**Unsaturated Soil Mechanics**
**Dr. T.V. Bharat**
**Departments of Civil Engineering**
**Indian Institute of Technology, Guwahati**

**Week – 07**
**Lecture – 19**
**Steady-State Flow Through Soils**

Hello everyone, today we will discuss a new topic Flow Through Unsaturated Soils. So, we have knowledge on flow through saturated soils in our basic soil mechanics which we learnt from our basic soil mechanics. Now let us understand the flow behavior in unsaturated soils.

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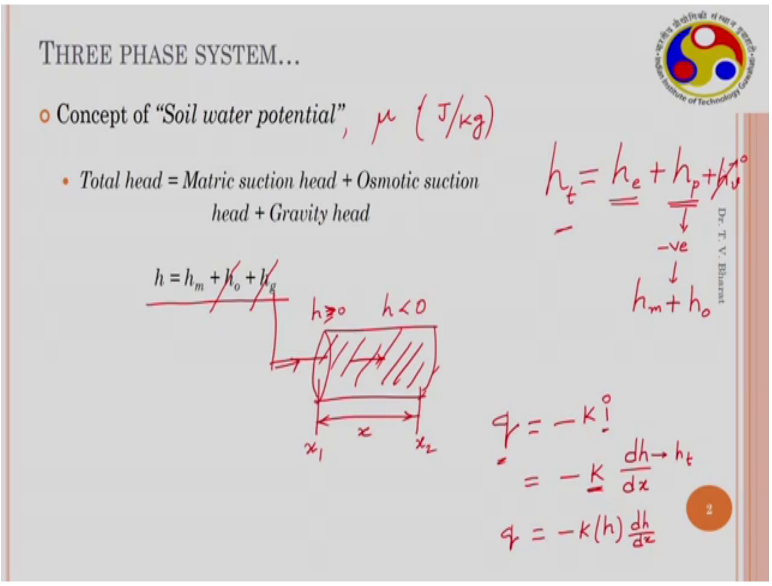


Earlier, we have already discussed the concept of soil water potential. The soil water potential which is indicated by mu has units of joule per kg. We have seen how the water potential decreases due to inclusion of salts and how the water potential decreases due to unsaturation, we have seen all of them and we have said that the total soil water potential consists of matric potential that is due to the matrics of the soil and also osmotic potential that is due to the presence of salts. And now the flow take place due to the or total head difference between two different points within the soil mass.

And what is a total head? Earlier we have seen in our basic soil mechanics we study that the total head consists of the elevation head plus the pressure head and plus the velocity head we always ignore because velocities of the flows through soil is very less.

So, generally we ignore this. This is the total head we consider in basic soil mechanics, but apart from this pressure head is a negative this pressure head is negative in unsaturated soils. So, because the soil water potential the soil water pressure is negative. So, this is negative, but this consists of coming from matric suction head plus the osmotic section head. The matric suction head is coming from the due to the matrics due to the capillary reaction etcetera which we have considered earlier.

And the osmotic section which is due to the presence of salts. So, this is gravity head or elevation head. So, therefore, the total head should consists of matric suction head, osmotic section head plus gravity head under this total head.

So, the flow takes places. So that means, generally if you take horizontal soil column initially the soil is completely dry. So, then now when the flow takes place through the soil mass or when there is a water reservoir connected to this then the head here is 0 because water reservoir is connected more than or equal to 0 because if you maintain some particular head then this is more than 0 and within the soil mass the head is negative.

So, because there is a gradient or the total head difference from point one to another point. So, along this length x there is a head difference because of which the flow takes place. So, here the elevation head does not make any role I mean the elevation head does not come into picture because so, it is a horizontal flow. So, the elevation here and the cognitive dissonance here it is the same. So, therefore, elevation head or gravity head does not make any role.

If you do not have any salts present in the in the soil mass then osmotic head also does not play a role. Only the matric suction head at point x 1 and matric suction head point at x 2 or any other points within the soil mass are different this values are different. So, therefore, flow takes place in this direction.

So, here the head is either equal to 0 or slightly more than 0 everywhere else. So, the head is less than 0 because its negative head matric suction head is negative. So, therefore, there is a flow that takes place from higher head to lower head flow takes place. So, therefore, we cannot we can use the Darcy's equation that is q equals to minus K i the water flux is proportional to the hydraulic gradient. We were earlier seen that the hydraulic gradient we can utilize provided we use total head here; so, d h by d x. So, therefore, d h by d x is a gradient and here h indicates the total head which is mentioned here.

So, now what is a k; K is hydraulic conductivity as we have mentioned that the hydraulic conductivity is a functional form, K which again depends on the head only times dh by d x. So, this is how we can utilize. So, then this is a generalized Darcy's law which can be utilized far flow through unsaturated soils.

