Unsaturated Soil Mechanics
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Week - 07
Lecture – 20

Steady-State & Transient Flow

Hello everyone let us look at the new topic Steady flows under the influence of gravity.

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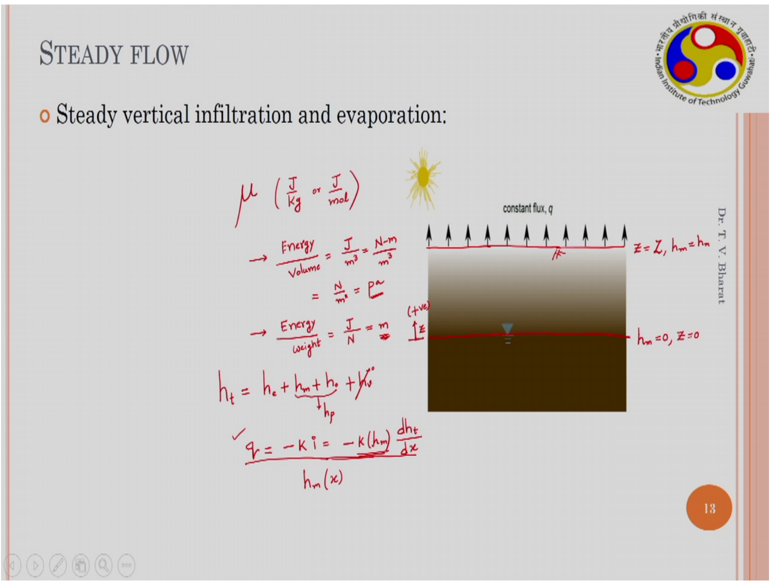

Before discussing the Steady vertical infiltration and evaporation, let us recapitulate what we have understood from the previous lecture; we have seen that, the flow takes place from higher chemical potential to the lower chemical potential. So, the chemical potential which is, described using symbol mu which has units of Joule per kg or Joule per mol. So, when this chemical potential is described as, energy required per unit volume. So, this is written as Joule per meter cube so, as Joule is Newton meter.

So, which will have units of Newton meter per square or Pascal so, that is the reason why the chemical potential often we represent as the matric suction or osmotic suction we represent with pressure terms Kilo Pascals and, this also when we represented as Energy per unit, mass or weight. So, Joule per weight when it is written in terms of Newton so, this is, this has units of meter. So, when we talk about matric suction head osmotic suction head or the total head which has units of meters. So, either you could represent in terms of Pascals or in terms of meter the conversion we have already seen the conversion is simply by multiplying gamma w here. So, the if you multiply with unit weight of water there is a kilo Newton per meter cube we will get the units for Energy per volume.

So, as we have seen the total head controls the flow total head consists of the elevation head matric suction head, plus osmotic suction head. So, these two are considered as pressure heads in our earlier, basic fluid mechanics plus velocity head we ignore in our soil mechanics because, the velocity gradients are negligible. So, when there is no chemicals present in the poor solution the osmotic head component is 0 and if we conduct a horizontal flow then the elevation head component is 0.

So, total head consist of only matric suction head in that particular case. So, this we have seen for flows we have utilized darks law q equals to minus k i, the k we started writing in a functional form as k of hm, and i gradient is the total head gradient dht by dx if it is horizontal flow. If, it is a vertical flow you can write corresponding, independent variables like z. So, this is a modified form of darks law for unsaturated flows and we have seen that the head distribution with respective space is highly non-linear even if, you consider a linear form of hydraulic conductivity function.

So, then, let us now, look at what happens to steady vertical infiltration and evaporation. So, as earlier said in the field, it is possible to have steady flows, depending on the boundary conditions of the system. So, if the water table is located somewhere here and, this is a ground surface so, here at the water table the there is a matric suction head component is 0. So, if this is a different state the z there is a elevation is considered from here onwards z is also 0 and, we can considered of sign convention as positive of word. So, whenever there is a evaporation that is taking place.

So, then we considered the flux is positive and if, the infiltration is taking place then, we can considered flux to be negative we can, choose any sign convention here at z equals to some height say z capital Z, the h m is equals to very high h m are very large value of matric suction head. So, for the vertical flows, same equation we can consider the Darcy’s, modified Darcy’s law we can consider.

