r 1 and r 2 both are concave, but in this case the r 2 which is a liquid cylinder that is available here in the liquid bridge is a is a convex. So, it should be minus r 2, then this equation turns out to be T s times 1 by r 1 minus 1 by r 2.

So, this is expression for liquid bridge in soils and this is a pressure drop because of that, so when the soil grains when they come close to each other because when water tries to evaporate, when the water is lost between the soil grains, so these particles would try to move close to each other that when the distance between these two grains decrease. So, the r 1 r 2 also change accordingly. Then the pressure drop across the interface would change that could be estimated using this particular expression.

So, so far we have seen that when you have a capillary when you have a capillary or when you have a curved interface what is a pressure drop across the interface we have seen and after that when you have a capillary rise, the when you have a capillary rise in a capillary tube. Then we have seen what is a pressure drop which is again dependent on not only it depends on not only the capillary tube diameter; it also depends the contact angle.

So, in case of 2 grains 2 soil solid grains which are because of which there is a liquid bridge that is forming between these 2 grains, then this is the expression we have seen this could be possibly applied for understanding the soil behavior, unsaturated soil behavior.

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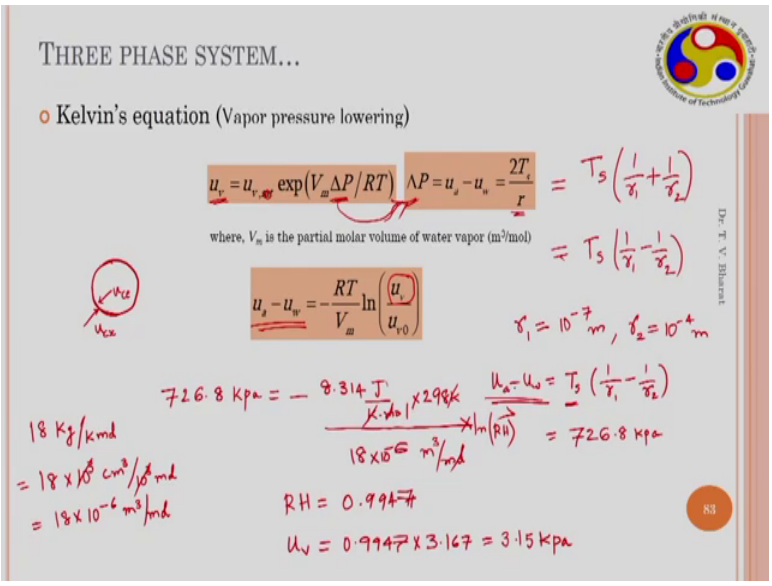

One more important concept in physical chemistry that is a directly applicable for our soil mechanics is a Kelvin's equation or vapor pressure lowering. This is very important concept for even to estimate one of the important state variables of unsaturated oils such as a suction total suction estimation. So, according to the Kelvin's equation as we have seen earlier that the vapor pressure of liquid depends on temperature as temperature increases the vapor pressure increases and also on presence of solute in solvent or presence of salts in water. So, because of the presence of salts in water the vapor pressure decreases.

And also there is an important parameter that is important state variable that is pressure. So, apply pressure on the liquid also influences the vapor pressure. Further we have seen that curvature at a water interface that gives rise to this is convex. So, the curvature at the air water interface gives rise to a pressure drop across the interface. Therefore, vapor pressure above the curved surface would be expected to be different from the vapor pressure above the free surface flat surface.

So, this is an important point. So, the because there is a pressure drop across a curved surface the vapor pressure would be different because vapor pressure depends on the pressure applied on the liquid. So, therefore, Kelvin's equation relates this change in the vapor pressure with the curvature; curvature or the capillary tube radius etcetera; capillary tube radius or a curvature. These two things could be related using Kelvin's equation.

So, u v is the vapor pressure and this is the vapor pressure near the flat surface or with the saturated solution, vapor pressure u v naught is a vapor pressure equilibrium when it is interacting with a flat surface and exponential of v m is a partial molar volume of water vapor which has units of meter cube per mole and delta P is change in pressure or pressure drop across the interface divided by R T; R is gas constant t is temperature in Kelvin. So, the delta P here could be estimated from different equations. So, far we have discussed could be from Laplace expression which is delta P is equals to 2 T s by r; Laplace expression when used for curved surfaces or it could be for angular plus equation when it is non spherical curved surfaces. So, then it would be T s times 1 by r 1 plus 1 by r 2.

So, this is an angular plus expression, if this becomes spherical. That means, r 1 and r 2 both are 1 and the same then, this boils down to the Laplace expression 2 T s by r. So, this is T s into 1 plus r 1 plus r 2 r. If you have if you are assuming toroidal approximation if you are using toroidal approximation for understanding unsaturated soil behavior, then this could be T s times 1 by r 1 minus 1 by r 2. So, anything the pressure drop using different expressions could be obtained and if you substitute you would understand: what is the change in the vapor pressure, what is the change in the vapor pressure.

So, this can be simplified as u i minus u w is equals to minus R T by v m times the log u v by u v naught. So, for a one particular case when r 1 is when 1 particular case when r 1 is a 10 power minus 7 meters and r 2 is r 2 is 10 power minus 4 meters. So, in this particular case you can estimate what is a pressure drop that is a T s into 1 by r 1 minus 1 by r 2 using toroidal approximation; this is here this is 7 to 2.75 milli Newton per meter at standard temperature and if you substitute r 1 and r 2, you would get the value of value is equal to 270 sorry 726.8 kilo Pascal; this is a pressure drop.

So, you can estimate for this particular pressure, this is a pressure we estimated and for this particular pressure drop, what could be the vapor pressure above the surface above that curvature; one can estimate. So, here you can substitute this value. So, this is 726.8 kilo Pascal is equals to minus r is 8.314 joule per Kelvin mole times 298 298 Kelvin divided by v means partial molar volume of water vapor which is 18 kg per molecular mass is kg per kilo mol.

So, the molecular mass is 18 kg per kilo mole which can be written in terms of volume. So, therefore, eighteen times this could be 10 power 3 centimeter cube per kilo 10 power 3 mol. If you write in terms of metal cube, this is 18 times 18 times ten power minus six meter cube per mol. So, you can substitute 18 times 10 power minus 6 meter cube per mol into log this is nothing but R H right. So, u v by u v naught or u v at is R H. So, therefore, R H is if you simplify here joule can be written as a Newton meters.

So, here kilo Pascal can be written as kilo Newton per meter square and after simplification, the Kelvin gets canceled and here you have meter cube per mole; the mole gets cancelled. Here Newton meter, you get Newton per meter square. So, here you have kilo Newton per meter square when you simplify you multiply with 10 power 3, then you get Newton per meter square, then when you simplify this you get 0.9947 is R H or 99.47 percent.

So, at standard temperature the R H u v sat saturated vapor pressure is 3.167. So, therefore, u v could be 0.9947 times 3.167 kilo Pascal which is equals to 3.15 kilo Pascal. So, this is how the vapor pressure above the curved surface can be estimated. So, we use this particular technique to measure the vapor pressure above the curved surface from that we obtain what is a u a minus u w or pressure drop across the interface. So, for estimating the one of the state variables that is suction or u a minus u w, we measure the u v and we estimate this particular value using Kelvin's equation.

Thank you.