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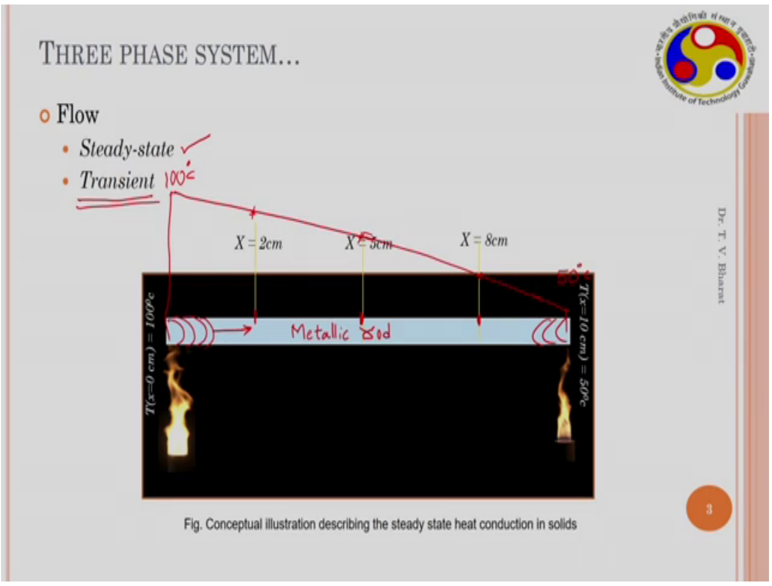


Now, we have we can see flow through any media can be whether steady state or transient. For example, if you take a metallic rod this is the metallic rod it is a metallic rod which is heated at both the ends. At one end the temperature is a constant temperature is maintained which is a 100 degrees and at another end the 10 centimeter away from the first boundary 50 degrees temperature is maintained. So, under these constant heating conditions at the two extreme boundaries so, there is a heat flux you can expect entertaining even cross section with the metallic rod. So, therefore, there is a conduction of heat that takes place.

So, from this end to the other end because here the more energies given here the less energies given. So, therefore, there is a flow that takes place from left to right in this particular case. So, assume that there are temperature sensor placed on the metallic rod at different special distances say x equal to 2 centimeter, 5 centimeter and 8 centimeter.

So, when the conduction of heat takes place you can measure the temperatures at these locations using the sensors in real time. When you are measuring the temperatures you would see that the temperatures would vary with a time. So, here the temperature is slowly increases because when the conduction of heat takes place from both side it take place initially because a temperature is 0 degree Celsius and your 0 or 20 degrees at room temperature and when you are heating from both sides conduction takes place and there is a because of this conduction of heat the temperatures would change with time at this locations.

But after some time you would observe that the temperatures would not change at these locations and there would be constant because here this will be 100 degrees maintained at any given time and here 50 degrees will be maintained at any given time. So, after this particular time the temperatures at any given location are constant. So, that particular state is called Steady state. So, which is time in variant the values of temperatures are time in variant.

So, therefore, steady state is will be achieved after certain time and then the value which you are talking about here it is a temperature is time in variant, but during the transient state that the time or the temperature values change.

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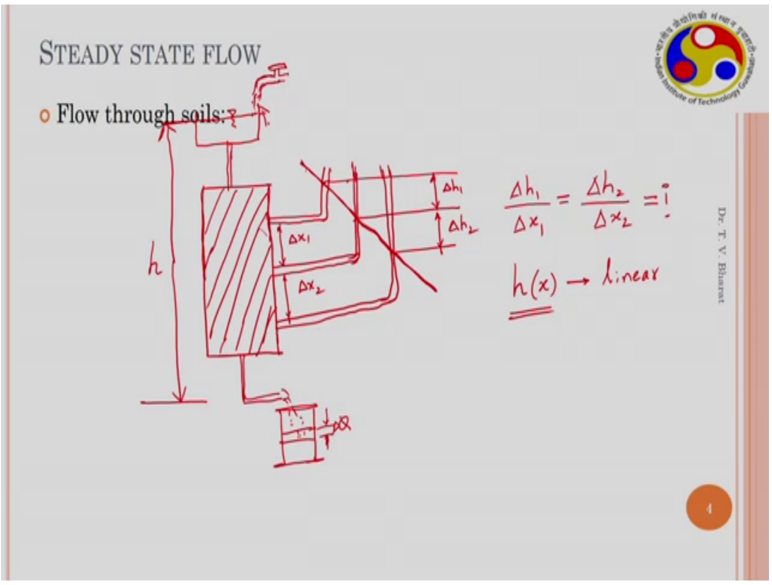


This particular phenomenon we see in soils also. In soils, generally when we conduct hydraulic conductivity tests on coarse grain soils and fine grain soils. We utilize this concept because in coarse grain soils when we have a soil column. So, this is the soil column. The soil column is connected to one reservoir. So, in the reservoir there is always a flow that takes place. You can connect to a pump. So, water always drops in and then you have a outlet.

So, you can maintain certain particular head always. So, this is soil mass and you connect to outlet. So, water comes out from here. So, now, there is a certain head which is maintained throughout the test initially the soil is fully saturated you saturate the soil and then after saturation you maintain certain head and here you can measure the flux.

