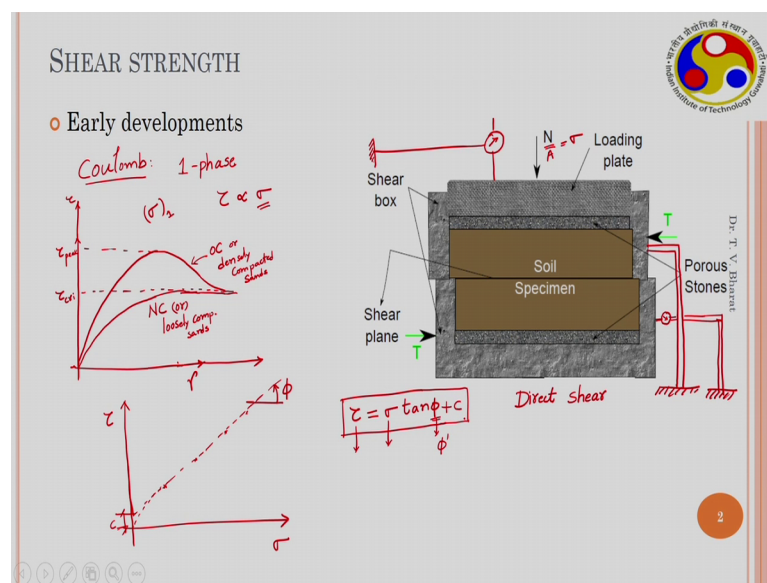


Unsaturated Soil Mechanics
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Week – 08
Lecture – 23
Shear Strength of Unsaturated Soils

Hello everyone. Let us start a new topic Shear Strength of Unsaturated Soils.

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So, shear strength is a very important concept in soil mechanics or in geotechnical engineering because, to support foundations in soils or to support any structures on the soil through foundations, the shear strength of the soil should be sufficient to withstand any loading conditions. Shear strength has interesting historical development. So, let us briefly understand those historical developments and then go in to shear strength of unsaturated soils. So, shear strength the concept of shear strength has been derived or brought into existence in the modern soil mechanics by Coulomb through his work inspired from Leonardo Da Vinci of 16 century.

So, Coulomb in the 18th century has given friction loss that made the understanding of shear strength of soils. So, it was Coulomb in the 18th century gave the friction laws based on the shear strength the soil has been derived and he looks at soil as single phase, he does not differentiate soil and rock. So, generally if you take a soil mass and

the failure of the soil mass along a slip plane, he is observed it could be there for rock or soil it should act in the same manner.

So, a single phase system, similar to the single phase system he considers, soil is like a entire soil is one particular (Refer Time: 02:23) he has considers, just like a rock mass and he proposes the shear stress is proportional to the total stress of the soil. So, based on this the direct shear operators has been developed, so in the direct shear operators, which is very popular shear stress operators which we use after in soil mechanics laboratory.

So, in direct shear operators soil mass is compacted and it can be it is like two half system and one half can be moved against another half. Generally, the one half can be fixed, supplying the shear stress other half can be moved related to the other half. So, normal load of N can be applied. So, divide by the cross sectional you should get the total stress which is acting on soil mass.

So, this is a total stress that is acting on a soil mass this is the total stress. A simply the stress which is acting on a soil mass and shear is applied and soil moves relative to other half and dial gauge can be loaded and which can be connected again to a fixed system. So, that this dial gauge can record what is the amount of strain that is taking place. Similarly one more dial gauge can be loaded, kept on the system, kept on the top of the surface.

So, therefore, vertical movement during shear operation and horizontal movement or the shear strain can be recorded at any given instance. So, this led to shear stress versus shear strain diagram for different soils. For OC soils, for normally consolidated soils they exhibit this kind of behaviour. They normally consolidate clays or loosely compacted soils, soils or sands; they exhibit this kind of behaviour and over consolidated clays or densely compacted.

