Unsaturated Soil Mechanics
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Week - 09
Lecture - 27
Extended M-C Criterion – II

Hello everyone, we have been discussing the interpretation of suction controlled direct shear test data using Extended M-C Criterion. So, let us discuss how to interpret the suction controlled triaxial test data using extended MC criterion in this lecture.

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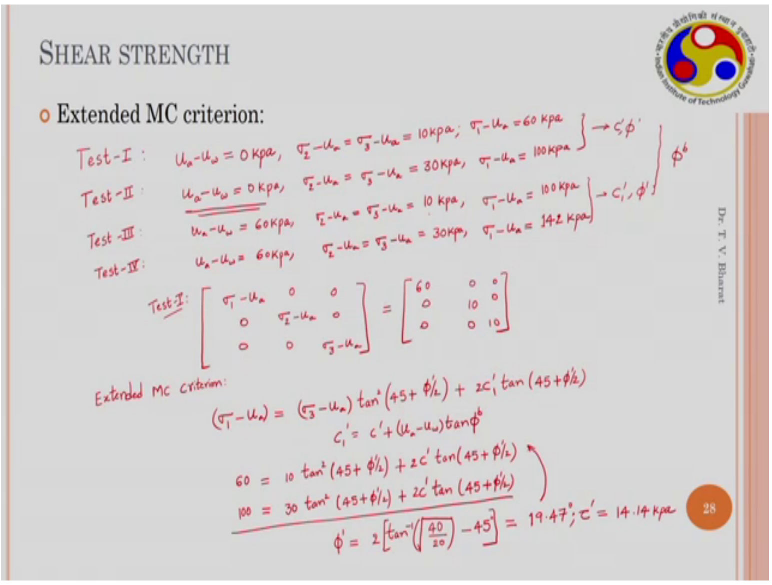

So, in triaxial test setup, which is a suction controlled, here we independently controls u a u w. So, therefore, you can controlled the suction and you can also control the all round pressure and the deviatoric stress. So, then you apply the deviatoric stress, so that you get the shear failure and you get the deviatoric stress at failure and pore water pressure etcetera. Then you can strength of the soil at different suction values using extended MC criterion so, let us solve one simple problem.

So, here we have some synthetic data that is a data representative of extended triaxial test suction controlled triaxial test. So, you have series of tests conducted say test I, test 1 consists of u a minus u w, so where the suction is maintained to be 0 kilo pascal. So, essentially the test is conducted at fully saturated state. And the all round pressure, so there is a sigma 2 minus u a, which is nothing but which is equals to sigma 3 minus u a, because this is radial applied in triaxial setup, so which is equals to 10 kilo pascal. And then the failure of the soil took place, when the major principal stress value of 60 kilo pascal.

Similarly, another test is conducted test II. Here the test is again conducted at saturated state u a minus u w is equals to 0 kilo pascal and the similarly the sigma 2 and sigma 3 or sigma 2 minus u a and sigma 3 minus u a both are increased. So, then the failure took place, when sigma minus 1 a 1 sigma minus u a of 100 kilo pascal. So, this is one set of data conducted at saturation state, so that we can get up you can obtain C and phi values.

Another set of test data conducted say test III, where u a minus u w is maintained to be 60 kilopascal and sigma 2 minus u a and sigma 3 minus u a both are at 10 kilo pascal. So, this is similar to the first test, except that the suction value is a non-zero or it has some suction value, the test is conducted at unsaturated state. So, obviously we expect that when the suction is increased, even though all round pressure is same, you expect to fail at higher load, so this would fail at 100 kilo pascal.

Similarly, another test is conducted test 4, where the suction is maintained to be constant 60 kilo pascal, but the all round pressure is changed to same value as test 2, so that is 30 kilo pascal. So, as the all round pressure is increased by keeping the suction value constant, then definitely the sigma 1 minus u a increases, so this value becomes 142 kilo pascal. So, it failed at higher major principle stress. So, higher deviatoric stress it fail so, this is another set of data.

So, from the first set of data, one can obtain the C value and phi value. And from this data, again you get C 1 dash here C dash and you get phi b. Now, here again you get phi dash so, using these two test data, you can get phi b. So, this is how we get, let us understand how we can solve this problem. This also can be represented in a matrix form, the test results one can be represented as sigma 1 minus u a, when does sigma 2 minus u a, sigma 3 minus u a, this is 0, this is 0 is equals to 60, 10, 10. So, it can be represented in this manner also, this is the test 1, either way we can represent the data in either way.

So, now the extended MC criterion given by Fredlund et al that is a sigma 1 minus u a is equals to sigma 3 minus u a times tan square 45 plus phi by 2 plus 2 C dash tan 45 plus phi dash by 2. Here, this is C 1 dash; this becomes C dash, when matric suction is 0, so, this is the expression we use for saturated soils also. So, when this is when you have suction in the soils, when this is defined for unsaturated soils, this is C 1 dash, where C 1 dash is equals to C dash plus u a minus u w times tan phi b fine.

Now, first test data we can substitute the values of sigma 1 minus u a that becomes 60 kilo pascal and sigma 3 minus u a is 10 kilo pascal, then tan square 45 plus phi dash by 2 plus 2 c. Here, anyways the u a minus u w is 0, so I substitute the value of 0, then it C 1 dash is nothing but C dash. So, this is 2 C dash tan 45 plus phi dash by 2.

