Unsaturated Soil Mechanics
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Week-11
Lecture – 31
Concept of "Suction Stress" - III

Hello everyone, today let us discuss more details about Suction Stress characteristic curve. And how to determine the Suction Stress characteristic curve and further to evaluate the effective stress of unsaturated soil from suction controlled direct shear test data and suction control tri axial test data. Suction stress characteristic curve is a characteristic curve represents the variation of suction stress with whether matric suction or normalized volumetric water content. The suction stress consumes energy that is required for the change in energy of soil water due to several physical chemical factors and capillary effects are consumed in to suction stress, suction stress.

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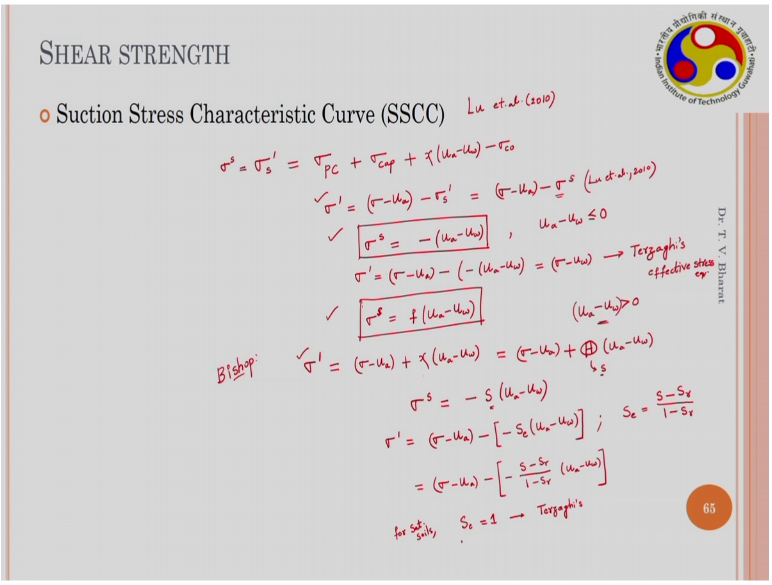

So, the suction stress is the suction stress equation as we have seen is a sigma s dash is equals to it consists of stresses due to physical chemical forces plus the stresses due to capillary forces plus. So, due to the effect of surface tension and this is a cohesion intercept which we use at saturated state. So, this we started writing it as so, this one the effective stress can be written as sigma dash is equals to sigma minus u a that is net normal stress minus sigma s dash. In further works of Lu et al this suction stress is represented as sigma s as suffix sigma s in this manner.

So, this can be written as sigma minus u a minus sigma s. So, accordingly the sigma s is the suction stress is equals to minus of u a minus u w. If u a minus u w is less than or equal to 0 which means that when the pore water pressure is positive; so, negative suction means it is a pore water pressure is positive. So, that is normal the consolidation test or any other test where the pore water pressures are more than 0. Any consolidation or shear strength tests where the pore water pressure is positive. So, in that particular case the suction stress is equals to the pore water pressure itself.

So, therefore, when you substitute this 1 here sigma s here so, this sigma dash becomes sigma minus u a minus minus u a minus u w. So, therefore, this is equals to sigma minus u w. So, this is the effective stress equation given by Terzaghi itself. So, this boils down to the Terzaghi's effective stress equation. So, this work is from Lu et al in 2008 2010 sorry. So, in the previous work of Lu and Lycos they have modified or expanded the Terzaghi's effective stress equation by incorporating physical chemical forces. And if it is unsaturated soil then the all the capillary forces are also included. So however, in their further work they again approximated their suction stress characteristic curve equation into a simplified manner where this boils down to Terzaghi effective stress equation for saturated soils.

So, for unsaturated soils where u a minus u w is greater than 0 then sigma dash is a function of u a minus u w this we called suction stress characteristic curve, sorry sigma s is function of u a minus u w. So, this is for saturated soils this is for saturated soils where the pore water pressure is positive. And this is for unsaturated soils where the pore water pressure is negative. So, therefore, the suction is above 0 you have suction in the soil. So, in the Bishop's approach as we have said earlier Bishop in 1950 in 1950's he has given the effective stress equation that is sigma dash is equals to, sigma dash is equals to sigma minus u a that is net normal stress plus xi into u a minus u w. So, here he says that xi varies in the same manner as normalized volumetric water content. He often assumes the value of xi to be normalized volumetric water content or this could be degree of saturation also.