Sands exhibit this kind of behaviour and there they reach critical state at one particular strain. So, this is τ critical and this is the τ peak, this is τ peak. So, this is exhibited in the system, but initially the direct shear apparatus meant for only dry sands, this is actually meant for dry sands and however, we using this for measuring shear strength of different soils, the clay soils and (Refer Time: 06:08) soils and also including some moisture. So, generally this is not meant for moist soils, there are dry soil, dry sands soils

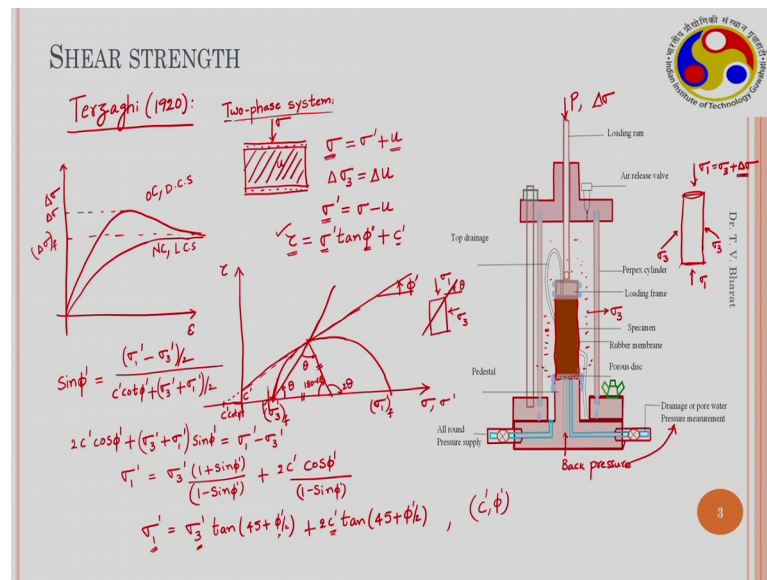
are generally used in this particular operators. So, this lead to shear stress versus shear strain and this is under one particular normal stress applied.

Similarly, the test can be repeated with different normal stress; therefore, one can obtain the relationship between normal stress and shear. So, you get as the normal stress increases, the shear stress peak stress increases. So, based on that either you get this kind of a linear relationship between tau and sigma. However, there are there is lot of debate why this does not follow through origin and there are some schools of thought they believe that it should a non-linear behaviour something like this and it should not be linear. But let us assume that it has cohesion and an angle of internal friction phi, but generally for sands the line passes through origin. In case of clays you have a cohesion term that is coming into picture and in unsaturated shear test also you get cohesion term and we will discuss about that little later.

So, generally this c cohesion term is absent in case of sands. So, essentially this lead to tau is equals to sigma tan phi, this is the relationship you obtain which is attributed to Coulombs friction loss this is on the failure plane. So, this phi is a material constant angle of internal friction and sigma is normal stress or total stress, which is a stress state variable and tau, is a shear stress and this is also state variable. So, here one can add cohesion to make it more generalized, some schools of thought they do not mind adding cohesion term along with this.

So, this is essentially to attribute whatever the non-linearity or to attribute the intercept which is coming into picture while plotting tau versus sigma data. So, this is what we get.

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So, however, Terzaghi called Terzaghi 1920, he realises that the total stress concept should be revised because he looks at the soil as two phase system; soil solids and water. When you have such a system, the strength of the soil which is exhibited during the application of the loading can differ, depending on the boundary conditions. So, for example, if I have a soil system are solids plus water system, your consolidation test also we have seen that when you apply a load.

So, initially there is no settlement that is because, the hydraulic conductivity of the soil is very less that water does not come out. So, depending on the drainage condition soil exhibits different strengths. That is because, when you apply normal load the poor water pressure can be built up to the same value as you applied to the applied normal stress and soil does not fail and soil behaves in different manner altogether. And if the water is allowed to come out through the boundaries then, it exhibits different strength. So, then shear strain effects started coming into picture and so two phase system so then, the modification to the total stress concept has come into picture.

The plain normal stress is taken up by the soil solids that is a sigma dash effects to stress plus poor water pressure. So, when the drainage is absent, there is no drainages then what is loading condition that is applied, there is normal stress which should be equal to the u and sigma dash is absent. So, in triaxial test so, the concept of triaxial test come out

where, we can control the drainage very well, in the direction operators we cannot control the drainages.

So, therefore, generally we do not use soils with certain good amount of moisture in it for shear strength testing. So, we take soils in triaxial test operators and here sorry the triaxial test operators is designed for testing soils construing drainages conditions. So, essentially the soil is seen as a two phase system, so fully saturated conditions. So, soil sample is prepared of standard sample 38 mm by 76 mm length 38 mm diameter and 76 mm length.

