Unsaturated Soil Mechanics
Dr. T.V. Bharat
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Week - 10
Lecture - 28
Extended M-C Criterion – II

Hello everyone, today let us discuss some more details about Extended More Coulomb Criterion given by Ferdinand at all. So, here some issues related to extend more coulomb criterion will be discussed.

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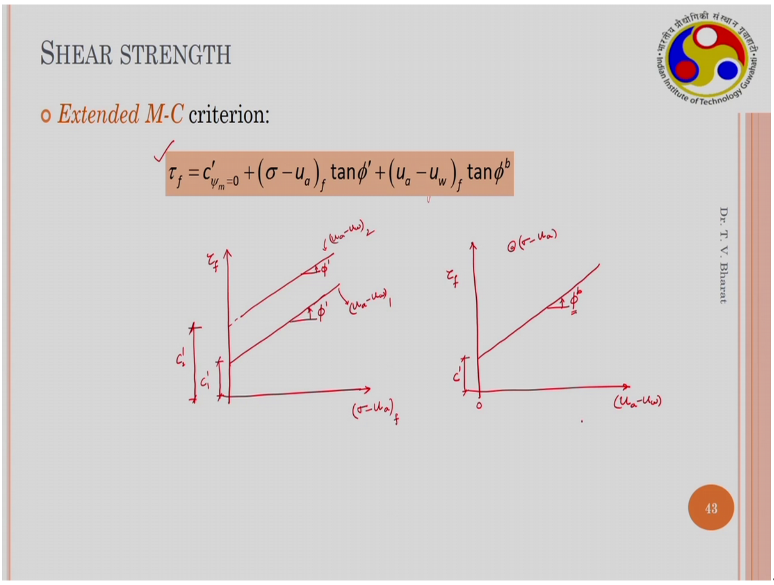

This is the Extended More Coulomb Criterion given by Ferdinand at all. Here the tau f the shear strength is written as c dash plus sigma minus ua at failure times tan phi dash plus ua minus uw at failure times tan phi b. So, when we represent this equation graphically, tau f on y axis and sigma minus ua on x axis.

At any given suction value, you expect linear relationship like this as angle phi dash and this intercept is c 1 dash, this is not c dash. If, the metric suction for this particular data is 0 then this is nothing, but c dash. The angle of internal friction does not vary if you tested with another matrix suction, then you get another relationship with different intercept, c 2 dash or something and this is also essentially phi dash.

So, this is with 1 ua minus uw and this is with another ua minus uw. So, you get another intercept c 2 dash. So, the same equation, this equation can be represented in this manner; so, in two dimensional on tau f and sigma minus ua at f. On the other hand, if I represent tau f with respect to ua minus uw with matric suction at a given sigma minus ua non net normal stress, then I expect the relationship to be a linear and which is something like this. And the angle is phi b at ua minus uw this 0,'this is nothing, but c dash.

This is what so, far we have discussed, and we said that the, it has great advantage in representing shear strength profile, shear strength envelop graphically and which can be understood clearly. However, there are some issues in representing this whole equations. In later, some researchers found that the phi b is no longer constant, phi b varies with matric suction.

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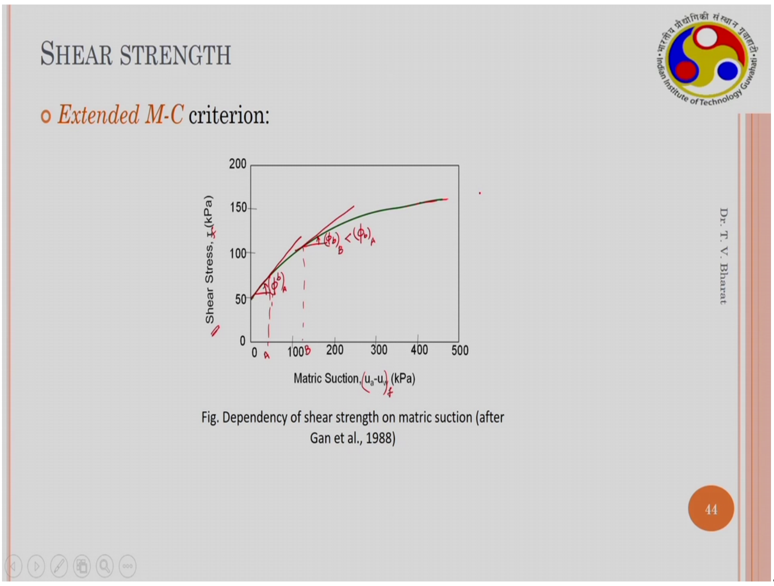

Fig. Dependency of shear strength on matric suction (after Gan et al., 1988)

So, this is a somebody Gan et al in 1988, they observed that the shear stress tau f versus matric suction data where is in this particular manner. So, interesting in that the phi b is highly non-linear initially takes phi b and as a, suction increases the phi b value decreases. This phi b at this particular point say b is less than at this particular point. Similarly, it observed that it nearly approaches to 0 even. And some more research in 1989 by Escario et al.