From the 2nd test data, again we can write this one as. Solving these two equations, we get if you subtract the 1st equation from the 2nd equation, then we get phi value. Phi dash value is nothing but 2 times tan inverse square root of 40 by 20 minus 45, then this is equals to 14 19.47. So, phi dash is 19.47, if we substitute this value in this, we get C dash is equals to 14.14 kilo pascal, of course this can be substituted into the other equation also. Either way you get the C dash the cohesion intercept, which is 14.14.

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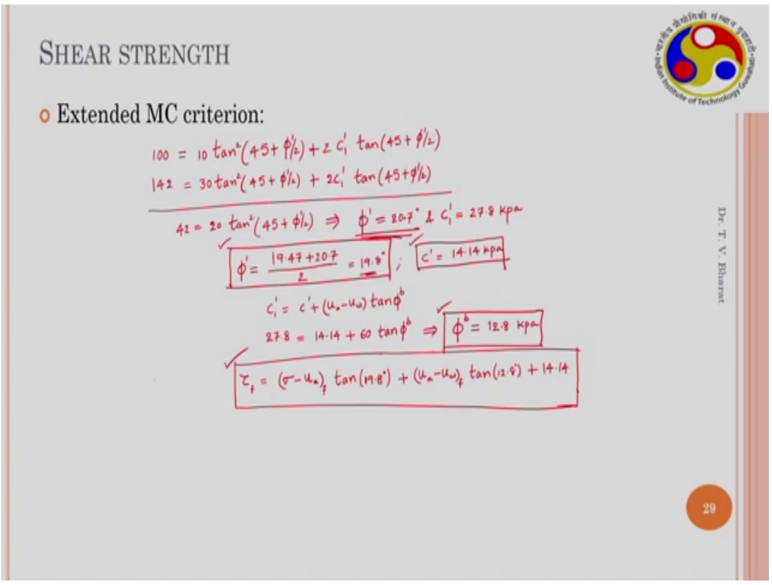

Similarly, we can use the other test data test 3 data when we use, so we get 100 is equals to 10 tan square 45 plus phi by 2 plus 2 C 1 dash, here you have an intercept, because the suction value is non-zero tan 45 plus phi dash by 2. Similarly, the 4th data here this is 142, which is equals to 30 tan square 45 plus phi dash by 2 plus 2 C 1 dash, these are same C 1 dash, because at the same suction value, the tests are conducted plus phi dash by 2.

When this is solved, you get this expression so, when you solve for phi dash, phi dash is 20.7 degrees. And when you substitute either of these equations, then you get C 1 dash, which is equals to 27.8 kilo pascal. So, if you observe, if you notice the phi dash value phi dash value from the first two test data, you got 19.47 so, from the last two test data, you got 20.7.

So, when we take average value, so this is 19.47 plus 20.7 divided by 2, this gives a value of around 19.8 degrees and C dash anyways we have 14.14. So, we got two of the strength parameters is estimated. We can estimate phi b dash by using those two expressions, where the C 1 dash is equals to C dash plus u a minus u w tan phi b.

If we utilize this expression, C 1 dash is known that is a 27.8, C dash is also known that is 14.14 plus u a minus u w, when the intercept is C 1 dash is 60 kilo pascal that is into tan phi b. So, when we solve this, we get phi b, which is nothing but 12.8 kilo pascal. So, phi b is less than phi dash in this case. So, these three are the strength parameters of the extended MC criterion given by Fredlund et al so, this is how we interpret the triaxial test data.

And then the you can write the expression that is a tau F is equals to sigma minus u a F tan phi dash is 19.8 plus u a minus u w F tan phi b is 12.8 degrees plus C dash, C dash is 14.14. So, this is the modified MC criterion for this particular soil. So, therefore, at any given stress state sigma minus u a and u a minus u w, the strength of the soil shear strength of the soil can be estimated using this particular expression. So, the stress state of the soil is defined, then we understand the behaviour.

If the stress state changes with time due to some monsoon season, during monsoon season generally the water infiltrates into the ground or infiltrates into the soil slopes, then the suction values decrease. When these suction values decrease, how the strength changes can be evaluated using this particular expression. So, the same thing can be solved graphically similar to what we did for direct shear test data, let us examine, how we do it.

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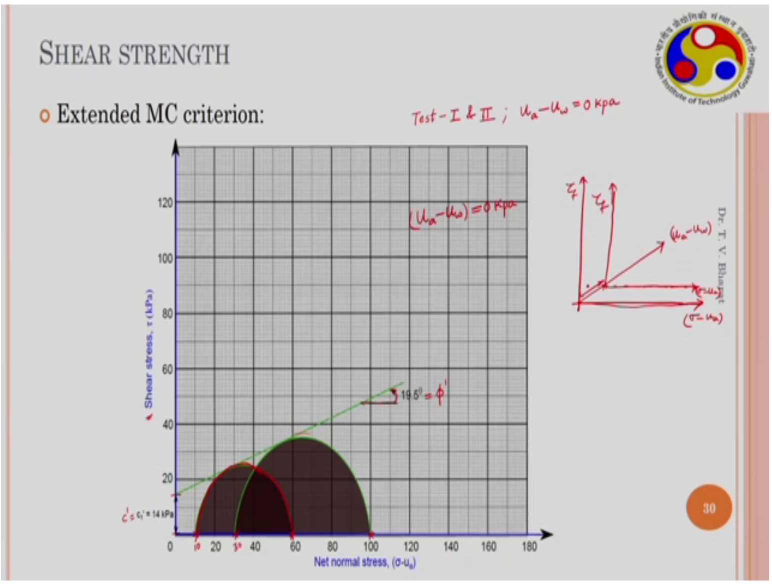

So, here in the extended MC criterion, we have all the test data given so, this is a using test I and II data sets. This is at one particular u a minus u w that is equals to u a minus u w is 0 kilo pascal. So, the three-dimensional diagram, there is a three-dimensional surface failure surface can be now approximated tau f and this is sigma minus u a and other axis is u a minus u w. So, this is a three-dimensional thing, we use to plot earlier.