And the samples are fully saturated through the back pressure valve ok. So, the sample is placed on porous disk and the sample is surrounded by the flexible membrane so that it does not directly interact with the water, which is available all around. So, here the water is allowed around the sample and now by applying pressure in this water, we can maintain all round pressure. So, σ_3 all round pressure can be maintained around the soil sample and back pressure valve is connected and through which we can saturate the soil sample.

So, here once the soil is completely saturated, so whatever the amount of σ_3 you apply, here, the porous pressure within soil sample would be same. So, whatever the σ_3 you apply, $\Delta \sigma_3$ whatever the change you make all around the sample that would be equal to Δu , in the beginning in the test. And to may, to apply the shear stresses on the soil loading ram is placed and load is applied vertically. So, this deviate the stress from the all round pressure or consolidation pressure and which shears the soil sample. So, as you can control the drainage valve, so this the back pressure valve, it is a same. So, in this particular case as you can as you can control the drainage and we can conduct different test all together.

So, here as of σ' , the affect to stress concept has been brought in by Terzaghi, which is $\sigma' = \sigma - u$. σ' affects to stress is a one which controls the share of the soil. So, therefore, Coulombs equation is modified to be $\tau = \sigma' \tan \phi' + c'$. So, this is effective stress and this ϕ' and c' are the effective metal constants are ϕ' is a effective angle of internal friction and effective cohesion. So, τ_w we do not use τ_w because, water does not contribute to share. So, this is τ is equals to τ only.

So, this is the equation that is modified based on Terzaghi's inputs and therefore now, depending on the drainage conditions the soil exhibits different strengths and the consolidated generally different test can be conducted in this particular set up. So, you can apply all round pressure σ_3 and you can consolidate the soil to whatever the extent possible or whatever the extent you want to consolidate and after that shearing is started, so that the shear stress are applied on the soil and so on the soil sample you can apply σ_1 on one direction and σ_3 on the other direction.

This is in radial direction. So, if the soil sample is shown like this, so this is in radial direction which is acting. So, this is σ_1 , this is σ_3 . So, the σ_1 is nothing but σ_3 plus $\Delta\sigma$. So, this is a deviator stress. So, soil fails under this combination of σ_1 and σ_3 . So, essentially you get with deviatoric stress, the actual strain can be measured. So, here also you can obtain for OC soil and NC soil are loosely compacted and densely compacted sands are coarse grained, soils loosely compacted sands this is a behaviour that is executed.

So, you have $\Delta\sigma$ at peak, but we consider the critical states critical values. So, this is a $\Delta\sigma$ at failure, if you consider this is a $\Delta\sigma$ at failure under certain strain values then, here as the failure plain direction is not known. So, we cannot directly use a Coulomb's principle we have to utilise Mohr theory as well. So, this is a Mohr Coulomb concept. So, the Mohr Coulomb concept you have τ_1 versus σ or σ dash on the x axis. So, then one test with σ_3 has initially the soil state is this, initially soil is consolidated to say σ_3 . Now, keeping the σ_3 constant $\Delta\sigma$ is increased, that means, this is σ_1 .

And which is continuously increased and increased to one particular value where it fails. So, finally, this is the this is the value at failure. So, we can conduct many more tests so that we can obtain the failure plain, this could be somewhat like this. So, here this is angle of internal friction, ϕ dash on this axis this is cut at say one particular point here and this is c dash and if you join the centre here and pool for planes is this because the on the soil sample, σ_1 is acting on horizontal plain and σ_3 is acting on vertical plane.

So, σ_1 is acting on horizontal and σ_3 is acting on vertical plain, this is pool for planes. So, p so front pole for plains, if you draw a line joining the tangent, so, this is a

failure plain, we can call theta. So, there is a point at which this is a failure plain direction. So, this is a failure plain direction, it should be same as theta. So, now, as this side is same as this side, this radius of the more circle and this side opposite angle is theta. So, this therefore, it should be theta 2 and this should be 180 minus 2 theta and this is 2 theta. And this is the $c \text{ dash cot } \phi$, this from this value is $c \text{ dash cot } \phi$.

