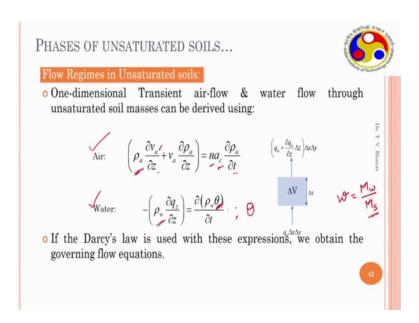
## Unsaturated Soil Mechanics Dr. T.V.Bharat Department of Civil Engineering Indian Institute of Technology, Guwahati

## Week - 01 Lecture - 03 Phases of Unsaturated Soils-II

Hello everyone. Earlier we have discussed about the state variables and unsaturated soils such as density of air, density of water, and some material constraints such as the viscosity of air and water. As I mentioned in the previous lecture that the density of air plays very important role in governing the flow through unsaturated soils.

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So, if you consider for flow regimes, if you considered one-dimensional transient air flow and water flow through unsaturated soil masses using physical law, it is mass one of the physical loss that is mass conservation principle, one can derive this expression for air flow where this is a rho a is a density of air, and this is velocity of air and this is porosity, which is volume of void (Refer Time: 01:33) total volume. And this is air content this is volume of air by volume of voids which is 1 minus s r degree of saturation and this is density of air again. So, here the z and t are the independent variables.

If you observe this expression which is derived from conservation of mass, the density of air is one of the dependent variable. So, as it is a dependent variable which governs the

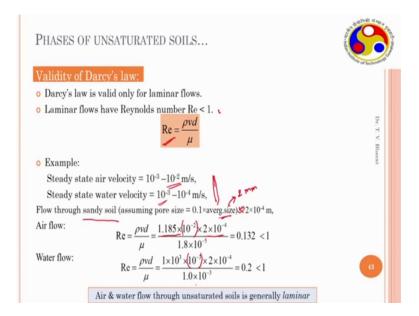
flow which can be derived by considering one volume element through which there is a flow there is taking place. And this is the flux that is entering into this volume element, and this is a flux which is coming out. So, we can balance, this you get this expression.

Similarly, one can drive for water flow simple mass conservation principle where you have a flux q z dou q z by dou z times rho w this is density of water equals to dou rho w theta by dou t. Here rho w is again density of water, and theta is the volumetric water content volume of water per total volume. We express the water contents in gravimetric basis in soil mechanics that is the gravimetric water content is mass of water by mass of solids this is usually what we do. And we take a soil sample either in the laboratory are filled and put it in oven and measure its weights weighs it, we weight and we get the ms mass of solids.

And when we deduct from the initial mass, we get the mass of water; and this is of we get the gravimetric water content. However in unsaturated soils will be using volumetric water content, we represent the water content in volume basis, because it is easy to estimate because when the water content changes significantly using indirect estimation methods we get the theta estimated. So therefore, we use theta. And this is how one can express. And rho w is relatively insensitive when it compare to the rho a. So, therefore, this expression is actually derive in terms of thetas instead of rho w. So, these two expressions are based on physical laws. If you combine Darcy's equation, if Darcy's equation is valid, if you represent this flux in terms of velocities and gradients, the sorry, if you represent this flux in terms of gradient and simplify, you get an expression in terms of thetas in terms of theta volumetric water content variations.

So, this is called a Richard's equation, we will discuss this expression little later when we deal with flows through unsaturated soils. As of now I want to bring your attention to the importance of different state variables that exist in unsaturated soil mechanics and how they play role in controlling the flow behavior through unsaturated soils. So, here in these two expressions need to be combined, these two expressions need to be combined with Darcy's law are Darcy's law, so that we get the governing expressions for flow. Now, the question is whether the Darcy's law is valid for unsaturated soil flow or not.

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So, Darcy's law we all know that it is valid only for laminar flows. So, laminar flows we define based on Reynolds number where Reynolds number is less than 1. So, Reynolds number is defined as the density times velocity times multiplied by the pore diameter divided by the mu dynamic viscosity. So, here rho is in kg per meter cube; velocity in meter per second, and d is in meter, and mu is kg per meter second. If you substitute, Reynolds number is a non-dimensional coefficient, non-dimensional parameter.

