Unsaturated Soil Mechanics
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Week - 02
Lecture – 05
Capillary Phenomenon in Unsaturated Soil-I

Hello everyone. So, far we have discussed the properties of individual phases such as air and water and after that we have seen how much water is present in air has a vapor. And the dependency of temperature we have seen. Now, we try to understand how much air can present in water. So, that is the air solubility in water.

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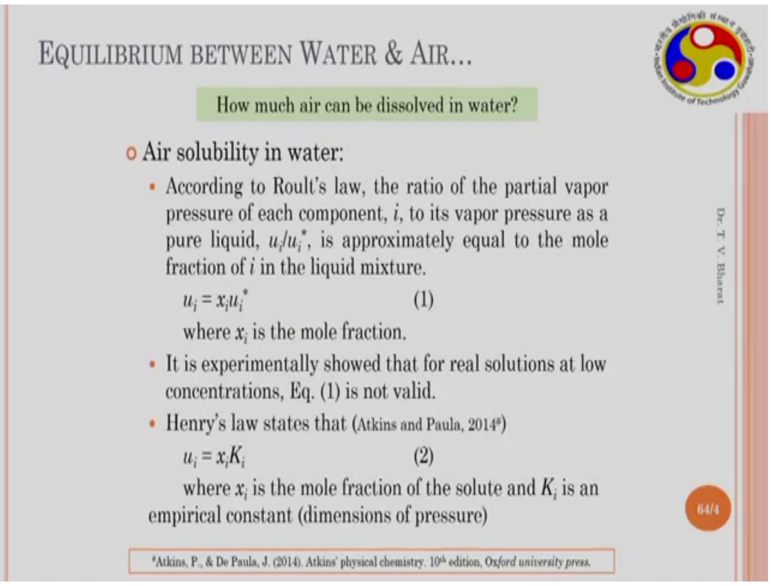

EQUILIBRIUM BETWEEN WATER & AIR…

How much air can be dissolved in water?

- Air solubility in water:
  - According to Roult's law, the ratio of the partial vapor pressure of each component, *i*, to its vapor pressure as a pure liquid, u_i/u_i^*, is approximately equal to the mole fraction of *i* in the liquid mixture.
    $u_i = x_i u_i^*$ (1)
    where x_i is the mole fraction.
  - It is experimentally showed that for real solutions at low concentrations, Eq. (1) is not valid.
  - Henry's law states that (Atkins and Paula, 2014[a])
    $u_i = x_i K_i$ (2)
    where x_i is the mole fraction of the solute and K_i is an empirical constant (dimensions of pressure)

[a]Atkins, P., & De Paula, J. (2014). Atkins' physical chemistry. 10th edition, *Oxford university press.*

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According to Roult's law the ratio of partial vapor pressure of each component I to its vapor pressure as a pure liquid that is u i by u i star is approximately equal to the mole fraction of i in the liquid mixture. This is a little complicated because we are not used to have the chemistry background, because we do not have much chemistry background in soil mechanics.

It states that u i's x i times u i star, where x i is the mole fraction however, it is experimentally showed that for real solutions at low concentrations this particular equation is not valid. So, we have to embrace some other equation other law that is Henry's law which states that the partial pressure of a component I is equals to mole fraction times a proportionality constant. So, this is an empirical constant which is obtained from experimental data. So, now so, after that these empirical constants are called Henry's law constants, so this the details of these laws and expressions can be found in Atkins physical chemistry book the reference is given here.

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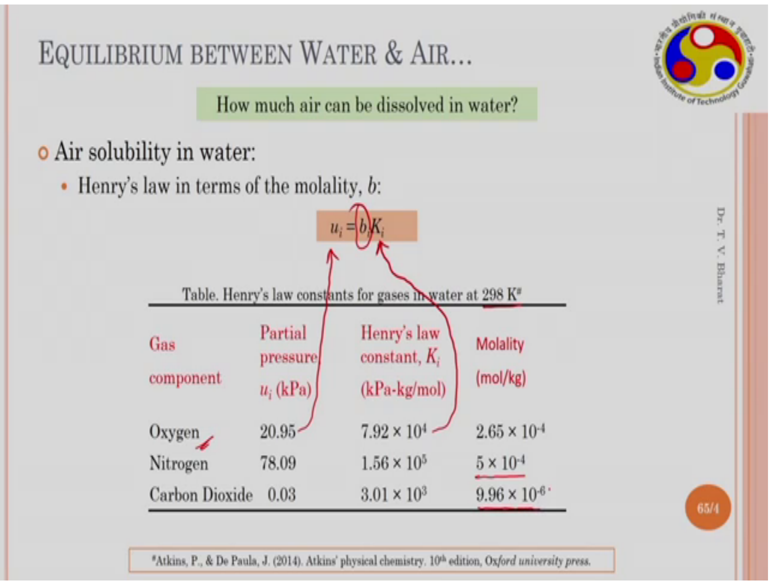

EQUILIBRIUM BETWEEN WATER & AIR...

How much air can be dissolved in water?

- Air solubility in water:
  - Henry's law in terms of the molality, b:

$$u_i = b_i K_i$$

Table. Henry's law constants for gases in water at 298 K#

Gas component	Partial pressure u_i (kPa)	Henry's law constant, K_i (kPa-kg/mol)	Molality (mol/kg)
Oxygen	20.95	7.92×10^4	2.65×10^{-4}
Nitrogen	78.09	1.56×10^5	5×10^{-4}
Carbon Dioxide	0.03	3.01×10^3	9.96×10^{-6}

#Atkins, P., & De Paula, J. (2014). Atkins' physical chemistry. 10th edition, Oxford university press.

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So, the air solubility in water using Henry's law can be expressed in terms of molality as this u i the partial pressure equals to molality times the k, k here is the Henry's law constant. In this table for different gas components the Henry law constant are given here. For example, for oxygen the partial pressure is 20.95 if you consider the total pressure as 101.325 kilo Pascal. So, based on the mole fraction you can consider that the you can obtain the oxygen partial pressure as 20.95 kilopascal.