And if you take $\sin \phi \text{ dash}$, $\sin \phi \text{ dash}$ and this is σ_1 , minus $\sigma_3 \text{ dash by } 2$ divided by so, this is again $c \text{ dash cot } \phi$ plus this is σ_3 f these are all f at failure, $\sigma_3 \text{ dash}$ this is the effect of stress we are plotting. So, $\sigma_3 \text{ dash plus } \sigma_1 \text{ dash}$ $\sigma_1 \text{ minus } \sigma_3 \text{ by } 2$, so for therefore, it is $\sigma_1 \text{ plus, } \sigma_1, \sigma_3 \text{ by } 2$ So, if you simplify this, this comes out to be $2 c \text{ dash, } \cos \phi \text{ dash plus } \sigma_3 \text{ dash plus } \sigma_1 \text{ dash equal to } \sigma_1 \text{ dash minus } \sigma_3 \text{ dash}$.

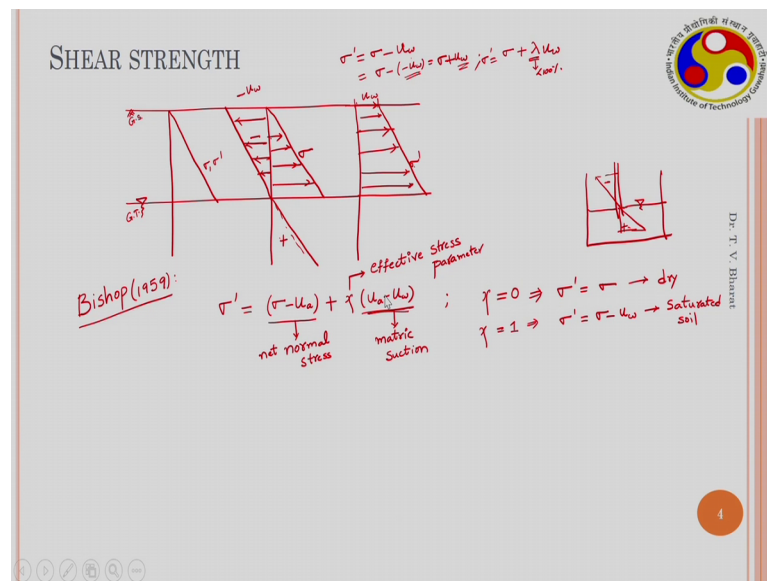
We simplify this sorry this is $\sin \phi$, $\sigma_3 \text{ dash plus } \sigma_1 \text{ dash times } \sin \phi \text{ dash}$ equal to $\sigma_1 \text{ dash minus } \sigma_3 \text{ dash}$. If you simplify this we get $\sigma_1 \text{ dash}$ is equals to $\sigma_3 \text{ dash times } 1 \text{ plus } \sin \phi \text{ by } 1 \text{ minus } \sin \phi \text{ dash plus } 2 c \text{ dash } \cos \phi \text{ dash}$ divided by $1 \text{ minus } \sin \phi \text{ dash}$ So, this is $\sigma_1 \text{ dash into } \sigma_3 \text{ dash tan square } 45 \text{ plus } \phi \text{ dash by } 2 \text{ plus } 2 c \text{ dash tan } 45 \text{ plus } \phi \text{ dash by two}$. So, essentially if we have two sets of σ_1 and σ_3 at failure, so this is from one test. If you have two sets of the data we can determine c and ϕ , if you do not have c cohesion it is for only pure sands then if you have one set of data you can determine what is the angle of internal friction.

This is using formula or graphically you can plot and then you can determine $C \text{ dash}$ and $\phi \text{ dash}$ it is are the affects of stress parameters. Similarly, here these effective stress parameters you can obtain when the drainage is allowed. So, when the drainage is open. So, then (Refer Time: 22:44) pressure dissipates at any given instants then you will get $C \text{ dash}$ and $\phi \text{ dash}$. So, if the drainage valve is closed during shearing operation. So, then the shear strength that is a exhibited is different and it exhibits s_u undrained shear strength data undrained shear strength values.

So, the either you get total stress parameters or affective stress parameters based on your analyses. You get either shears undrained shear strength or affect to stress parameters like this in a given test. You can you can get both total stress parameters and affective stress parameters if you conduct consolidated undrained test, but measuring porous pressure

during the shearing then you get both combination of affective stress parameters and total stress parameters, that we will useful for their analysis for long term analysis and short term analysis. The affective stress parameters are used for the long term analysis and undrained parameters are used in the short term analysis in case of clays. So, the Terzaghis concept of two phase system that is introduction of affective stress concept helped in understanding the soil behaviour during shearing and the influence of drainages conditions very well.