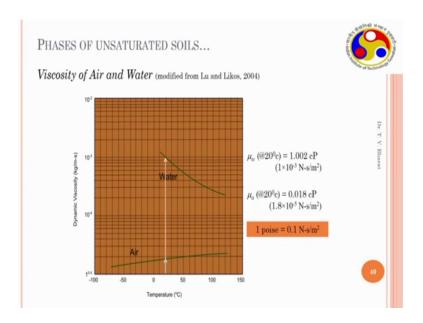
For example, if you consider the steady state air flow through soils and steady state water flow through soils. The air velocity may vary between 10 power minus 3 to 10 power minus 2 meter per second. And water velocity may vary between 10 power minus 3 to 10 power minus 4 meter per second. In sandy soils, for extreme cases, we are considering where these two may be possible if the flow is taking place through a sandy soil where the sand average diameter particle diameter would be 2 mm.

So, then pore size could be assumed as a one-tenth of the average size of the particle. So, if assume size of the particle as 2 mm, so the pore size can be approximated as 2 times 10 power minus 4 meter, so this is 0.2 meter. So, in when we express in terms of meter this is 2 in 10 power minus 4 meter. So, then the for air flow Reynolds number is rho into v into d by mu, where density of air is 1.185 kg per meter cube. And this is the velocity which we are assuming an extreme value are higher side that is 10 power minus 2 meter per second times this is the diameter of the particle 2 into 10 power minus 4 meter

divided by the dynamic viscosity of air is 1.8 times 10 power minus 5 kg per meter per second.

So, this is 0.132, Reynolds number is 0.132, which is less than 1 weight less than 1. So, the flow should be laminar. If you consider 10 power minus 3, then it should be 0.0132, so it is always less than 1. So, if you consider water flow, the Reynolds number when you estimate it is the density is 1 into 10 power 3 kg per meter cube, it is nearly 1000 times more, water is 1000 times denser than a times this is the velocity of water.

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We are considering 10 power minus 3 and times this is the pore diameter divided by this is a dynamic viscosity which we discussed earlier at the same temperature. This is a dynamic viscosity of air; and this is the dynamic viscosity of water. This is how it varies at 20 degrees. The dynamics dynamic viscosity of air is 1.8 into 10 power minus 5 Newton second per meter square per kg per meter second. And dynamic viscosity of water is 10 power minus 3 Newton second per meter square.

So, when we substitute this value, we get 0.2 as a Reynolds number which is also which is also less than 1. So, the flow is laminar. So, air and water flow through unsaturated soil is generally laminar. So, Darcy's law may be applied from Reynolds number point of view or laminar point of view. But then validity of its gradient, atelic gradient is proportional to the flux or not that we need to see. And we will discuss this, when we discuss the flow behavior of flow behavior through unsaturated soils. So far we have

discussed the air phase separately and water phase separately. And we consider the state variables of air and water and material properties separately

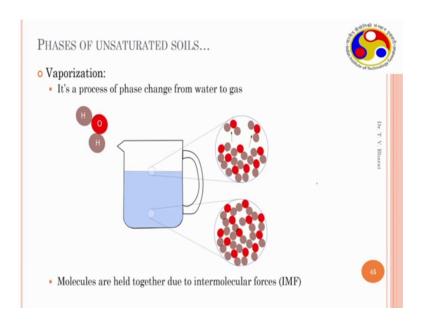
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So, now when you have two phases together there will a thermodynamic equilibrium between different phases. There will be a thermodynamic equilibrium between two different phases. So, water can exist in air as vapor; air can exists in water phase as solute. So, each phase is no means pure, because of the thermodynamic equilibrium there is a some component of water is existing in air and the some component of air may exists in water under different temperature and pressure conditions. So, knowledge on thermodynamic equilibrium between different soil pore phases that is air and water is important is important for understanding the soil behavior and predicting unsaturated state variables.

So, for example, we estimate the suction using one of such principles later on. So, section is one of the state variables; as of now this is sufficient for us to know. And I have not defined suction so far and we will discuss it little later.