So, if you multiply this, if you substitute u i here and Henry's law constant which is K i substitute here then you would obtain b i that is the molality. So that means, 2.65 into 10 power minus 4 moles of oxygen can be dissolved in 1 kg of water at temperature of 298 Kelvin and pressure 101.325 kilo Pascal. Similarly for nitrogen slightly higher 5 into 10 power minus 4 mol per kg and carbon dioxide is much less carbon dioxide is less 9.96 into 10 power minus 6 mol per kg.

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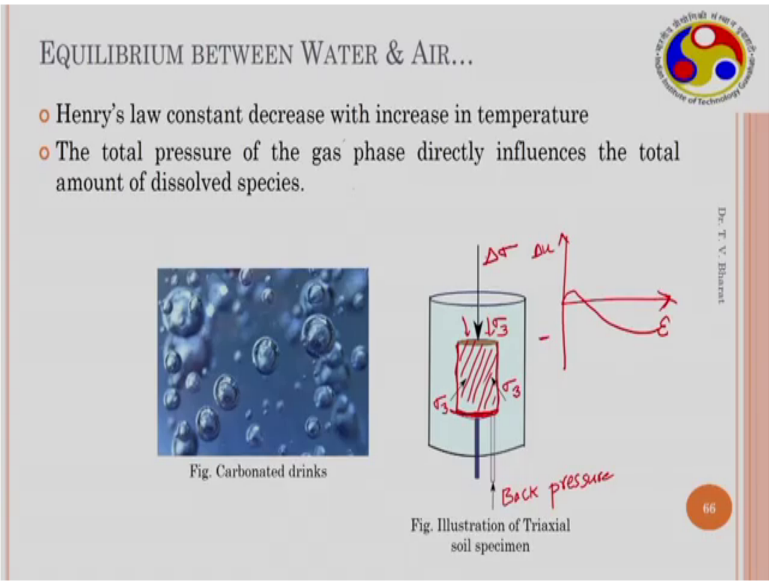

EQUILIBRIUM BETWEEN WATER & AIR…

- Henry's law constant decrease with increase in temperature
- The total pressure of the gas phase directly influences the total amount of dissolved species.

Fig. Carbonated drinks

Fig. Illustration of Triaxial soil specimen

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This is further what happens to this Henry's law constant with temperature and pressure. So, if you see the Henry law constant decreases with increase in temperature this is especially important for fishermen, they look for slightly colder water where you have more dissolution of oxygen is available. So, therefore, more fishes are likely to be found and they obtain, they catch the prey by this particular technique.

So, when it comes when it comes to the pressure what about the influence of pressure. The total pressure of the gas phase is directly influences the total amount of dissolved species that means, if you increase the pressure to increase the gas pressure then the more amount of individual gas components get dissolved in water. So, that we often see in carbonated drinks where carbon dioxide is dissolved in water at high pressures. So, this is a for they actually bubble up or phases up phases with carbon dioxide gas. So, we like to drink such rings, we will like to take such a rings because there is a carbonation that takes place. When you allow it to come to the normal pressure normal atmospheric pressure then this carbon dioxide the gas comes out. So, that forms small bubbles these are vapors.

So, this is of particular importance to us to being a soil engineers when you test the soil samples in triaxial setup, this is an illustration of triaxial test setup, where this is the soil sample a soil sample is placed in a case you have a water, the water pressure is the all round pressure you apply an all round pressure there is sigma 3 on the soil mass. So, even though pressure acts from top also. So, this is sigma 3. So, then you have axial load that you can apply or deviate (Refer Time: 06:33) stress can apply which causes shear failure of the sample and this is a platform on which the soil sits.

So, here soil has soil is confined in a membrane and this soil sits on a platform here you will have a porous stone, and this porous stone is connected to a pipe and you can apply backpressure, backpressure through this so that you can control the pore water pressure within the soil mass. So, initially when the sample is brought from the field when the soil is in partially saturated conditions. So, then you want to saturate the sample, then what we generally do is we apply a cell pressure of some amount say sigma 3 equals to some say half 50 kilo Pascal and then increase the back pressure to 50 kilo Pascal.

So, after that again you increase the back pressure 200 kilo Pascal of course, while increasing the cell pressure also to the same mode or the back pressure slightly less than the all round pressure. So, by increasing the pore water pore, pore pressure using the back pressure wall we are trying to dissolve the gas gases into pore water and so this is what we are doing we are this is this is how we are saturating the soil sample. So, at what pressure the gas components get dissolved at a given temperature could be understood using this Henry’s law.

However, the Henry’s law here what we discussed here he is not for understanding how much amount of gas component would get dissolved in pore water when you are applying a back pressure for standard triaxial test setups, but there is a bigger role that will play you know understanding on unsaturated soil mechanics that will be discussed little later.

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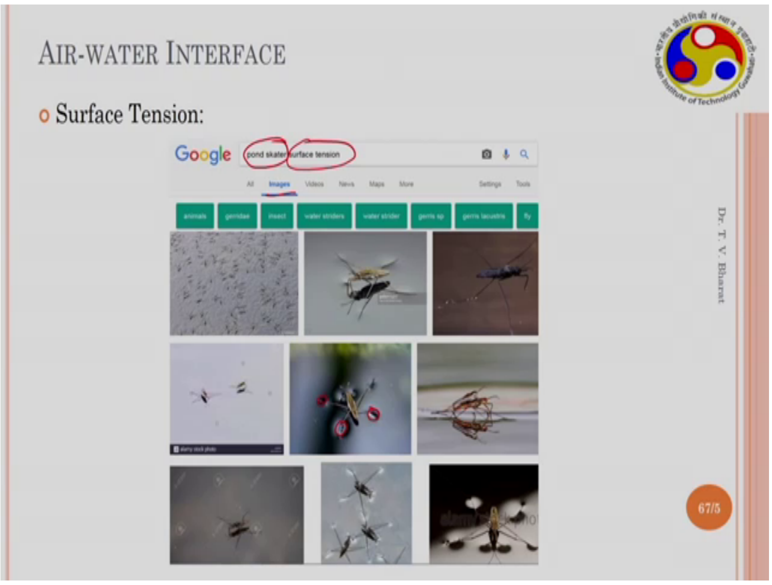

AIR-WATER INTERFACE

- Surface Tension:

So, coming to interesting phenomena that is air water interface; As of as of now we have completed understanding, as of now we have completed the phases of different phases of soil soils apart from soil solids you have pore water and pore air. So, individual pore air and pore water characteristics we have seen, after that the equilibrium between these two phases we have seen. Now, there is interface that exists at the conjuncture between the air and water when you consider two phase system. Often we wonder why several insects can freely float on waters.