So, flux of water which is flowing through it. So, generally the volumetric flux can be measured by measuring the volume of water which is collected in two different time intervals and from that we estimate the hydraulic conductivity using Darcy's law.

Now here this is the steady state experiment where the volumetric water flux or the flux which is coming out which is time in variant which does not change with time. And moreover if I connect a manometers a different depths within the soil sample what I can observe is that the head is higher in this particular manometer and head is smaller and head is smaller you see that there is a linear variation of head with depth for example, this variation is delta x 1 and this is delta x 2 where the manometers are located and this is delta h 1 and this is delta h 2. What we can absorb is that the delta h 1 by delta x 1is equals to delta h 2 by delta x 2, so, which is the hydraulic gradient.

So, the head drop across any given length of the soil specimen will be constant that is hydraulic gradient which is constant throughout this test when you maintain a constant hydraulic head across the soil sample. So, this is how we conduct a text and this is linear. So, the head varies a linearly with depth the head variation with x is linear in steady state saturated soils.

So, this test particular test is often conducted on saturated soil samples a on coarse grain soils because the volumetric flux or volumetric flow rate is significant. So, that we can measure what is amount of water which is coming out from the soils at any given two different time intervals can be determined.

So, from that we can estimate the hydraulic conductivity, but for fine grain soils time required for the flow to take place is very large because of the low hydraulic conductivity we utilize the concept of un steady state or transient conditions to estimate the hydraulic conductivity we connect to a Beirut instead of connecting it a constant reservoir we allow the head to drop with time and from this variation in the head we determine the hydraulic conductivity that is based on the transient flow concept.

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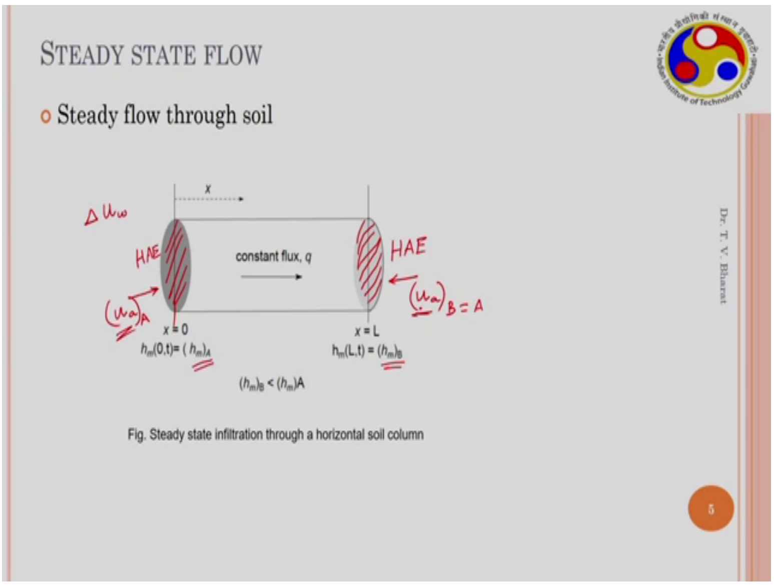

When it comes to unsaturated soils or partly saturated soils what can be expected? The question is whether steady state flow can take place through unsaturated soils or not is a big question. So, because often we feel that when you take a dry soil sample, when we connect to a water reservoir when water enters into the soil mass, so, the water when the water distributes the soil gets saturated. So, because of this saturations, so, the condition of the soil changes with time.

So, therefore, it can only be transient steady state flow may not be possible that is our understanding, but it is generally it is possible to have steady state flows through unsaturated soils. So, this is a particular example where we have a soil column a horizontal soil column where two different boundaries we maintain two different matric suction values. So, here at x equal to 0 we maintain one particular matric suction that is h m matric suction head A.

And at another section we maintain different matric suction value that is  h m d. So, how do you maintain is another question, we can maintain by through access translation technique we can maintain one particular suction here another particular suction here. Utilizing the steady state flow, the estimation of hydraulic conductivity of unsaturated soils we have already seen where either by using higher entry disk at this particular point. Here you can utilize higher entry disk and here also you can utilize higher entry disk.

Then you allow the water to flow under a certain head and also you give air pressure u a and both u w. So, the water pressure can be maintained delta u w can be maintained across this soils sample, but then  u a constant u a can be given. So, therefore, u a minus u w is maintained at these two boundaries this value will be different this value may be different. So, then by maintaining two different air pressures at two different boundaries and allowing the water to flow you can maintain two different matric suctions across the sample or you maintain the same value as this one same air pressure throughout, but you change the water pressure across the sample.

So, then there is a steady flow that takes place. So either way you can maintain two different suctions or two different total head values at two boundaries and because of this there is a constant flux that takes place through the soil sample. But this happens naturally many soils.

