Unsaturated Soil Mechanics
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Week – 08
Lecture - 22
Analytical Methods of Transient Flow II

Hello everyone, we have been discussing the transient flows through Unsaturated Soils. Extend where we can solve the exact formulation we have an exact formulation for the transient flows. So, the using the Boltzmann transformation, we can solve the theta form of Richards partial differential equation and we get an ordinary differential equation then we integrate it we get the equation for d of theta.

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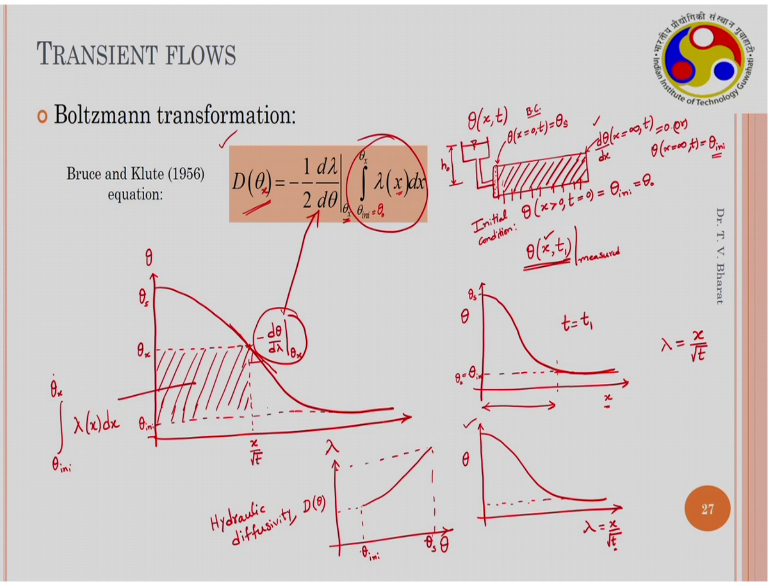

Diffusivity; here the diffusivity is equals to minus half times d lambda by d theta at theta x earlier I would have written theta i that is at given point into integral theta initial value that could be 0 or something theta naught theta naught may be 2 theta x that is it a given point and lambda of x dx this if you solve this here x is a integrational variable.

So, if you solve this we get D theta D variation diffusivity variation with theta d is a hydraulic diffusivity which has units of metre square per second we discussed earlier how to solve this particular question is a question. Now we need to conduct an experiment for obtaining theta of x and t. So, this can be done by compacting a soil sample into a column and connecting this column, this is how you (Refer Time: 02:24) or something and then which is connected to a reservoir. So, if you maintain a constant head of water and you take a sufficiently long column so, that on infinite boundary condition can be booked.

So, then this is this is a particular head that is maintained, positive head that is maintained there is h naught this is the initial condition. And in terms of theta initial condition is at this particular point that is at x equal to 0 at any given time, this is equals to the theta s volumetric water content. Here within the soil sample the initial condition is at any given x greater than 0 when time t equal to 0, this equal to initial value or you can write is theta naught or whatever. This is the initial condition and this is one boundary condition inlet boundary condition and other boundary condition that is at theta at x equals to infinity at any given time this is invariant; that means, d theta by is equal to 0 or with respect to is x is 0. So, there is no flux here. So, with respect to x this is 0. This is the boundary condition we invoked or simply this is theta at x equal to infinity at any given time this is simply 0 this simply is equals to theta initial.

So, this is the constant value therefore, if you differentiate it become. So, 0 it is 0 values now such an experiment can be conducted where we have long column. So, that this boundary condition is satisfied and always a constant head is maintained, then you can maintain then the theta is also become theta equal to theta s then after certain time, we can measure the water content at different points within the soil sample. Either by the placing sensors at different locations or by destructive technique, we can establish theta at any given x at one particular time interval this can be estimated.

So, the theta variation with respect to x at given any particular time t one can be estimated from the measurements. So, this is the measure data. So, knowing spatial location; so, this how does it is vary? So, this varies in this particular manner. So, the grammatical water content or theta volumetric water content. if you plot with respect to x. So, then it varies in this particular manner. So, this is equal to theta initial or theta naught and this is theta s.

So, beyond this point the water has not reach. So, wetting front somewhere here, but there is water distribution within this entire space from theta s to theta initial value. So, the sharp wetting front cannot be seen, you know that is only hypothetical and mostly that can be invoked in coarse grained soils without much loss of accuracy. So, this is volumetric water content versus x this is observation you get.