If you simply type surface tension and pond skaters if you give these two key words in Google and if you go to images you would find there are so many pictures of pond skater and other insects floating on waters. So, you would clearly see that the there is a depression that is formed and the legs of this insect did not cut across are go into waters, but you see that the water is acting as a membrane and it is floating on a membrane, but actually water exhibits such a behavior that is because of surface tension.

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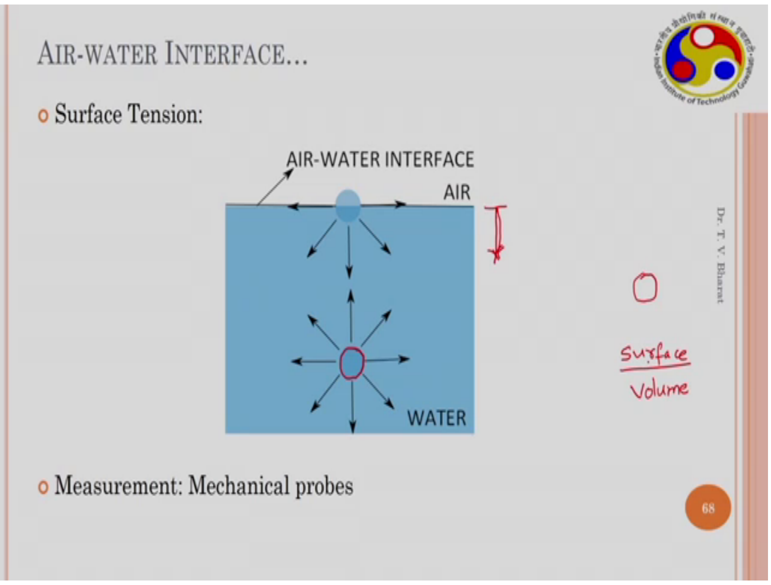

AIR-WATER INTERFACE…

- Surface Tension:

- Measurement: Mechanical probes

If you consider a microscopic view of a water when you have s many water molecules there are intermolecular forces there is a cohesion, cohesion property that has our cohesive forces that are acting between individual molecules. And if you consider a molecule at some depth below the air water interface some depth below the air water interface you would see that particles the molecule has on this molecule equal forces are acting from mol directions, because you have molecules this molecule is surrounded by this number of molecules and these molecules exert the same force from all directions. So, this is kind of stable.

But if you consider a molecule which is closed to the air water interface, so the forces that are acting from the air phase or absent. So, there is a net downward force acting on this molecule located at the interface. So, therefore, liquids tend to adopt shapes that minimize that surface area. So, that maximum number of molecules is in the bulk and hence surrounded by and interacting with neighbors. So, the droplets of liquid generally assume a spherical shape because it has a smallest surface to volume ratio the surface to volume ratio of spherical, spherical dimension are lowest smallest.

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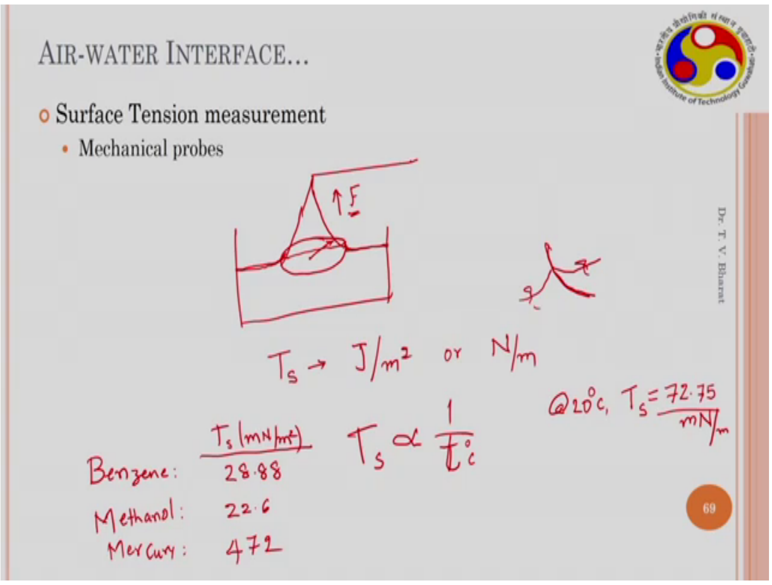

AIR-WATER INTERFACE...

- Surface Tension measurement
  - Mechanical probes

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So, generally this surface tension is measured using mechanical probes such as a dew wire rings where you have a small ring wire, wire which is circular which is connected with different wires which can be must in water. When you immerse in water if the due to the surface tension there is a curvature that do you observe. So, curvature on both sides in fact, from inside and outside inside of the wire and outside of the wire you find the curvature. So, essentially if you have the wire. So, there is a curvature there is a water surface and this is the wire.

So, known diameter of wire you consider and when you immerse and when you pull it up with some force you observe that the water tries to get pulled up the water near the air water interface, and the solid near the solid it gets pulled up and it breaks apart at one particular under one particular force applied. So, that force is estimated and from that the surface tension is measured surface tension is can be defined as the energy required to stretch a unit surface area at the air water interface. So, generally this is this is this as the dimensions of Joules per meter square or Newton per meter or milli Newton per meter. So, these are the general units that general units used for surface tension. So, therefore, surface tension is a is a maximum energy of fluid or liquid can store without breaking apart, surface tension decreases with increase in the temperature.

