Unsaturated Soil Mechanics
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Week - 02
Lecture - 04
Equilibrium between Air and water Phase

Hello everyone. We have been discussing the vapour pressure and the effect of salt content in water, the effect of salt presents in water on vapour pressure we have briefly seen towards the end of the previous lecture. And as we understand that the effect of salts in water on vapour pressure the lower, it lowers the humidity when you have salts in pure water when you add sodium chloride when you prepare a sodium chloride solution, when it compared to the pure water the vapour pressure drops because of which the relative humidity decreases.

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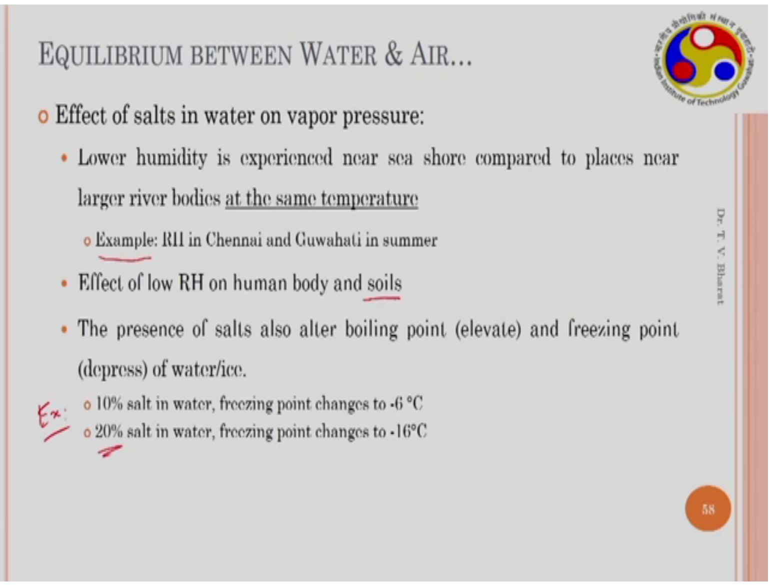

EQUILIBRIUM BETWEEN WATER & AIR...

- Effect of salts in water on vapor pressure:
  - Lower humidity is experienced near sea shore compared to places near larger river bodies at the same temperature
    - Example: RH in Chennai and Guwahati in summer
  - Effect of low RH on human body and soils
  - The presence of salts also alter boiling point (elevate) and freezing point (depress) of water/ice.
    - 10% salt in water, freezing point changes to -6 °C
    - 20% salt in water, freezing point changes to -16°C

Ex:

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So therefore, lower humidity is experienced near sea shore compared to places near the larger river bodies at the same temperature. So, here the example is given. If you compare the relative humidity in Chennai and Guwahati in summer, but it should be compared at the same temperatures. Chennai the temperatures could be as high as 45 degrees, 46 47 degree Celsius, but in Guwahati you will not experience more than 40 degree Celsius. So, therefore, a direct comparison may not be possible. But at the same temperature if you compare Guwahati which is the places where you are next to the Brahmaputra river huge large water body you experience more humid climate.

As we have seen in the previous lecture when the relative humidity decreases you experience drawing of your body because the sweat is taken by the atmosphere there is a mass transformation that takes place and the sweat is taken by the atmosphere when the humidities are low, and as it has enriched the saturated vapor pressure it can take more and more sweat and you feel dry. So, you feel thirsty very often, when you are sitting in air conditioned room or in dry climates on other hand if you are in very humid places.

So, the body sweat is not taken with atmosphere because of which the body temperatures increase. If the sweat is taken by the atmosphere due to the latent heat mechanism the body temperature cools down, but when that water is the sweat is not taken by the atmosphere in the body temperature keeps increasing that is dangerous, the body temperature increases with the atmospheric temperature that is dangerous. So, similarly the soils experience such kind of variations with the changes in the humidity, in humidity and temperature environments as our body experiences.

So, if you keep a soil mass outside in outside in the atmosphere you would see that the moist air is absorbed by the atmosphere or evaporation takes place the evaporation takes place until there is a chemical equilibrium between soil pore and atmosphere. So, the chemical potential of the soil pore should be equal to the chemical potential of the atmosphere, until then the there is a mass transformation that takes place mass transfer that takes place.

So, this is a very much important which we often experience with simple test also there is shrinkage limit test. In shrinkage limit test we evaporate the water until the volume stops changing then we determine the shrinkage element. So, these shrinkage limit are volumetric shrinkage behavior that is volume of soil sample to the mass of soil sample. It again varies, so the atmospheric conditions.

So, another important point is that the presence of salt also alter the boiling point and freezing point of water arise. So, for example, it is an example this may not be the exact values, but if you add 10 percent of salt in water the freezing point can change to minus 6 degree Celsius. So, if you add 20 percent salt the freezing point can change to minus 16 degree Celsius this is one of the important mechanism why we use salts in a cold countries to melt the ice. When the ice is deposited near your apartment we generally put I rock salt. So that which has a good concentration of NaCl and because of this salt tradition the freezing point changes. So therefore, ice melts, ice melt down.