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In 1959, Bishop the gentleman British gentleman called Bishop came up with, now it comes to unsaturated soils how the soil behaviour can be very well predicted are can be understood very well using the concept of negative for water pressure.

So, if we draw a profile, if we have a soil mass this is a ground surface and this is groundwater table. So, far we have assumed that groundwater table is either at the ground surface or there is no groundwater table exists. So, essentially we consider only two phase system, which consists of whether soil solids and pure air or soil solids and pure water. So, we then in real situation we see that the soil consists of three phases. In that scenario, let us assume that the pure this is the groundwater table and this is ground surface and how the in our earlier assumption where we considered that the if the water table is up to the ok, in the early assumption the what we did is we considered total stress total stress distribution. So, even in the soils it may very differently. So, this is what we

consider and effective stress the pure water pressure does not exist. So, there for this is σ or σ' profile, but however, if you correctly analyse the pure water pressure varies in this particular manner total stress.

Positive in the downward and negative in upward at the hydrostatic condition, this is what you have seen when you have a capillary inserted in water table. In beaker of water a small capillary is inserted a beaker of water. So, then there is a reason water table the pure water pressure distribution if required this is 0 here and sorry this is how it goes. And this is positive and this is negative. This downward positive because water is water level is here and this is positive and this is negative. Similarly at the hydrostatic condition this should be negative this the negative pure water pressure and this is positive pure water pressure so above this is the condition. And if you see your total stress variation this is how the total stress varies.

So, this is σ plot and this is a negative in this direction So, according to the effective stress principle the variation should be the effective stress principle σ' is equals to $\sigma - u$, u means pure water pressure. So, as we so far dealt with the two phase system in basic soil mechanics we generally, use u to indicate pure water pressure. So, here we have two pure pressures, one is water pressure and air pressure to distinguish that we use a suffix w here this is water pressure. So, if $\sigma' = \sigma - u_w$ then $\sigma - u_w$ is negative. So, then they should be $\sigma + u_w$.

So, that means, effective stress should be somewhat like this and it should be plotted from here onwards. So, here whatever the u_w you have that will be here. So, this is this value u_w . Because a σ is 0 there you have u_w and this is how it should vary σ' . But after we have seen that generally if you take dry clays or dry soils the magnitude of u_w is very high. So, negative pure water pressure or suction of the clays theoretical it can go to 10^6 kilo Pascal's nearly dry state or residual state. If this goes that much then the effective stress should be nearly infinity and soil should never fail, but then we see failures very often.

So, therefore, the contribution of u_w , the contribution of negative pure water pressure to the total stress for effective stress calculations, is not 100 percent. So, then what is the question is how much percentage of negative pure water pressure contributes for your effective stress calculation is a question. So, there are several proposals and there are

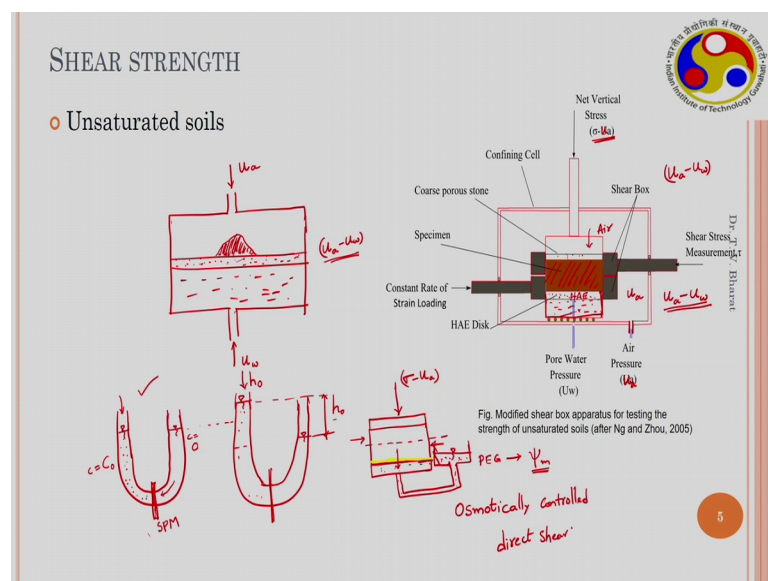
several theories put forward and finally, it is agreed that there is certain percentage which is less than 100 percent will contribute to the effective stress.