So, this is the at one particular time t equal t 1, that is observation. As we know the how theta varies with x and t, we can obtain lambda which is x by square root of t. So, we can also plot how theta varies with lambda because this variation is also similar though even though the x axis scaling would be different.

So, this may be similar because is simply x by square root of t, t remains constant because at one particular time interval for different x value you are obtaining the it will scaled and you will get the same qualitative similar data. So, this is the data we require for this particular analysis, I will draw this one here. So, this is lambda Boltzmann variable and this is theta. So, now, this is theta and. So, this varies in this particular manner. So, this is the observation. So, this is theta initial value. Now this integration means at any given point that is theta x at any given point x, x square root of t.

So, this area is simply integral theta initial that is this point to theta x and lambda of x dx. So, this can be simply obtain by numerical integration you can take discrete point because the data will be available. So, we can take discrete points and we can calculate the area under this particular curve, you know this hashed portion can be obtained for any given data and if this is obtained and at this particular point the slope can be obtained that is d lambda by d theta.

So, this is in fact, is slope is minus d theta by d lambda at theta x that is what you are getting here; because the slope of this one is d theta by d e lambda. So, if you inverse it, this value can be substituted into this and this portion area is coming directly can be substituted here. So, essentially by multiplying with minus half and the slope of this point at on the curve at this particular point theta x, and integration that is a area of the hashed portion should give you d theta at x at this particular point. Similarly discrete data of d at different theta x values can be obtained.

So, if you plot that so, that appears to be. So, on y axis you can plot D theta or simply D and x axis you can plot theta, and theta varies from theta initial to theta s. As theta increases the D theta value increases like this may be several orders of magnitude changes from here to here when theta changes as over a small value like say initial value of theta may be could be 0.1 or something and are 0.01 theta s could be say 0.4 or something. So, then it can change in this particular manner.

So, this is how the hydraulic diffusivity can be obtained using graphical technique it can be easily obtained. So, this is a very popular technique in soil science and this particular equation is called Bruce and Klute equation because of their contribution and the derivation of this particular expression, and graphical solution is also commonly used for obtaining the diffusivity.

So, the diffusivity helps in understanding how fast the water diffuses in the soil.

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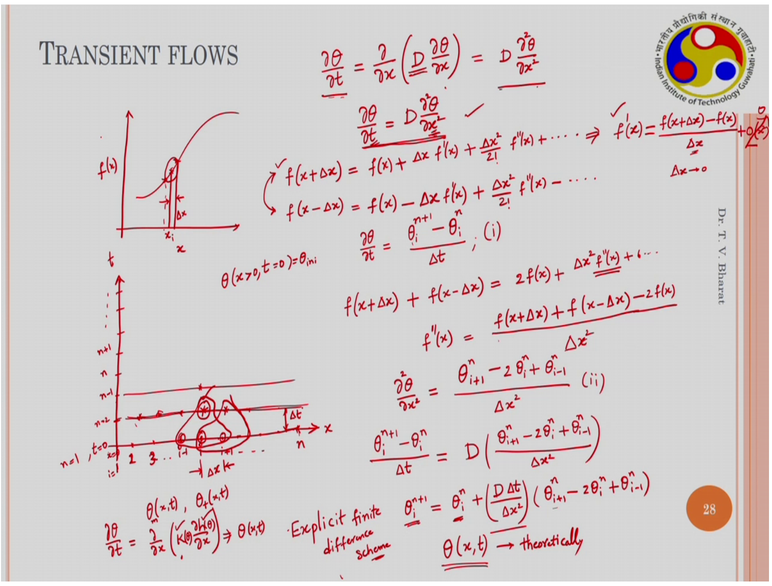

Often we may require to solve the original partial differential equation of either mixed form or theta form or head form depending on the situation and the Richards equation that is a that equation is theta form is dou theta by dou t is equals to dou by dou x of D dou theta by dou x.

So, this is the theta form of Richards equation and we earlier simplified this to dou square theta by dou x square times D. Assuming that the d variation with respect to x is negligible. So, this we applied in case of a small sample which is trusted in axis translation technique or (Refer Time: 12:35) plate operators using the method of multistep out flow technique for the estimation of hydraulic conductivity function.