So, approximately for at 20 degree Celsius are temperature small t degree Celsius at 20 degree Celsius 20 degree Celsius the surface tension of water is 72.75 milli Newton per meter. So, this also changes with different fluids. For benzene, for benzene the surface tension is all in milli Newton per meter square. So, for benzene its 28.88 which is less than water and for mercury for methanol this is a 22.6 for the mercury, this is very high 472. Surface tension changes with temperature surface tension changes with different fluids and for water this is 72.75 and for different fluid this is how it changes this is measured data using mechanical probes like (Refer Time: 15:49) rings.

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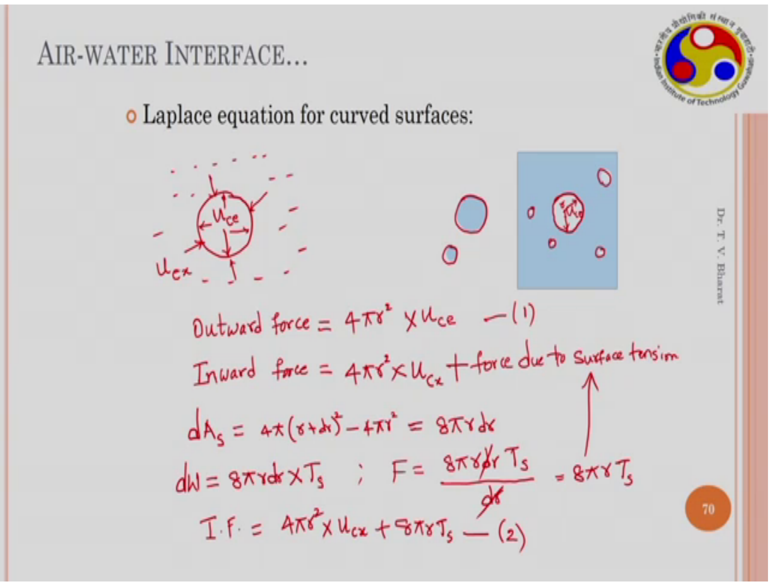

AIR-WATER INTERFACE…

- Laplace equation for curved surfaces:

Outward force $= 4\pi r^2 \times u_{ce}$ — (1)

Inward force $= 4\pi r^2 \times u_{cx}$ + force due to Surface tension

$dA_s = 4\pi(r+dr)^2 - 4\pi r^2 = 8\pi r\,dr$

$dW = 8\pi r\,dr \times T_s$; $F = \frac{8\pi r \cancel{dr}\, T_s}{\cancel{dr}} = 8\pi r T_s$

$I.F. = 4\pi r^2 \times u_{cx} + 8\pi r T_s$ — (2)

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So, often we see curved surfaces. You often see that there is a small air bubbles that are available in water air bubbles that are available in water, but these are not air may not be air bubbles there may be a vapor these are vapor bubbles because minimization of the surface area of liquid may result in curved surfaces like this liquids tend to adapt shapes that minimize their surface area. So, a bubble is a region this is a bubble or this is a bubble, bubble is a region in which vapor and possibly some air too trapped by a thin film.

So, at therefore, a cavity is a vapor filled hole in the liquid. Another hand you may have water droplet us that we often see from rainwater or from a sprinklers nozzles when they emit the water you see small droplet us of water. So, here you see again spherical shape of water small water droplet us. So, they usually shown spherical. Small droplet is a small volume of liquid at equilibrium surrounded by its vapor or air this is in normal atmosphere. Pressure on the concave side of an interface if you say this is a concave side is u c e or if I consider this is the bubble so pressure inside outside you have water. So, the concave side you have now here concave side you have air and outside this is water you have this is convex side. So, I am writing c e that is indicating concave and the c x indicating convex.

So, let us see the pressure equilibrium or force equilibrium let us see the force equilibrium in this particular case where when it is an equilibrium. So, the outward force is the outward force by this a which is vapor trapped inside is outward force is equals to the area that is 4 pi r square here r is the radius r is the radius of this bubble times the pressure that is u c e. So, if you consider the inward force, inward force so inward forces from all around. So, that is 4 pi r square again times from convex side plus there is a surface tension force 4 0 surface tension. The force due to surface tension can be estimated by considering the change in the surface area of a sphere due to some work done.

So, what is the change in the surface area? So, the change in the surface area dA s for example, it is trying to expand the bubble is trying to expand or it is contracting the change in the surface area is 4 pi r plus dr square minus 4 pi r square. So, there is equals to 8 pi r dr by neglecting the higher atom dr square terms here. So, because that will be too small a small increase in the surface area if you consider that is a due to as force into distance is the work the work that is opposing the stretch is because of the surface tension only that force is that force that is; so the work done when the surface is stretched by this amount is work done is equals to, so this much 8 pi r dr times the surface tension.

So, as work is nothing, but force into distance the force opposing stretching through a distance dr is force is equals to this work 8 pi r dr times t s divided by dr which results in 8 pi r T s. So, this is the first equation which we got there is outward force and while substituting this 8 pi r T s here. So, the inward force is: 4 pi r square into u cx plus 8 pi r T s. So, this is a second expression.