So, therefore, comparing this particular equation expression with this particular expression given by Lu et al sigma s suction stress becomes simply minus degree of saturation S times u a minus u w. So, therefore, it is an extension of as they have using the degree of saturation here it is an extension of Bishop's approach and expansion of Terzaghi's equation for unsaturated soils. So, therefore, effective stress sigma dash is equals to sigma minus u a minus of minus S e they have introduced to Lu et al introduced a term as c I will explain what it is times u a minus u w. So, instead of degree of saturation if we use a c which is defined as degree of saturation at any given water content minus degree of saturation divided by 1 minus S r.

So, this is defined in this particular manner c is equals to S minus S r by 1 minus S r. So, this is equals to sigma minus u a minus. So, minus of minus so, this is a same as Bishop's expression except that they have used normalized degree of saturation here. So, for saturated soils S c is equals to 1 for saturated soils S e equals to 1 which boils down to Terzaghi's 1 dimensional consolidation equation sorry Terzaghi's effective stress equation. And when I say S equals to 0 this is a dry condition which is sigma dash is equals to sigma minus u a. So, that is equals equivalent to the Bishop's expression for dry soils.

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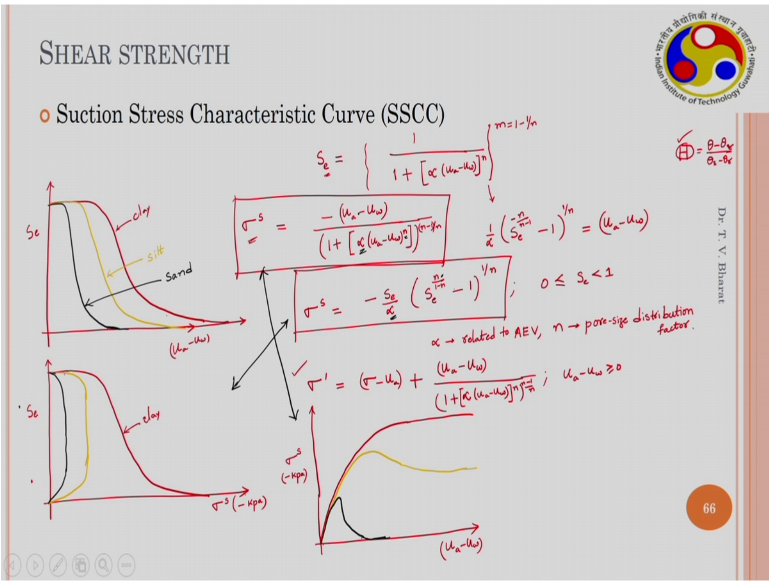

So, here the additional advantage they have given is that they assume that S e is a function of matrix suction and which assumes a form of which assumes a form of (Refer

Time: 09:27) equation. So, the (Refer Time: 09:29) equation which is going to be written as 1 over 1 plus alpha times u a minus u w whole power n whole power m, but they assume m to be dependent on n. So, this is 1 minus 1 by n. So, this is a form they assume to be existing between S e and u a minus u w. So, this S e could be normalized volumetric water content also that is a big theta which is theta minus theta s sorry theta r by theta r s minus theta r which is similar to this.

So, you can assume normalized volumetric water content also instead of S c. So, they assumed this form to be existing then the sigma s can be simplified as minus u a minus u a minus u w minus u a minus u w divided by 1 plus alpha u a minus u w whole power n whole to the power of n minus 1 by n. The expression for suction stress and as a continuous form it can be defined suction stress characteristic curve.

The dependency of suction stress on matrix action can be defined in a continuous manner using this expression. So, this is one expression and other expression is in terms of S e. So, S e here u a minus u w can be written as S e S e power n by n minus 1 and minus minus 1 whole power 1 by n 1 by alpha so, this is u a minus u w. So, when you write u a minus u w in terms of S e this is so, what you get and when you substitute this in the suction stress characteristic curve equation. So, we get minus S e by alpha S e power n by 1 minus n and minus c is taken in here this becomes 1 minus n minus 1 whole power 1 by n where S e varies between 0 and 1.