So, in that we made a similar approximation; however, the representation was in the head form or the suction was used as a dependent variable. Here is the dependent variable is theta volumetric water content, and this form there is a dou theta by dou t equals to D times dou square theta by dou x square exactly replicates the fixed diffusion equation. Fixed diffusion equation for ion movement within the any media porous media or any other media; so, this is exactly similar to that either this can be approximated like this or the whole equation need to be solved or mixed form need to be solved very often. So, that can be achieved by considering numerical approaches.

So, far we are trying to understand how analytically the problem can be addressed, but for complex problems and complex boundary conditions it is not possible to obtain analytical solutions. Moreover in the field we do not expect simplified boundary conditions and 1Dimensional flows very often. So, in that particular case the numerical solution can be advantageously used to solve these problems and address those field issues.

So, I will demonstrate how one particular type of finite difference approximation can be used to solve the partial differential equation, and this is the demonstration purpose only and extensive understanding on this particular finite difference or computational fluid dynamics requires a specialized training. So, here only its demonstration will be done. So, I take a simplest form of the expression that is dou theta by dou t is equal to D dou square theta by dou x square. This is a linear form of partial differential equation and the parabolic partial differential equation. So, I will demonstrate how a numerical solution for this particular case can be obtained. Generally in computational fluid dynamics or finite difference techniques we invoke the Taylor's series expansion for example, if you have a function f which is varying with x, if there is a function that exist like this, then at any given point x I can expand the function around this particular xi.

So, this difference would be delta x. So, around this particular region of delta x, I can expand this particular function. So, if I expand that I get f of x plus delta x. So, therefore, I can invoke Taylor's series expansion that is for f of x delta x this equals to f of x plus delta x times f dash of x plus delta x square by 2 factorial f double dash of x plus so, on and so, forth.

Similarly, for f x minus delta x; so, that is for this particular point. So, this was expression becomes a f of x minus delta x f dash of x plus delta x square by 2 factorial f double dash of x minus so, on and so, forth. Using these expressions, we can obtain first order difference expressions this is the differential equation will be expressed in the form of difference equation so, that we can generate algebraic equation to solve these equations and finally, solve.

So, as we are saying there is a small increment of delta x considered and around that we can expand this function f of x in this particular manner. So, therefore, what we could is, the theta variation with x and t we want and would not know how it various here there are 2 independent variables that is time and spatial variable x. So, now, what we assume is that we consider a grid a domain; so, a theta variation over a column length of say x centimetres. So, this is the domain that is shown, we divide this domain into small small cells are grid points.

So, now we have say, this particular thing is i, this is i plus 1, this is i minus 1. So, starting from 1 etcetera 2 etcetera up to say n number of points I can consider on this particular domain.

So, my intention is to obtain this is a spatial x and this is temporal. So, how the theta variation happens along this spatial length at any given time? So, there is t equals to 0, I want to understand. Here as it does not start from 0, this is 1, this is 2 and this is three and so, on and so, forth. So, here at x equal to 0 there is first point grid point and x equal to n that is the large grid point on this domain. Similarly in the time domain time t equal to 0 that is n th time step or n equal to 0 one time step.

So; that means, n equal to one time step, this is I equals to one time step. So, again we divide this into number of time intervals; so, n equal to 2 etcetera. So, this is n this is n plus 1 and this is n minus 1 so, on and so, forth. We are now getting the volumetric water content variation how it varies in the spatial domain of the problem how it varies at discrete points we can obtain similarly with time, how it varies at discrete time intervals what is a value we can obtain.

So, essentially we are changing in a continues form into a discrete form. So, that is what is called difference form we are getting from the differential equations. So, this is n equal to 2 and this is n equal to 3 or some other time interval. So, here what we are trying to do is, we express the theta to obtain this difference form.

Now, we can simplify the dou theta by dou t term there is a differentiation of this particular equation can be made in difference form by considering dou theta by dou t as the first first derivative of theta with respect to time can be obtained by considering this particular equation itself. So, from this particular equation f dash of x can be written as f of x plus delta x minus f of x divided by delta x plus some higher order terms.

So, if I consider a delta x terms which is very small, delta x approaches 0 very small delta x then this higher order terms can be ignored. So, when we ignore essentially the slope or the derivative of the function can be approximated into difference form like this.

So, now this particular dou theta by dou t that is the first order differential form, can be expressed as theta x plus delta x I can write or I can simply write that this is i plus 1 th value because x plus delta x if I consider at i th point then this is i plus oneth point minus theta at i th point divided by delta t.