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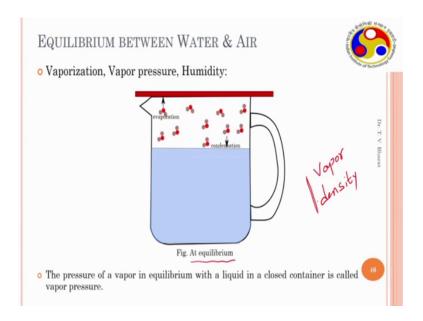


So, how does water exists in air phase? So, we need to understand the vaporization, we all know about this. It is a process of phase change from water to gas. When you consider beaker of water, water molecules exists, water molecules when you consider a beaker of water, you have water molecules which is designated as oxygen in red color and hydrogen atoms in a little light colour. And this is one water molecule brownish shade. So, if you consider water molecule that is designated as oxygen in red in color, and hydrogen atoms in brown color.

So, in this particular scenario, if I consider microscopic molecular viewpoint at these two locations in the beaker, so at this particular point when I zoom in trillions of trillions time, so this could be one possible molecule structure that may exists. The molecules here are held together due to intermolecular forces. So, when it comes to the air water interface, some water molecules will have sufficient energy to escape from the waterwater phase, and they going to air phase that is we observe as vapor. So, water molecules they leave from the water phase, when they have sufficient energy and they reach their phase.

On other hand, the molecules which are sitting here, they cannot escape unlike the water molecules which are place near the area water air water interface.

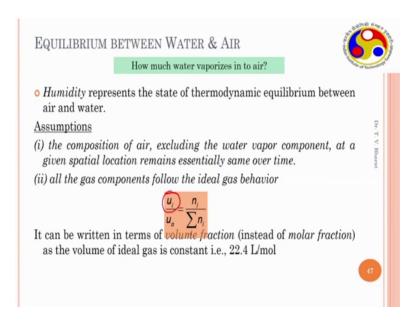
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So, if I close this container with a lid, then the water molecules which are in air phase due to evaporation cannot escape, and then they fall back to the water phase, and again they are held due to the intermolecular forces. And some of the molecules would go into the air face and some molecules drop into the water phase. This is called condensation; this is called evaporation. After some initial time, the number of molecules that are leaving the water-water phase would be equal to the number of molecules that are coming to the water phase; that means it reaches in equilibrium.

At equilibrium, so the both the number would be same. So, the pressure that is experienced by the container or the lid due to these vapor-vapor molecules or water molecules is called vapor pressure. And this is often represented with vapor density, vapor density, because number of molecules that are present in a given volume would be represented with the vapor density or vapor pressure.

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Now, the question is how much water vaporizes into air so that we with understand using thermodynamic principles. And the first assumption one would make is the composition of air excluding the water vapor component at any given spatial location remains essentially same overtime.

So, the composition of air remains same even though there is water vapor that exists in the atmosphere. And all the gas components second one is all the gas components follow the ideal gas behavior. So, there is a each gas component pressure to the pressure due to each, each gas component u i divided by the atmospheric pressure that is the total pressure is equals to the molar fraction of each gas divided by total molar mass. So, it also can be expressed in terms of volume fraction, because I have total volume of ideal gas is constant, it is 22.4 liter per mole.

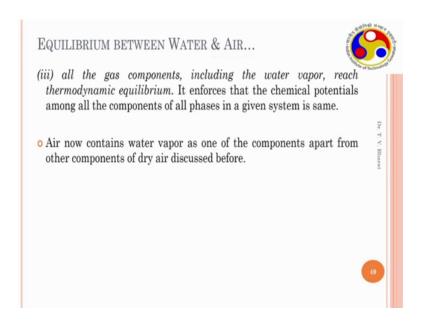
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So, in terms of molar fraction this could be expressed as this. So therefore, now we know different components of air, different gas components in gas components in air. So, we have oxygen which is about 21 percent nearly. So, this fraction volume ratio times the total pressure this one times the u a, if you assume u a is equals to 1 naught 1.325 kilo Pascal.