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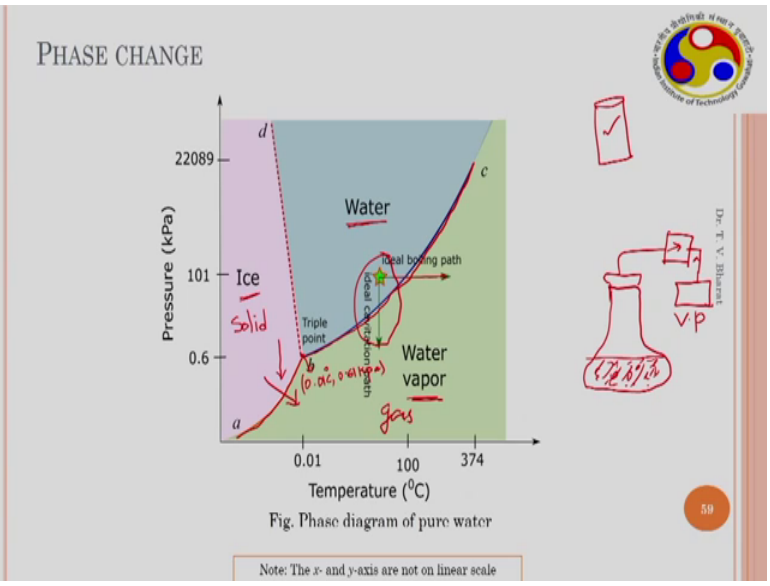

PHASE CHANGE

Fig. Phase diagram of pure water

Note: The x- and y-axis are not on linear scale

To understand the phase changes often you see that there is a change in the phase water becomes vapor and water can also become mice and so there is a phase transformation that often takes place due to the changes in the environmental conditions. So, the water can exist in 3 phases one has a liquid, another one as gaseous, phase another one is solid phase. So, for than at different temperatures change in the temperature or pressure can cause the change in the phase, phase transformation takes place.

For example, this for pure water when the temperature increases you have a water at this particular temperature when the temperature increases it follows ideal boiling path and it becomes vapor at that particular temperature 100 degree Celsius. So, it becomes at the atmospheric conditions like one not 1.325 kilopascal its absolute pressure then when your boiling when at the temperature reaches to 100 degree Celsius it boils and water becomes vapor.

Water also becomes vapor at a constant temperature, but when the pressure is decreased which is called cavitation. So, often you see when you have a beaker. So, beaker of water this is water. So, this is connected to a pipe pump. Pump exerts vacuum it connected to a vacuum pump vacuum pump, when you operate this so the pressure inside drops, so at the 20 degree Celsius itself you see that lot of bubbles would form and water starts to boil. So, this is cavitation. So, due to decrease in the pressure the water bubbles the vapor forms within the water which is called cavitation.

So, this is vaporization curve, this particular curve is a vaporization curve. So, if water crosses this particular curve under given pressure and temperature conditions it becomes vapor. And this is called sublimation curve where the ice can become directly vapor without going to the water phase. So, the solid phase can be change to the gas phase directly through this phase transformation by changing pressure or temperature this is often seen in soil mechanics, where you want to recover when you want to test a soil sample soil specimen without altering its structure without altering its particle structure, then what we do is we freeze dry the soil sample.

So, that the water directly become size and after that we follow the sublimation condition where the pressure is dropped under constant temperature. So, then directly it forms vapor and vapor escapes to the atmosphere you have the soil sample without changing its pore fracture. Otherwise if you follow this path where the temperature is dropped when you freeze dry. And then when your temperature is dropped then it becomes water are for example, otherwise if you have a saturated soil sample if you want to test its pore structure using some (Refer Time: 09:00) intrusion (Refer Time: 09:01) are scanning electron microscope to observe the particle structure or pore structure, then it is not possible to dry the soil sample because drying the soil sample alters the pore structure.

So, therefore, often we freeze right and follow the sublimation cross the sublimation curve and then make it to gas, and when the gas is skips you have the soil without altering its pore structure there is o v tested. So, this is a very important point and this particular point is equal to triple point where water can exist in 3 different phases. This point exists at 0.01 degree Celsius and pressure is 0.61 kilopascal, 61 kilopascal, and this is triple point or 273.15 Kelvin and 0.61 kilopascal.