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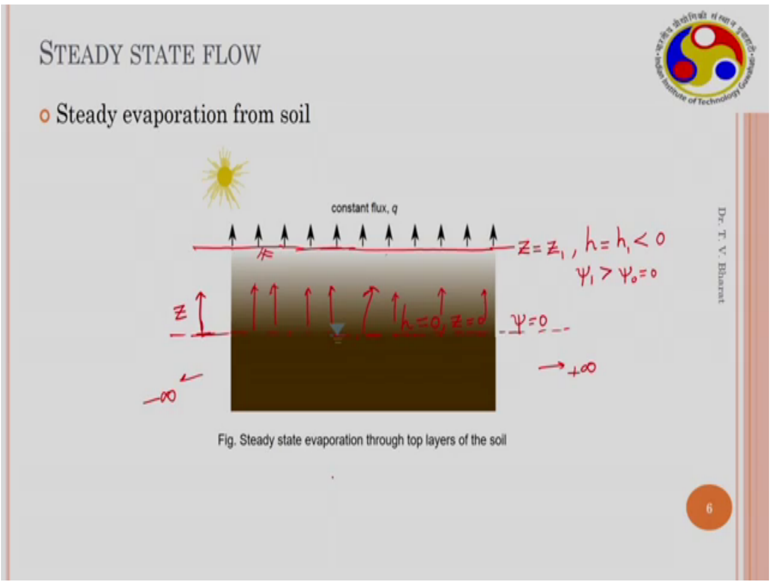


So, for example, a steady state evaporation from soil. The soil mass when the field where the groundwater table is located somewhere here. So, this is the ground water table location and this is a ground surface. So, now, because the groundwater table is located here it is a infinite source. So, this lateral extension is also infinity the lateral extension of the soil is also infinity. Now the water table is located at this particular level. So, now, due to sun or the heat that is available at the surface there is a constant evaporation rate that may take place in a given time period because of the constant temperatures that may be maintained for a given particular time period. So, then water evaporates from the soil and because we have a constant source here. So, the amount of water that is evaporated that would not significantly change the water levels within the soil mass.

So, therefore, here the head is always maintained constant that is h equals to 0 at z equal to 0. So, z is upward positive. So, at z equal to 0 the total head is equals to 0. Here and z equals to at some elevation z 1 you may have some h. So, this h is much smaller than h 1 which is much smaller than 0.

So, therefore, because as it dries the suction value increases or suction head decreases. So, therefore, the suction head is a negative value. So, if you are talking about in terms of suction it will be higher here it will be psi equal to 0. So, psi one is much high higher than psi 0 are 0, but suction head is a negative value therefore, which is much less than zero. So, it may be minus 300 centimeter are minus 1 meter or anything it could be depending on the evaporation rates and depending on the soil type.

So, therefore, depending on unsaturation level here at the ground surface and the soil type the values would vary. So, these are at the boundaries two boundaries therefore, there is a continues evaporation that takes place from the surface and there is a flow of water that takes place upward in upward direction from the ground water.

So, therefore, in this particular case if provided for a significant time the temperatures maintained constantly then you have a steady flow that takes place this within this vadose zone which is unsaturated. So, this is the unsaturated steady flow that can take place in this particular manner.

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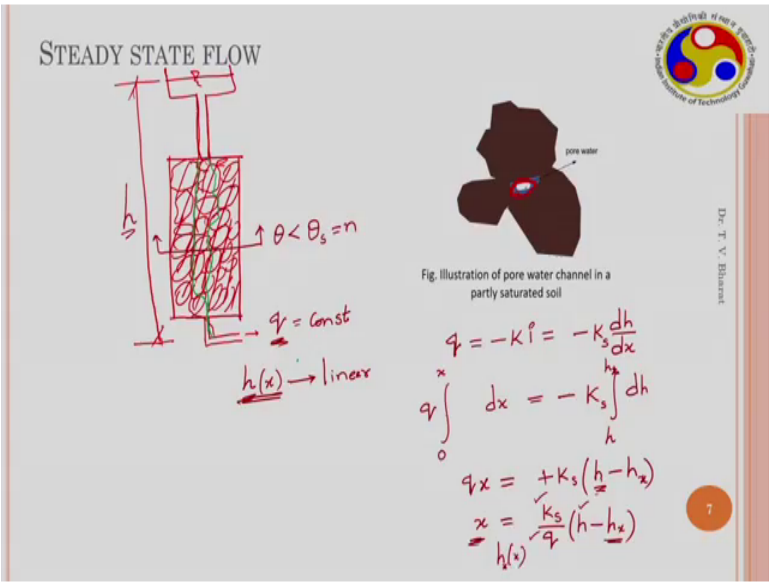


And often in infiltration also you may have steady state infiltration that takes place through unsaturated soils, but we may not notice for example, if you take soil column initially we maintain certain head to saturate the soil sample you connect to the reservoir you connect to a thin reservoir using thin pipe you connect to a outlet pipe. So, then you are maintaining certain particular head here also in this particular case.

So, what then what is a guaranty that the soil get saturated because you may have small pores within the soil small air pockets within the soil pore system to drive away this small air pockets from the soil pore system we require enormous energy are you require enormous energy to dissolve this air pockets into water for either to dissolve or to drive away from the soil water system.