Now, because we are plotting at one particular u a minus u w, so you can now consider tau f here and sigma minus u a here and this is at one particular u a minus u w that is u a minus u w in this particular case is 0 kilo pascal. So, as is 0 kilo pascal, you can directly plot it here itself, this is 0 now. So, directly you can plot it here, so that is what we have done, the tau f or the shear stress on y-axis and net normal stress sigma minus u a on x-axis.

So, when this is plotted, now we have two test datasets. So, two test data sets, one is 1 sigma 1 minus u a sigma minus u a is 60 kilo pascal, this is 60 kilo pascal and sigma 2 minus u a and sigma 3 minus u a both are 10 kilo pascal, so this is 10. So, this is one more circle and similarly, the other test data that is a sigma 1 minus u a is 100 kilo pascal and sigma 2 minus u a are sigma 3 minus u a is 30 kilo pascal, so this is second more circle.

So, you draw a line joining to joining these 2 circles so, this is the line you get and the angle of this line is phi dash angle of internal friction. The intercept here is C dash itself, because u a minus u w is 0 here. So, the C 1 dash is equals to C dash that is a 14.14 or 14 kilo pascals so, this graphically we can directly get this data.

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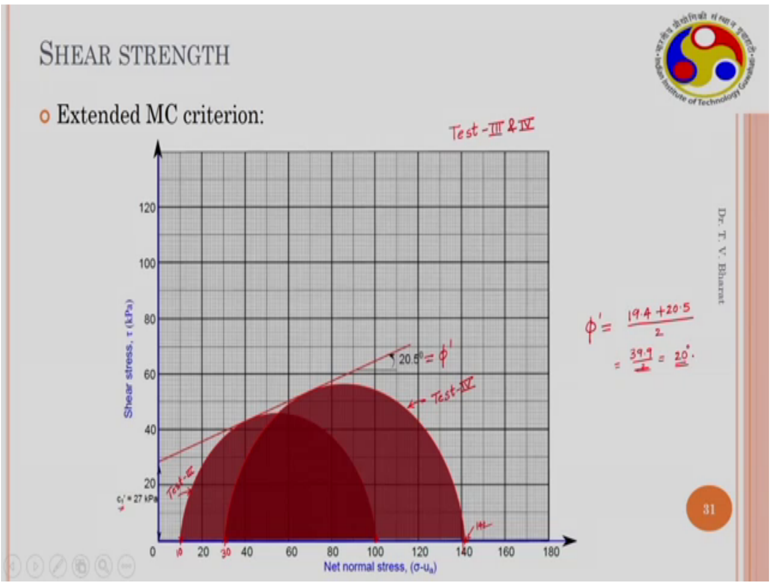

Second, using 3rd and the 4th test data that is test data III and IV we can obtain shear stress versus net normal stress by recognizing the values of sigma 1 minus u a and sigma 3 minus u a. Sigma 3 minus u a value is sorry, for this particular data that is test data III, this is a test data 3 test III. Test III data the sigma 3 minus u a data is 10 kilo pascal and this value is 100 kilopascal, sigma 1 minus u a is 100 kilo pascal. And similarly, the test IV, the sigma 3 minus u a is 30 and sigma 1 minus u a is 140 142 kilopascal so, this is 140 and this is 142.

So, now again we join a line to these two circles so, this is a failure plane. So, this failure plane once it is drawn, the angle is this is angle of internal friction that is 20.5 degrees and the intercept is C 1 dash, this is 27 kilo pascal. So, now you got C 1 dash and phi dash. Now, you can take average value of phi dash, earlier you got a 19.47 or 4, now 20.5. So, we take the average and we get the average value nearly 20 nearly 20 degrees we get.

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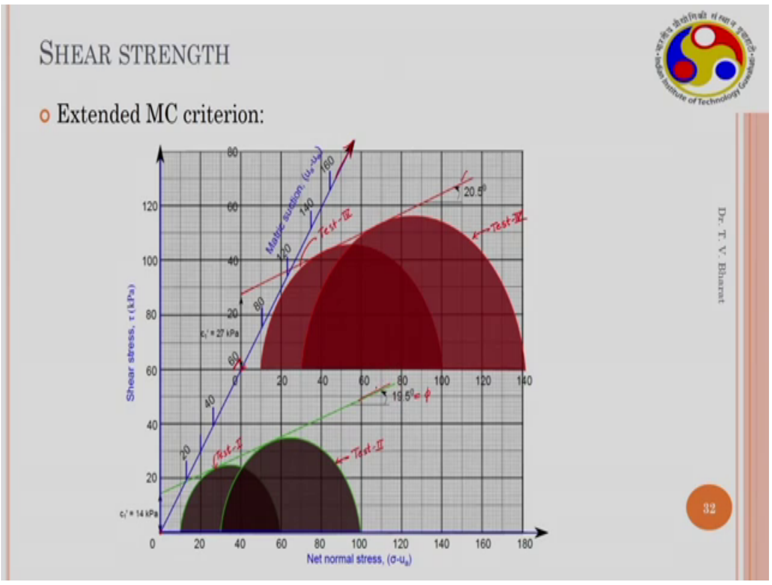