So, this is another expression in terms of S e. So, if soil water characteristic curve data for any given soil is known under applied under any given stress state either applied mechanical stress or something. So, then the suction stress characteristic curve or the effective stress of the soil can be determined by simply substituting these parameters. Here alpha is the bubbling pressure or is related to, is related to bubbling pressure are Air Entry Value AEV and n a is pore size distribution factor.

So, essentially the SWCC of the soil can be directly utilized to determine the soil suction stress characteristic curve. So, from that we can obtain the effective stress of the soil. So, this is a very useful. So, then you can obtain the effective stress by simply summing up sigma minus u a minus sigma s anyway minus sign you have there. So, this is simply plus u a minus u w divided by 1 plus alpha u a minus u a minus u w whole power n whole power n minus 1 by n.

So, here u a minus u w is greater than or equal to 0. So, sigma dash this is a effective stress equation. So, for several soils when the S c versus u a minus u w is plotted so, this can be plotted as like this for goes to 0. This is for clay soils, this is for silt, this maybe for sand. The Suction Stress Characteristic Curve can be drawn u a minus u w and S e for clay soils. So, this is how the SSCC varies for clay soils for silt soils. As we have seen yesterday the Suction Stress Characteristic Curve decreases and goes to 0 sorry this is sigma s in negative kilo Pascal. Then for sandy soils this goes somewhat like this.

The suction stress characteristic of increases and nearly become constant and then goes to 0 because, the capillary effects are dominant only at some particular value and beyond that that is after the air entry value. After the air entry value the capillary effects are predominant and beyond that again as a degree of saturation goes to 0 approaches 0 the capillary effects are absent and suction stress approaches 0 for sand soils and silts. Similarly if this is plotted in terms of u a minus u w and sigma s; so, sigma s sorry suction stress on y axis and u a minus u w on x axis. So, suction stress increases and becomes nearly constant for clay soils and which increases and decreases beyond certain suction value for silt soils, for sandy soils this increases and decreases and goes to 0 at higher section values.

So, this is how the suction stress characteristic curve varies for different soils these are the solutions this is the data obtained from closed form solutions for different by assuming different values of alpha and n. Here the clay soil has very high value of air entry value. So, alpha is accordingly adjusted and some n value is assumed. And similarly silt value the alpha value slightly got changed because the air entries slightly smaller and sand air entry is smaller. So, that is how these three curves are generated on SWCC using (Refer Time: 17:57) equation in using m and n dependent equation.

So, once this is derived and there c versus sigma s is directly obtained by substituting these values into this equation and for any given S c which has c changes from 0 to 1. So, knowing the alpha and then n parameters from SWCC equation. For clay sand silt and sand these parameters are substituted and then sigma s is plotted in this manner. Similarly, this third figure sigma s versus u a minus u w is obtained using this particular equation by again substituting alpha and n parameters into this equation and u a minus u w is varied over a wide suction range and then sigma s is plotted in this manner.

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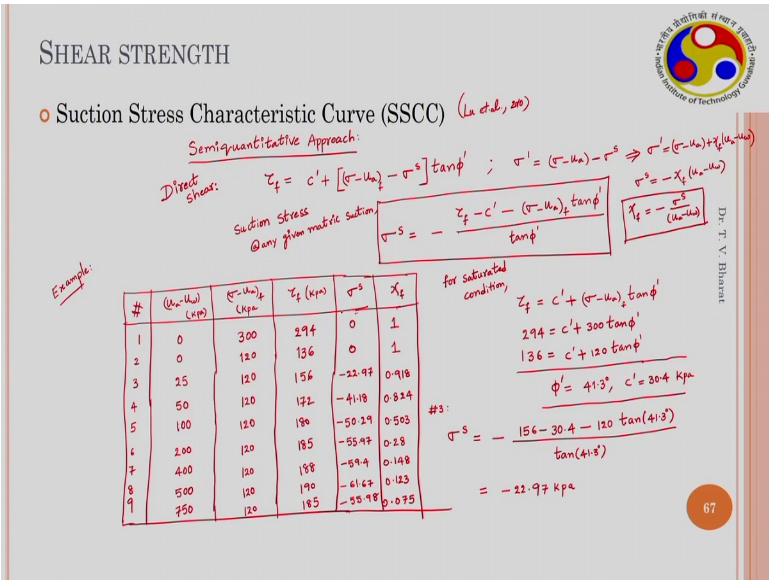

So, they have proposed two approaches for estimating the suction stress characteristic curve. One is a semi quantitative approach. So, the Lu et al in 2010 they have come up with two approaches Semi quantitative Approach. This approach is used when soil water characteristic curve data is not known only the shear strength data from suction controlled direct shear test and suction control tri axial tests are known then this particular approach could be used to derive the effective stress variation within the soil mass the suction shows variation with either suction or volumetric water connected.