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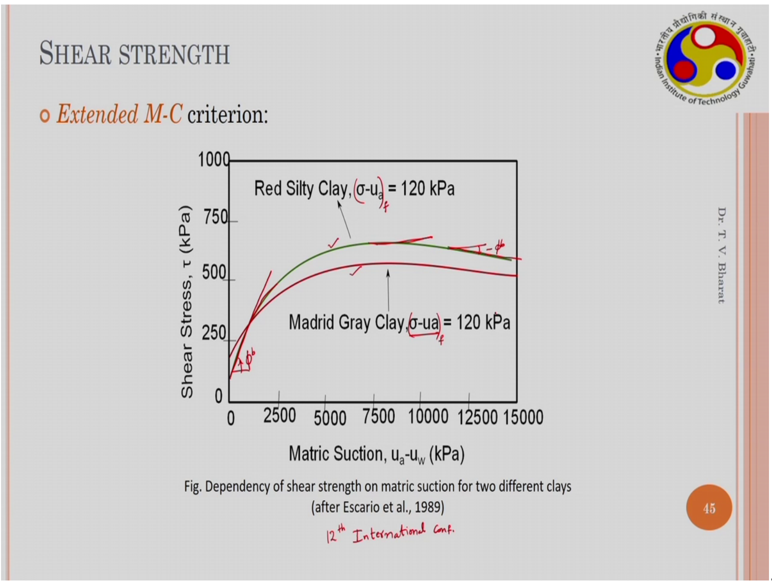

So, this work is published in a conference in a twelfth international conference; Conference on Internationals Conference on Soil Mechanics and Foundation; Foundation Engineering in 1989. The title of the work is strength and the formation of partly saturated soils.

So, in this work we showed that when they found the shear strength of the soil with different matric suctions at a given sigma minus ua, the sigma minus ua is kept constant which is 120 Kilopascal. For two different soils, one is a red silty clay and another one is Madrid Clay, Gray Clay. So, the strength profile varied in this manner. So, this is for red silty soil, this is for Madrid Gray Clay. So, initially it has certain phi b then as the suction increases the phi b starts decreasing. Similarly, Madrid Gray Clay also exhibited the same behaviour. Interestingly, phi b becomes 0 at one particular point and even started decreasing; it becomes negative negative phi b at very high matric suction values. So, phi b is no longer constant.

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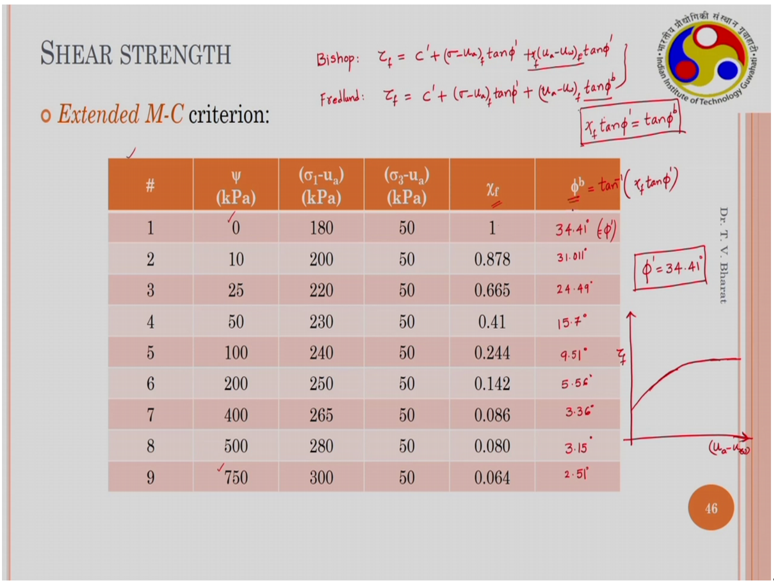

Then in earlier discussion using Bishop's effective stress principle, when we are discussing that the shear strength can be represented as c dash plus sigma minus ua, f tan phi dash plus ua minus uw, f tan phi dash.

So, this is from the Bishop and Ferdinand 1 tau f is equals to c dash plus sigma minus ua tan phi dash plus sorry, here this should be X i f X i f ua minus uw f tan phi dash. So, this is ua minus uw and tan phi b. If you compare these two equations, the X i f ua minus uw is just replaced with tan phi b and because of this the representation of the shear strength envelope is very easier. And we could see that the shear strength envelope varies with an angle phi b when it is a represented with tau, tau f versus ua minus uw. And when it is represented with tau versus sigma minus ua, the angle of internal friction is phi dash.

So, only the representation change; however, the, this essentially the X i f tan phi dash when it got replaced with tan phi b, the Ferdinand and Bishop relationship, the difference between these two relationship is simply understood by replacing X i f tan phi dash with tan phi b. So, this is the earlier data we used for trixial test data, to analyse the triaxial test data using Bishop's effective stress and principle.