So, this is a less than 100 percent. So, now, the Bishop in 1959 has come up with he has come up with 2 stress variables 2 independent stress variables, but in a lamped manner. He has proposed effective stress equation which is $\sigma' = \sigma - u_a + \psi u_w$. So, here $\sigma - u_a$ is normal stress, net normal stress and this is matric suction.

And this ψ parameter is the effective stress parameter, which where is between 0 and 1. If ψ becomes 0, ψ becomes 0 the equation becomes $\sigma' = \sigma$. So, this indicates the soil is completely dry. So, if u_a is used as gauge pressure, then this is $\sigma = \sigma' + u_a$. Say if ψ is equals to 1, then this equation becomes $\sigma' = \sigma - u_w$, which indicates completely saturated soil. This is completely dry. So, $\psi = 0$ indicates completely dry soil and $\psi = 1$ indicates fully saturated soil. So, generally for unsaturated soils this ψ parameter varies between 0 and 1.

And this led to controlling matrix suctions in the shearing test. So, net normal stress is a applied on the soil and how to control the matrix suction is another question, that is how the research began during that time.

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So, then modified shear box test have been developed. So, here in the modified share box tests, so the important inclusion is that the HAE disk, Higher Entry Disk is added. So, this a shear box and you have 2 halves here and soil is specimen is here and here one coarse porous stone is placed and here HAE disk is placed higher entry disk is placed. And this whole setup is confined in a one closed volume air tight volume and which is connected to one air pressure valve and air pressure can be controlled in this particular manner. So, here the air pressure can be controlled in the system.

And the air can go through this coarse porous stone, but then this higher entry disk which is saturated with water chamber here and air cannot enter into it. This is closed and this is completely closed, air cannot enter into it. So, the soil specimen you can take initially saturated soil which is indirect contact with HAE disk and should have a hydraulic equipment in the beginning. Then so through the coarse porous stone, the air can enter, but then the air exert some pressure on the air can enter into it and then it can desaturate the soil sample and it cannot air cannot enter into the HAE disk until the air pressure minus water pressure $u_a - u_w$ is equal to the air entry of the HAE disk.

So, this is exactly similar to what we use in access translation technique. So, in access translation technique you have a HAE disk, higher entry disk. And you have a water chamber; you can control the water pressure. So, generally we keep it we keep this value close to the atmospheric pressure and this is connected to the air pressure valve and u_a is applied here. So, here the $u_a - u_w$ is applied on the soil system here. If you place a soil here this porous disk is completely saturated and this is the soil mass as specimen, specimen is placed here.

So, initially if you keep a saturated soil specimen here, so under the application of $u_a - u_w$ that is a load or suction load is applied then, soil starts bleeding and water comes out and so by measuring what is our amount of water that is come out from the soil mass we can determine the water content and here (Refer Time: 35:15) you are controlling the suction. So, that is how we established soil water characteristic curve.

So, in this particular case, in direct shear operators the same technique is applied. Here the higher entry disk is placed above this the soil sample is placed. So, then you have a water chamber, now here coarse porous disk is place so that air can enter out and. So, by controlling $u_a - u_w$ you can maintain one particular matric suction within a soil

sample. So, by maintaining on particular matric suction under this control matric suction your controlling $u_a - u_w$, similarly the $\sigma - u_a$ is also controlled.

So, under particular net vertical stress $\sigma - u_a$ you can apply in the soil system and under this particular controlling two stress variables $u_a - \sigma - u_a$ and $u_a - u_w$ you can conduct the shearing test right. So, this is how the shear test direct shear test is modified by in cooperating the suction control technique there is access translation technique into this one and shear stress at different control net normal stress and matric suction values can be obtained.

We will discuss the results little later, we will only discuss the; now we will discuss the how the (Refer Time: 36:42) test is modified So, this particular setup is also modified for exerting different stress, this is one particular way of doing it, this is by controlling using the access translation technique or this also can be done by, osmotically controlling the matric suction within a soil mass.

So, earlier I have discussed this particular system, this particular methodology, generally if you look at the principal if you have a U shape tube you have a water level same on both sides initially and if you keep one semipermeable membrane, which is like a filter which you keep and then add some salts and the salt molecule size is such that, it cannot pass through the semipermeable membrane. Semipermeable membrane is placed here, then salt is added on this side, so then if the semipermeable membrane is not present the salt must have diffused the salt solution here high concentration C equal to C_{naught} and here concentration is 0, had it been that the semipermeable membrane is not present.