So, this can be represented in this particular manner. The first test data is this test I and this is test II and this is test III and this is test IV. So, this test I and test II both are conducted at u a minus u w of 0. So, this is u a minus u w axis, the z-axis so, when u a minus u w is 0 on this axis, this particular test data is conducted.

So, tau f versus sigma minus u a at u a minus u w is equals to 0 so, this is the data so, this is the failure plane that is angle of internal friction is 19.5. Similarly, at u a minus u w value of 60 kilo pascal, this tests are conducted. So, this is represented in this manner and this line is the failure plane line that is 20.5 degrees and which is nearly parallel to this line, because the values are very close right.

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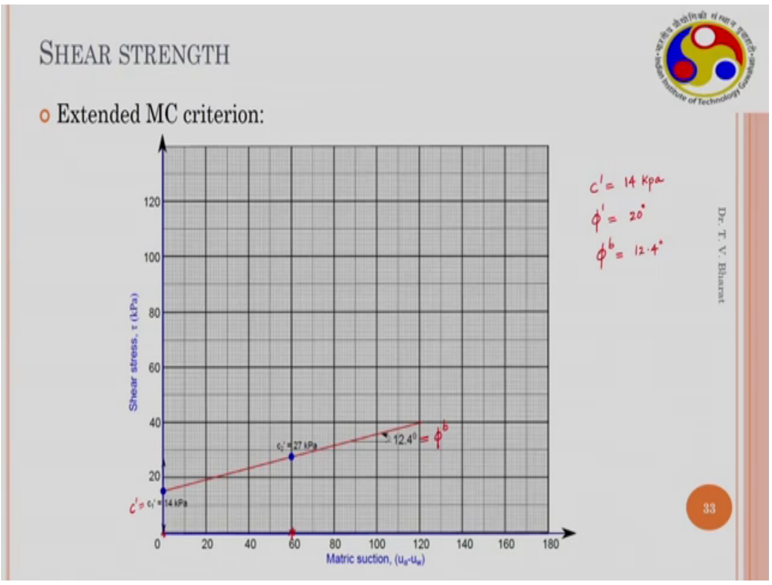

Now, as we have the values of C 1 dash equals to C dash from the first two test data, which is which are that are conducted at 0 kilo pascal u a minus u w. Though, so the matric suction is 0 correspondingly, the values of value of this one is 14 kilo pascal. So, corresponding to matric suction 0, the C 1 value is C 1 dash is equals to C 1 that is 14 kilo pascal. And corresponding to matric suction 60, the C 2 dash value is 27 kilo pascal, so another intercept.

So, now if you join these two lines, you get phi b that is 12.4 degrees. The intercept, in fact is anyways it is starting at matric suction 0, so you got C dash. So, therefore, C dash is estimated, which is 14 kilo pascal. And phi dash is estimated, which is 20 degrees and phi b is estimated, which is 12.4 degrees. This data, which is obtained, graphically is very close to the analytical technique. This data is very close to the data obtained from the analytical technique.

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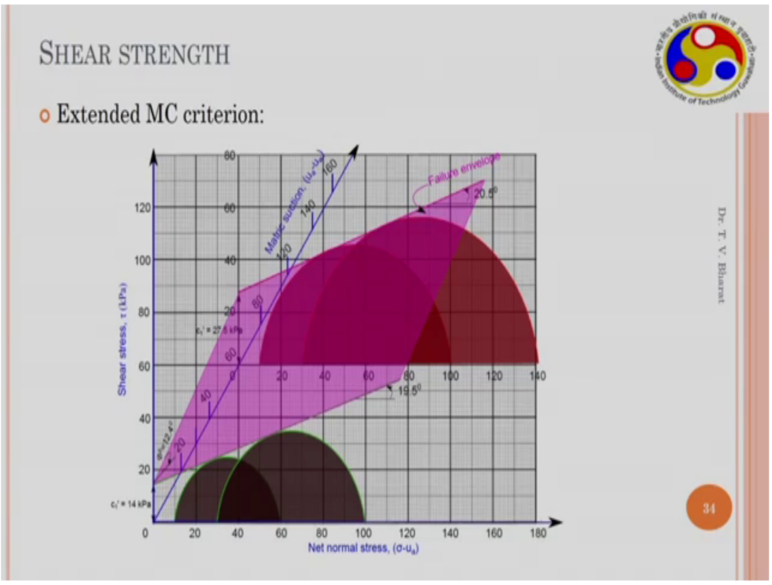

Now, knowing these three parameters estimated, now you can draw the failure surface. So, the extended more MC criterion gives a surface so, this is the surface, the pink coloured one is the surface failure surface. So, here earlier data which we have drawn from the test 1 and test 2 and test 3 and tests 4 so, this is estimated to be 19.5, which is phi dash. And this 20.5 is also phi dash, you can average it and give the values value of phi dash has 20, so you can in fact, take phi dash as 20 in both the cases, but here it is as it is drawn.