So, where the X i f estimated for different matric suction values. So, as a matric suction varied from 0 to 75 kilopascal the X i of varied from 1 to 0.064 So, in this particular case, I get phi b from X i f. If, I estimate phi b from X i f knowing the angle of internal friction for this particular soil; so, the angle of internal friction 34.41 which is constant which is does not vary then phi b is nothing, but tan inverse of X i f tan phi dash, so, we write this. So, here X i f is 1 then phi b equal to phi dash so, that is 34.41. So, this is same as phi dash and in this particular case this is 31.011 and it decreases 24.49, 15.7, 9.51, 5.56, 3.36, 3.15 and 2.51.

So, the strength parameter phi b varied from phi dash to very small value such as 2.51. So, if we can draw this, tau f on y axis and ua minus uw on x axis, phi b as tau f increases this value increases and which becomes nearly constant or 0. So, this is how the experimental observations are also. So, therefore, the phi b by replacing X i f tan phi dash with tan phi b, the equation slightly got changed, but this is the same representation as effective stress principle given by Bishop. So, not much improvement except that graphically this can be represented very well.

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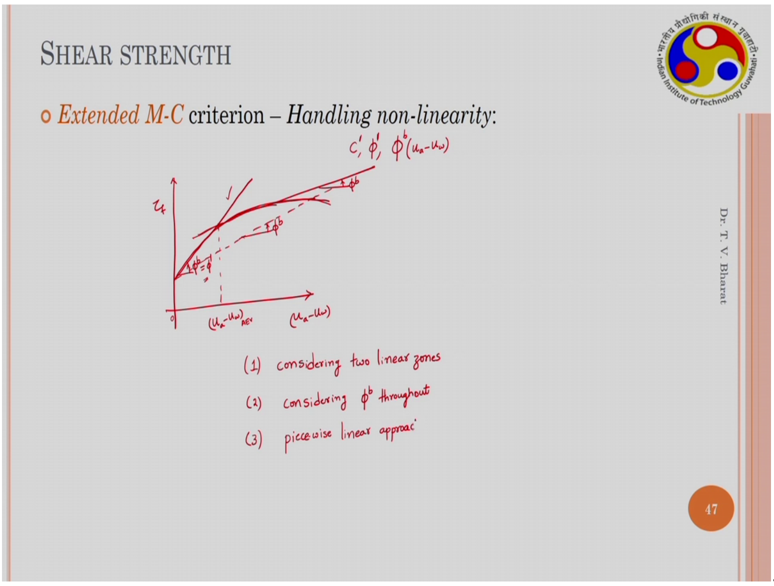

So, now it is very clear that phi b is no longer constant. So, therefore, using the Ferdinand's approach, we get the strength parameters like c dash phi dash and the phi b which is again a functional from which is dependent on the matric suction ua minus uw. So, phi b is no longer constant, the function is non-linear. How to handle this nonlinearity in phi b? Ferdinand suggested that three ways, it can be resolved. Perhaps, the tau f versus ua minus uw, when you have this data up to air entry value. So, this ua minus uw at air entry phi b can be approximated as phi dash. So, beyond that this non-linear portion can be ignored and which can be replaced with a straight line. So, this can be some phi b then using two linear portions, this is one linear zone, linear zone from here to here and this is another linear zone this from here to here.

So, this is how it can be approximated and then phi dash and phi b both can be used in this manner; that is one approach in first approach considering two linear zones. Second, he, he suggested conservative solution where you can consider from ua minus uw is equals to 0, you can consider the entire profile varies with an angle phi b. So, this is highly conservative solution. So, considering phi b throughout this is a constant approach.

So, generally the observations from experiments; so, it is understood that the phi b value is nearly equals to phi dash in the beginning. As a matric suction increases the phi b starts decreasing and it approaches 0 and even it goes to negative, is very high suction values. So, therefore, Fradland suggest that the phi b can be replaced with phi dash up to the air entry matric suction. And beyond that a linear approximation can be used for the estimation of for the representation of strength envelop. The second approach he considers phi b us constant, when you consider phi b us constant it is less than phi dash, so, this is a conservative approach.

And third one, he suggests to consider piecewise linear. So, the entire non-linear curve can be approximated as linear by considering several segments we need segment the profile can be considered as linear and this can be used in the design. So, piece wise linear approach,so the third one is piece wise linear approach. Let us try to understand with a simple problem, how to address this particular problem using piecewise linear approach?