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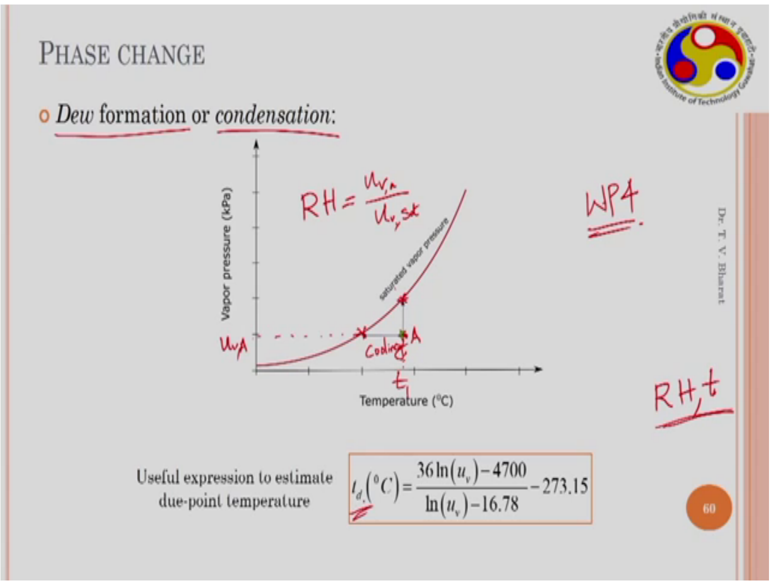

PHASE CHANGE

- *Dew* formation or *condensation:*

Useful expression to estimate due-point temperature

$$t_d(^{\circ}C) = \frac{36\ln(u_v) - 4700}{\ln(u_v) - 16.78} - 273.15$$

So, this phase change phenomena is very important in soil mechanics especially unsaturated soil mechanics are many experimental facilities which determine the determine or control the state variable such as suction or directly or indirectly depend on these principles, the directly use is principles.

So, let us look at the dew formation or condensation. So, this particular phase transformation you are looking at. So, one of the important state variable measurement that is matrix suction or the suction measurement instrument called WP for or dew point potentiometer is based on this particular principle. If you take if you consider the present atmosphere this may have a particular temperature and particular vapor pressure. So, the particular vapor pressure maybe. So, that is u v this is point A.

So therefore, they should be it should be u v A. So, this is at particular temperature t 1. So, t 1 the relative humidity RH is equals to nothing, but u v A divided by u v, u v sat saturated vapor pressure. So, at this particular point this is a saturated vapor pressure at this particular temperature this is a saturated vapor pressure and this is the vapor pressure so RH is the ratio of relative humidity to the saturated vapor pressure: Vapor pressure at this particular point to the saturated vapor pressure at this particular temperature that is RH.

So, when there is a sudden decrease in the temperature keeping the vapor pressure constant then it follows a cooling path it is a it follows a cooling path, then you see that there is a formation of dew or another path is possible where you increase the temperature, you keeping the constant temperature you increase the vapor pressure then it reaches the saturated vapor pressure. So, either way it is possible to achieve dew point this is often encountered. So, where we see that there is sudden decrease in the temperature that causes of formation of dew or condensation and rain drops would fall that is nothing but the atmospheric vapor gets deposited as dew.

So, these are very useful expression to estimate the dew point temperature knowing the vapor pressure knowing the RH, R knowing the relative humidity. For example, RH is given, relative humidity is given and temperature is given. So, using this you can estimate what is the dew point temperature. For example, say RH is equal to 90 percent and temperature is 40 degree Celsius. So, then RH is known you know the saturated vapor pressure knowing the temperature from the Tetens equation. So, then you can estimate what is the u v. So, knowing the u v if you substitute you will know; what is the dew point, dew point temperature.

So, when the temperature drops to dew point temperature are below dew point temperature you see that there is a formation of dew.

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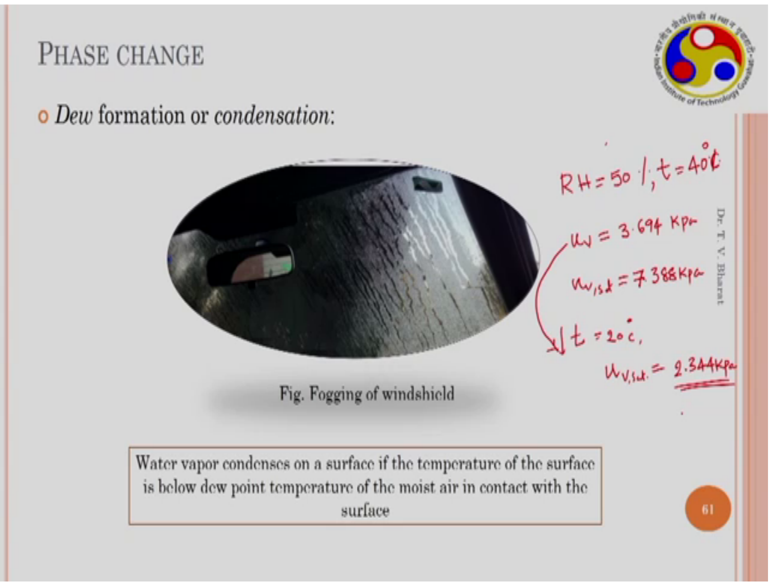

PHASE CHANGE

- *Dew* formation or *condensation:*

Fig. Fogging of windshield

Water vapor condenses on a surface if the temperature of the surface is below dew point temperature of the moist air in contact with the surface