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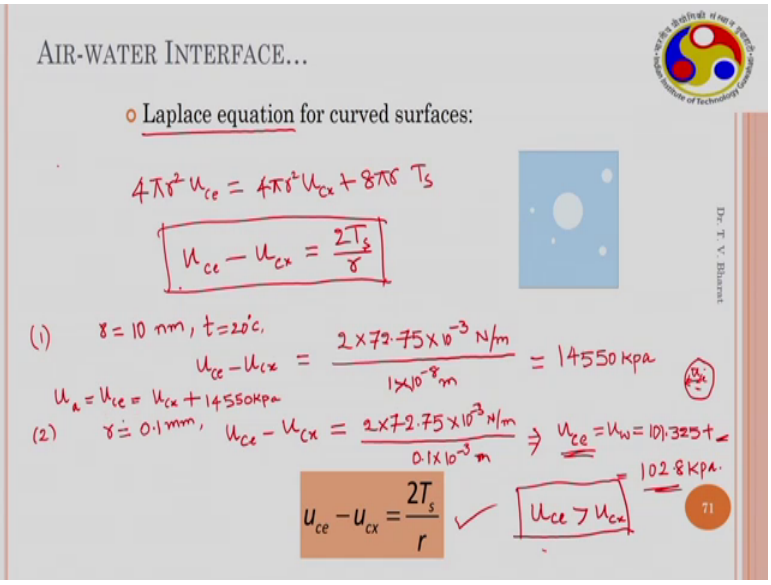

AIR-WATER INTERFACE…

- Laplace equation for curved surfaces:

$$4\pi r^2 u_{ce} = 4\pi r^2 u_{cx} + 8\pi r T_s$$

$$\boxed{u_{ce} - u_{cx} = \frac{2T_s}{r}}$$

(1) $r = 10\ nm,\ t = 20^\circ c,$

$$u_{ce} - u_{cx} = \frac{2 \times 72.75 \times 10^{-3}\ N/m}{1 \times 10^{-8}\ m} = 14550\ kpa$$

$u_a = u_{ce} = u_{cx} + 14550\ kpa$

(2) $r \doteq 0.1\ mm,\quad u_{ce} - u_{cx} = \frac{2 \times 72.75 \times 10^{-3}\ N/m}{0.1 \times 10^{-3}\ m} \Rightarrow \underline{u_{ce}} = u_w = 101.325 + \ldots = 102.8\ kPa.$

$$u_{ce} - u_{cx} = \frac{2T_s}{r} \quad \checkmark \quad \boxed{u_{ce} > u_{cx}}$$

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Equating the first expression and first one and second one expressions at equilibrium would result in 4 pi r square u c equal c is equals to 4 pi r square u cx plus 8 pi r T s the simplification gives you u c minus u c x is equals to 2 T s by r this is this expression you get after simplification. So, this is called Laplace equation, this is a Laplace equation. So, when you have a curved surface this is applicable for this case also this is where you have water inside and outside you have air. Here in this case you have air or vapor inside and outside you have water. So, the for both cases this equation is applicable. So, at equilibrium this equation is valid.

So, the pressure differentiation across an interface across air water interface can be found can be understood using this particular expression. So, this expression tells you that on the difference of pressure becomes 0 when r becomes infinity that means, for flat surfaces so the difference is 0.

So, generally the small cavities when r is small the pressure difference will be higher so that we can estimate what could be the pressure variations in the cavities. So, if you consider a spherical droplet of size radius is equals to radius is equals to 10 nanometer this is very small droplet; so a very small cavity at 20 degrees. So, you can consider T s as 72.75, 72.75 milli Newton per meter. So, therefore, you can estimate this value this is equals to the pressure difference u c minus u cx is equals to 2 times 72.75 milli Newton's therefore, into 10 power minus 3 Newton per meter divided by r 1 into 10 power minus 8 meter. See if I simplify this I get 14,550 kilopascals. So, this is nearly 15 mega Pascal pressure, difference across a small tiny nanometer size cavity.

If the if the diameter of diameter or radius of the cavity decreases then the pressure drop across this interface decreases. This especially important and important in mechanical engineering where you often see when you have a ship propellers when there when you have ship propellers when they circulate they create pressure difference at different regions at different regions. Set few regions a smaller pressure is generated when the pressure at that particular region drops below the vapor pressure of water then cavitation takes place.

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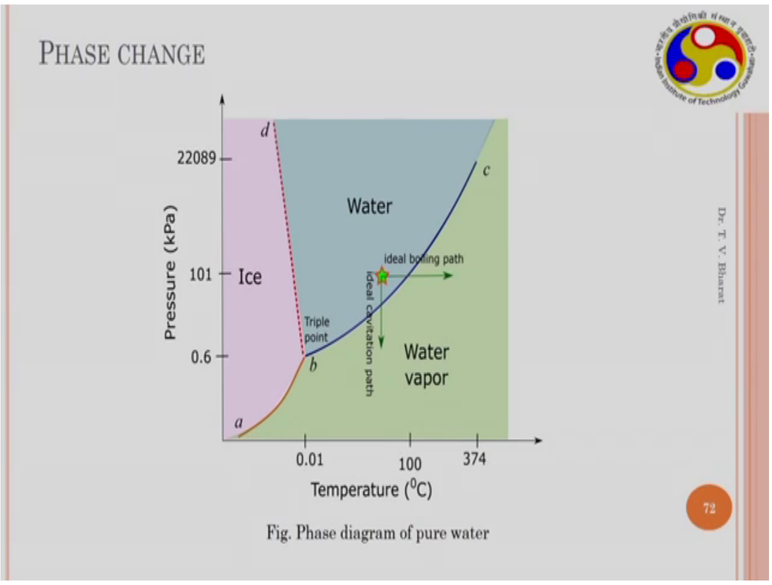

PHASE CHANGE

Fig. Phase diagram of pure water

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We have seen earlier under given temperature when the pressure drops this is a absolute pressure at temperature given temperature at 20 degrees it will may be 101.325 kilopascal. If the pressure drops then water becomes water vapor. Same thing happens when in either pipes or in the propellers when they are circulating there are few regions that are developed where you have pressure dropping below the vapor pressure of water. So, in that particular case the cavitation occurs if the absolute pressure at any given particular point is generally in waters you may consider it could be slightly higher than the 101.325 kilo Pascal because down the somewhere down below the waters.