If the energy is not sufficient to do either of this then the pockets will remain and the water flows through this channels this forms a channel and a pre path and simply water flows through. So, still with given time you may have a constant flux of water. For example, if you take coarse grain soils like assume that you may have big grains like boulders are gravel large particles if you are if you have in the system.

So, then they need not be a guaranty when the entire system need not get saturated, but if it finds a free path then water can come out. So, this is the free path this may be a free path and water can come out.

So, similarly it may form few free paths. So, in elsewhere the soil will be unsaturated, but there are few free paths through which the water can come out. So, because of which the flux may be constant with time, but then the soil is partly saturated soil is not completely saturated all the pores are not utilized are all the pores are not completely saturated if this condition arises even under constant flux the steady state flow can take place through unsaturated soil mass.

If the water content here theta is less than theta s saturated water content are porosity this situation can arise in the soil mass. So, if you start changing the head if you increase the head to higher values this flux may be different again now. So, because when your increasing the gradient the some of the pores may get saturated and the flux rates also will change.

So, as we see in our Darcy's law. So, how do we detect whether this is due to the increase in the hydraulic gradient or due to increase in the water content. So, what we could notice is probably will I will explain this in a minute how the head distributes with space is a question this head distribution with spaces is linear for saturated soils and whether it is a linear or non-linear we can absorb for unsaturated soils.

For saturated soils the variation is linear that we can verify also theoretically why it is linear. For example, if you consider the Darcy's law that is q equals to minus K i which is minus K d h by d x. So, if this can be written as q and d x minus K dh. So, this is the saturated soil this is for this is K s this is independent of hydraulic gradients.

So, it can be taken out and now the flux also can be taken out because the flux is constant with depth because its depth and time because its steady state flow. So, the q can come out then this is how it varies if I use boundary values at the boundary at x equal to 0 to sum x value. And here at the boundary you may have higher head to some lower value say h 1. So, then if it varies in this manner this is simply q x is equals to minus K s and h minus h one. So, h is constant or it can be written as h x. So, therefore, x is equals to simply K s by q times h minus h x.

So, the variation of x with h are variation of h with x is linear. So, this can be simply verified a substituting the value of K s and a flux and a constant head which may be maintained at given time given boundary. So, you can verify how h x verifies with x are this could be written in terms h x also h x in terms of x also can be written. So, this is essentially linear. But what about unsaturated soils is a question. So, that can be verified.

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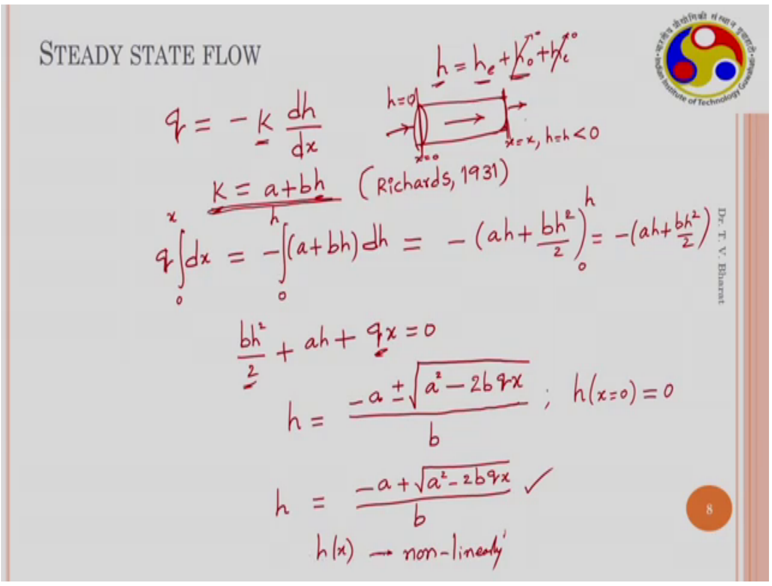


So, the same generalized form of Darcy's law is used that is q is equals to minus K d h by d x. So, here the h is the total head which consists of matric suction head osmotic suction head and as well as the gravity head or elevation head. If you consider a horizontal flow the elevation head is 0 and osmotic suction head that can be included if that is present, but generally when the salutes are not present this is also 0. So, essentially the total head is because of the elevation head alone.

So, therefore, here the K is a functional form. So, now, K needs to be substituted some value. So, what K values we can substitute. We can use a simple expression for K which is simply a plus b h. So, the K varies with head in this particular manner as a head increases K decreases. So, in this particular manner it varies and here this is given by Richards in 1931. So, this is an simple empirical relationship.

That can be utilized for understanding how the head varies with space in steady state horizontal unsaturated flows. So, when you substitute you get q d x you can integrate from 0 to say x. So, here minus a plus b h we integrate this from say for example, you have a column here at x equal to 0 and this is at x equal to x some value x. So, now, at x equal to 0 assume that you have a head of 0 and at this particular point the head is h.