So, then here from here to here, here the intercept is 27.5 kilo pascal from this point at 60 kilo pascal u a minus u w. So, this is a 27.5 kilo pascal intercept and here the intercept is 14 kilo pascal. So, line joining these two with respect to the axis, which is parallel to this one matric suction line in between these two the line, which joins the two intercepts. And another line, which is parallel to the matric suction line, the angle made between these two lines is your phi b, so that phi b is 12.4 degrees.

So, for this surface, this angle or this angle is 20 degrees that is phi dash and this angle with respect to the u a minus the line parallel to u a minus u w is phi b. So, once phi b is known and phi dash is known, we can draw the failure surface. So, this is the failure surface for the extended MC criterion using triaxial test data.

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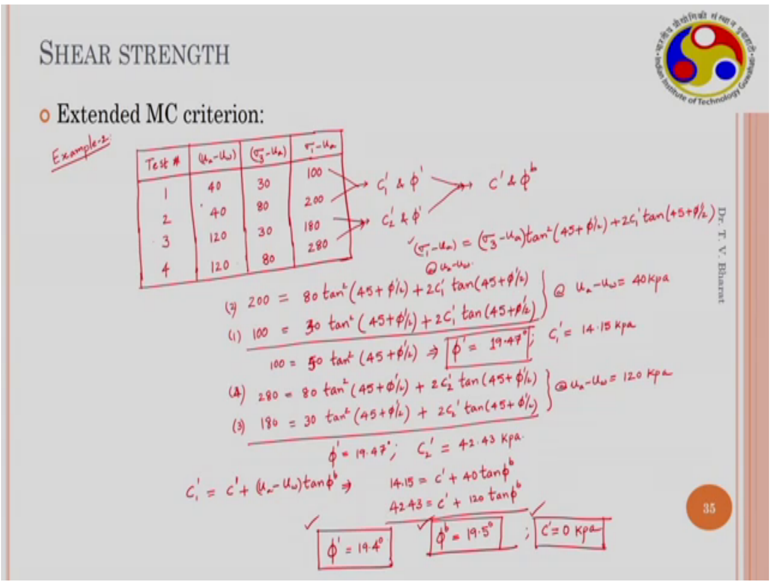

Let us solve another problem, so, here the data is summarized in this manner. So, test data and u a minus u w, so which is controlled and sigma 3 minus u a, which is maintained and sigma 1 minus u a, which is observed. So, now test 1, the u a minus u w is maintained to be 40 and the sigma 3 minus u a is maintained to be 30 and this sigma 1 minus u a is observed to be 100. For test II, this is again 40 u a minus u w is 40 and sigma 3 minus u a is 80, so at higher all round pressure the test is conducted by maintaining the same matric suction, therefore the sigma 1 minus u a should now increase, this value is 200.

Then now, the matric suction is increased to 120 kilo pascal and the sigma 3 minus u a is now 30, then sigma minus u a should be higher than 100, which is observed to be 180. And the IV test with the same matric suction, but increasing the all round pressure 280, the same value as the second test now they should be higher than 200, because the matric suction is higher than the second case, so this is 280 so, this these are the observations. Here there is one interesting aspect that is a reason why I am actually discussing this particular example so; here no test is conducted at fully saturated state.

In case if of Bishop's effective stress, Bishop's extended MC criterion, Bishop's modified Mohr Coulomb criterion to estimate c phi values and psi f, one needs to conduct the tests at fully saturated state, at least two tests should be conducted. If you have cohesion intercept also, then minimum two tests need to be conducted, so that you can estimate cohesion and an angle of internal friction. And when these two values are estimated, other tests can be conducted at higher matric suction values, so that you can obtain psi f value at corresponding matric suction value.

However, in the in this case, even though you have cohesion intercept, you do not need to conduct the tests at fully saturated state without knowing the test data at fully saturated state, you can obtain the entire Mohr Coulomb failure, you can obtain the failure surface. Let us see how we have obtain? So, briefly I discussed this using these two test data sets using these two test data sets, you can obtain C 1 dash and phi dash. And using these two test data sets, you can obtain C 2 dash and phi dash. So, using these two test data, you should be able to get C dash and phi b, let us see how we get.

So, from the first two sets of data, we get when we substitute in the MC criterion, there is sigma 1 minus u a is equals to sigma 3 minus u a tan square 45 plus phi by 2 plus 2 C 1 dash tan 45 plus. This is at one particular u a minus u w, so, here when we substitute sigma 1 minus u a from the 2nd test data, this is 200 is equals to 80 tan square 45 plus phi by 2 plus 2 C 1 dash tan 45 plus phi dash by 2. From the 1st test, this is 100 40 tan square 45 plus phi dash by 2 plus 2 C 2 dash, because this is conducted at one particular suction, which is not same as the suction value, which is used in the first two test data Ist test sorry, here the this is C 1 dash, because this is conducted at the same suction.

So, now when you simplify this simply 100 and this is 40 tan square 45 plus phi dash by 2. Thus, if you solve for phi dash, you get phi dash is equals to 19.47 degrees, which is same value as we obtained in the previous case. So, you can obtain the C 1 dash C 1 dash is equals to 14.15 kilo pascal. Just in the previous cases, you can directly substitute phi dash will phi dash value in one of these expressions, you can get c 1 dash, which is 14.15 kilo pascal.