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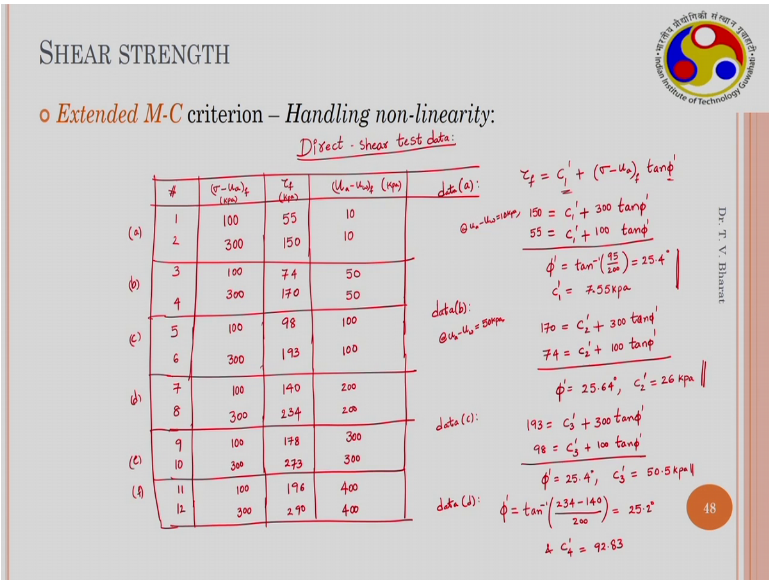

Direct Shear test data is considered for demonstrating how, how to estimate the strength, strength parameters using extended mc criterion where the nonlinearity in phi b is considered? So, Direct Shear test data, so, here several tests are conducted.

So, this is the test number test number 1, 2 like that and sigma 1, sigma minus ua. So, this is a net normal stress at failure and these are all 1 kilopascal. And the other one is tau f, this is at kilopascal. So, tau f is a shear stress at failure and the matric suction ua minus uw at failure. So, this is also in kilopascal. So, the first data whether net normal stress of 100 kilopascal applied and ua minus uw is 10, then the measured shear stress at failure is 55 Kilopascal. Similarly, under the same condition of same matric suction when the sigma minus ua is increased to 300 so, the tau f value is increased to 150.

Another set of data is considered, this is varied 100 and 300 only in all the test, only the matric suction is increased. Then the observed shear strengths varied as 74 and 170. And similarly, the 5th and 6th stress data is also shown. Here, also the sigma minus ua is varied 100 and 300 and matric suction is 100, then my tau f is 98 and 193. And another set with a matric suction as 300, here this is varied 100 and 300 so, then this is 178 and 273, these are the observations and last set 11 and 12.

So, this is 400 kilopascal and then this is 100 and 300. So, that tau f values are 196 and 290. So, this is a data observed from suction control direct shear test data; however, this is synthetic data which is generated using certain strength parameters for demonstration.

So, here considering the first two sets of data; so, this is one set of data with matric suction 10 and this is with 50 and this with 100, this is with 200, this is with 300 and this is with 400.

Now, considering the first set of data there is data set a then tau f is equals to c 1 dash plus sigma minus ua at failure times tan phi dash. This is a expression, if you expand this c 1 dash this is c 1 dash plus ua minus uw times tan phi b.

So, here anyways the phi b is not known. Now we express this as one intercept unknown intercept which is also not known, and phi dash is also not known; that is how we solve other state variables are known tau f and sigma minus ua f are known. So, when you substitute form data a. So, this is a 300 is equals to c 1 dash plus 150 tan phi dash and this is 100 is equal to c 1 dash plus 55 tan phi dash. When we solve this, you get phi dash which is tan inverse of 95 by 200, which is 25.4 degrees and c 1 dash is equals to 7.55 kilopascal.

As this test is at ua minus uw of 10 Kilopascal. So, the c 1 dash is equals to c dash, this is one set. And similarly, using data b that is at ua minus uw is equals to 50 Kilopascal. So, this is 170 is equals to c 2 dash, this intercept is different because at higher matric suction value plus 300 sorry, here there is a mistake in writing tau f is 150 and sigma minus ua is 300. And here this is 55 and this is 100 and here this is tan phi dash and here this is 74 is strength shear strength and c 2 dash plus 100 is the net normal stress and tan phi dash. Again this is solved phi dash; we obtain as 25.64 which is nearly same as earlier phi dash and c 2 dash is equals to 26 Kilopascal from the second set.

So, these are the one strength parameter and one intercept, we got. Similarly, using data c, we get 193 is equals to c 3 dash plus 300 tan phi dash and this is 98 equal to c 3 dash, 100 tan phi. So, phi dash is 25.4 and c 3 dash is 50.5 kilopascal. And data d phi dash is tan inverse of 234 minus 140 by 200 which is equals to 25.2 and c 4 dash is equals to 92.83.