$RH = 50\,\%, \; t = 40^\circ C$

$u_v = 3.694\ kPa$

$u_{v,sat} = 7.388\ kPa$

$\downarrow t = 20^\circ C,$

$u_{v,sat} = \underline{\underline{2.344\ kPa}}$

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So, this is often we encounter on car windshield car. Windshields often get fogged, fogged up and we switch on the defogger for clearing the condensation. So, this is because suddenly if the temperature outside it changes then it temperature drops then you say that the condensation that happens inside the car, car windshield and a vice versa it may happen. So, water vapor condenses on a surface if the temperature of the surface is below dew point temperature of the moist air in contact with the surface. Consider this example where you have say RH is equals to 50 percent, RH equals to 50 percent and temperature is 40 percent 40 degrees, 40 degree Celsius, temperature is 40 degree Celsius then the u v is 3.694 kilopascal and u v sat from the Tetens expression is 7.388 kilopascal.

Suddenly if the temperature drops to 20 degree Celsius then the u v sat for 20 degree Celsius is 2.344 kilopascal, which is less than the current u v. So, therefore, it causes fogging. So, your windshield inside gets fogged.

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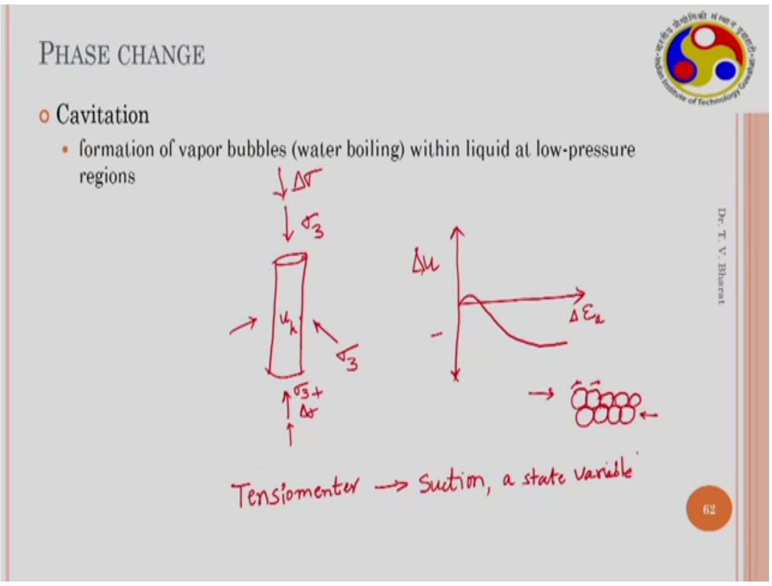

PHASE CHANGE

- Cavitation
  - formation of vapor bubbles (water boiling) within liquid at low-pressure regions

Tensiomenter → Suction, a state variable

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Similarly, there is another phenomenon that we often observe is cavitation. I just mention that this is a cavitation path. Cavitation happens cavitation is nothing, but formation of vapor bubbles within liquid at low pressure regions. So, as I just explain when you have a water that is pressure is reduced by applying vacuum then you see that the water boils are vapor forms vapor bubbles form. The same thing we observe in traxial set up where we have a soil sample where we apply some all round pressure all round pressure sigma 3 and we apply additional deviated stress sigma delta sigma.

So, when you are conducting a drain test consolidated drain test the pore water pressure is generally excess pore water pressure generally gets dissipated and when you are not maintaining any particular pore water pressure inside then pore water pressure maybe zero. And then in that particular case you see that sample dilates if the initial compaction state is such that the soil is a over consolidated clay or densely compacted sand and soil dilates. However, if it is a consolidated undrained test where the drainage is not allowed water is not allowed to go in inside the sample or outside the outside to the sample. So, in that particular case the changes in the pore water pressure if you observe for the over consolidated clays, with axial strain change in axial strain you see that there is change in the pore water pressure which is actually there is a negative pore water pressure that develops.

So, this in particular in particularly for densely compact sand or over consolidated clays where the negative pore water pressure develop because when you consider particles to be of spherical in nature and particles are compacted well because this is densely compacted sand. When you are shearing these top layer, so particles have to climb up the bottom particles because of which it wants to suck the pore water it wants to suck the water to fill in the gaps. But however, the water movement is not allowed therefore, there is a negative pressure that develops, because of the development of the negative pore water pressure which causes formation of bubbles or cavities full form or cavitation occurs.

So, to avoid that often we maintain a particular back pressure a back pressure equal to some is often maintained if back pressure of 100 kilopascal is maintained then cavitation would not occur, because when the negative pore water pressure develops which compensates the back pressure compensate that form the whatever the negative pore water pressure that develops in the sample. So, these concepts are very much relevant to even basic soil mechanics, but you would see many more application of the cavitation phenomenon and unsaturated soil mechanics especially in the application of tensiometer. Tensiometer is one set up for measurement of suction a state variable in unsaturated soil mechanics.

As we have seen that water can exists as vapor in air phase, so in that particular case the density of air changes earlier we have estimated the density of air using ideal gas flow.