So, it varies from 0 to h is negative. So, this is negative this is much smaller than 0 that is what I said because it is 0 and this is negative value the flow takes place from left to right. So, now, this can be integrated. So, if you integrate this minus a h plus b h square by two and the integration varies from 0 to h. So, this is simply minus plus b h square by 2 only here on the side it is simply q x. So, therefore, the equation is b h square by 2 plus a h plus q x is equals to 0 this is a quadratic equation.

We have a solution for this per head h is equals to minus b that is minus a here in this case plus or minus square root of b square that is a square here minus 4 ac four time this one times this one. So, it will be 2 d q x divided by 2 a. So, two times a is b by 2. So, therefore, it is simply b. So, this is the solution. If you use the boundary conditions one of the boundary condition that x equal to 0 the head value at x equal to 0 is equals to 0.

So, therefore, if you substitute x equal to 0 this value becomes simply a. So, minus a plus or minus a divided by b to make the h equals to 0 this should be plus then only h will become 0. So, therefore, the correct solution is minus a square root of a square minus 2 b q x is a solution for this particular one.

So, this is highly non-linear h varies with x non-linearly. Even though the form we assume for hydraulic conductivity with respect to head is a linear variation. So, hydraulic conductivity varies linearly with hydraulic head even then the hydraulic head variation with space is special distance is non-linear. We can solve simple example.

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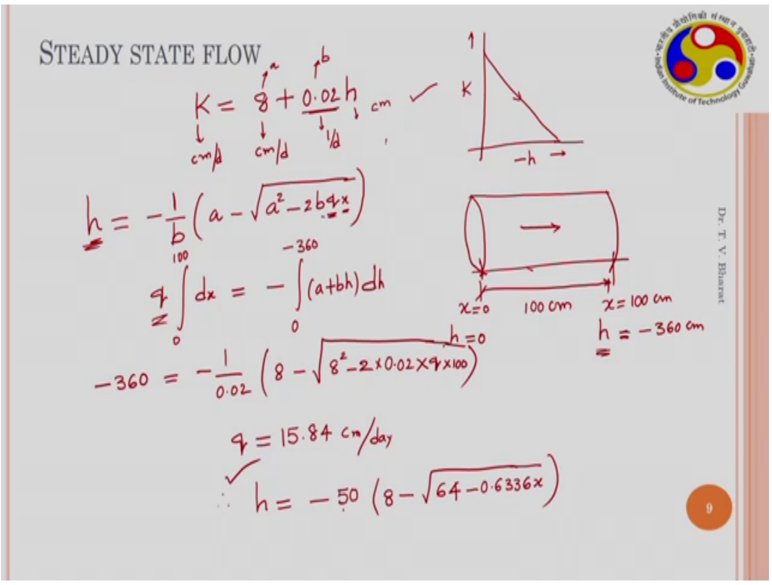


So, for example, the equation that is given is K equals to 8 plus 0.02 h. Here h is negative therefore, as a negative head increases the hydraulic conductivity decreases. So, this is 8 in centimeter per day. So, therefore, this also in centimeter per day and h and this is this coefficient this is a in the earlier equation and this is b, this has units of per day. So, therefore, these are centimeters and this is 1 over d. So, therefore, it has units of centimeter per day right dimensionality is fine.

So, if you have this particular equation, this particular equation known for in hydraulic conductivity which is varying in this particular manner as a either minus h is increasing in this manner or h is decreasing in this manner.

So, then hydraulic conductivity drops like this. This is a constitutive relationship or this is a relationship between K and h. If so, then you can determine for again this is a constitutive relationship or this a hydraulic conductivity variation that is given a soil mass soil column which is taken the soil column which is considered has the considered soil mass soil column is 100 centimeter long the boundary condition here at x equal to 0 is that h is equals to 0 so that means, it is completely saturated and at x equal to 100 centimeter the head is equals to minus 360 centimeter.

So, under this condition head is 0 here head is a negative minus 360 therefore, the flow should take place from left to right. So, this is how it should change.

Now, if this varies in this particular manner then what is the suction head variation with x that is the question at steady state? So, here somehow you are maintaining a constant suction head that is h is equals to 0. Here you are consistently maintaining a constant suction head that is equal to minus 360 centimeter then how the suction head varies with depth within this column is a question at steady state.

So, we can utilize our equation h is equals to minus 1 by b times a minus square root of a square minus 2 b q x. So, here minus 1 b is taken out. So, that is why this is this form. So, here everything else is known, but the flux is known then we can establish the relation between h and x. So, we can utilize one of the boundary condition that is given are we can integrate and then do it. We can again integrate the q, q anyways can be taken out q d x is equals to minus a plus b h d h. Here x is varying from 0 to 100 and h is varying from 0 to minus 360.

If you solve this expression you will get q otherwise you can also substitute the value of h at x equal to 100 centimeter when you will get q. If I do that h is equals to minus 360 is equals to minus 1 by b is 0.02 and a is 8 minus square root of 8 square minus 2 times b is simply 0.02 times q x is 100 centimeter.