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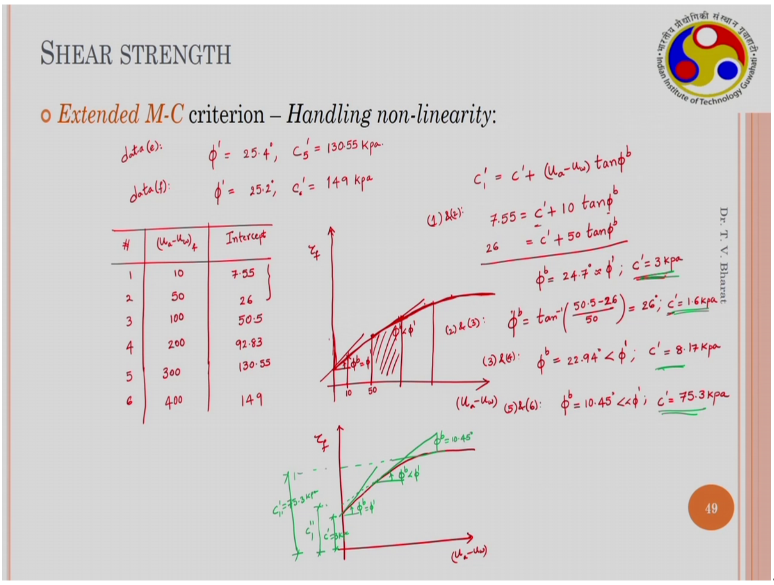

Similarly, the data e when we were take and we consider phi dash is 25.4 and c 5 dash is 130.55 kilopascal. Data f phi dash is 25.2, c 6 dash is 149 Kilopascal. So, from this you, we get angle of internal friction which is nearly 25.4 degrees even after averaging it and which is nearly same. So, the intercept values have changed from 7.55 Kilopascal to 149 Kilopascal.

So, if this data is given for ua minus uw at f and the intercept values. When it is 10 Kilopascal, this is 7.55 and this is 50 then this is 26, if this is 100 then this is 50.5. And if this is 200, this is 92.83 and this is 300 this is 130.55 and when this is 6 when 6 point that is 100, 400 kilopascal suction this is 149. So, the intercept if you plot tau f versus ua minus uw so, the first intercept at 10 kilopascal small value of suction is 7.55 and then at 50, this is 26, 50.592, 130 and 149.

So, essentially this curve, the envelop if you see this is how it varies. So, this is the exactly the experimental data also showed that the it is highly non-linear. So, initially at the first point, this angle is equals to phi dash. As a suction increases the phi dash phi b decreases phi b is less than phi dash and nearly approaches to 0, right. So. So, this phi b also can be estimated by considering two data points. Here, we consider p square is linear. So, we take two data points, this is 1, this is 10 kilopascal and this is 50 suction and you have a 100 kilopascal.

So, we take different zones, in each zone we consider that the the variation of tau f with ua minus uw is linear. So, when this assumption is valid then we can consider two sets like this and we consider c dash or c 1 dash is equals to c dash plus ua minus uw times tan phi b. So, using this expressions we can estimate what is phi b. So, when we estimate c 1 dash is 7.55 for 10 kilopascal. So, that is c dash plus, so, this is 10 tan phi b. So, here two unknowns we have c dash and phi b. And from the second point this 26 intercept is 26, this is c dash plus this is 50 tan phi b.

So, when we solve this, we get phi b is equals to 25, 24.7 degrees which is equal to phi dash. And when you substitute c dash comes out to be 3 kilopascal a very small value, this is from 1 and 2. So, using 3 and 4, we can also use second data set and third data set as well. So, we use this one and this one and we can use this one and this one as well.

So, when we use 2 and 3, so, we get phi b as tan inverse of 50.5 minus 26 by 50 which is equals to 26, just slightly higher and c dash comes out to be 1.6, this has decreased. The c dash which is cohesion intercept which should be constant got decreased. And if you take data 3 and 4 phi b value is 22.94. So, this data we took.

And this is lesser than phi dash and c dash is 8.17 kilopascal. And if we take 5 and 6, phi b comes out be 10.45 decreased very much, much less than phi dash. And c dash is very high 75.3 kilopascal. The interesting observation is that c dash is a cohesion intercept at ua minus uw is equals to 0. So, this cohesion intercept should be constant for any given soil; however, this is also changing along with phi b that is, because this is non-linear. So, if I redraw this curve tau f versus ua minus uw, this is the non-linear behaviour, we have observed. So, initially the angle is phi dash only and as it starts decreasing this angle is decreasing this phi b is less than phi dash.

So, intercept value starts increasing. So, this is the c, this is the c dash and now the new the intercept is some other intercept, this is may be c 1 dash you can put or c 1 double dash. So, this intercept starts increasing because a phi b is getting decreased as phi b is getting decreased. At one particular point, the phi b when becomes 0 at that particular point you have the cohesion intercept very high value. So, this value is as high as 75.3 kilopascal. So, this c dash is just 3 kilopascal and the c 1 dash where phi b is very small, where phi b here is may be 10.45 degrees, when this becomes 0 then this further increases the cohesion intercept becomes very high.

So, because this curve is highly non-linear when the cohesion intercept is not constant which also started varying with phi b. So, as this phi b is highly non-linear and we are approximating linear in a given range of matric suction values and estimating using the Extended MC Criterion by piece wise linear approach, the estimated values may be erroneous, if this is highly non-linear.