So, you may have some value, but then because of the pressure drop at that particular region because of the water propeller. So, it may reach to a vapor pressure equal into

3.25 kilo Pascal or any value at the vapor pressure at that particular temperature then cavitation takes place and then small cavities would develop at that particular region. So, these cavities would flow towards the higher pressure region. So, when they flow towards the higher pressure region they have to break when they collapse they released huge energy because the pressure drop across this interface is used. So, it releases huge energy when they collapse. So, this causes this gives a huge impact on the machinery surfaces propeller surfaces. So, because of that the machinery gets corroded or eroded to the surfaces often get eroded this is a serious problem.

And in geotechnical engineering also when we have interaxial setups we have seen, especially this particular point I have already mentioned earlier interaxial setup when the soil is over consolidated or densely compacted sand if you have then when you are when you are shearing the sample in undrained manner or negative pore water pressure develops or pore water pressure decreases. This graph we have seen earlier. This is which, this is which strain this is a change in the pore water pressure a change in the pore water pressure decreases this is negative. If this decreases if this value decreases to the pore absolute pore water pressure decreases below the vapor pressure of water then cavitation takes place and cavities would form in the system and then this no longer behaves like behaves like a saturated soil sample. So, to avoid that usually we maintain some back pressure.

Similarly if you consider water droplet us water droplet us which are coming out from nozzles a sprinkler nozzle for this is the first case where we have discussed the cavity problem. Then the second problem is with a sprinkler nozzle where the radius of the water droplet you may consider as a point one millimeter radius of the sprinkler nozzle is 1.1 millimeter. So, at same temperature what is a pressure of water inside. So, that is the question. So, you can use the same expression. So, here u ce minus u cx is equals to 2 times 72.75 into 10 power minus 3 Newton per meter divided by. So, the radius is 0.1 times 10 power minus 4 sorry 10 power minus 3 meter.

So, therefore, u v so, this is the water droplet and outside you have air. So, in the water droplet you have the curvature u ce, u ce is the water pressure is equals to at the solute pressure atmospheric pressure is 101.325 kilo Pascal plus there is a additional value so that gives 102.8 kilopascal. So, one thing you need to note down is that the pressure always here is this is u ce here, here this is a u ce you can estimate. For example, here u ce if you want to estimate you need to know what is the u cx, at concave what is a pressure, outside the water pressure plus this value. So, always u ce is more than u cx the pressure on the concave side is always more than the pressure on the convex side, so always.

So, you can use this simple expression and to estimate the pressure on the concave side then you find out which side is concave. In this particular in this particular case the concave side is air pressure that is inside. So, in this particular case the concave side is water, so water pressure. Here in this case the water pressure is higher in a water droplet in this particular case the air pressure is higher which is a cavity. So, with this one can find out what is a pressure drop across a given interface. So, then we often see that when you have a thin capillary and insert if you take in thin capillary and put it in a beaker of water you often see that the water raises up.

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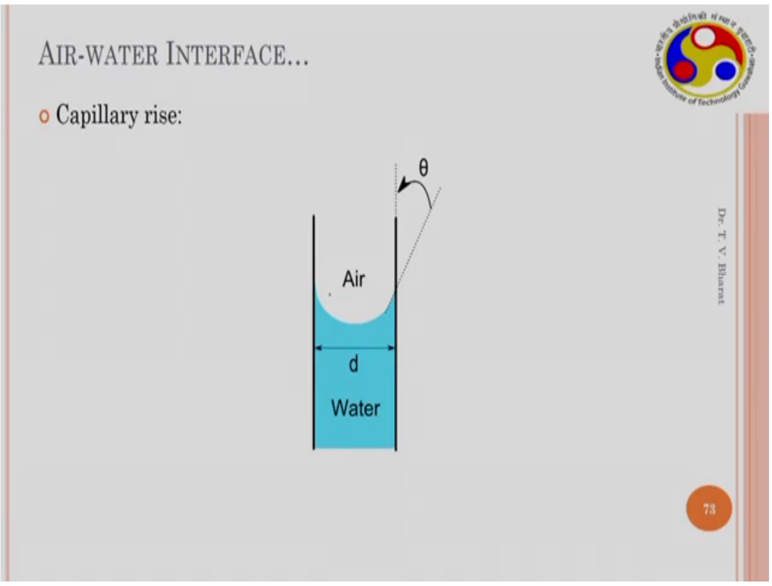

AIR-WATER INTERFACE...

- Capillary rise:

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So, raising of water in a capillary tube is again due to the surface tension and also the solid also plays a role this particular case if a capillary tube. If a glass tube is immersed in water then water tries to adhere to the glass surface and to minimize energy, water has to water can occupy water can wet the glass surface as much as possible with a thin layer. So, water price to key creeps up. So, when water tries to keeps creeps up due to the adhesive forces. So, other water molecules would follow because of the cohesive forces. So, in turn it forms a curved shape like this because water molecules near to the surface wants to adherent and then form a thin layer. So, water tries to pull up and it eventually forms a core surface like this.

So, how much it can raise up to?

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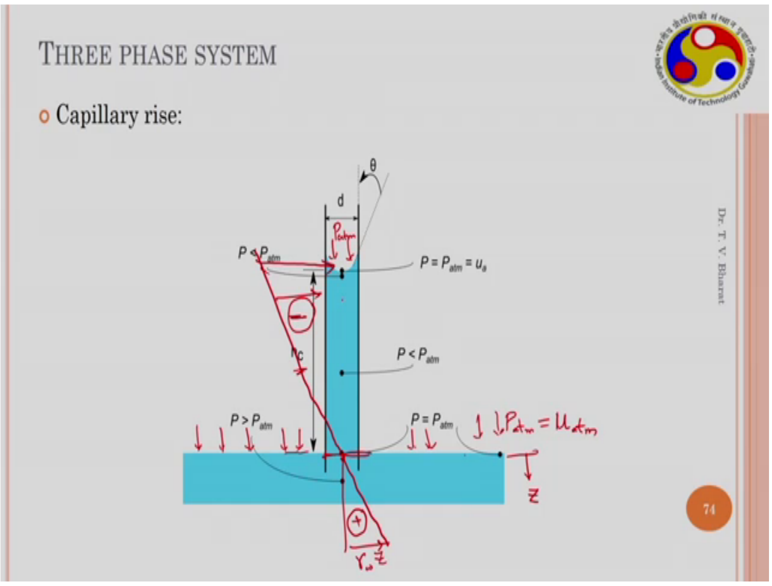

THREE PHASE SYSTEM

- Capillary rise:

If glass tube is immersed in a water beaker, so then you see that there is a rise of hc this is a capillary raise height. So, at this point you have a equilibrium at this particular this is an at equilibrium this is a hc capillary rise. So, now, if you try to understand what is the pressure at different places at this particular level in water the pressure is equal to the atmospheric pressure, these are atmospheric pressure that is existing that is atmospheric pressure u atm, u atm or P atm this is atmospheric pressure that is acting all over here and here P atm.