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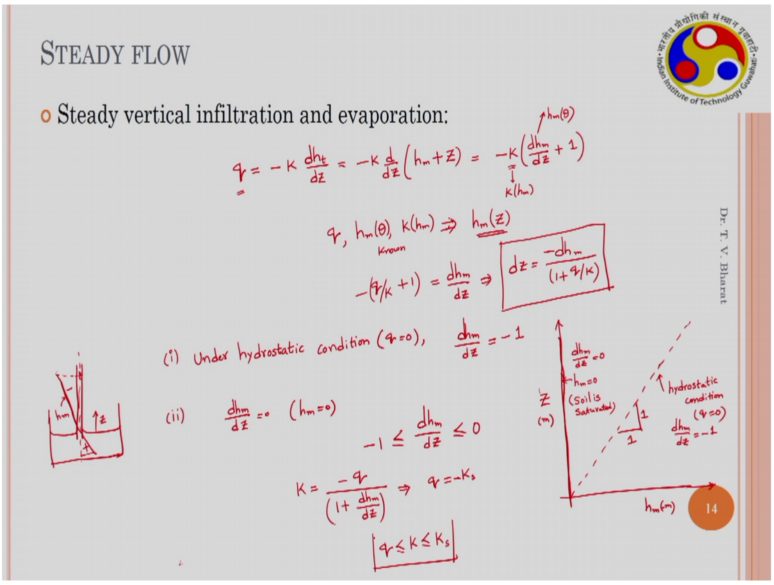

So, for the Steady state vertical flows, the q the flux is equals to minus k dh t by dz. So, as this is total head, which consists of matric suction head plus z if you considered of four positive. So, this is equals to minus k, dhm by dz, plus 1. Here q is constant and hydraulic conductivity needs expressed in functional form k of hm, k of theta and hm also need to be expressed in functional form. So, therefore, if q a steady flux either steady infiltration and steady evaporation that is taking place and the soil water characteristic curve that is the of theta.

And, k of or k of theta that is a hydraulic conductivity function if, all these three things are known then the variation of matric suction head with elevation can be obtained if, these three things are known. So, then we can obtain the variation of matric suction head with the elevation further this can be simplified for the estimating the variation of dz; dz equals to this can be written as this can be simplified as minus q by k and minus 1. So, I will write it this way is equal to dhm by dz or dz equals to dhm, minus dh m by 1 plus q by k.

So, when you have a hydrostatic condition the hydrostatic condition is that, the hydrostatic condition happens when q is equals to 0, under hydrostatic condition, that is when q is equals to 0 the dh m by dz. So, the variation of matric suction and with elevation is equals to minus 1. So, which is similar to I will discuss this so, 2nd thing, when the soil is completely saturated then dhm by dz is 0, because itself is 0. So, therefore, the dhm by dz, has bounds, from minus 1 to 0. So, what exactly this indicates for example, when you plot the z scene meters and the matric suction head in meters. So, then the dhm by dz equals to minus 1 is so, this follows 1 is to 1 line. So, as a matric suction head is negative so, therefore, so, this is negative meters. So, then this is a dhm by dz equals to minus 1 are this is simply one is to one line it follows. So, this is a hydrostatic condition. So, in the hydrostatic condition the flux is 0 so, therefore, dhm by dz is equals to minus 1.

So, this is just similar to when we have a capillary tube inserted in a capillary in a beaker of water. So, then the hydro variation so, this is linear, this is positive and this is negative. So, here this is z so, this is so, the dhm by dz is minus 1 this is hydrostatic condition. So, similar to this the hydrostatic condition exists when there is a flux is 0.

On other hand when the soil is fully saturated the itself is0 so, it follows this line this particular line. So, this is equals to 0 or soil is fully saturated so, soil is fully saturated therefore, dhm by dz is equals to 0 then we can derive conditions for the limits for k hydraulic conductivity, what should be the values of hydraulic conductivity or what values of hydraulic conductivity we can consider for the determination of this elevation. So, for the dhm by dz equals to minus 1 this can be written in this form. So, k equals to from this particular equation k equals to minus q by 1 plus dhm by dz.