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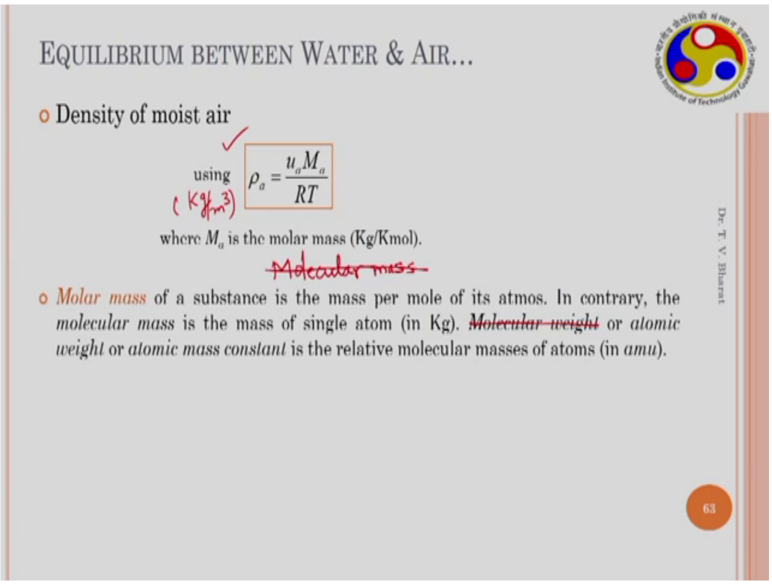

EQUILIBRIUM BETWEEN WATER & AIR…

- Density of moist air ✓

using $\boxed{\rho_a = \frac{u_a M_a}{RT}}$ (Kg/m^3)

where M_a is the molar mass (Kg/Kmol).

~~Molecular mass~~

- *Molar mass* of a substance is the mass per mole of its atmos. In contrary, the *molecular mass* is the mass of single atom (in Kg). ~~*Molecular weight*~~ or *atomic weight* or *atomic mass constant* is the relative molecular masses of atoms (in *amu*).

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So, what happens to the density of air even vapor is present? So, as we have used this particular relationship earlier using the ideal gas flow where the density of air is equals to the u a; the atmospheric pressure times the molar mass of air divided by gas constant times the temperature in Kelvin. Here I said the molar mass has units of gram per mol or kg per kilo mol. So, in many text books they often their using their defining in many text books their defining ma as molecular mass, where defining ma as molecular mass not molar mass so, but then molar mass is of a substance is the mass per mol of its atoms. So, it has units of kg per kilo mol.

In contrary the molecular mass is the mass of single atom it should be measured in kilo grams or if you want to use molecular weight or atomic weight or atomic mass constant is the relative molecular masses of atom which is measured in atomic mass unit it does not have it is a dimensionless parameter. So, therefore, uses of molecular weight are molecular mass is wrong because first of all its not dimensionally correct, it should be molar mass only because the when u a is substitute in kilo kilopascal or kilo Newton per meter square then molar mass in kg per kilo mol or in Joule per Kelvin mol and temperature in Kelvin then this as units of kg per meter cube. This we have seen in the previous lecture, one of the previous lecture. Please make a note of this.

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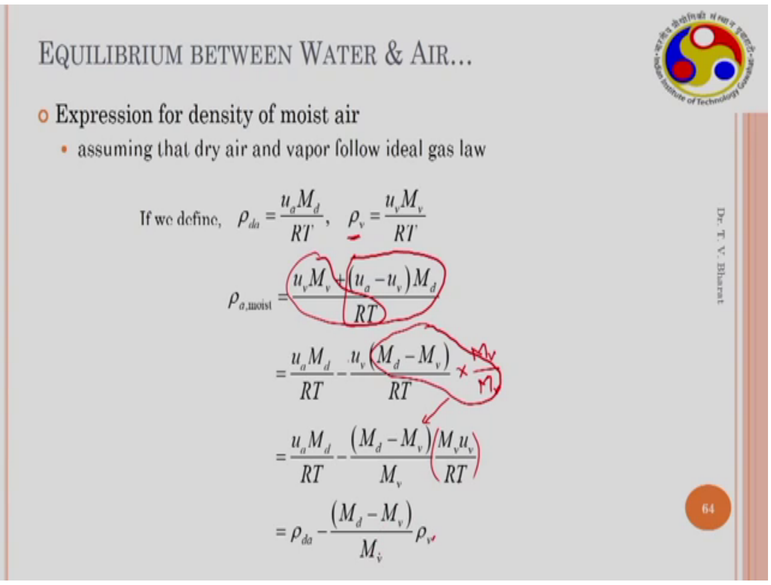

EQUILIBRIUM BETWEEN WATER & AIR…

- Expression for density of moist air
  - assuming that dry air and vapor follow ideal gas law

If we define, $\rho_{da} = \frac{u_a M_d}{RT}$, $\rho_v = \frac{u_v M_v}{RT}$

$$\rho_{a,moist} = \frac{u_v M_v + (u_a - u_v) M_d}{RT}$$

$$= \frac{u_a M_d}{RT} - \frac{u_v (M_d - M_v)}{RT} \times \frac{M_v}{M_v}$$

$$= \frac{u_a M_d}{RT} - \frac{(M_d - M_v)}{M_v} \left(\frac{M_v u_v}{RT}\right)$$

$$= \rho_{da} - \frac{(M_d - M_v)}{M_v} \rho_v$$

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So, let us derive the expression for density of moist air. Assume that the dry air and vapor follow ideal gas law when we define rho d a that is a dry atmospheric air that is equals to u a atmospheric pressure times M d molar mass of dry air divided by RT and rho v is for vapor density is equals to vapor pressure times molar mass of vapor divided by RT.