So, the pressure just at the interface within water is equal to the pressure outside that is atmospheric pressure. And here also just below this just below this line within water this is atmospheric pressure, but slightly above this level slightly above this level the pressure is negative and the negative pressure increases, increases and it reaches again here this is a negative and at the interface just here is equal to the atmospheric pressure to come to equilibrium.

So, if you plot the pressure distribution within water we know that the pressure distribution is like this sorry the pressure distribution is like this if density is gamma w.

So, this is gamma w times Z, this is positive. But above this if you see this is a negative pressure that develops and it becomes 0 at the interface. So, this is negative. So, this is positive and this is negative. So, there is a negative pressure in water, water is there is a negative water pressure that exists in the column. So, how much rise it is possible?

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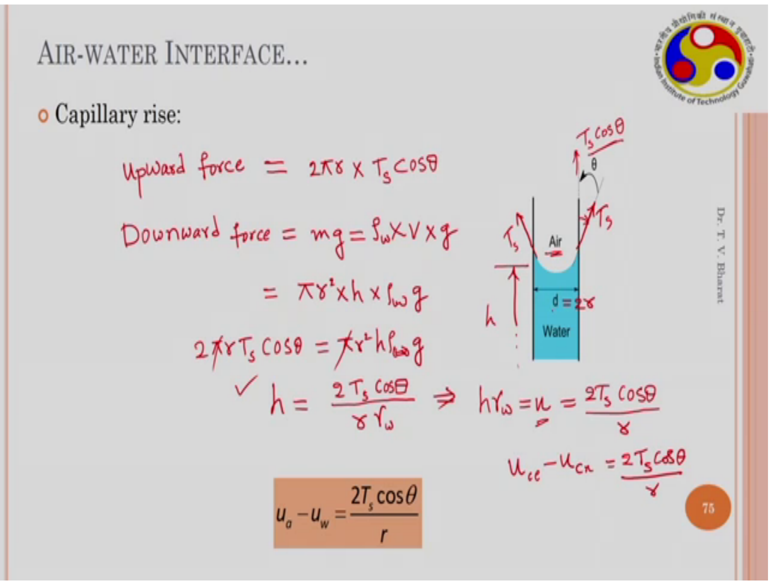

AIR-WATER INTERFACE...

- Capillary rise:

$$\text{Upward force} = 2\pi r \times T_s \cos\theta$$

$$\text{Downward force} = mg = \rho_w \times V \times g$$

$$= \pi r^2 \times h \times \rho_w g$$

$$2\cancel{\pi} r T_s \cos\theta = \cancel{\pi} r^2 h \rho_w g$$

$$✓\; h = \frac{2 T_s \cos\theta}{r \gamma_w} \Rightarrow h\gamma_w = u = \frac{2T_s \cos\theta}{r}$$

$$u_{ce} - u_{cr} = \frac{2T_s \cos\theta}{r}$$

$$\boxed{u_a - u_w = \frac{2T_s \cos\theta}{r}}$$

So, that we can estimate, so if you consider surface tension forces T s are acting here in this in this direction. So, if you resolve the forces this is a angle of contact or contact angle theta. So, then if you resolve the forces into vertical this is T s cos theta and this is a T s sin theta, that they get cancel and the vertical component pulls the liquid upward this T s cos theta contributes in pulling the liquid upward. So, the total of upward force if you consider, total of upward force if you consider acting along the circumference of the water surface acting along the circumference of the liquid surface. So, total of upward force is 2 pi r r is the radius d is 2 r. So, this is radius 2 pi r times T s cos theta.

So, the downward force is the weight of the water column, so which is a weight mass into time mass times g acceleration due to gravity. So, this is this can be written as density of water times volume density is mass by volume, so volume times g, density of water times volume of liquid times g. So, if you ignore the cone shape the this region.

So, the total height, the total height is h then this is a pi r square h pi r square is the area times the height h is the total volume times rho w times g. So, if you equate the upward force and downward force then you have a 2 pi r T s cos theta equals to pi r square h rho w g, sorry. So, if you simplify this you get at equilibrium you get h is equals to 2 T s cos theta divided by r gamma w. So, this is the equation for capillary height which is equals to 2 T s cos theta by r gamma w. This can be written in terms of pressure as also. So, if you take gamma w the other side this is the h gamma w which is a pressure is equals to 2 T s cos theta by r. So, this pressure is pressure across the interface. So, this is again you can write this as u ce minus u cx is equals to 2 T s cos theta by r.

So, in this particular case the ce is concave side is air. So, convex side is water. Concave side if you see a has pressure atmospheric pressure that may be 101.325 kilo Pascal that means (Refer Time: 40:36) given temperature. So, water has pressure less than the air pressure if you consider the absolute pressure, if you consider the gauge pressure then air pressure is 0 and water pressure is negative.