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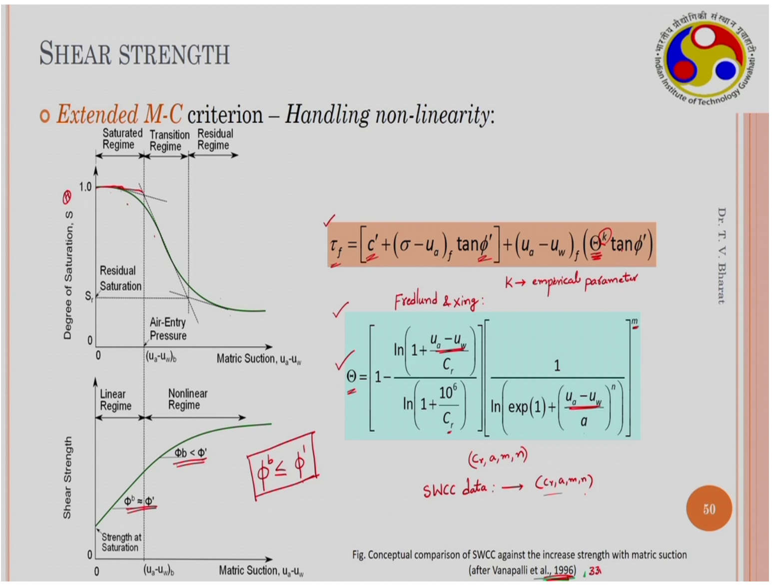

So, in 1996 so, monopoly at all they have come up with new model for the prediction of phi b for handling the non-linearity in phi b. So, he suggested this particular relationship this monopoly in 1996. So, this is a journal paper in which is published in Canadian geotechnical journal in 1996.

So, in issue 33 so, the work is the title of the work is model for the prediction of shear strength with respect to soil suction. So, in this particular paper they have come up with a new equation tau f equals to c dash plus sigma minus ua at failure tan phi dash plus ua minus uw at failure times theta power k into tan phi, this is the normalised volumetric water content. Here, the assumption is that if this is the variation of degree of saturation either you can represent with degree of saturation sr are big theta. So, then theta versus matric suction ua minus uw, if the variation is shown by this line. Then, so, the variation in shear strength versus matric suction follows this particular variation.

As the degree of saturation value is nearly constant or this value is up to the air inter value this degree of saturation is nearly 1 or very close to 1 then phi b should be equals to phi dash. As the pores get de saturated once the air enters into the largest pore of the soil system, then de saturation starts very quickly. And when the de saturation takes place the capillary forces are accounted where the, the contribution of suction for the strength starts decreasing. So, therefore, phi b becomes less than phi dash and as it approaches the residual saturation zone, then the phi b nearly approaches to 0. So, this is the conceptual comparison of soil water characteristic curve and shear strength envelop for a particular soil by monopoly at all.

So, here the interesting observation is that the phi b is always less than or equals to phi dash, phi b would never be more than phi dash. So, this is the assumption, we do not know whether this assumption is valid or not, because we have lack of shear strength data on different soils. So, the already we have discussed the limitation of estimation of shear strength of the soils. If you take highly plastically, first of all saturation itself is very difficult then once you saturate and bring it to any other given suction by applying air pressure in axis translucent technique and decreasing it's water content and maintaining certain ua and uw.

So, therefore, we can maintain particular matric suction in the soil sample. And then allowing the drainage to take place you shear the soil sample then measure the shear strength parameters then estimate. So, from the controlled shear stress variables, then we estimate the shear strength parameters. So, this particular compacted plastic lays exhibit very high air inter values such as more than 1500 kilopascal 3000 kilopascal or so. So, therefore, over a wide suction range axis translucent technique has a limitation to control the matric suction beyond 1500 kilopascal, because of the limitation coming from the pore size of higher entry dicks.

So, therefore, the testing is not possible. Moreover, this plastic lays take enormous time for the shearing operation consolidation shearing, etcetera. So, therefore, it is very difficult to estimate and we do not have enough data to validate any particular model. So, based on available data on different from different researches. So, different models have been widely used. So, here therefore, this is assumed that phi b is less than or equals to phi dash. And now the modified form of equation is tau f is equals to c plus sigma minus ua f tan phi dash plus ua minus uw at failure times, this theta power k this k into tan phi dash, this k is the empirical parameter. So, the k is empirical parameter which can be assumed to be any value based on by fitting this equation on the shear strength profile.

So, here big theta relationship with ua minus uw can be obtained by Ferdinand and Xing formula. As we discussed earlier the Ferdinand and Xing equation, 1994 could be used to obtain a smooth functional form between normalized water content and ua minus uw. So, here this is 1 minus log 1 plus matric suction divided by Cr divided by log 1 plus 10 power 6 by Cr, Cr is 1 fitting parameter times 1 by log exponential of 1 plus matric suction by a air inter value whole power n and whole power m. So, Cr a m and n, these four parameters are estimated by knowing the matric suction versus degree of saturation data; that is soil water characteristics data.