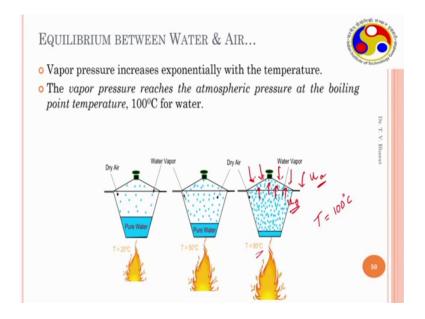
So, this is 21.23. Similarly, nitrogen which has a volume fraction of 78.1 percentage. So, this would be 78.1 times 1 naught 1.325 kilo Pascal at standard temperature, this would give 79.14. Similarly, each individual gas component pressure vapor pressure could be partial pressure of it each individual gas component could be estimated in this manner. And if you do not have any vapor pressure, the summation should be equal to this; but if you have vapor pressure, the partial pressure of all different gas components will get reduced.

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So, third one is the all the gas components including the water vapor reach thermodynamic equilibrium. It enforces that the chemical potentials among all the components of all phases in a given system same. Air now contains water vapor as one of the components apart from other components of dry air as discussed before.

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So, now this vapor pressure depends on different state variables. The vapor pressure depends on temperature; it depends on the purity of water; it depends on the solid water; depends on the interaction between the solid and water; it depends on many other factors.

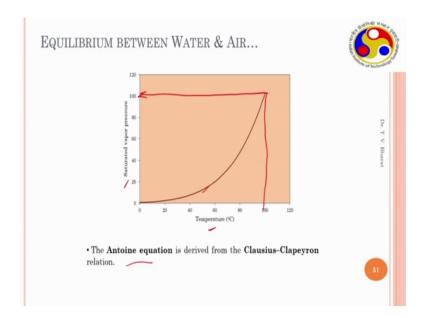
So, let us look at the how the vapor pressure varies with temperature. So, if you consider a container which is closed with the lid. If you take pure water at some temperature say 20 degrees, so maybe the ambient laboratory temperature is ambient room temperature maybe 20 degrees. So, in that particular temperature, there are some water molecules that escape from the pore water so pure water and it has certain vapor phase in the air, in the closed container.

So, these blue color dots represent the water molecules; and this is pure water. And white colored thing is air dry air. So, now, when you heat further to probably 50 degree Celsius, more water molecules would escape into the air phase because you are giving additional energy to the molecules to escape. So, they break inter molecular forces; and they have sufficient energy to get away from the; they may reach the given energy would reach sufficiently to break the intermolecular forces and then get away into the air phase. Similarly, when you reach, when you heat it to further, when you increase the temperature further, more number of more number of molecules would go into air phase and this is a condition.

So, as and when you start increasing the temperature so more and more water vapors go into the air phase so that the vapor pressure would start increasing. At 20 degrees the vapor pressure would be around 3 kilo Pascal, 3.17 kilo Pascal. And as a temperature increases, it increases; and when it reaches temperatures, which 100 degree Celsius which is the boiling point boiling point for water. Water vapor pressure reaches 1 naught 1.325 kilo Pascal: that is equivalent to the atmospheric pressure, and which can uphold, which can uplift the lid, which is available here.

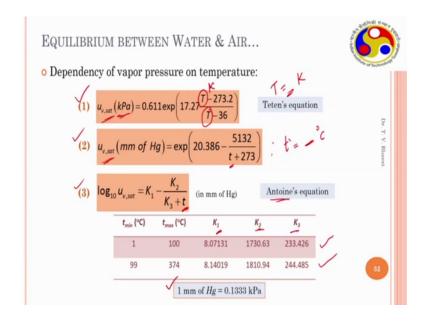
So, you have atmosphere pressure which is acting. This is atmospheric pressure u a. And the vapor pressure the vapors with exact pressure on this lid and which this when this pressure compensates this u a, this is vapor pressure. When u v equals to u a, your lid can be uplifted slightly if u v become slightly more than u a and then this lid will be uplifted that we often see. When you close a container with a lid, and when you boil a water or anything when it reaches the boiling point, you would see that the plate oscillates, the lid oscillates.

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So, the saturated vapor pressure would change with temperature in this particular manner. So, at 100 degree Celsius, nearly reaches 1 naught 1.325 kilo Pascal atmospheric pressure. So, this variation with temperature is approximated by approximated and empirical relationships are available based on Antoine equation Teten's equation and many other things so which Antoine equation is derived from the Clausius-Clapeyron relationship. To understand these relationship one can refer any standard physical chemistry text books like Atkinson.