So, for the direct shear testing the Mohr coulomb failure criterion is tau f equals to this is for direct shear tau f equals to c dash plus sigma minus u a plus minus sigma s tan phi dash because effective stress is sigma minus u a minus sigma s. So, this is the effective stress the total stress or net normal stress minus the suction stress is the effective stress. So, the Mohr coulomb failure criterion is tau f equals to c dash plus sigma dash tan phi and sigma dash is substituted here then this is the equation.

So, therefore, the suction stress can be obtained by for any given matrix section. At any given matrix suction the suction stress is sigma s is equals to minus tau f minus c dash minus sigma minus u a at failure tan phi dash by tan phi dash. So, this equation can be used for determining the suction stress characteristic curve if the soil water characteristic curve is not known. So, let us solve an example problem. When the suction control direct shear when the shear test data from suction control direct shear tests are obtained on some silty soil this is a generated synthetic data.

So, number of tests performed and the suction value control sigma minus u 8 failure that is net normal stress which is which can be controlled and tau f which is measured. So, here the first number of tests are 9 to saturated two tests are conducted at saturated condition. So, suction varied in this manner in these tests. So, the net normal stress varied in the first two tests which are saturated tests. So, that different shear strength will be obtained. So, from that the from that the strength parameters of saturated soils can be obtained for remaining tests the net normal stress is maintained constant only the matric suction is varied.

So, this test data is used this test data were used earlier for demonstrating the Bishops approach for the prediction of xi f calculation. So, same test data is used here. So, here. So, for saturated tests and soil is saturated for saturated condition. So, tau f equals to C dash plus. So, this is simply sigma, sigma dash or you can write sigma minus u a and suction stress is 0 u as tan phi dash.

So, therefore, solving these two equations 294 is equals to C dash plus 300 tan phi dash and 136 is equals to C dash plus 120 tan phi dash. Solving these two we get phi dash is equals to 41.3 degrees and C dash is equal to 30.4 kilo Pascal. So, knowing these two shear strength parameters so, the effective stress parameters; so, shear strength drain parameters phi dash and C dash. So, we can obtain the suction stress at any given matrix section value. So, the sigma s is equals to minus here if you are given the suction stress characteristic suction stress values. So, this is a 0 here and 0 here and for the third data test three suction stress is minus tau f is 156 minus C dash is 30.4 minus sigma u a is 120 tan 41.3 divided by tan 41.3.

So, this value comes out to be minus 22.97 kilo Pascal. So, we can substitute the data for all the values minus 22.97and for this is minus 41.18 and this is for minus 50.29 minus 55.97 minus 59.4 minus 61.67 minus 55.98. So, these are the suction stress values. And we can also compute for comparison xi f because this equation is quite similar to the Bishops equation. So, the Bishops equation if you compare that is sigma minus u a plus xi f u a minus u w. If you compare these two equations sigma s is nothing, but minus xi f u a minus u w. So, therefore, xi f is equals to minus sigma s by u a minus u w.

So, here u a minus u w is 0 xi f is nothing, but 1 this is a maximum value is 1. So, this is 1 and here. This is a minus 20.97 divided by minus 25 this is 0.918 and similarly if you get all the values 0.824 and 0.503 and 0.28 0.148 0.123 and 0.075. So, this is a xi f data and. In fact, if you compare our earlier calculations where we you estimated the bishops effective stress parameter using the same data and if you compare the xi f data is same. So, therefore, it is not a big modification we have done here using suctions stress characteristic curve only the representation of effective stress is more effective here in this case.