So, if you solve this you get q that q is q value is equals to 15.84 centimeter per day. So, therefore, h is equals to minus 50 times 8 minus square root of 64 minus 0.6336 x. So, this is a expression, this is how h varies with x.

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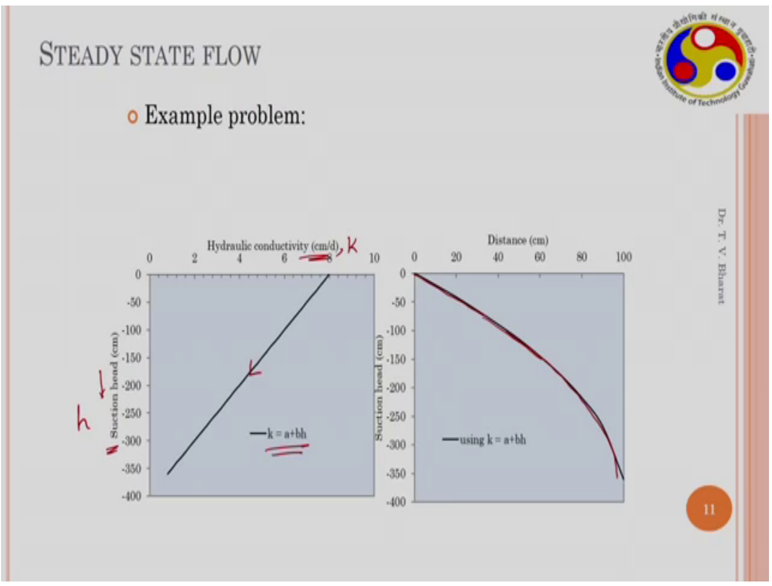

So, therefore, this is a expression for hydraulic conductivity K versus suction head h. So, as a suction head this can be seen from this manner.

So, hydraulic conductivity decreases with increases in suction head ok. As a suction head this is dropping which is becoming negative and negative more and more negative the hydraulic conductivity decreases.

So, this is the expression we have utilized if you utilize then this is the variation for suction versus distance this is highly non-linear. So, initial some portion is linear, but again  this is highly non-linear. So, these are non-linear expression fine.

So, even though the hydraulic conductivity variation with suction head assume to be linear the suction head variation with special distance within the soil column for steady state horizontal flows is non-linear you can also assume another function like you can assume K is equals to K s times exponential of alpha h; alpha h which is given by Gardner in 1958.

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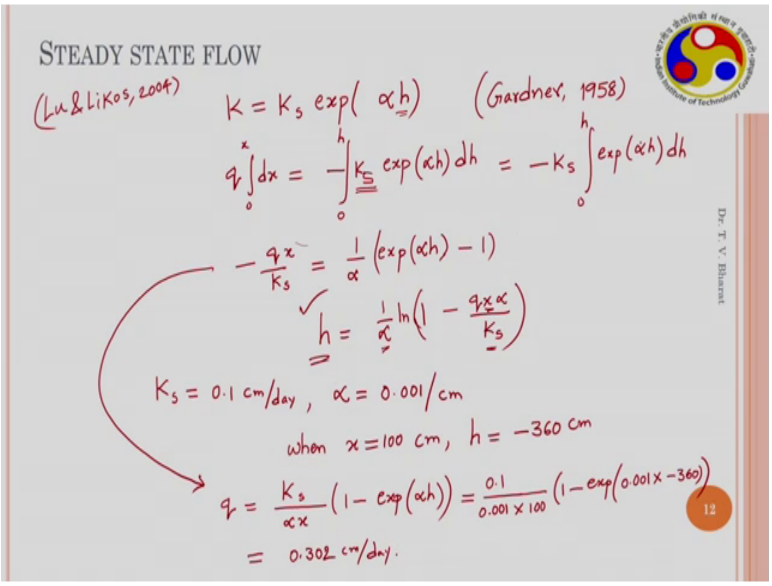


So, similarly we can derive expression which is this particular solution is given in Lu and Likos textbook. When you substitute you get a q d x which is varying from 0 to x is equals to minus for K if you substitute this expression K s times exponential of alpha h d h and here which is varying from 0 to h. So, K s anyways is a constant. So, you can take it out. So, minus K s integral 0 to h exponential of the alpha h d h. So, this is minus q by K s x is equals to the integration of this one is 1 by alpha exponential of alpha h.

So, if you simplify this 1 by alpha h minus 1. So, if you simplify this expression and write expression for h this can be written as 1 by alpha again because when this goes here it becomes minus q x alpha by K s and 1 if it is taken that side this is expression you get and if you take natural log and this is a expression and alpha fit is taken out then 1 by alpha.

So, the head variation with x is highly non-linear again, here it is logarithmically varying. So, here also you can give some boundary values and provide some values for K s and alpha parameter then you will get how the head varies with x you can observe quantitatively. Let us take one simple example you assume K s saturated hydraulic conductivity to be 0.1 centimeter per day. And alpha is equals to 0.001 per centimeter. So, these are the units because here head has units of centimeter, see if this has units of 1 over centimeter. Then this does not have any units, K s should be have same units as K. So, therefore, both have centimeter per day.