If you have soil water characteristic data which is measured in the laboratory; so, from that you can obtain these set of parameters Cr a m n by any optimisation technique, you can fit this equation on the data, then you can obtain these parameters. So, then the smooth functional form is obtained. So, then for any given shear stress parameters like sigma minus ua and ua minus uw, the variation of theta can be obtained for any given ua minus uw. And then by at least knowing three different tau f by obtaining at least having three equations, we can solve c dash phi dash and k.

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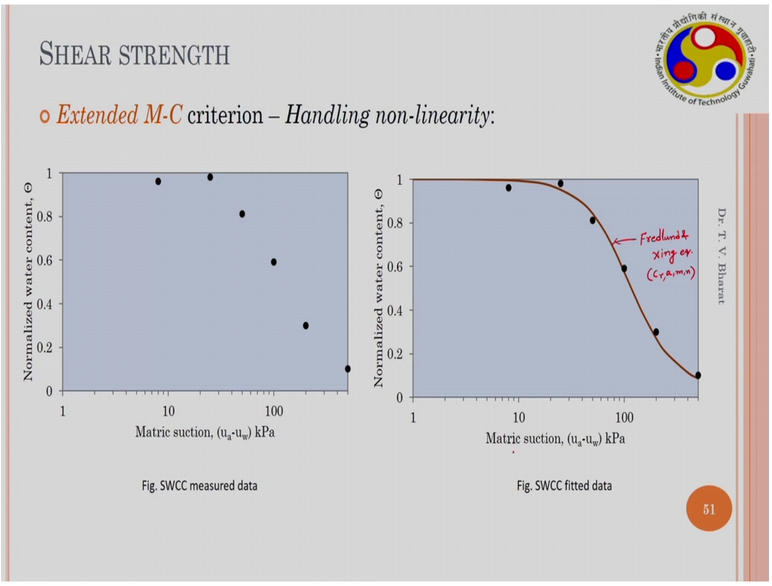

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For example, let us assume this particular data, while you have normalised water content versus matric suction. Here, are the data of normalised volumetric water content at nearly 10 kilopascal and 25, 50, etcetera you have. And once, you have this data, you can utilize

Ferdinand and Xing equation to fit the relationship; this is Ferdinand and Xing, Xing equation which is fitted by optimising Cr a m n parameters. So, once you have continues function then for any given matric suction, you can obtain the normalised volumetric water content.

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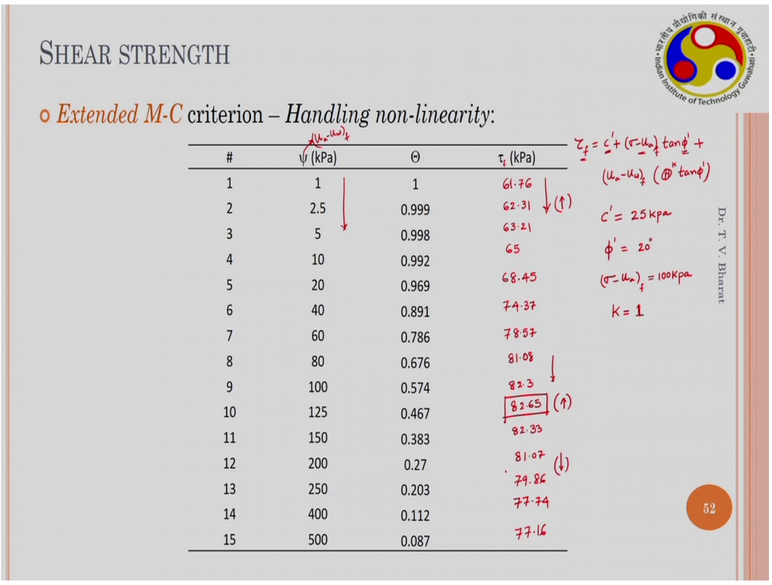

So, once you have this data psi versus theta continuously over a wide range of suctions, then you can estimate tau f values by utilising tau f equals to c dash plus sigma minus ua, f tan phi dash plus ua minus uw f into theta power k tan phi dash. Here, if the strength parameters c dash and phi dash are known, c dash is 25 kilopascal and phi dash angle of intersection is 20 degrees. And apply net normal stress at failure for all these stress is 100 kilopascal. If this data is known, then we can substitute c dash phi dash and sigma minus ua, these trusted variables are known and strength parameters c dash and phi dash are known, then you can estimate tau f by assuming k equals to 1.

If we, if we assume k 1 then tau f varies in this manner for the first data point you get 61.76 and 62.31, 63.21, 65, 68.45, 74.37, 78.57, 81.08, 82.3, and 82.65, 82.33, 81.07, 79.86, 77.74, 77.16. So, if you observe the shear strength of the soil increases with increase in the matric suction, this is matric suction ua minus uw at failure. So, as a matric suction increases, the Shear strength of the soil increases to maximum value of 82.65. and beyond that the shear strength of the soil decreases. So, this is realistic phenomenon, because the Shear strength of the soil at fully saturated state would be less and fully dry state would be less too; they may not be equal, but shear strength of the soil at one particular matric suction value will be highest.