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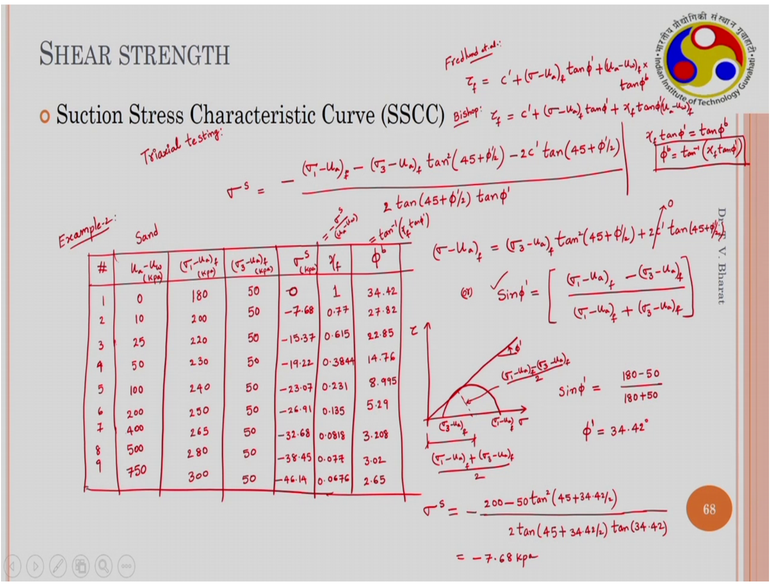

Similarly, in the semi quantitative approach for tri axial testing; if you have a data from suction control tri axial test then sigma s can be represented as minus sigma 1 minus u a at failure minus sigma 3 minus u a at failure tan square 45 plus phi by 2 minus 2 C dash and 45 plus phi dash by 2 divided by 2 tan 45 plus phi dash by two times tan phi minus. So, this is the expression can be used to determine the suction stress if the major principle stress at failure and minus principal stress at failure are the net normal stress in the vertical direction and net normal stress in the horizontal direction or radially it directions are known at failure. And we can determine the suction stress. So, let us solve another example for this. So, the test data so, here also we utilize 9 test data.

So, this is a matrix suction in kilo Pascal. This is a varied in this manner 0 10 25 50 100 200 400 500 750. So, as the matrix action is only the saturated state at the saturated state only one tests are conducted. So, therefore, this must be for sand for sandy soil and sigma 1 minus u a at failure this is 180 these are observed data. So, sigma three is maintained constant. So, we can obtain the sigma s we can calculate the xi f and we can also estimate the phi b. So, that we can compare Bishop's approach and Fredland's approach we with SSCC.

So, now, using the first test data either to obtain the anglophone intersection either you can use this equation. So, here anyways c is 0 because it is sand you can directly use this expression or you can use sin phi minus is sigma 1 minus u a at failure minus sigma 3 minus u at failure by sigma 1 minus u at failure plus sigma 3 minus u a at failure. Because, when you are plotting tau versus sigma and if this is a failure envelope and this is more circulant this is a failure envelope and this is angle of internal friction phi dash and if you take sine. So, this is a sigma 1 minus this quantity is sigma 1 minus u a minus sigma 3 minus u a whole divided by 2 when this is major principle stress at failure and this is a minor principle stress. And this quantity is equal to sigma 1 minus u a plus sigma 3 minus u a by 2.

So, if you take sine then this psi divided by this one so, you get this expression. So, if you use this expression 180 minus 50 by 180 minus 50 by 200 minus 50 sorry 180 plus 50 from the first expression. So, you get phi dash which is equals 34.42 degrees. Once phi dash is obtained you can calculate sigma s suction stress is equals to minus sigma minus u a is sigma 1 minus u a is 200, 200 minus sigma 3 minus u a is 50 tan square 45 plus 34.42 by 2. And anyways to see this is whole thing is cancelled for sand and divided by 2 and 45 plus 34.42 by 2 times and 34.42.

So, if you solve this you get a value of minus 7.68 kilo Pascal. So, there is a suction stress. So, suction stress minus 7.68 kilo Pascal. So, first the first value this is 0. So, similarly if you get all the other values this is minus 15.37 this is minus 19.22 and this is minus 23.07 this is minus 26.91 this is minus 32.68 this is minus 38.45 minus 46.14. So, that xi f can be obtained in the same manner which is a nothing, but minus sigma s by u a minus u w for the first data for saturated states xi f is 1 for other values this is a 7.68 by 10.