So, this is in the time time domain. So, now, I can solve this particular form in the n th level itself n could be 1 and i could be 1 in this particular case like wise or i could be 2 in this particular case because i equal to 1 because boundary condition. So, i equal to 1 becomes the boundary condition what happens at the boundaries we already know the information theta at x equal to 0 that is equals to theta s that information is known. So, this can be approximated to this now we have second order differential equation on the right hand side dou square theta by dou x square how to obtain difference form for this particular equation.

So, we can use these 2 equations, when you add these 2 expressions what you get is f of x plus delta x plus f of x minus delta x is equal to 2 f of x, these 2 terms get cancelled plus. So, this is simply delta x square f double dash of x plus some higher order terms.

. So, now, if I write the expression for f double dash of x, this is simply f of x plus delta x plus f of x minus delta x minus 2 f of x divided by delta x square. This is called central difference and this is called forward differencing etcetera, but I am not discussing the details of what is central difference etcetera etcetera central difference higher order terms. This is the second order approximation or these things need not be discussed here.

So, the dou square theta by dou x square term can be approximated as based on this theta, x plus delta x is i plus 1 minus 2 theta i f of x is theta i plus theta x minus delta x is i minus 1 divided by delta x square.

So, now, if I substitute this one when this one in the differential equation, what we get is theta i plus 1 n minus theta i n divide by delta t is equals to d theta i plus 1 minus 2 theta i plus theta i minus 1 by delta x square. If I simplify this is an explicit form of final difference approximation. So, here I can write for theta i plus 1 sorry this one is not theta i plus 1.

So, this is at essentialist at same i th level. So, this is at i and n plus 1 only time step is varying because we are actually doing the difference operator on the time steps. So, this is n plus 1. So, here sorry for the mistake; so, this is n plus 1. So, therefore, this is n plus 1 can be written as theta i n plus D delta t by delta x square times theta i plus 1 these are all at n th level this is my approximation I am considering at n th level these values.

So, minus 2 theta i n plus theta i minus 1 n I choose to consider them at n th level. So, now, if you closely observe theta i at n plus 1 for example, if n is equals to 1. So, at the second interval i th level this point is calculated using theta i at n th level. So, that is this particular point and theta i plus 1 at n th level. So, that is this point and theta i minus 1 at n th level so, this is this point. So, using these three different points this point is calculated.

So, as we have initial condition in this particular case is, theta at any given x greater than 0 when time t equal to 0 is equal to theta initial. So, therefore, we have the knowledge of theta along this grid. So, knowing these three terms, we can obtain theta in the next time step. So, this is delta t and this is grid point between 2 different points is delta x.

So, essentially for any small delta time increment, what happens to theta how theta distributes along the spatial domain can be obtained spatial distance can be obtained. So, the next time level again. So, similarly for this particular point utilized these three points, for this particular point theta i plus 1 theta i plus 2 utilized these three points known points. Similarly you can obtain all points on this particular n th n th n equal to 2 th level.

Then once you obtained all these values, you increase a next level increment again this point can be obtained by utilizing these three known points. Similarly you march in time from n equal to one th level to 2 th level to third level etcetera you march in time. So, this therefore, this is the final difference techniques are called time marching techniques. So, we march in time and step by step we estimate how theta value changes which spatial distance for different time intervals and now any given theoretical time what is the theta value can be obtained along the spatial length.

So, this is a simply the explicit finite difference scheme we call. So, we can one can obtain the stability and convergence of these particular expressions or equations and based on how this d delta t by delta x square changes, one can obtain the convergence and stability criteria can be developed; how error propagates if you change the delta x or delta t etcetera can be studied. And this is one type of scheme in the finite difference, and there are other schemes like implicit scheme where if you put n plus 1 th values here.

So, then so, if you put n plus 1 th values here, then this becomes implicit scheme. So, because we have n plus 1 on either side of the equations only n th term only one particular data point is there then it needs to be solved by numerically it will be more rigours because it is implicit scheme.

So, by gauss elimination or such kind of techniques we utilized to solve these expressions. To theoretically obtain theta of x and time theoretically, so, in inverse problems essentially you may want to estimate what is a D value knowing theta of x and t experimentally. So, then in particular case we utilizing optimisation techniques.

Because earlier I have shown that if you have theta measurements x t and theta theoretical then the difference between these 2 data points can be taken, and error can be computed and this error can be minimized to by utilizing several optimisation techniques. So, now, I will demonstrate one way of estimating the theta of x t for a known data of hydraulic conductivity functions and soil water characteristic curve.