So, here when the hydrostatic condition occurs or when q is equals to 0 the flow does not take place and dh by dhm by dz equals to minus 1. So, when fully saturated condition exists then q becomes ks minus ks it depends on whether you are constraining infiltration or evaporation this negative sign is for infiltration or evaporation. So, the maximum value of k is equals to ks so, k varies in this manner k should be less than equal to ks and, greater than q. So, in that range of the k can be considered.

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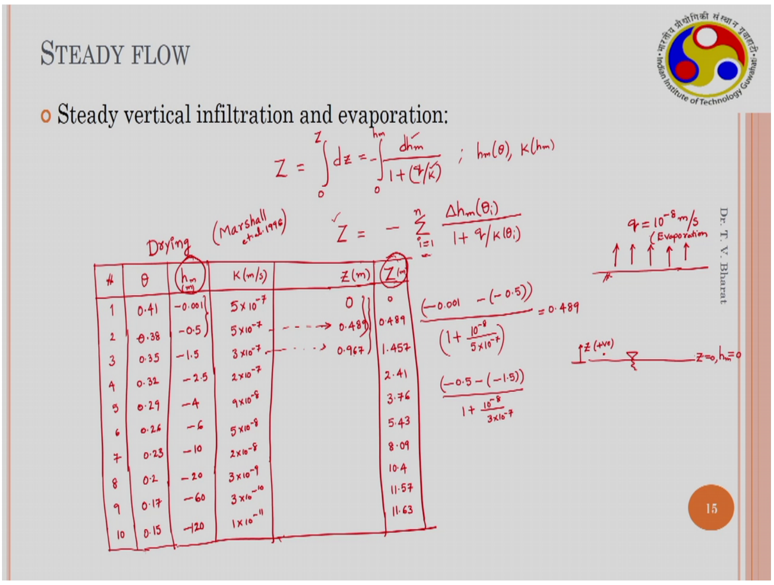

So, further the equation can be solved for estimating the variation of, matric suction head or totals, total head with respect to elevation by integrating this particular equation dz equals to minus dhm by 1 plus q by k. So, this particular equation can be integrated if, we have the functional form well establish to functional forms for of theta and k of theta so, this negative sign if I take it out.

So, this is a varying from say 0 to some depth Z and this is varying from 0 to hm and this integration can be solved if we have the functional forms for k and hm of theta and k of theta or k of are known then, this can be solved. If, the (Refer time: 13:4) discrete data are available then this can be solved by writing the summation this can be written as minus sigma I equals to 1 to n delta hm of theta I divided by 1 plus q by k of theta i. So, let us solve one problem to understand, how the matric suction head varies with elevation.

So, let us assume the following data, the variation of theta with matric suction head in meters is known and the hydraulic conductivity data is also known the when the is close to 0.001 that is, near the near the phreatic line. So, this is the ground surface and this is z equals to0 so, at this point the is equals to 0. So, this is a very small value equals to 0.001 meter and the corresponding theta is 0.41 so, that means, this is equals to the theta as saturated volumetric water content.

So, then k is equals to ks that is 5 into 10 power minus 7 meter per second and similarly other data when the head increases the matric suction head or matric potential increases. So, the theta value decreases 0.38, when the matric suction head increases the theta value decrease. So, 0.35, for minus 1.5 and this is 0.32, this is minus 2.5, 0.29 minus 4, 0.26, minus 6, this is 0.23, minus 10, 0.2 for minus 20, 0.17 for minus 60, 0.15, for minus 120.

The corresponding hydraulic conductivity data is also available which is, 5 into 10 power minus 7, 3 into 10 power minus 7, 2 into 10 power minus 7, 9 into 10 power minus 8, 5 into 10 power minus 8, 2 into 10 power minus 8, 3 into 10 power minus 9, 3 into 10 power minus 10 and one into 10 power minus 11. So, this data is the drying data which represent hypothetical data for well structured (Refer time: 17:10), the data is taken from Marshall text book, 1996 from this reference the data is taken and slight modification in the data might be there this is hypothetical data.

So, from this we can estimate because we have a continues distribution of variation in matric suction head with volumetric water content or vice versa and the corresponding hydraulic conductivity is also known. This data can be obtained by conducting pressure plate test on a given soil initially when the soil is taken at fully saturated state and when the ua the air pressure is gradually increased.