Therefore 0 or 7 7 the 0.615 this is point 3844 is 0.231 0.135 0.0818 0.0770 0.0676. You can obtain phi b also phi b is tan inverse of xi f tan phi dash. So, that is because the

Fredland et al expression for the strength envelop is tau f equals to sigma minus u a plus tau f equals to C dash plus sigma minus u a f tan phi dash plus u a minus u w f times tan phi b. If you compare this expression and Bishops approach tau f equals to C dash plus sigma minus u a tan phi dash plus xi f tan phi dash and into u a minus u w if you compare these two expressions xi f tan phi dash is equals to tan phi b.

So, phi b is nothing, but tan inverse of xi f tan phi dash. So, this is what is utilized here. So, after utilizing this we can write the phi b values. So, phi b values varied in this manner 34.42 27.82. So, the phi b is decreasing as the suction is increasing. 22.85 14.76 8.995 5.29 3.208 3.02 and 2.65. So, phi b continuously decreased. So, we can see the variation.

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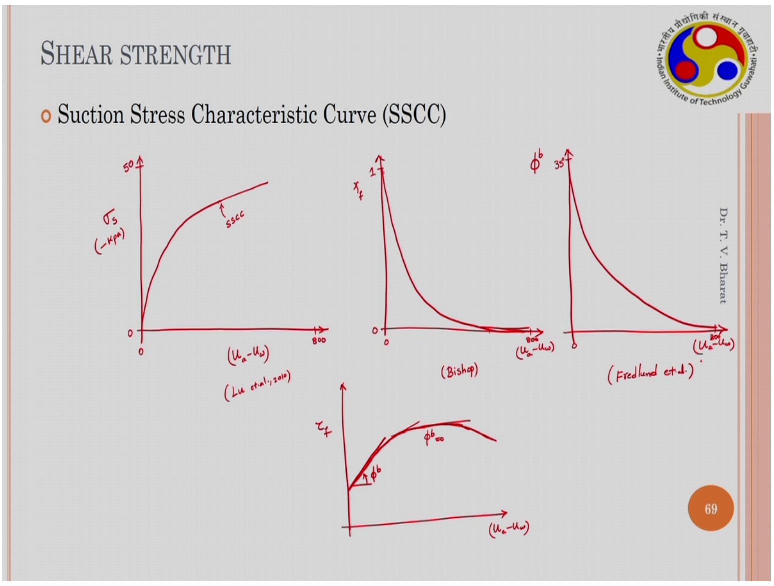

We can see the variation if the suction stress is plotted sigma s negative kilo Pascal with respect to u a minus u w we see that when this is varying between 0 and 50 and if the suction is varying between 0 and 800 because the suction maximum value is 750. So, if the range is this much then this varies in this particular manner. So, this is a typical SSCC curve for the given data and the xi f is plotted for the same variation of u a minus u w xi f varies from 0 to 1 and this is 0 to 800 and this decreases in this manner it approaches to 0 somewhere.

So, this is a Bishops approach this is a Lu et al or L u and Lycos and this bishop and you have Fredland and Morgenstern or Fredland et al where u a minus u w variation with phi

b. And here it is 0 to 800 and this is 0 to 35 maximum angle which is equals to nearly phi dash. So, then this also decreases in this manner so, this is a Fredland et al or Fredland Morgenstern. So, even though there are different expressions and different researchers have attempted to understand the effective stress of unsaturated soils.

So, essentially there are different parameters in each model either suction stress or xi effective stress parameter or phi b. So, these values vary in the same manner and the approaches are nearly same except there are minor variations. So, because of these the representation becomes different from each model. So, in the first representation that is Lu et al. We determine the suction stress and by adding the suction stress to the net normal stress this becomes effective stress. And in the other approach we determine the xi f suction effective stress parameter.

So, the effective stress parameter indicates the contribution of suction to the shear strength of the soil. And Fredland and Morgenstern and Fredland and Fredland et al they have proposed a similar approach where phi b is considered. So, this because of this representation of failure envelope using tan phi b; so, the representation of failure surface is very clear and easy to understand. However, because the phi b itself varies with suction this adds more difficulties in determining the phi b. We require huge datasets to determine the phi b parameter similar to phi f and similar to any other similar to even suction stress.