So, the moist air density is equals to this is a vapor density this is density of dry air that is present in the moist air. So, you are separately considering density of vapor, density of dry air that is present in the moist air. Then this is the expression you get here the density of dry air that is present in the moist air can be written as atmospheric pressure minus the vapor pressure. So, then this expression can be simplified to this where you can write u a times M d by RT minus u v times M d minus M v divided by RT. So, this can be further simplified by multiplying with M v in the denominator and numerator and denominator then this M d minus M v can be written as this and this M v times u v divided by RT can be written as rho v further. So, this is simplified to rho da the dry atmospheric air minus M d minus M v by M v times rho v.

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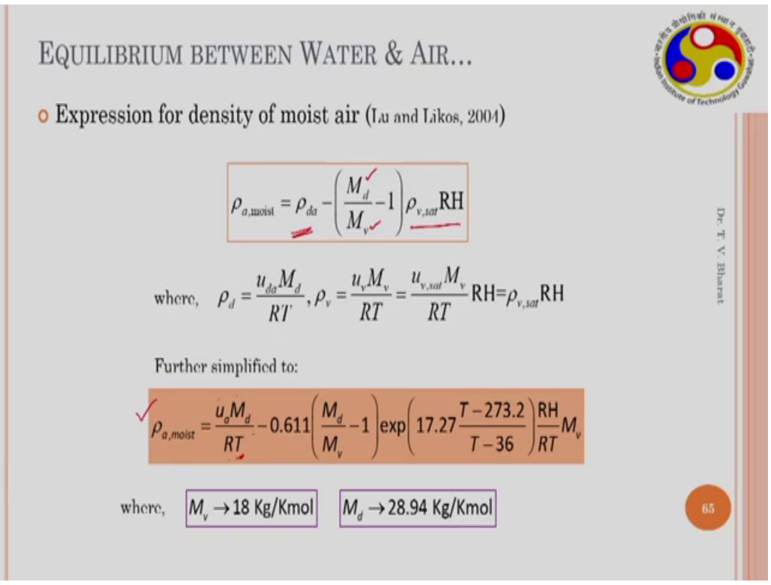

EQUILIBRIUM BETWEEN WATER & AIR…

- Expression for density of moist air (Lu and Likos, 2004)

$$\boxed{\rho_{a,moist} = \rho_{da} - \left(\frac{M_d}{M_v} - 1\right)\rho_{v,sat}\mathrm{RH}}$$

where, $\rho_d = \frac{u_{da}M_d}{RT}, \rho_v = \frac{u_v M_v}{RT} = \frac{u_{v,sat}M_v}{RT}\mathrm{RH} = \rho_{v,sat}\mathrm{RH}$

Further simplified to:

$$\rho_{a,moist} = \frac{u_a M_d}{RT} - 0.611\left(\frac{M_d}{M_v} - 1\right)\exp\left(17.27\frac{T-273.2}{T-36}\right)\frac{\mathrm{RH}}{RT}M_v$$

where, $\boxed{M_v \rightarrow 18\ \mathrm{Kg/Kmol}}$ $\boxed{M_d \rightarrow 28.94\ \mathrm{Kg/Kmol}}$

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So, therefore, the expression for density of moist air is rho da density of dry air dry atmospheric air that means, if you have completely dry air there is no vapor that is present then the density is this much minus molar mass of dry air divided by molar mass of vapor. Molar mass of dry air is 28.94 kg per kilo mol we have used earlier and molar mass of vapor is 18 kg per kilo mol. So, divided M d by M v minus 1 times rho v sat times RH the rho v is written as rho v sat times RH because from the relative humidity definition we can write rho v sat times RH.

So, where different variables are defined here, so it can be further simplified because rho v sat can be obtained from the Teten's expression. So, from that we can write what is a rho a moist where you have u a atmospheric pressure and molar mass of dry air and RT. So, this is for a given condition this is atmospheric condition that is one not 1.325 kilopascal, M d is a 28.94 kg per kilo mol, R is 8.314 Joule per Kelvin mol and T temperature is may be standard temperature say 20 degrees then it is 273.15 plus temperature 20 degree Celsius in Kelvin. Minus 0.611 this is again 28.94 divided by 18 minus 1 exponential of 17.27 times temperature in Kelvin temperature in Kelvin RH would is required and RT we know the substitution and M v is know which is the 18.