Similarly, this is 1 and this 2, when you use sorry this is 2 and 1 so, this is 2, and this 1. And similarly, when you use 4, 280 is equals to 80 tan square 45 plus phi by 2 plus 2 C 2 dash, because these two are conducted at u a minus u w of 40 kilo pascal. And this one is 180 30 sorry this is 30, so this is 50, so that is why this is half, and you get the same value phi dash. So, here this is 30 tan square 45 plus phi dash by 2 plus 2 C dash tan 45 plus phi dash by 2. These two are u a minus u w of 120 kilo pascal; this is 4th and this

3rd data. So, when we solve this, we get the same phi dash, which is 19.47 degrees, and C 2 dash is 42.43 kilo pascal.

So, from the expression, that C 1 dash is equals to C dash plus u a minus u w times tan phi b. Using this expression, you can substitute C 1 dash that is 14.15 is equals to C dash plus u a minus u w is 40 tan phi b. And in the second expression, C 2 dash is 42.43 is equal to C dash plus 120 tan phi b. When you solve this, you get phi b is equal to 19.5 kilo pascal, which is same as your phi, phi is nearly same and the C is 0 kilo pascal.

So, when the intercept this is intercept is 0 that means, this is sand, the test data is for sand. So, you got phi dash, which is same as 19.45 or 5 kilo sorry degrees, sorry this is degrees. So, you have phi dash, phi b dash and C dash, all these thee strength parameters could be estimated just by utilizing four test data sets. So, these test data sets are essentially we obtained by maintaining one particular suction value, two test data sets could be obtained by changing the all round pressure and another set of data can be obtained by maintaining different matric suction, but changing the all round pressure. So, if we have such data, we can obtain all the strength parameters estimated. Quickly let us see how graphically this could be done.

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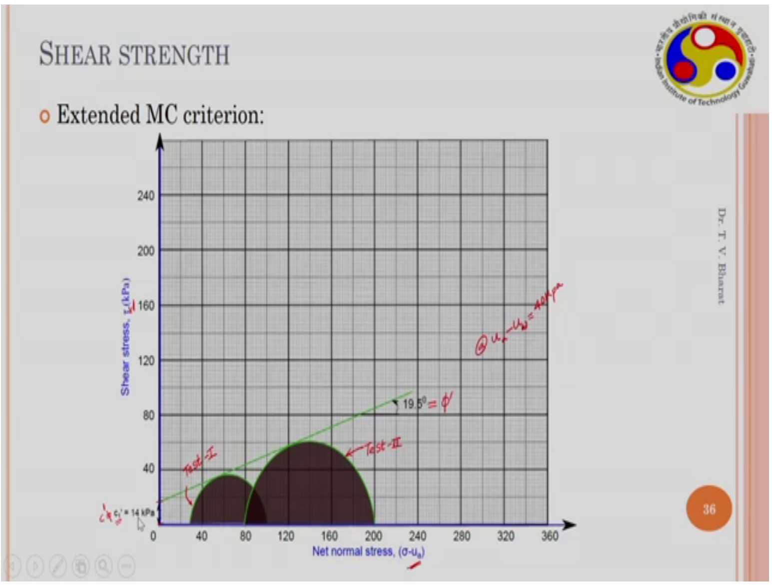

So, here as we have done earlier, the net normal stress that is sigma minus u a and tau f are plotted, this is at one particular u a minus u w that is u a minus u w is 40 kilo pascal. So, at 40 kilo pascal, when we use the test data 1, this is for test data 1 and this is test data 2, then we obtained the angle of internal friction that is 19.5 degrees. And the intercept, C 1 dash this is not equal to C dash, because this test is conducted at one particular matric suction value, so this is 14 kilo pascal.

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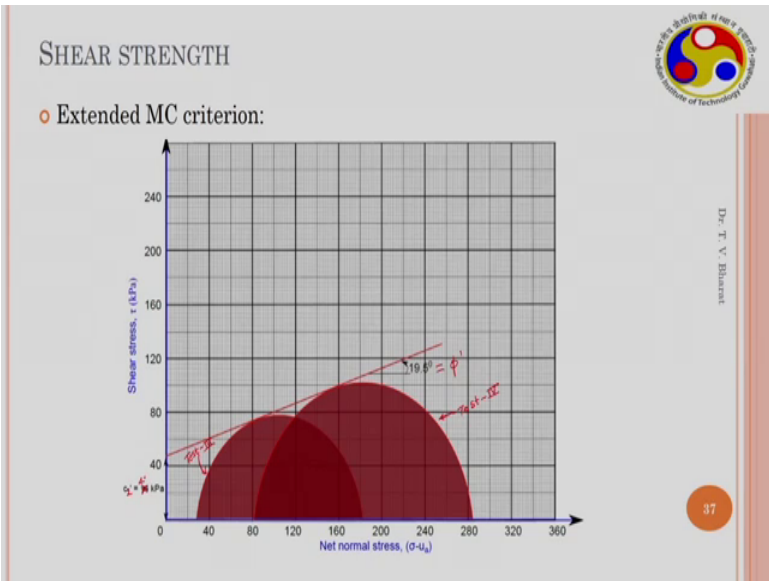

Then the other two test data were used, where this is test data III and this is test IV. So, the angle of internal friction is same that is 19.5 degrees. This is C 2 dash which is not 14; here we could see that this is 42, so these are around 42.5 so, this is 42.5 kilo pascal. So, the angle of internal friction is the angle of internal friction value is same for both the cases, but the intercept value changes from the first set of data and the second set of data because of matric suction value change changed.