So, if the soil water characteristic curve data is not known. So, this phi b variation that is decreasing with increasing suction is similar to tau f versus u a minus u w variation. So, for a given net normal stress the tau f may increase and becomes nearly constant and starts decreasing this is the observations from experiments this decrease like this. So, this itself is phi b. Phi b increases at the nearly same angle as same angle as phi dash, but later on starts decreasing and nearly becomes 0 phi b is 0 here and even it decreases negative value it approaches it gives a negative value too. So, that is a observation that is what is indicated here phi b is plotted with respect to suction that is what is indicative.

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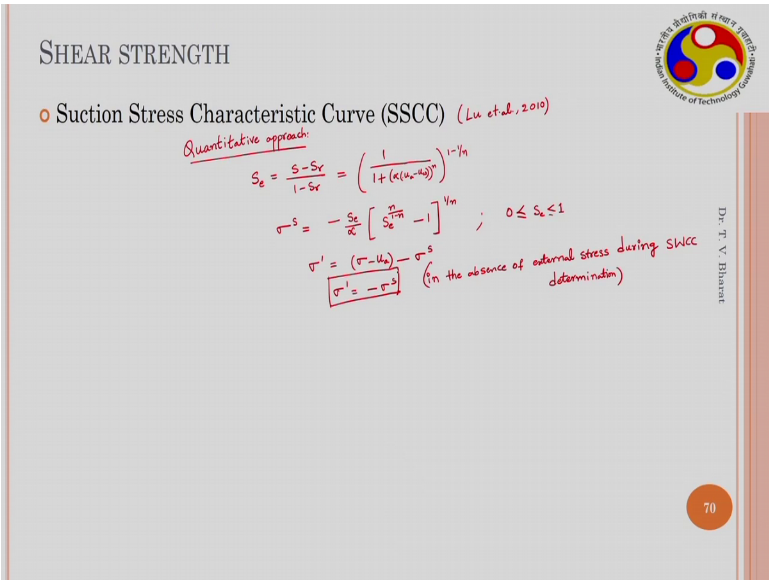

Lu et al further came up with quantitative approach where the S e is determined. And which is assume to be related to psi or matric suction has determined using (Refer Time: 47:49) expression. And then sigma s is determined this quantitative approach is useful sigma s is equals to minus S c by alpha S e times n by 1 minus n minus 1 whole power 1 by n S e varies between 0 and 1. So, then sigma dash that is a effective stress is determined in this manner; in the absence of any external stress on soil specimen during SWCC test.

So, this is absent. So, therefore, sigma dash is equal to minus sigma s itself in the absence of, in the absence of external stress during SWCC determination. So, effective stress is directly equal to the minus suction stress. So, this approach is proposed when soil water characteristic curve data is known. So, then the soil water characteristic curve data can be fitted with (Refer Time: 49:20) model with m and n parameters related. And, using alpha and n we can determine the suction stress and the effective stress can be determined.

So, this is how the suction stress characteristic curve determined and effective stress of the soil in unsaturated state are determined. So, this is a completion of shear strength topic and I will summarize what we have learned so far.

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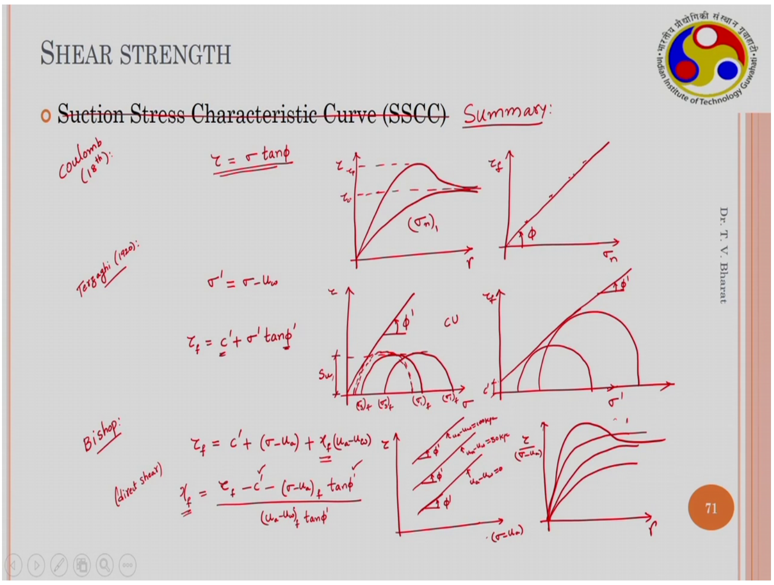

In the beginning historical developments we have seen where the Coulomb in 18th century. Coulomb in the 18th century proposed the Shear Strength model that is tau equals to sigma tan phi. So, the total stress controls the shear strength of the soil that is his approach where he proposed a one phase model such as a he has he considers soil as nothing, but weak rock mass. So, he does not distinguish between soil and rock only the difference between these two is the strength of the material changes from rock to soil.