Then if you substitute this in this expression the same boundary conditions if you use that at x equal to 100 centimeter you have head value of minus 360 then you can obtain q. So, therefore, when x is equals to 100 centimeter, the h is equals to minus 360 centimeter.

So, therefore, if you are substitute h is equals to 1000 you can write for q from this expression. So, q is equals to K s from this expression we can write K s by alpha x into 1 minus exponential of alpha h. If you substitute K s equals to 0.1 centimeter per day and alpha is 0.001 times x is 100 centimeter times 1minus exponential of alpha is 0.001 times head is minus 360. If I substitute the q value comes out to be 0.302 centimeter per day.

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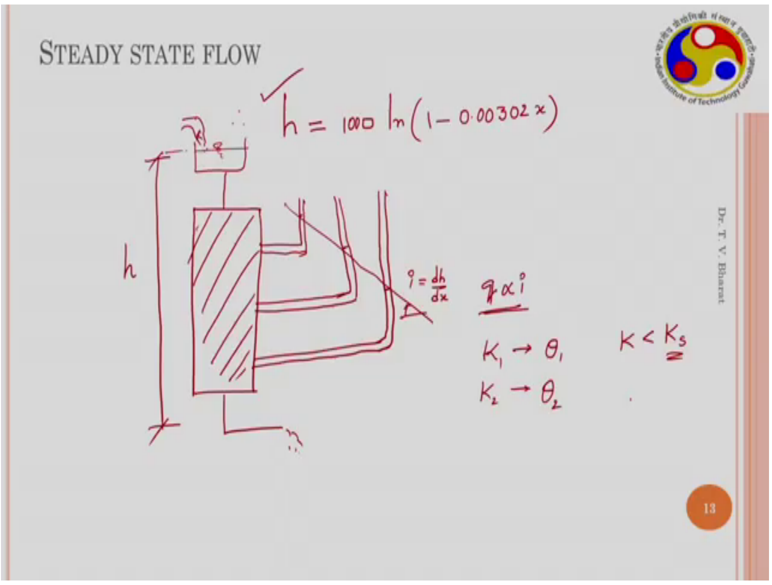


So, therefore, the expression for h is equals to 1000 log 1 minus 0.00302 x. So, this is the expression for head with x. So, which is again a non-linear expression.

In summary the steady state flows in unsaturated soils also can take place depending on the boundary conditions and type of soil. If you have a highly course grained soils like gravels and a sands there may be some path ways developed within the soil mass and a remaining soil may become unsaturated and using the pore water space using some free paths the water can flow and you may get constant flux.

So, even though you get constant flux it does not guaranty that the soil is completely saturated. So, as I said if you take a soil mass and connect to a reservoir constant water head reservoir here water flux can be maintained constant water head is maintained.

So, now, this is the soil sample. So, now, which is connected to outlet pipe where water comes out. So, you have a particular head that is maintained constant head that is maintained across the soil sample. Now in this particular case when you connect two manometers.

The variation in the head with distance if this itself is non-linear that means, the soil is unsaturated. If this variation is linear with depth this is the head variation with depth the slope is i that is d h by d x right the head variation with the space is non-linear that means, the soil is still unsaturated. Moreover as we increase the head the flux may increase, but the hydraulic conductivity should remain constant. so, that means, the variation of q with respective hydraulic gradient should be darkly dependent because K s is independent of hydraulic gradient. So, when the K s is remain same estimated hydraulic conductivity remains same with change in the gradient then the soil is saturated.

So, if that is not satisfied, but you are getting a constant flux that means, your soil is partly saturated, but there is a steady state flow that is taking place through your soil sample. So, in this particular case you can estimate the hydraulic gradient and corresponding moisture content can be determined by destructive technique.

You can take soil sample and then measure it is weight and then put it in oven and again you measure the weight you obtain the (Refer Time: 47:18) water content knowing the density you can estimate the volumetric water content. So, therefore, there is one hydraulic conductivity which is estimate to corresponding to given theta.

So, now by varying the head you can maintain different volumetric water content you will get different values of K. So, this way you can obtain hydraulic conductivities which are less than the saturated hydraulic conductivities. So, to obtain the saturated hydraulic conductivity is generally very high head is required to be maintained. So, that all the air pockets will be taken out from the soil system and the soil will be saturated completely.

So, this is how we can obtain the saturated hydraulic conductivity. So, the unsaturated hydraulic conductivity estimation is also done using the similar way by maintaining particular water content are particular suction head within the soil sample a constant flux is obtained and it is measured and based on that the hydraulic conductivities are estimated. So, these hydraulic conductivities are unsaturated hydraulic conductivities.

So, this way one can estimate the hydraulic conductivities in unsaturated state, but it gives a immense information that even though the hydraulic conductivity variation is assume to be varying linear with suction head the suction head variation with depth within the soil mass in a horizontal flow steady state condition is non-linear.

Thank you.