Because a once the d diffusivity is known are knowing actual equation is that this is a dou theta by dou t is equals to dou by dou x of K dou h by dou x. So, here h of theta if this function and k function are known. So, then you can obtain theta of xt by numerically like what I have shown here.

So, such a methodology is utilized in one of the freely available software called HYDRUS 1D; which is developed by the same authors who developed ret c r etc which I earlier discuss. So, this is a HYDRUS one Dimension 1D and HYDRUS 2 D and 3D is also available, but they are paid software they are commercial and here you can choose one particular name and you can write some description.

So, then you have this free processing window you can select the main process in that the water flow that is what we want to solve through unsaturated soil and vapour flow we have not discussed and we have solute transport. Solute transport and many other options we transport and root water taken many other options. Here we will concentrate on water flow alone and this inverse solution is nothing, but what I discussed earlier that if you want estimate the hydraulic conductivity function soil water characteristic curves, knowing the volumetric water content distribution with time and space, you can estimate those functions by inverse solutions. So, the inverse solution means that.

Then you select next. So, then length units can be selected as in centimetres metres or millimetre and number of soil materials. So, it can handle layering system or non-homogeneity. So, you can select number of layers and each layer you can give what is the difference soil material that is available; and whether the flow is taking place vertically and horizontally that can be seen here that can be understood because here it mentioned that decline from vertical axis if you put one; that means, it takes vertical flow and if it is 0 then it simulates horizontal condition.

So, here I was saying the number of soil materials can be considered for non homogenity. So, the depth of the soil profile by default is 100 centimetres and here you can change if you have any other values then if you select next. So, here the time information you have units per time seconds minutes hours.

days and years and the default values days and I keep the days initial time is 0 and final time is 100 days. So, you may choose may be seconds or minutes and then this is the initial time and final time and you can consider time variable boundary conditions also.

So, for every one time step it gives the output print options and print regularly it you can play with these options available. So, number of iterations you can select may be 100 you can select or you can give the. So, you can select maximum number iterations to be 100 and these are tolerance etcetera and if I go to next.

So, here the choice for hydraulic models is given, here if you consider single porosity model your van Genuchten Mualem this is what we discussed in our one of the lectures. And with air entry value if you enforce minus 2 centimetres then the stability will be satisfied. So, it will be smooth it seems as it mentioned in the manual and you have modified vVn Genuchten and Brooks Corey and Kasogi models, and you have other dual porosity models also and you can consider hysteresis you are considering of cyclic effect of drying and wetting.

So, if you consider a simple problem and here theta r are residual water content, and the q r means theta r residual water content is 0.078 there is a default value, and theta s is 0.43 this is the porosity or saturated volumetric water content is 6.43, alpha is 0.036 and n is 1.56 and Ks is this value 0.017 centimetre per minute which is considered.

So, these are the values for loam soil and you can also select difference soils from the menu. Here you can select boundary conditions for these particular cases you have constant pressure head constant flux and atmospheric boundary condition with surface layer etcetera etcetera. These are under pressure boundary conditions.

So, you can select either pressure boundary conditions because is essentially you have a mixed form to solve and you can input the boundary conditions in terms of head or water content. So, if you change into water content boundary conditions then you have constant water content at the upper boundary, our boundary this is theta s you know constant water content theta s saturated water content or constant flux etcetera and you can go and lower boundary constant water content constant flux or free drainage etcetera we can select.

So, we give in water contents only constant water content we give, and next do you want to run profile application save data yes. So, this is the, your mesh and you can consider. So, the initial input so, far it is not taken anything. So, this is the initial input and if you close this. So, this is how the grid is considered; here the theta variation is not considered at all and here we need to give input. So, here theta is given as. So, this is not theta this is a head. So, it is wrongly considered we go to previous. So, here we go and go for options.

So, here we go for the pressure head conditions itself, because it is not the taking water contents now at present. So, pressure head consideration if you take and then if you consider the initial conditions then, there is the initial conditions is selected here. So, then we can proceed and initial condition is that at x is equal to 0 or z equal to 0 the head is 0. And all other points this is minus 100 centimetres. So, this is an initial condition next ok. So, continue and then profile information. So, here it is not able to generate anything.

So, this is the pressure head profile and this is a water content information at one particular time interval. And more information can be obtained if you increase a time and if you run the run the software the run the software for different time intervals and you get the data, and you can analyse how the water content changes depth it can be clearly seen.