Then, the outflow data can be utilize to estimate the hydraulic conductivity by estimating the slope of the variation in the ua minus uw from initial state to the final state. And the amount of water that has come out from the system from that we come to know what is a volumetric water content from the slope and estimating the diffusivity we can obtain hydraulic conductivity.

Similarly, (Refer time: 18:25) data are also estimated simultaneously so, this way we can obtain the entire data for drying test. So, as this is a drying data we can utilize this for understanding the evaporation rates assume that there is a flux of there is a constant flux that exists from the soil. So, there is q equals to 10 power minus 8 meter per second so, this is a evaporation.

As we are considering upward positive, evaporation is positive and this q can be substituted directly. Now, this can be solved for the first data so, this is at depth is equals to 0 itself and for the second data, we can obtain the values minus 0.001 meter minus of minus 0.5. So, there is a difference between these two, divided by 1 plus q should be substituted positive because this is a evaporation. So, 10 power minus 8 divided by k is 5 into 10 power minus 7 so, when we solve this we get 0.489. So, that is, 0.489 so, this is at one point there is at this point, but we require to sum it up. So, when we sum of these two you will get 0.489. Similarly, for the other set of data second test of data this is minus 0.5, minus of minus 1.5, divided by 1 plus 10 power minus 8 divided by 3 into 10 power minus 7.

So, this is, 0.9677 summing up of all these three gives 1.457. So, this gives 1.457 because here, summation is there. So, we are calculating at discrete points at, 0.1, 0.2, 0.3 and at any given point what is a total elevation we are getting this value. So, further so, this can be estimated to be 2.41, 3.76, 5.43, 8.09, 10.4, 11.57 and 11.63 so, this is the data complete data. So, here, in the previous case when we know the values of matric suction head at any given point x explicitly we could estimate what is the hm value.

However, in this case, we solve it implicitly we estimate the elevation at which we want to estimate how much matric suction head is existed. So, this is how we estimate the z from known values of theta and k. So, then, z and can be plotted together to understand the variation of with respect of z so, this a evaporation data.

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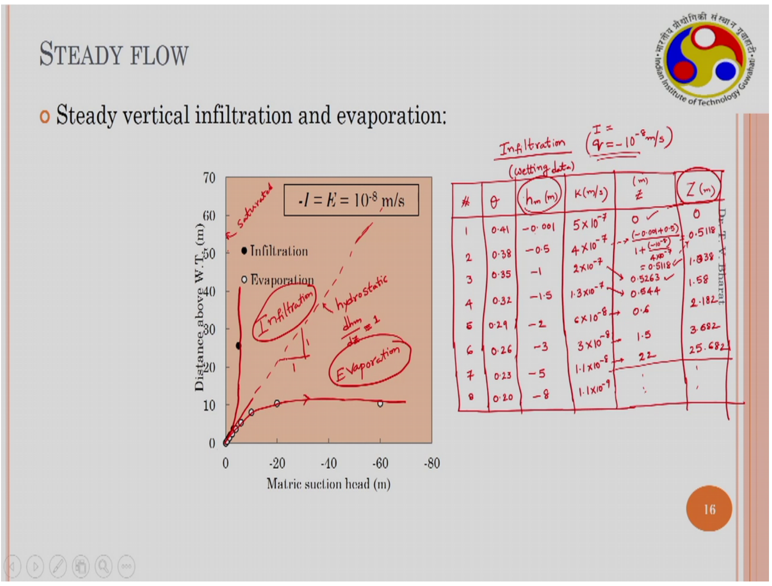

Similarly, we can utilize infiltration data. Here is the same Infiltration flux the water is entering into the soil so, this is a 10 power minus 8 meter per second; however, as we considered a downward negative so, this is minus. So, this is infiltration is a minus 10 power minus 8 per second. So, if you have data so, this is wetting data, so, this is how the soil water characteristic how data varied, the hydraulic conductivity data are also available so, which are given here, this continuously decrease as a suction head increases.

So, this wetting data also can be obtained from pressure plate operators where initially the sample should be at dry state then as a ua value decreases at equilibrium when you apply very high ua value the soil will be at a dry state. And as you decrease the ua value the water starts entering into the soil from the higher entry porous disk which saturates the soil sample under control suction.