So, where the contribution of matric suction to the shear strength of the soil is maximum. So, that is the reason why we need to understand how the shear strength varies with matric suction by several tests. If shear strength is maximum at fully dry state then the experiment are not required and all the soils will be stable in completely dry state, but that is not the case.

So, because of the surface tension contribution the shear strength of the soil is maximum at one particular point. As the water content increases, so, the influence of surface tension get reduced and shear strength decreases. Similarly, as more and more water is taken out from the soil again the bonding between different particles due to the surface tension also is lost and shear strength decreases. So, therefore, shear strength values increases up to this value and after that it starts decreasing.

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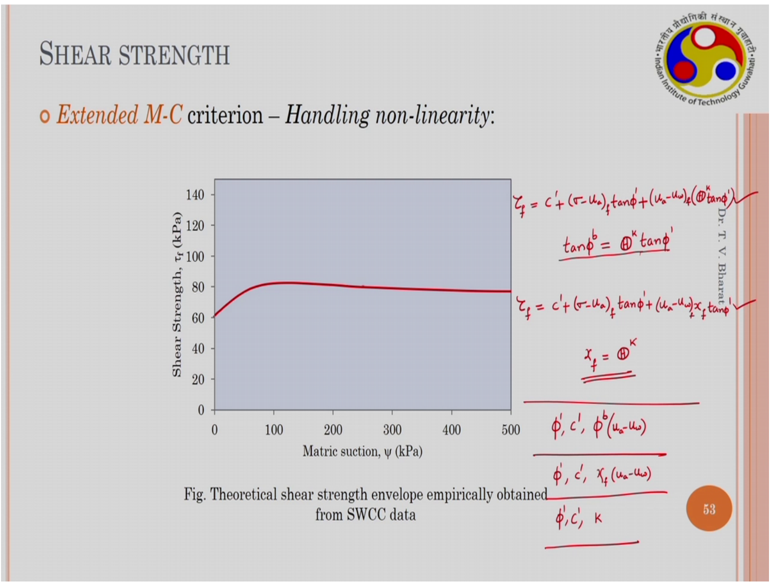

So, if we plot shear strength versus matric suction, this is what we observe, the with matric suction, the shear strength increased to certain value and then it started decreasing. And if you test at very high values then you understand that it comes down and comes to very small value. Now, the interesting aspect is here the equation that was proposed by monopoly at all; that is tau f is equals to c dash plus sigma minus ua f tan phi dash plus ua minus uw f times 5 power k tan phi dash. So, essentially the replaced tan phi b with theta power k times tan phi dash. Now, let us try to understand how this is different from Bishop's approach? In Bishop's approach, it is given tau f is equals to c dash plus sigma minus ua tan phi dash plus ua minus uw times psi f tan phi dash.

So, if you compare this equation and this equation, psi f is substituted to be theta power k. If k is equals to 1 xi f equals to the normalised volumetric water content that is an empirical expression which is already available by criterian, etcetera and many other have worked on that. So, essentially, this again boils down to the same effective stress principal that is given by Bishop, so, this is not significantly different. However, in this particular case for the estimation of phi b or in this particular case it is estimation of k, we require data consisting of same matric suction values, but changing the net normal stress or changing the principle stresses so, that we can determine this phi b values.

However, due to highly non-linear nature of this particular expression given by Ferdinand; that is very difficult the, the obtained strength parameters may be not accurate. By approximating tan phi b with theta power k and tan phi dash this again voice down to Bishop's effective stress approach.

So, in that particular approach, so, the way the xf xi f is estimated by conducting series of experiments and in the same manner it should be conducted in this particular case also. Here we have one more variable that is instead of xi f unknown, here we have k unknown. How k varies is also need to be understood whether k is constant or k also varies with matric suction; this is not very clear and it needs to be verified thoroughly.

So, as in the earlier lectures, I have mentioned that the advantage of Ferdinand at all expression is that we can have limited data sets by probably varying two different matric suctions and conducting tests by varying net normal stress values. So, that if you have four sets of test data, we can obtain all the strength parameters, but that is not no longer valid. So, that is, because the strength parameter phi b is no longer constant and which is highly non-linear.

So, this is just similar to the way the xi f varies. So in fact, all these approaches are similar to what originally the Bishop has proposed. So, the way the tests are conducted for the estimation of xi f, in the same manner series of tests are need to be conducted at over wide range of matric suction values, so, that accurate values of strength parameters can be obtained. So, now, the phi dash c dash and phi b which is function of ua minus uw this is Ferdinand approach. And phi dash c dash and xi f which is again function of ua minus uw which is Bishop's approach and phi dash c dash and k, k is this empirical constant which is approach and these are all similar. In fact, and you require same type of data to evaluate these strength parameters. So, the equation slightly differ by different symbols, but the essence of all these equations are one and the same.

Thank you.