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So, these are the empirical expressions that are available. This is the Teten's equation. Here u is at the vapor saturated vapor pressure is equals to 0.611 times exponential of e of 17.27 times t minus 273.2 by t minus 36. Here t should be substituted in Kelvin it is a Kelvin temperature in Kelvin. And then you get u v sat saturated vapor pressure in kilo Pascal. So, this is the second expression for estimating the saturated vapor pressure to approximate this expression to approximate this relationship, so which is u v sat is equals to exponential of 20.386 minus 5132 divided by t plus 273. Here small t indicates that the temperature should be substituted in degree Celsius.

So, here the temperature should be substitute in degree Celsius. Here capital T it should be substituted in Kelvin. So, here the saturated vapor pressure you will get in millimeter of hg, millimeter of mercury. And one can convert to kilo Pascal using this expression 1 millimeter of hg is equals to 0.133 kilo Pascal approximately. Similarly, the third expression which is Antoine's equation as I said this is a derived from Clausius-Clapeyron expression which is stated here.

Here the log 10 u v sat is equals to k 1 minus k 2 by k 3 plus t t should be in degree Celsius. And again u v sat you will get in millimeter of hg here k 1, k 2, k 3 are Antoine's equation constants for different temperatures the coefficients are given in this particular table. So, for the temperature changing from 1 to 100 degree Celsius, these are the coefficients. And from 99 to 374, these are the coefficients. So, at 100 degrees Celsius, you can use both the expressions. So, you can use either of these set of equation constants to obtain the u v, sat. So, these are the three expressions could be used to estimate the saturated vapor pressure at an given temperature.

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EQUILIBRIUM BETWEEN WATER & AIR...

o Dependency of vapor pressure on temperature:

$$t = 20^{\circ}\text{C}, \quad T = 293.15 \text{ k}$$

(1)  $U_{v_{1},\text{Sak}} = 0.611 \text{ exp} \left( 17.24 \frac{293.15 - 273.2}{293.15 - 36} \right) = 2.33 \text{ kpa}$ 

(2)  $U_{v_{1},\text{Sak}} = \left( \exp\left( 20.3\%6 - \frac{5132}{20 + 273} \right) \right) = 2.353 \text{ kpa}$ 

$$\times 0.1333$$

(3)  $U_{v_{1},\text{Sak}} = \left( \left( \frac{9.07131}{20 + 273} - \frac{1730.63}{233.426 + 20} \right) \right) \times 0.1383 = 2.33 \text{ kpa}$ 

$$25^{\circ}, \quad |V_{v_{2},\text{Sak}}| = 3.16 \text{ kpa}$$

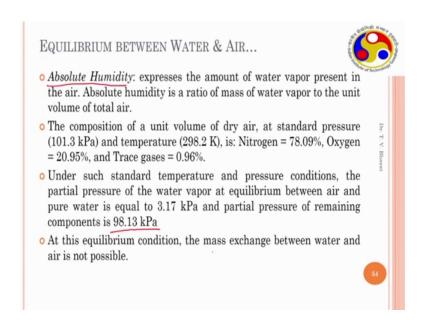
$$|00^{\circ}, \quad |U_{v_{2},\text{Sak}}| = 101.33 \text{ kpa} = 104.33 \text{ kpa} = 104.33 \text{ kpa}$$

So, if you see at temperatures set t equals to 20 degrees Celsius, the capital T would be 293.15 Kelvin. From the first expression, Teten's expression that is u v sat is equals to 0.611 times exponential of 17.27 times 293.15 minus 273.2 by 293.15 minus 36 would give you a value of 2.33 kilo Pascal's. From the second expression, u v sat is equals to exponential of 20.386 minus 5132 divided by 20 here temperature should be substituted in degree Celsius plus 273 is equals to 2.353 kilo Pascal.

So, here you get to 2.353 kilo Pascal, when you multiply this whole thing with 0.1333 a conversion factor to bring this value from millimeter of hg 2 kilo Pascal; so this whole expression you to be multiplied with this to get 2.353 kilo Pascal. So, using the third expression, u v sat is equals to which is the 10 power which is log 10, so 10 power k 1, so k 1 is 8.07131 minus k 2 is 1730.63 divided by k 3 is 233.426 plus 20 degree Celsius. So, this expression should be again multiplied with 0.1333 then you get 2.33 kilo Pascal.