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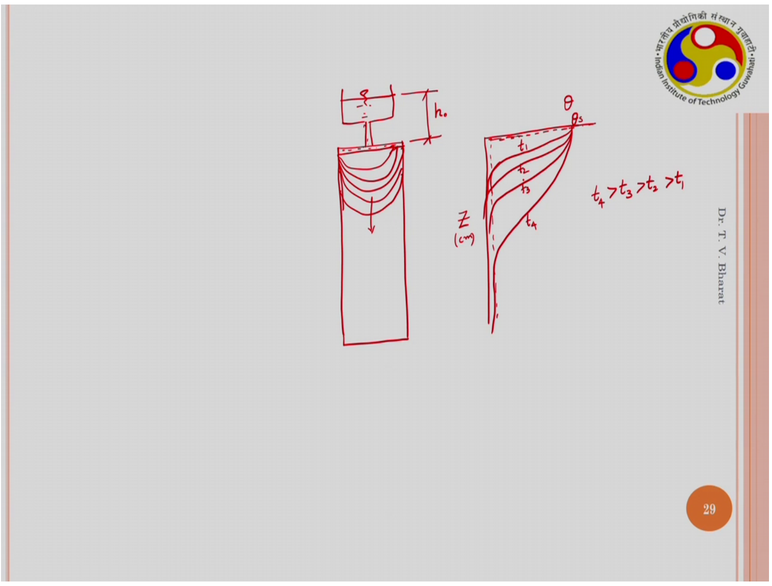

And here if you put realistic values, so, the variation in the water content can be very clearly seen. So, in the particular case what we cloud see that, when soil column and you maintain some certain pressure head. Say you can have your porous plate here and connect to a reservoir, water reservoir you can maintain certain head it could be a constant head.

So, then the water flow takes place into the soil in this particular manner. So, this is how the water front advances with in the soil due to head gradient as well as elevation or the gravity effect, this is how it advances in this particular direction. So, the distribution of water with depth; so, for examples this is depth Z in centimetres and the volumetric water content if you plot.

So, here the initial condition is that theta s here and theta i or theta initial value here everywhere else. So, you have. So, here it is theta s and this is how it is slowly advances with time and slowly it reaches like this is how the profile advances. So, volumetric water content increases in this particular (Refer Time: 42:27) this is the time t 1, this is t 2, and this is t 3 and this is t 4 here t 4 is more than t 3 more than t 2 more than t 1. So, this is how the volumetric water content increases with in the soil with time such kind of a profile can be obtained using such free software like HYDRUS 1D. So, here our discussion on a steady state flows and transient flows is complete for our course.

So, far I have discussed only the water flow or water movement through unsaturated soils under steady state and transient conditions. However, in unsaturated soils there cloud be vapour flow that can take place because of the thermal gradients are any other humidity variations or density of the air variation in air density across the soil length and there cloud be vapour flow that takes place. And similarly in unsaturated soil the air also can get dissolved in water and this air can diffuse through water.

So, those 2 cases also prevail in unsaturated soils and those we did not discuss much in our course.

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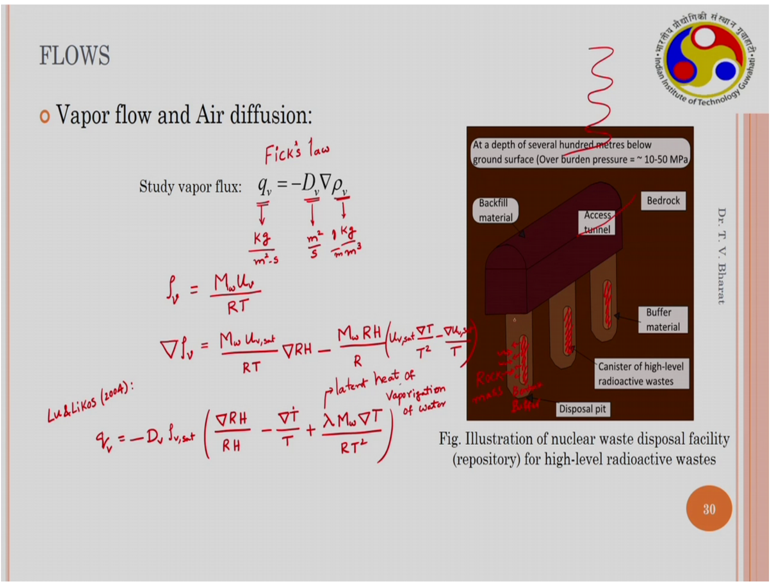

In unsaturated soils the vapour flow can take place the steady vapour flow for unsaturated soils can be can be described using Fick's law.