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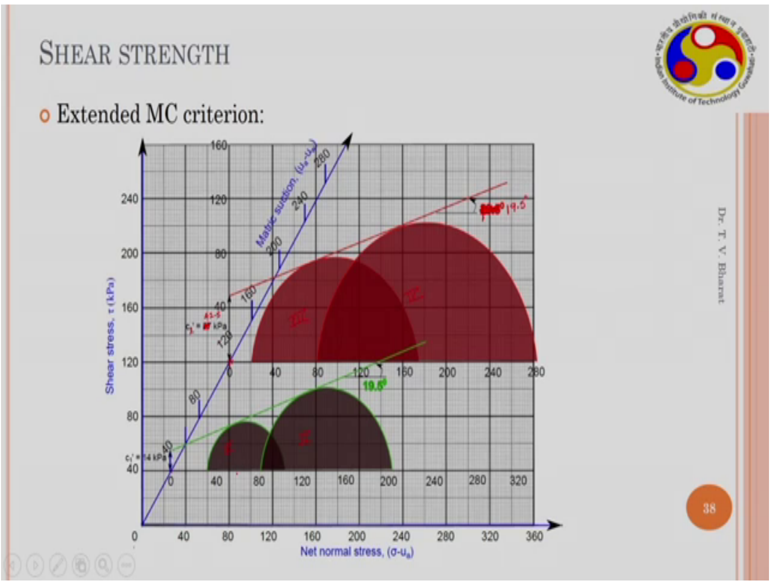

Now, here this is 14, and this is not 27, this is C 1 dash, and this is C 2 dash, this is 42.5 kilo pascal and here 19.5 and this is also 19.5. Sometimes, the measurement errors would be there while plotting. And here this is a test data I, II, III and IV so, this is if you observe, which is on an axis of this is the matric suction axis.

So, this is at a value of 40 kilo pascal so, this is 40 and this is 80, 120, 160. So, here again at 120, so that is another matric suction value, where the tests were conducted at 120, so this is drawn so, this is how now, we locate the test data of all the four test data on the three-axis plot.

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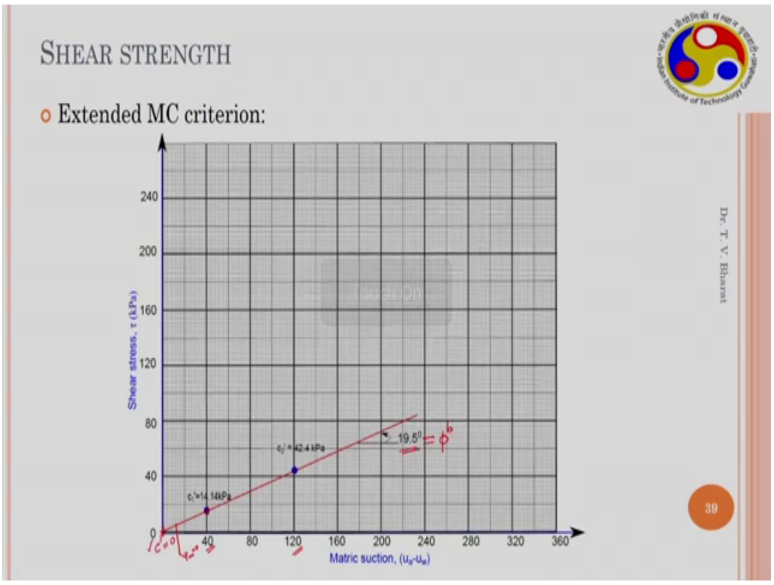

Now, once we have the phi dash, now phi dash in the cohesion intercepts, the first intercept value 14.14 kilo pascal or 14 kilo pascal, which is obtained at matric suction value of 40. And another intercept is obtained that is 42.5 at 120 kilo pascal. So, when these two test data are plotted on tau f versus matric suction plot so, when you draw the straight line joining these two points, the line passes through origin. So, the cohesion intercept C dash is equals to 0, which is at matric suction of 0 so, this we obtain for graphically. Now, the angle of internal the angle, which indicates phi b is 19.5 degrees so, this data is how we get phi b estimated.

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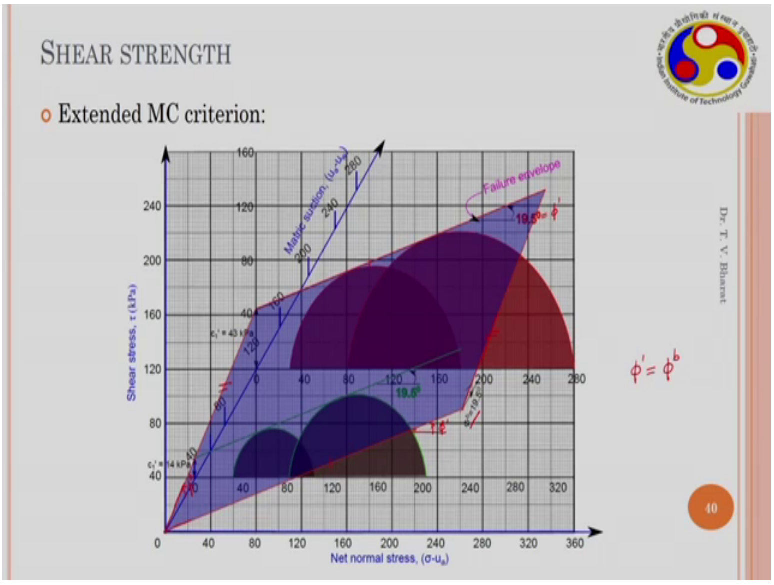