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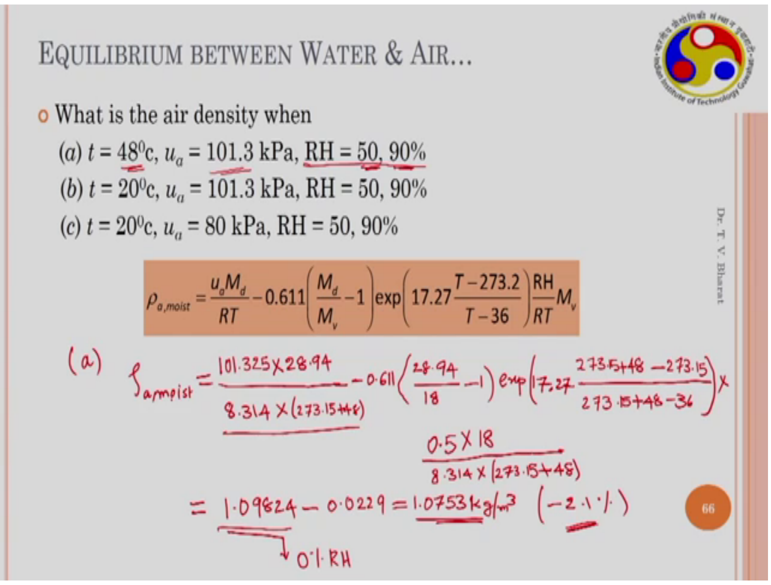

EQUILIBRIUM BETWEEN WATER & AIR…

- What is the air density when
  - (a) $t = 48^0\text{c}$, $u_a = 101.3$ kPa, RH = 50, 90%
  - (b) $t = 20^0\text{c}$, $u_a = 101.3$ kPa, RH = 50, 90%
  - (c) $t = 20^0\text{c}$, $u_a = 80$ kPa, RH = 50, 90%

$$\rho_{a,moist} = \frac{u_a M_d}{RT} - 0.611\left(\frac{M_d}{M_v} - 1\right)\exp\left(17.27\frac{T-273.2}{T-36}\right)\frac{RH}{RT}M_v$$

(a)

$$\rho_{a,moist} = \frac{101.325 \times 28.94}{8.314 \times (273.15+48)} - 0.611\left(\frac{28.94}{18} - 1\right)\exp\left(17.27\frac{273.15+48-273.15}{273.15+48-36}\right) \times \frac{0.5 \times 18}{8.314 \times (273.15+48)}$$

$$= \underbrace{1.09824}_{0\%\ RH} - 0.0229 = 1.0753\ \text{kg/m}^3 \quad (-2.1\%)$$

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So, from this one can estimate how the moist density changes with temperature, the little humidity and atmospheric pressure. First let us understand how the air density changes with relative humidity when temperature is maintained 48 degrees and atmospheric pressure is 101.3 kilopascal. So, the relative humidity 50 degrees may simulate the condition of dry climate may be near Hyderabad or some places where you have a dry climate. And 90 degrees may simulate a condition of Chennai closed to Chennai environment where temperature can go to 40 degrees and relative humidity may be 90, to 90 percent. And this is near mean sea level, so this is a pressure is 101.3 kilopascal.

So, in the first case rho a moist is a u a is 101.325 kilopascal types times M d is a 28.94 kg per kilo mol divided by R is 8.314 Joule per kg Joule per Kelvin mol into temperature 298 Kelvin minus 0.611 times M d 28.94 by 18 minus 1 into exponential of 17.27 and temperature is temperature is 273.15 plus 48. So, here the temperature is temperature is 273.15 plus 48 minus 273.15 divided by same again 273.15 plus 48 minus 36 bracket close times RH relative humidity is 0.5 here it should be substitute in decimals not percentage. So, 0.5 times M v is vapor 18 divided by RT is 8.314 times temperature again 273.15 plus 48. So, which gives a value of 1.09824 this is a density of dry air minus 0.0229 which is 1.0753 kg per meter cube.

So, eventually there is a change of, there is a decrease of 2.1 percentage of density. So, the density of moist air in this particular case with relative humidity of 50 percent is 2.1

percent lighter than the dry air. So, with the increase in moist air this is this condition is this particular value is corresponding to 0 percent RH, completely dry air. So, if you substitute for RH equals to 0 then you get density of dry air.

So, therefore, with increase in the RH with increase in the RH the density of moist air decreased. Though this value is very small negligible value 2 percent, but then this place an important role in the flow of air from one place to another place due to changes in the RH similarly changes in the temperature also we will see very soon. So, if I erase this whole part here, so corresponding to 50 degree, 50 percent of RH when RH is 50 percent, when RH is 90 percent you would see that in this particular case the density of moist air is 1.0753 kg per meter cube and with 90 percent this is 1.057.