The salt must have diffused from left hand to left hand side to right hand side, salt concentration should have become same. But however, as semipermeable membrane separates these two chambers and the salt corrupt migrate through this semipermeable membrane. So, water has to move from right hand right side to left hand side and dilute this solution. So, therefore, at equilibrium if you see, so at equilibrium oilier level was somewhere here and now the level is here and the level on the right hand side decreases.

So this is the condition there is a osmatic head of h_{naught} will be available here. So, that means, to make these two level same, you need to apply had cosmetic head of h_{naught} to bring it to this particular level. So, this concept is utilised in modifying the

direct shear test to conduct unsaturated shear stress shear strength stress. So, essentially you will have a soil mass.

And initially it is saturated and then now this is connected to a porous stone and then which is connected to a reservoir of test solution polyethylene glycol, the molecular size of this polyethylene glycol PEG is much larger than the semipermeable membrane. A semipermeable membrane will be placed somewhere here then, this is loaded in the same manner as it was earlier. So, this is the load that is applied or sigma; sigma minus u_a .

So, here the PEG solution is placed in contact with soil, so as the molecules of the PEG cannot enter into the enter through the semipermeable membrane, the water will be drawn out from the soil mass and it moves down and then what the essential PEG solution gets dilute diluted. So, PEG concentration gets changed, and water content in the soil system decreases, which causes by which causes particular matrix suction maintained within the soil mass.

So, this can be obtained because the knowing PEG concentration and what is the size of these molecules and what is the semipermeable membrane size etcetera this can be obtained and standard using the standard techniques this can be obtained and the matric suction within the soil sample can you maintained. After that this is sheared in the same manner. So, either using access translation technique or osmotic technique osmotically controlled control direct shear test.

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SHEAR STRENGTH

○ Unsaturated soils

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w)$$

$$\tau = \left[(\sigma - u_a) + \chi(u_a - u_w) \right] \tan \phi + c'$$

$\chi(\theta)$

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Fig. Modified triaxial cell apparatus for testing unsaturated soils (after Ng and Chen 2005)

So, osmotically control test also can be conducted. Either way, the matrix suction and normal stress are two independently maintained and the strength of the shear stress can be conducted. Similarly, the triaxial test is also conducted in the same manner where, you have a triaxial test is also conduct in the same manner where, we control the matrix suction within a soil sample by applying, so air pressure and we place higher entry disk here. So, this the soil sample and this is a higher entry disk and coarse porous disk is placed here and you have air pressure control and so these is air pressure line, so through which air pressure is control within the soil mass and air cannot enter into the higher entry disk until the $u - u_w$ is equal to the air entry of the higher entry disk.

So, this is how the soil actually starts bleeding and it attains particular it decreases what? The water content of the soil sample decreases when the air pressure is increased.

And at equilibrium it attains one particular suction value that is $u_a - u_w$ and net normal stressing can be applied by applying the pneumatic stress, shear stress can be by applying one you can anyways control the σ_3 all round pressure of the in the soil sample σ_3 and by controlling these 2 we can shear the soil sample by applying pneumatic stress on a soil sample. So, this is similar to direct shear operators and whatever the test setup.

Essentially the access translation or osmotic concepts are brought into the triaxial and direct shear operators for modification or for controlling the matrix suction within the soil sample.

So, under this control matrix suction environment $u_a - u_w$ and $\sigma_3 - u_a$, so the affective stress can apply. So under this controlled environment, so the tau now will is becomes the equation, so essentially the Bishops effective stress per have effective stress is $\sigma_3 - u_a + \psi(u_a - u_w)$.

So if you write in terms of tau, this is $\sigma_3 - u_a + \psi(u_a - u_w) = \tan \phi + C$. So, here you have another affect (Refer Time: 44:50) to estimate and C and ϕ also you have. So these are material constants. So, ψ is actually function of now moisture content theta or matrix suction.

So, these 2 stress variables are controlled independently in this all this stress and the shear strength of the soil sample are the shear strength of soil sample can be determined

by understanding the by estimating the material constant like C dash ϕ dash and ψ variation with θ . Then you can estimate how the shear strength varies with moisture content.

Thank you for this lecture and will continue for discussing how the saturated soil can be obtained from modify direct shear operators and modify triaxial stress operators using Bishops concept of effective stress.

Thank you.