Now, we have all the data phi dash, phi b, and C dash. So, now this data this line is a this line is with phi b sorry this line is with phi dash, this line with horizontal makes phi dash 19.5. And this line joining these two planes is phi b that is also 19.5, for this particular case, in this particular case, the phi dash is equals to phi b. So, here this line joins from origin and at a 40 kilo pascal of matric suction, the intercept is 14.

And at 120 kilo pascal matric suction, the intercept is 42.5. And this line with respect to horizontal line, with respect to the matric suction line parallel to this. This angle is again phi b, in this case, phi b is equals to phi dash. Therefore, this is a 19.5 and this line is parallel to this line these two are parallel. And again, this and these two are parallel and this is phi dash and again this is phi dash. So, this represents a failure envelope, which is a surface so, this particular surface can be drawn due to the model given by Fredlund et al.

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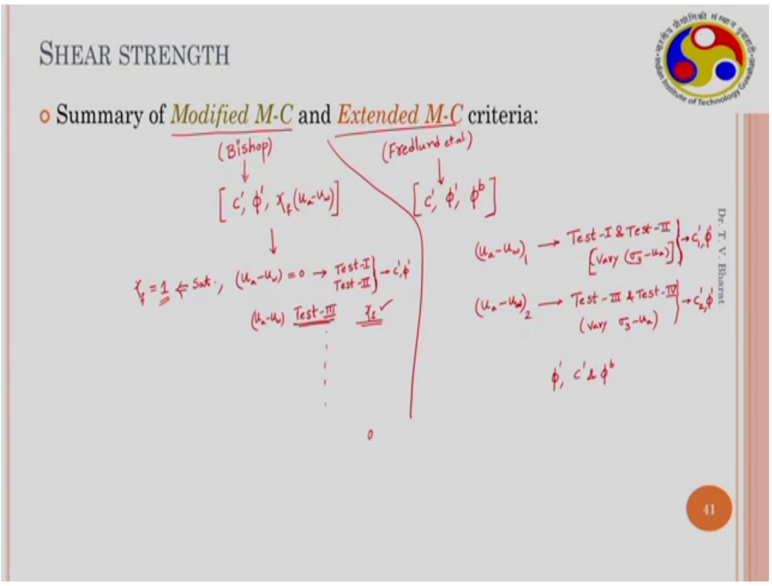

So, in summary, if we compare the modified M-C Criterion, which is given by Bishop and extended Mohr Coulomb criterion, which is given by Fredlund et al, there is a slight difference. Here the strength parameters are C dash, phi dash and psi f, which is a functional form, which is function of u minus u w. In the Fredlund et al, the strength parameters are C dash, phi dash and phi b. So, these are the strength parameters in Bishop and these are the strength parameters in Fredlund et al.

The tests the triaxial tests are direct shear test, we need to conduct these tests sorry to estimate these strength parameters, we require suction control direct shear test data or suction control triaxial test data in a specified manner. For example, in case of Bishop's model, we acquire the test data at u a minus u w is equal to 0 that is at fully saturated state. At fully saturated state, we require 2 sets of data minimum because, from these 2 test of test data, we can obtain C dash and phi dash. If you have sand, then anyways the cohesion intercept does not exist at saturated state, so one test data would be sufficient.

And then you need to conduct series of tests. Test III, IV like that series of test data to obtain the values of psi f. Psi f in this case is 1, but other cases psi f need to be estimated psi f can be estimated using the equations expressions for different suction values. When you maintain different suction values in different tests, you can obtain psi f value. Psi f value changes from one at saturated state to nearly 0 at nearly dry state.

In Fredlund et al case, we do not require series of test data, we just require four test data, two test data should be conducted at one particular u a minus u w, two tests need to be conducted test 1 and test 2. So, in these two tests, you can vary all round pressure sigma 3 minus u a, if it is a triaxial test, or you can vary the net normal stress, if it is a direct shear test. And conduct another two tests, test III and IV again by varying sigma 3 minus u a or net normal stress in direct shear and conduct these two tests.

So, from this test, you obtain the cohesion intercept to one particular cohesion intercept at u a minus u w and the angle of internal friction. And from these two tests, you get another cohesion intercept and angle of internal friction. So, by solving these two expressions for cohesion intercepts, we get C dash, phi dash and phi b by solving these two expressions for cohesion intercept, we get C dash and phi b. Anyways we already got the phi dash, so we get all the strength parameters.

We do not require to conductive in the test set fully saturated state in case of Fredlund et al model that is extended MC criteria. So, this way, we can obtain the strength parameters using modified MC criterion and extended MC criterion by conducting different set of tests under suction control in suction control triaxial or suction control direct shear test.

Thank you.