So, this way we can get the wetting data and the amount of water there is flowing into the soil from that we can estimate the hydraulic conductivity. So, here also the elevation z at 0, this is the matric suction is close to 0 and this is theta is sarcastic and this is saturated hydraulic conductivity so, other values can be estimated. So, for this is minus 0.001 and minus of minus 0.5 divided by 1 plus q should be here negative. So, 10 power minus 8 by 4 into 10 power minus 7, then, when we solve this value is equals to 0.5118 and for this is a 0.5263 and this is a 0.544 and this one is 0.6 and this one is 1.5 this is 22.

So, when you use a summation when you get the summation so, the capital Z this is 0, this is 0.5118 same as this and this is 1.038. So, the summation of this one this one and this one and this is 1.58 and this is 2.182, this is 3.682, this is 55.682. So, beyond that it is not required because, the q value is 10 power minus 8 meter per second so, here, the k value becomes 10 power minus 9. So, beyond this value of q the values are not important and if you considered you get 9 numbers so, we stop it here so, then if, you we can plot hm versus z for the infiltration test.

So, the evaporation data is going somewhat like this and the Infiltration data follows, follows like this so, the hydrostatic line is this. So, this is 1 is to 1 line so, this is hydrostatic, along which the dhm by dz is minus 1.So, the evaporation data is constrained in this part and infiltration data is constrained in this part. So, this is infiltration and this is evaporation. So, this is completely saturated so, your infiltration data will be in between completely saturated and hydrostatic condition. So, in between that is infiltration and this is evaporation.

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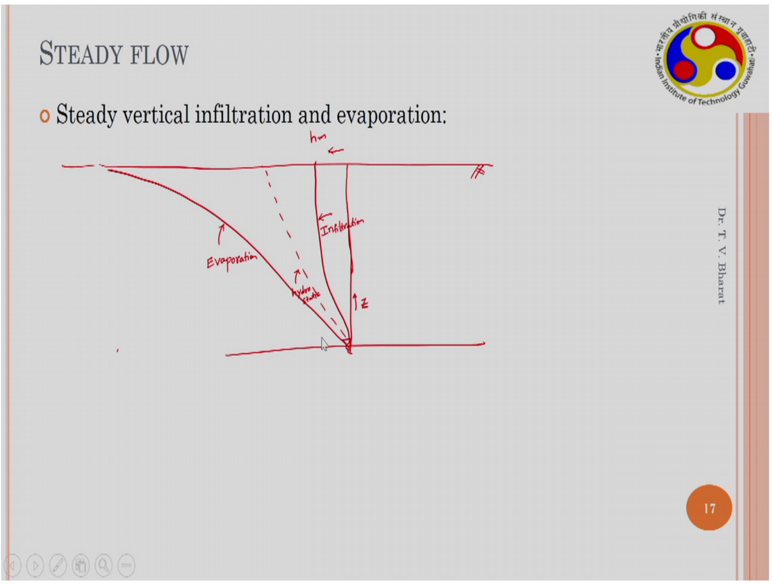

So, that means if this is the ground water table. So, the phreatic line and this is a ground surface, and, this line shows the elevation that is z upward positive now, when you have Infiltration. So, the variation of matric suction head is if the matric suction head can be shown in this manner.

So, the matric suction head variation is somewhat like this, but another hand this may be your hydrostatic condition this is the evaporation, this is infiltration. These are all for steady state conditions so, this is how this can be visualized very well in this particular manner. Now, let us understand the transient flows in unsaturated soils.

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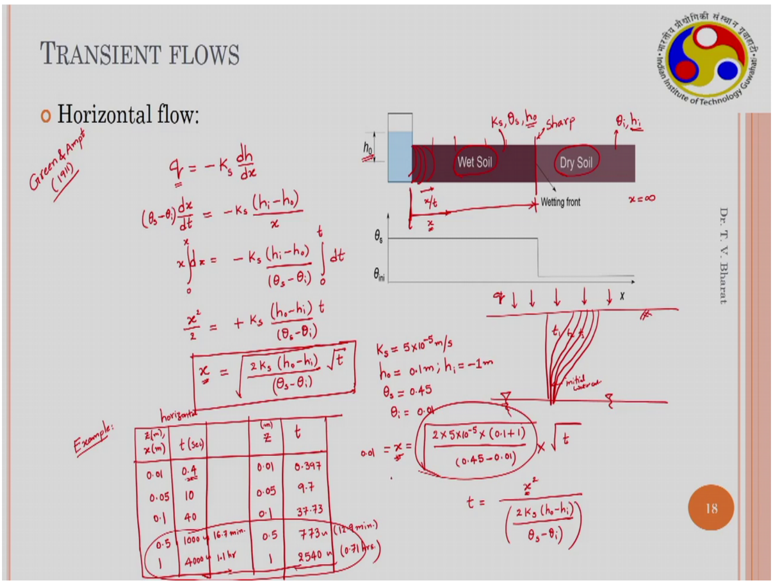