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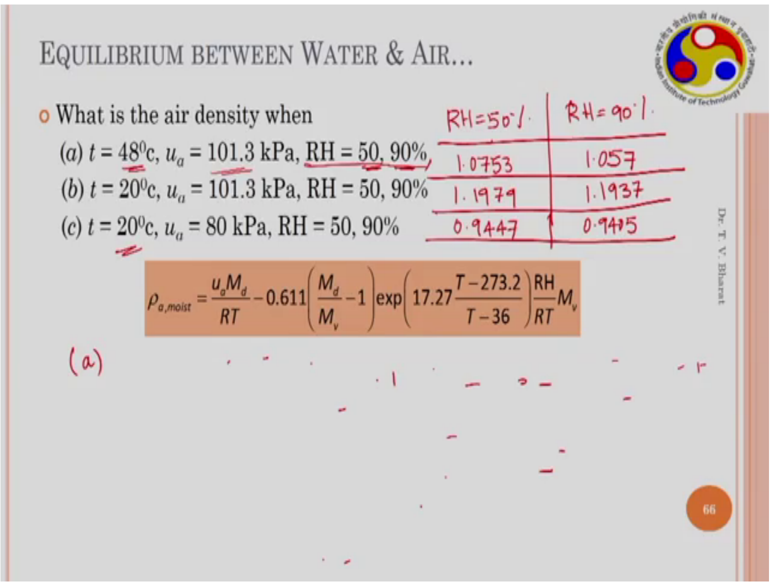

EQUILIBRIUM BETWEEN WATER & AIR…

- What is the air density when
  - (a) $t = 48^0\text{c}$, $u_a = 101.3$ kPa, RH = 50, 90%,
  - (b) $t = 20^0\text{c}$, $u_a = 101.3$ kPa, RH = 50, 90%
  - (c) $t = 20^0\text{c}$, $u_a = 80$ kPa, RH = 50, 90%

RH=50%	RH=90%
1.0753	1.057
1.1979	1.1937
0.9447	0.9415

$$\rho_{a,moist} = \frac{u_a M_d}{RT} - 0.611\left(\frac{M_d}{M_v} - 1\right)\exp\left(17.27\frac{T-273.2}{T-36}\right)\frac{\text{RH}}{RT}M_v$$

(a)

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So, the dry air is 1.09824 and 50 percent is 1.0753, 90 percent further decreases to 1.057. So, with increase decrease in the temperature from 48 to 20 degrees, so then the RH becomes 1.1979 and RH is 50 percent this value is a 1.1937 when RH is 90 percent. And when the temperature is maintained constant at 20 degrees, but atmospheric pressure is decreased to 8 kilopascal then for RH equal to 50 percent. So, this value is 0.944 7 kg per meter cube and at 90 percent RH this is 0.9405 kg per meter cube. So, from this you can see that at a constant temperature and pressure with increase in RH the density of moist air decreases because more and more moist air would go into the atmosphere as the molar mass of vapor is smaller than the molar mass of dry air more and more vapor goes into the atmospheric air then it decreases the density of air.

Similarly, when temperature decreases, temperature decreases from 48 degrees to 20 degrees you would see that there is increase in the density or other way around when the temperature increases which decreases the density of air under the same RH. And atmospheric pressure in contrary at the same temperature and then RH condition when the atmospheric pressure is decreased the density of air decreases.

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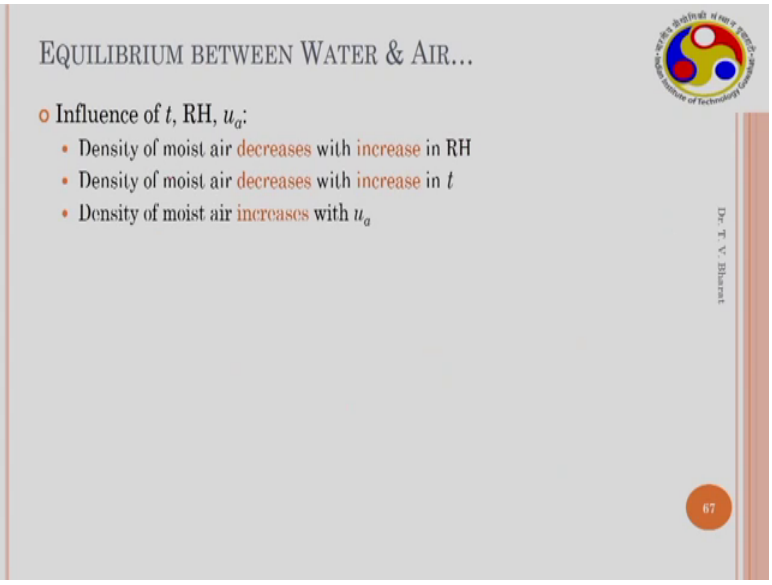

EQUILIBRIUM BETWEEN WATER & AIR…

- Influence of t, RH, u_a:
  - Density of moist air decreases with increase in RH
  - Density of moist air decreases with increase in t
  - Density of moist air increases with u_a

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So therefore, this is in summary the density of moist air decreases with increase in RH when temperature and atmospheric pressure are constant. Similarly the density of moist air decreases with increase in temperature when RH and atmospheric pressure constant and density of moist air increases with increase in atmospheric pressure and when RH and temperature are constant.

Thank you.