So, Ficks law for steady vapour flow. So, here this is a vapour flux which has units of kg or metre square second. So, this is a diffusivity which has units of meter square per second and this is a density of vapour, which has units of kg per meter cube. So, the expression is q is equals to minus Dv del dou v. So, this metre square per second in 2 kg per metre cube. So, the metre square and here dou by dou x comes because of the from del operator will get dou by dou x. So, this is 1 by metre also.

So, therefore, the units are satisfied on both sides. So, this particular expression the rho v we have earlier seen the density of water which can be written as molar mass of water are moist air times Uv vapour pressure divided by RT gas constant and temperature. So, the variation in density of vapour is essentially due to change in RH and change in temperatures which can be expressed as M w Uv sat times RH you get.

So, therefore, divided by RT times change in RH minus M w times RH divided by R times Uv sat del T by T square minus del uv sat by capital T. So, this is a expression for del rho v once you simplify this and substitute in this expression. So, the expression for steady vapour flow has given in Lu Likos is q v is equals to minus D v rho v sat times del RH by RH minus del T by T plus lambda Mw del T by RT square. So, this is expression from Lu Likos textbook for study vapour flux is this, here the variation of relative humidity and variation of temperature could be considered here this lambda is latent heat of vaporization of water.

So, specially these study vapour flux conditions are encountered in the nuclear waste repository which I mentioned in the beginning of the course that you have a copper canisters this is a copper canister, which contains the nuclear waste these are all copper canisters which are place at different spatial distances and this is a compacted back in a pit this is placed and which is backfilled with the bentonite.

So, this is bentonite backfill; these are all bentonite backfills and. So, this is an access tunnel below depth of may be several thousand meters. Below the ground surface you come down by tunnelling several tunnels and you reach this access tunnel and you make pit here and the disposal pit you place the canister and then backfill it with betonies this is called buffer bentonite buffer. So, this is generally buffer material contains bentonite alone and then which is compacted very high densities, and then once it is compacted and this access tunnels are filled with backfill material again bentonite plus other materials also kept and this is buried.

So, now once this is placed this canister is hot at higher elevation elevated temperatures, because of the radioactive elements and the temperatures are expected to be around 60 to 80 degrees or even more than that sometimes and surrounding the rock mass this whole thing as a rock mass. So, which is saturated generally at that levels; this is completely saturated.

So, the water movement takes place in to the bentonite buffer material, and this bentonite is initially (Refer Time: 49:38) stretch. So, this is unsaturated condition heat conduction takes place in opposite direction. And water movement takes place in this direction generally the coupled phenomenon if you solve because there is a vapour flow there is heat conduction taking place, there is a water flow there is taking place, and it exerts some pressure around the around the surroundings the bentonite exerts pressure around the surroundings that is the mechanical pressure that is exerted. So, hydro thermo mechanical studies are often conducted by coupling all this analysis together.

So in the, this particular cases this is very important; however, as this course is for preliminary audience. So, these advanced issues are not discussed in this particular course. Similarly often we see that the air gets dissolved in water when you apply very high pressures etcetera this dissolution takes place that we have discussed in the beginning of the course. So, when the dissolution take place this air which is dissolved in water also travels from higher pressure to lower pressure. So, that is the diffusion that takes place within the water.

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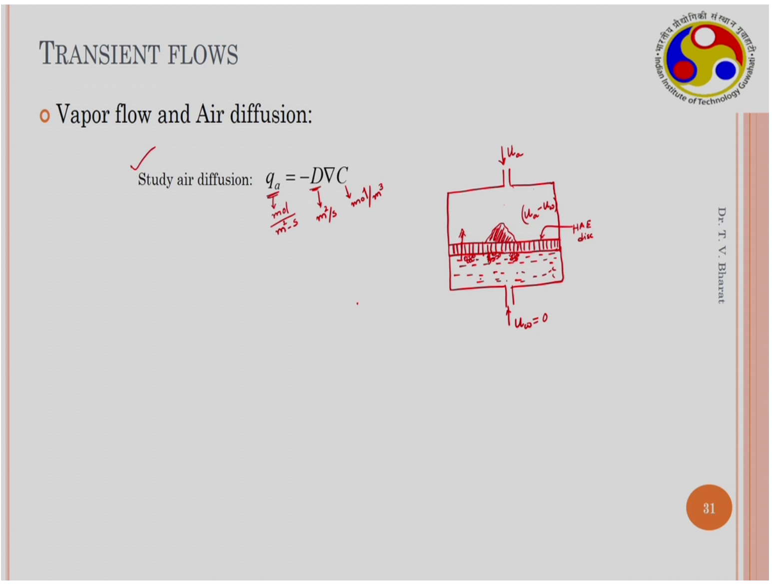