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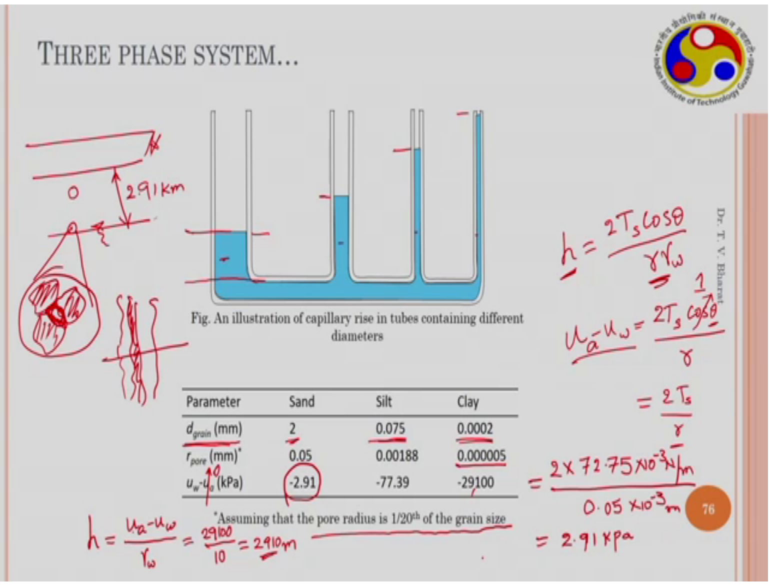

THREE PHASE SYSTEM…

Fig. An illustration of capillary rise in tubes containing different diameters

Parameter	Sand	Silt	Clay
d_{grain} (mm)	2	0.075	0.0002
r_{pore} (mm)*	0.05	0.00188	0.000005
u_w-u_a (kPa)	-2.91	-77.39	-29100

*Assuming that the pore radius is 1/20th of the grain size

$$h = \frac{2T_s \cos\theta}{r\gamma_w}$$

$$u_a - u_w = \frac{2T_s \cos\theta}{r}$$

$$= \frac{2T_s}{r}$$

$$= \frac{2 \times 72.75 \times 10^{-3}\,N/m}{0.05 \times 10^{-3}\,m}$$

$$= 2.91\,kPa$$

$$h = \frac{u_a - u_w}{\gamma_w} = \frac{29100}{10} = 2910\,m$$

So, therefore, as from this equation we see that the capillary height is 2 T s cos theta by r, if you change the radius of the capillary the height capillary height changes for a given fluid, at a given temperature and for a given material if you consider glass. So, then the contact angle would be same. So, therefore, for this particular experiment where you have glass tubes of different diameter are considered, so then the capillary height changes with change in the radius. These are particularly importance to us because when you have a pore structure. So, this is a soil sample with a soil sample this is a pore space these are solid soil solids and this is pore space.

The pore space is again very small and when this pore space comes in contact with water then water raises. So, when you have a ground surface and you have you may have water table somewhere here ground water table. So, then at any given point if you consider or at the interface if you consider this may have this kind of a structure pore structure. So, because of this pore small pore structure or pore size you have here it the pore, pores can act as a capillaries and the water can raise just like u c here.

So, let us consider one ideal example where the grain diameter, diameter of the grain soil grain is a varied 2 mm is considered for sand and for silt it is considered 75 micron. So, this is 0.075 mm and the clay size is considered as 0.2 micron micrometer. Generally less than 2 microns are clays so here therefore; 0.2 micron is considered fine clay size. So, then pore radius is considered to be one-tenth of the one-tenth of the radius of the grain. So, here grain diameter is 2 then radius of the grain is assumed to be spherical then it is 1. So, here the pore radius is assumed to be one-twentieth of the grain size. So, you have a pore radius as 0.05 mm and for silt this is the value and for clay this is the value. So, I am using this equation when you sorry this is here you have gamma w.

So, using this particular expression when you write in terms of u u a minus u w this is a simply 2 T s cos theta by r. If you consider theta to be equal to 0 then cos theta is equals to 1 that is perfect wetting. So, when perfect wetting takes place we are assuming that perfect wetting takes place between soil grains and water then this equation simplifies to 2 T s by simply r. So, when the r is known one can estimate what is the u a minus u w that is a pore water pressure drop across the interface.

So, here for this particular case this is a 2 times 72.75 millinewton. So, this is a 10 power minus 3 Newton per meter divided by r, r here is 0.05 millimeter. So, this is 10 power minus 3 meter. So, in this particular case this value comes out to be 2.91 negative. So, if you consider sorry u w minus u a is negative, but u a minus u w is positive 2.91 kilopascal. So, if you consider gauge pressure that means, u a is 0, so then there is a negative pore water pressure that builds up within water in the pore water just like the water above this level pressure is negative here. So, here in the pore water the pressure is negative u w is negative.

So, the negative pore water pressure is how much which is a 2.91 kilo Pascal in case of sand and which is about 77.39 kilo Pascal for silt, for clays this about 30 mPa, 30 mega

Pascal negative pressure that is built up in water. So, other way around it means that in terms of h if I express this is nothing, but this u a minus u w minus divided by gamma w is equals to this value divided by 10 should give you value in meters for this particular case of clay this is a 29100. So, that is 2910 meters, that is nearly 2.9 kilometers. So, water above this level should exists for 2.91 meters sorry kilometers, kilometers. So, that is a theoretical value you got 2.91 kilometers.

But practically it would not happen because of the low hydraulic conductivity of clay etcetera that we will discuss when we discuss the flow issues. So, eventually theoretically you can get a value of 2.9 kilometers above the groundwater table you can have capillary rise provided you have a clay particles can be the play pore structure can be idealized as simply the capillary tubes of diameter of radius of this much then you will have 2.91 kilometers. So, that means if you have 2.91 kilometer long tube height tube then water will be filled in. So, the pore structures if you consider. So, these are the pore structures. So, in between you will have soil grains. So, this is the, this is the idealization. So, this is at any given cross section. So, this is the pore structure. So, this can be this is approximated as simply diameter, diameter equals to this much.

So, the essentially the point important point here need to be considered is that negative pore water pressure is developed in the soil system due to the surface tension. So, this negative pore water pressure is one of the important state variables of unsaturated soil mechanics that needs to be measured and we will discuss more in the next lecture.

Thank you.