So, he proposes that the shear strength varies with sigma the total stress or the total stress. So, therefore the direct shear test for obtain direct shear test were designed and the tau versus sigma plots could be obtained by conducting several tests by varying the net normal stress on the soil, normal stress on the soil. So, the shear stress and shear strain profiles are obtained for heavily compacted sand and loosely compacted sands. So, this is a critical shear and this is a critical shear and this is a peak shear.

So, this is said one net normal stress this is one net this is at 1 normal stress. So, if this is plotted with normal stress. So, when you get the data if you fit with a straight line fit with a linear line you get angle of internal friction phi dash and you had an angle of internal friction phi dash.

So, therefore, the shear strength of the soil can be determined using the shear strength is assumed to be dependent on directly total stress. So, the angle of internal friction of the soil can be determined directly. So, here it should be used phi there is no phi dash when this was proposed. Then later in Terzaghi in 1920 came up with effective stress principle that is sigma dash should be equals to sigma minus u w. So, here mostly the stress is on, mostly the stress is on two phase system soil solids and water; he considered soil solids and water. And if you have a two phase system the sigma dash effective stress is equal to sigma minus u w sigma minus u w.

So, then the pore water the importance of pore water pressure in the determination of shear strength of the soil came into picture and the tri axial tests were designed to control the drainage. Then by controlling the drainage valve drained and un drained tests are conducted and in the drain tests essentially you get same size the Mohr circles. Essentially when the sigma 3 is maintained particular value and sigma 1 is varied and the soil fails in this manner.

So, these are un drained test and if the sigma three is increased and the radius of the Mohr circle does not change. And this is another half essentially, you get shear strength of the soil in an un drained manner the un drained shear strength of the soil are obtained. So, which are used for short term analysis. So, if the in the same test if the pore water pressure is measured at the failure. So, then pore water pressure is measured then effective stress envelopes also can be obtained. And all these things will merge into one single Mohr circle then you get effective stress parameters.

So, you can determine effective stress parameter and then you can determine the drained parameters and un drained parameters in the same tests called consolidated un drained test. Similarly the drain test can be conducted where tau f and sigma dash variation can be obtained. So, for clay soils there is an intercept which is seen this is equal to tensile stresses. So, and this is an intercept and this is angle of internal friction. So, using the C dash and phi dash the equation is modified now as C dash plus sigma dash tan phi dash.

So, C dash and phi dash are the shear strength parameters. Shear strength parameters are material constants and sigma dash is effective stress. And Bishop later on in 1950s came up with two stress state parameters or two stress state variables for understanding the shear strength of unsaturated soils. So, here sigma dash plus sigma minus u a plus xi f u a minus u w. So, the xi f is the effective stress parameter which indicates what is a contribution of u a minus u w on the on the shear stress or shear strength of the soil.

So, there were several tests conducted by controlling the suction in the direct shear test and tri axial tests, where the data obtained in this particular manner sigma minus u a if the tau and sigma minus u a is plotted for saturated state. So, this is a failure envelope obtained this is with phi dash and this for u a minus u w is 0 and this is for another. Similarly for other envelopes for different u a minus u w say if 50 kilo Pascal and this is for another u a minus u w. Angle of internal friction nearly constant hardly varies within 4 to 5 degrees.

Similarly, the tau by sigma minus u a if it is plotted with shear strain. So, this is how the stress strain curves varied for different matric suction values, for different matric suction values. So, therefore these can be analyzed using Bishop's effective stress principle, extended effective stress principle this data can be very well be analyzed by determining xi f parameter for direct shear.

And similarly, for tri axial test we have seen how to determine. Here for different u a minus u w values initially the tests are conducted at completely saturated state and then C dash and phi dash are obtained. Then, when the u a minus u w's are varied that xi f parameters are obtained. Then xi f relationship with u a minus u w are obtained. This relationship is used to define the stress state of the soil very accurately.