The transient flows are time variant flows earlier we have construct steady state flows where the flow is time invariant; that means, the flux does not vary with time at it any given point or time the flux is constant. But, here in the transient flows the flux varies with time, this condition is before the steady state is achieved. So, in the earlier case where I was discussing you have a groundwater table and you have a ground surface a steady in flux is taking place that is a steady infiltration is taking place into the ground.

So, then initially the water content variation is varies in this manner this is initial water content and this increases with time and then at steady state it may establish somewhat like this so, this is at steady state. So, with time this is t 1 this is t 2 and this is t 3 like that with time this varies and, it may achieve Steady state after certain time. So, before it achieve steady state the variation of moisture, the variation of flux the flux also varies with time and that is the transient conditions.

Most frequently we found transient conditions in the field because of the boundary conditions would vary with a time and also in case of fine grained soils it takes enormous time to achieve steady state even though the boundary conditions remain constant. So, you have a constant flux here maintained and you have a groundwater table here so, but then it takes enormous time for the flow to take place and a significant time to establish steady state.

Similarly, when the evaporation is taking place and the evaporation fluxes are same due to atmospheric conditions and you have a groundwater table is here but then takes enormous time to achieve the steady state within the soil system if soil is fine grained soils in coarse grained soils it quickly achieves the steady state. So, therefore, transient flows are very important in unsaturated soils let us understand how to find out the variation of flux or how to find the water movement in soil in a transient condition.

So, the first attempt there was made was by Green and Ampt, to understand transient flows in 1911. So, Green and Ampt, they have assumed that for example, when a laboratory test is conducted where this is a soil column horizontal soil column let us, first consider the horizontal flows when you consider a horizontal column, which is connected to a water reservoir.

So, where, elevation of h naught is maintained the water level of the water head of h naught is maintained, then water percolates into the soil and which travels with the speed of expertly. So, x is a distance at any given point so, here the x is wetting front distance from the water reservoir.

So, this is x equal to 0 and this is x equal to infinity so, infinite column when it is taken and when the wetting front moves. So, the wetting front location is x and a the wetting front moves at a speed of x by t and, the assumption is that the wetting front is a very sharp boundary, there exists a sharp wetting boundary this is sharp. So, another assumption they made is beyond the sharp wetting front soil is completely dry and before this wetting from the soil is completely wet.

So, therefore, the hydraulic conductivity of the soil can be shown to be ks and the volumetric water content of the soil can be considered as theta s that is porosity and here whatever the initial state you have theta i and k is anyway 0 so, that can be considered. So, this assumption they make and Green and Ampt the derived analytical solutions for horizontal flow and vertical flows for these conditions they have utilize the Darcy's law.

That is q equals to minus k dh by dx, here hydraulic conductivity can be considered as ks because, we are considering the soil is completely wet, before the sharp wetting boundary. So, therefore, this minus ks dh by dx so, here the flux that is volumetric flow rate by unit cross suction area can be written as, the velocity that is dx by dt a variation of this is a movement of moisture within the soil.

So, the velocity with that which the wetting front is moving in the soil and this is moves through force. So, the porosity term should be added that is theta s, but minus theta i because initially some moisture content is there in the soil then that is theta i that is equals to minus ks and this is a initially some i would be there kh i. So, that is a initial suction head and this is a here the head is positive that is h naught.

So, this is negative head, this is positive head there is this h naught only h naught divided by dx so, there is x. So, therefore, now we can rearrange this terms x dx is equals to minus ks hi by hi minus h naught divided by theta s minus theta i, integration of dt. So, here, x varies from 0 to say x and, during the time 0 to t. So, then this is a x square by 2, which is equals to minus or this can be written plus ks, if this can be written as h naught minus hi, by theta s minus theta I, then it will this whole thing will be positive and you have time.