So, that is expressed by again Ficks law the study air diffusion equation becomes q a is a air flux is equals to minus D diffusivity times delta C. So, q a is a air flux which has units of a mole per metre square second and the diffusivity has units of metre square per second, and this is air concentration which is expressed in mole per metre cube. So, this is the study air diffusion equation; now this equation has many applications especially we discussed the axis translation technique earlier. In the axis translation technique you have higher entry porous discs disc which separates the water reservoir

So, this is the water reservoir. So, here water pressure can be applied, and your soil sample will be here and this is porous disc. So, you have air chamber.

So, here you apply a required air pressure. So, here there is air pressure that is maintained here water pressure is maintained generally this air pressure is very high you know as high, as higher than the atmospheric pressure. So, here due to the water pressure that is exerted you maintain a u a minus u w pressure that is suction that is controlled within the chamber because water pressure is u w and this is air pressure is generally water is equal to that atmospheric pressure that is 0 if we consider it to be gives pressure. So, then in this particular case if you have dissolved air in the water; so, this is air pockets some air pockets are here available.

So, generally this happens when you are when you do not flush the higher entry disc very often and then some air pockets may be left over and those will be available on the water side, and sticking on to the higher entry porous disc and as shown here. So, now, when you are conducting a test at equilibrium you have a air pressure that is maintained here and there is a water pressure that is maintained nearly close to 0.

So, if there is a pressure difference that happens then, these air pockets would refuse through this higher entry porous disc. So, this is a air we will diffuse through this higher entry porous disc. So, because there is a pressure difference between 2 chambers across this higher entry disk, the air diffuses through this porous disc and because of this pressure gradient.

And that diffusion can be estimated using this particular expression if the flow is steady state. If vapour flow are air diffusion is not study are time dependent, then we need to combine this expressions with mass conservation and transient equations can be developed and those transient equations can be utilized for understanding the variation in flux with time.

So, in summary we have so, far seen the study state flows and the transient flows through unsaturated soils. In study state flows we started off with how the hydraulic head that is the total head which consist of matrix suction head and osmotic suction head and the elevation head in case of horizontal flows this elevation head is negligible or 0.

So, then the matrix suction head plus the osmotic suction osmotic suction head, the summation or the combination would influence the flow behaviour in the absence of solute in your soil system then only the matrix suction head that plays a role. So, then this matrix suction head how it varies with spatial distribution for study state flows we have seen.

For study state unsaturated flows the suction head varies non-linearly with distance. Even though if you consider a linear form of hydraulic conductivity function. In case of vertical flows we require the data of hydraulic conductivity function and soil water characteristics, for obtaining the spatial distribution of matrix suction head profiles or total head profiles.

And we have derived implicit expressions and we have also seen how to solve these implicit expressions and coming to the transient flows, then we have discussed the transient flows in transient flows we have used the simplified expressions given by green and ampt initially for horizontal flows and vertical flows.

Ah Here how the sharp wetting front moves with the time was obtained and this is well collaborated for a coarse grained soils because of the assumption made in this particular analysis. As these are very old approaches they try to get analytical solutions and this they had many simplifications and they got analytical solutions. And after that we moved beyond this particular analysis and we have seen that how Terzaghi got influenced in giving the capillary rise height or rate of capillary rise in soils based on this green and ampt analysis.

And beyond that we moved to the proper analysis or considering the mask conservation. We have derived Richard's expression and this Richards expression we have solved using Boltzmann transformation and the diffusivity variation with respect to volumetric water content can be obtained using this particular form of equations called Klute and Bruce.

And the final difference analysis of mixed form and other forms of a Richards equations can be obtained for accurate analysis, and we have demonstrated the how the equation can be approximated by taking the theta form and the software that is a hydrous 1D is demonstrated and other flows like a vapour flow and air diffusion are briefly discussed.

Thank you.