So, the value is nearly same from three different expressions. You would see that at 25 degrees the u v sat is nearly u v sat is around 3.16 kilo Pascal; and at 100 degree Celsius, u v sat is 1 naught 1.33 kilo Pascal from Antoine's expression. But from the first expression, you will get around 1 naught 2.2; and second expression gives 1 naught 100.72; and third expression gives 1 naught 1.33 kilo Pascal. So, this reaches this is a nearly atmospheric pressure. So, this is what happens at boiling.

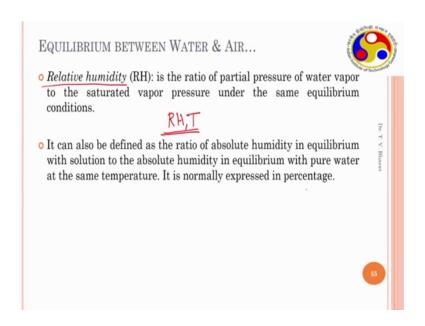
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So, this is absolute humidity; absolute humidity is the vapor density so which is expressed as the amount of water vapor present in the air. So, the absolute humidity is a ratio of mass of water vapor to the unit volume of total air. So, this is a vapor density. So, the composition of unit volume of dry air at standard pressure and temperature is for nitrogen gas, it is 78.09 percent; oxygen is 20.95, and trace gas is 0.96 percent. Under such under such standard temperature and pressure conditions, the partial pressure the water vapor at equilibrium between air and pure water is equal to 3.17 or 3.16 kilo Pascal. And partial pressure of remaining components is 98.13 kilo Pascal.

So, at this equilibrium condition, the mass exchange between water and air is not possible anymore unless until you change the temperature or other conditions probably then mass exchange would change until unless you change the temperature or other conditions, the mass exchange is not possible. So, humidity as we I just said this is the absolute humidity is the density of vapor or vapor density. So, this does not mean anything actually because it does not give much information, the absolute humidity does not give much information whether outside ambient conditions are very humid or not, or is it tolerable or not nothing will be extracted from the absolute humidity.

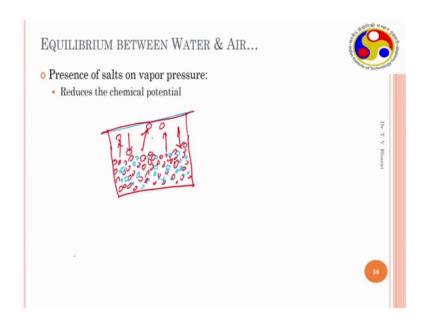
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So, we define something called relative humidity, which is the ratio of partial pressure of water vapor to the saturated vapor pressure under the same equilibrium conditions that means the water vapor pressure when it is when if you to consider a closed container, when you have two phase system water and then air phases. So, water is pure then the vapor pressure is saturated vapor pressure under this ideal condition. However, due to several factors such as due to present of presence of solute in water or due to some interaction between solid and then water are different-different conditions, the chemical potential of the water phase decreases.

So, because of that the vapor pressure is smaller than the saturated vapor pressure. So, therefore, the relative humidity defines how what is the vapor pressure, how close a vapor pressure is to its saturated vapor pressure under a given temperature. So, that is how the atmospheric conditions are defined. So we defined that; atmospheric conditions using relative humidity and temperature. These two state variables define the atmospheric conditions. So, relative humidity can also be defined at defined as the ratio of absolute humidity in equilibrium with solution to the absolute humidity in equilibrium with pure water at the same temperature. It is normally expressed in percentage.