So, then you can write this one as x equals to 2 ks, into h naught minus hi, by theta s minus theta i, whole square root and square root of t. So, this is an analytical expression that is derived for understanding the movement of moisture within the soil in the horizontal column when the test is conducted.

So, we can solve one example problem, here x in meters, t in seconds. So, you can put x as 0.01 meter this is distance from the water reservoir or where the water reservoir and soil interface exist. So, from that at different locations when the water approaches or when the water appears that is what you are estimating for the water movement to 0.01 meters and for the water movement for 0.05 meters 0.1, 0.5 and 1 meter, how much time it takes can be estimated by substituting into this equation when the properties of, this soil are given.

The assume that the ks saturated hydraulic conductivity is 5 into 10 power minus 5 meter per second and h naught is 0.1 meter, 0.1 meter head is maintained. And theta is saturated volumetric water content is 0.45 and theta i initial volumetric water content is 0.01 close to 0. So, now, when we substitute x is equals to square root of 2 times ks is 5 into 10 per minus 5 into 0.1minus hi is not given hi is say minus 1 meter. So, minus 1 meter head is available. So, this is, 0.1 minus of minus so, plus 1 meter divided by theta s is 0.45 minus theta i is 0.01 times.

So, this is, time and x is given that is 0.01 so, time can be calculated which is 0.4 seconds. So, this whole thing is constant now, when x is varying the t can be estimated by calculating the x square by 2 ks, h naught minus hi by theta s minus theta i, this whole thing is constant when x is varying t can be estimated in this manner. So, t is 0.4 seconds when x is equals to 0.05 meters. So, then this is 10 seconds and this is 40 seconds and this is 1000 seconds which is approximately 16.7 minutes and this is 4000 seconds so, this is around 1.1 hour. So, this is how we can estimate, how the water movement are at what rate the water migrates in the soil.

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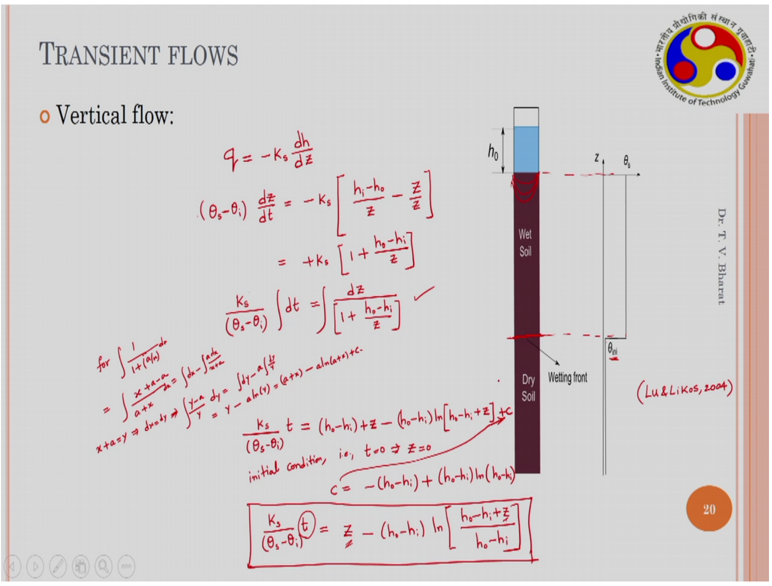

Similarly, for the vertical flows, we can write in the same manner q is equals to minus ks dh by dx. So, dh by dz in this case. So, this is similar to the previous one; theta s minus theta i times dz by dt, is equal to minus ks here, the matric suction head component that is a h i minus h naught by z and apart from that the elevation due to gravity this is and the (Refer time: 42:29) infiltration that is taking place. So, downward negative we have consider so, minus z by z.

So, here infiltration we have consider, that is why this is minus z by z. So, this is nothing, but minus ks 1 minus if you take inside then, this is plus 1 minus, again you can construct plus and you can inter change this terms h naught minus hi by z. So, when we rearrange the terms for integration. So, this could be ks by theta s minus theta I integral dt is equals to dz by 1 plus h naught minus hi by z integration.