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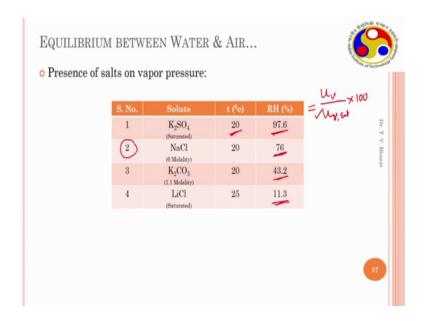


So, when you have salts present in water, the chemical potential decreases. Why, for example, if I take a container which has water, so there are so many number of molecules present in this container; these are the water molecules that are present. So, in this particular system many number of molecules which are existing at the air water interface can escape into air, because they have access to the air phase and they can escape into the air. But when you add salts to the water phase, that means, when you have a solute which is present in the solvent, so these are the molecules of solute that are present in the solvent

So, if this is the condition, the number of water molecules now they are exposed to the air phase got reduced. Because of that the number of molecules that would leave to the atmosphere now get reduced that would decrease the vapor pressure. Again when you have a closed container, when there you close this, so when you close this with a lid, so now, it reaches one equilibrium where number of molecules that would be leaving would be same as number of molecules that would be coming and dropping number of molecules that would condense and reach the water phase.

So, in this particular situation, the molecules that are reaching the water phase may also now get interacted with the solute particles solute ions. So, in the water phase and due to the again bondage between these two molecules, the water molecules will be held in the water phase and number of molecules that are available in the air phase would get reduced which decreases the vapor pressure.

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So, therefore, when you can consider some solute in the water phase, the relative humidity would reduce because relative humidity is nothing but the vapor pressure is nothing but the vapor pressure divided by the saturated vapor pressure. Saturated vapor pressure is known from Teten's expression are Antoine's expression, one can estimate for any given temperature. So, u v, how much it got reduced because of the presence of salts can be obtained using chemical equilibrium and how much chemical potential is dropping can be estimated from different ways and this could be found.

And here we have tabulated some values when you have these solutes present. When you have K 2 So 4 in saturated state when you have then at 20 degrees temperature, we have the RH value of 97.6 percentage. And when you have Na Cl with 6 molality concentration at 20 degrees which gives relative humidity of 76 which reduces the vapor pressure considerably, when you have K 2 CO 3 with 1.1 molality at 20 degrees which gives relativity humidity of 43.2 percentage; and lithium chloride at saturated state at 25 degrees, which gives very small value of relative humidity 11.3 percentage. So, relative humidity can change when you when the solute changes.

This could be employed from the second point that when you have Na Cl or relative humidity is dropping, so that means, when you have a pure water or when you have salt

water. So, you have vapor pressure above the salt water should be less than the vapor pressure above the pure water. So, from this one could understand that the people who are living in on seashore will experience lesser humidity compare to the people who are living near large water bodies, pure water bodies like you can compare one person who is living at living in Kanpur or somewhere where you have a Ganga waters and are Guwahati where you have a Brahmaputra huge water body and the humidity's here are very high; compared to the humidity which we experience may be near Chennai.

Chennai of course, everybody would feel that the humidity is too much are relative humidity is close to 100 percent, they feel so because temperatures reach 45, 46 that is the reason why they feel more humid. However, in Guwahati near the Brahmaputra river, it is more humid compared to the seashore at Chennai at the same temperature. Here in Guwahati, temperatures would not cross 40 even then you feel very humid you exert lot of you exert lot of energy, because you feel lot of humid. So, the skin if you considered as a pore structure, which has pores and water can come out when the temperature rises, we sweat.

So, when there is a humid climate where the vapor pressures are close to the saturated vapor pressure, so the atmospheric cannot take any more water from your body. So, then you feel that it is sweating, because the sweat that has come out he is not taken by the atmosphere, you feel more humid or you are sweating. On other hand, if you enter into a air condition room, you feel that you are dry, because the relative humidity drops to 70, 60. So, because of that the vapor pressure is smaller much smaller compared to the saturated vapor pressure. Therefore, more water from your skin will be taken by the atmosphere. So, because of that reason, you feel dry you feel thirsty very often.

So, this could be a wonderful analogy that could be used in unsaturated soil mechanics to estimate several state variables such as suction, we estimate using such principles. The analogy which I just discussed is useful to understand what exactly happens when RH and temperature changes. Such principles are useful in unsaturated soil mechanics, because when we want to estimate the state variable such as suction something called matrix suction, which I have not introduced, which I will discuss little later. So, that state variables those state variables can be estimated using such principles.

So, if we can measure the vapor pressures of soil pore system directly, then we can relate to the suction because suction is related to the vapor pressure. So, then we can understand the behavior of soil. So, there is a reason why we have been discussing all this fundamental principles of different phases and thermodynamic equilibrium between air phase and water phase.

Thank you.