So, as we know for integral of 1 by 1 plus ayx dx, are which can be written as integral x by a plus x. Whether, you can add x plus a minus a and then write this one expression as integral dx minus adx by x plus a or you can substitute x plus a for y and then dx equal to dy and then you can write the same expression in the same manner. So, that if we write and then this becomes simply y minus a by y dy. So, this is, integral dy minus a integral 1 by y dy so, this is integration is y minus a log y so, this is a plus x minus a log a plus x plus c.

Similarly, if you write for the same this expression so, this is ks by theta s minus theta i, t is equals to this is h naught minus hi plus z minus h naught minus hi times log, h naught minus hi plus z plus c constant. Now, using the initial condition it is t is equals to 0 the z is equals to 0 because, the wetting front is at z equal to 0 only it has not started at so, therefore for the vertical case so, this is what we have considered.

So, this is a h naught same head which is maintained and the flow is taking place in this manner. So, the sharp wetting boundary is this and theta equals to theta initial beyond this and before this theta is equals to theta s. So, this, all expressions are available Lu and Likos for a horizontal and vertical flows in this text book. So, for t equals to 0 and z equals to 0; that means this is here only the flow.

So, then if we consider that then, c is equals to minus h naught minus hi plus h naught minus hi log h naught minus hi when we substitute this c into this we get, ks by theta s minus theta i, t is equal to z minus h naught by hi, log h naught minus hi plus z divided by h naught minus hi so, this is for the vertical flow. If, we assume any value of z we can calculate what is the time that is required or how much time it takes for reaching that particular point we can solve for the same problem instead of x we consider z in meters then how much time it takes.

So, this is horizontal and let us say let me write it again. So, for the z same depth as a horizontal distance that is 0.01, 0.05, 0.1, 0.5 and 1 meter. Now, let us calculate what is the time required if, I substitute the values in the previous equation and estimate the time, I get these values I am reporting it here 0.397 seconds which is very close to 0.4 seconds. So, the influence of gravity is not available at this particular depth and this is a 9.7 which is very close to 10 second and this is a 37.73 and this is 773 so, which is equals to 12.9 minutes.

So, 3.7 minutes saved because, of the gravity and this is 2 5 4 0 which is close to 0.71 hours. So, the influence of gravity can be seen at higher depths or it higher elevations at higher depths or at higher distances so, the due to the gravity the flow is faster.

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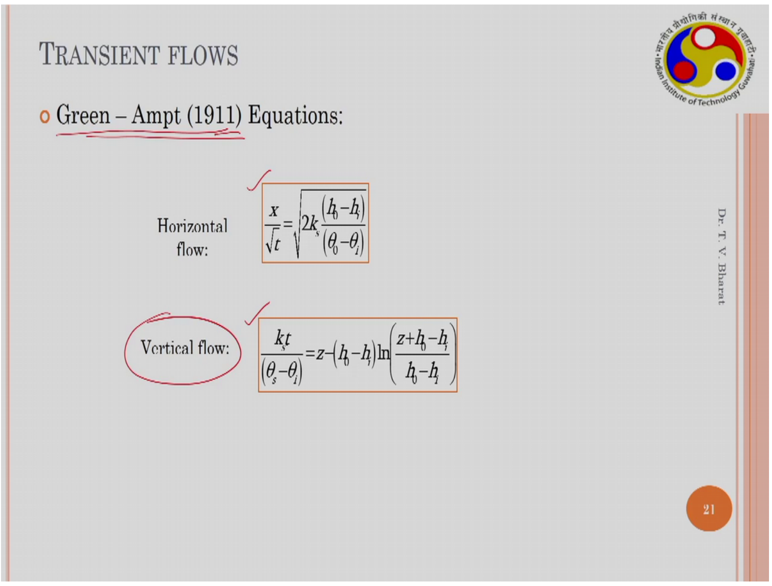

So, this could be easily simulated and therefore, Green and Ampt in 1911 he is provided of equations for horizontal flow and vertical flows. And in fact, Terzaghi capillary rise rate or rate of capillary rise estimation, the derivation is based on this particular expression of Green and Ampt Terzaghi has given later on in 1920's or 30's. In theoretical soil mechanics textbook that derivation is influenced by Green and Ampt that is what we are can understand from the way the gradient Ampt has derived solution for vertical infiltration for transient conditions. So, we will see it in the next class.

Thank you.