Unsaturated Soil Mechanics
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Week - 05
Lecture - 14
HCF Determination

Hello everyone. Let us discuss the Hydraulic Conductivity Function Determination because along with the soil water characteristic curve we require another function which is hydraulic conductivity function. Earlier we have seen the HCF give us the relationship between the variation in hydraulic conductivity with amount of moisture present within the soil. So, as the moisture content decreases the hydraulic conductivity decreases because the available flow channels decrease.

So, therefore, when an infiltration is taking place through initially partly saturated soil or unsaturated soil there is increase in the moisture with time therefore, the hydraulic conductivity increases and which will become equal to the saturated hydraulic conductivity, once the soil completely become saturated. Similarly when there is an evaporation taking place through a fully saturated soil system reduction in the moisture content, the hydraulic conductivity of the soil decreases and which becomes nearly 0 when the soil water content reaches the residual state.

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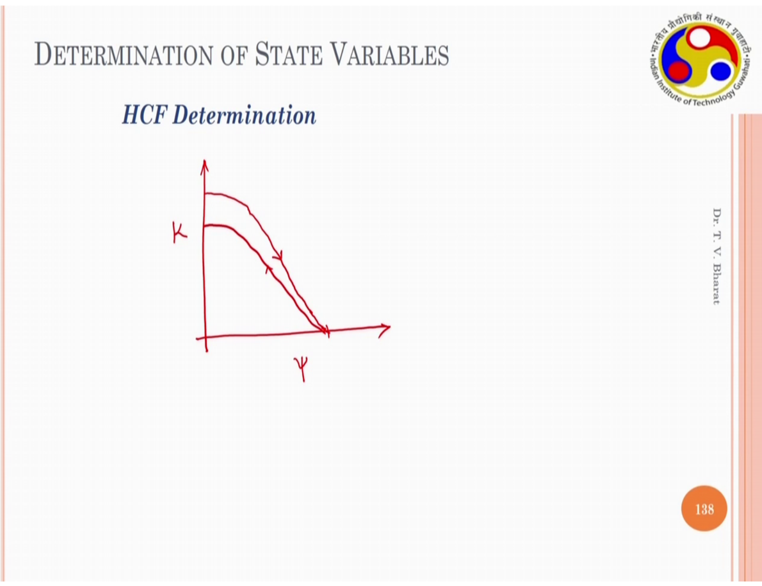

This we have seen earlier where the relationship between hydraulic conductivity and suction has been plotted, where hydraulic conductivity is plotted on y axis and the matric suction which is inversely related to water content is plotted on x axis. As we have seen the hydraulic conductivity decreases with increase in the suction and beyond the air, air entry value it decreases to great value and nearly become 0 at very high suction value.

So, this is a drying case and similarly you may have wetting case. So, therefore, you will have a hysteresis in hydraulic conductivity function. How to determine hydraulic conductivity function? Because the hydraulic conductivity of saturated soils we generally determine in the laboratory using falling head technique or the constant head technique depending on the type of soil we have. But as we see that the hydraulic conductivity of fine grained soils is very low therefore, we generally determine using variable head technique.

But when you have a unsaturation in soils, that is initially soil is partly saturated, and then you allow the water to go in first the sample gets wet, right. So, for a given suction or for a given water content what is the hydraulic conductivity in partly saturated determination? The determination of hydraulic conductivity at any given constant water content or constant suction value is very important aspect because we need to establish such a relationship for understanding the flow behaviour. So, any evaporation that is taking place through saturated soil mass or infiltration through a dry initially dry soil sample, the flow rates can be determined using the soil water characteristic curve and hydraulic conductivity function.

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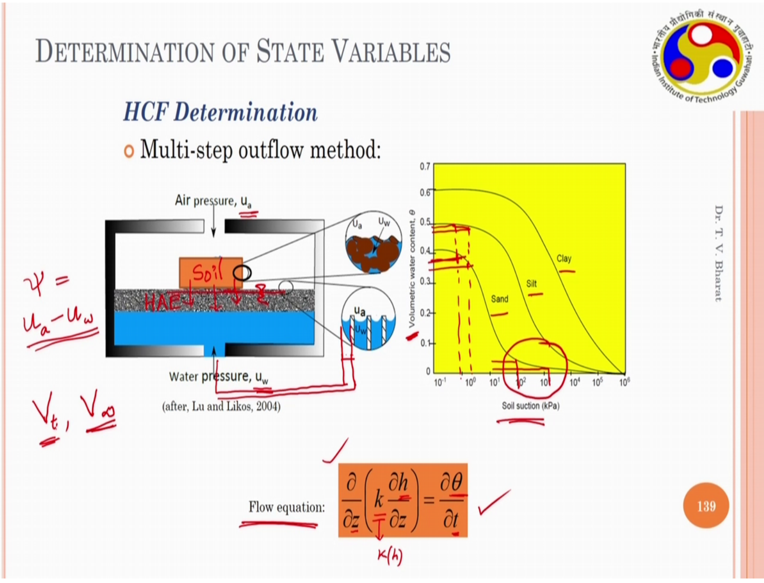

So, we will not discuss one of such laboratory techniques where we determine the hydraulic conductivity function. The technique is called multi step outflow technique, which is similar to the technique utilizes the concept of the increase in the air pressure for maintaining certain suction, which we used in axis translation technique. Earlier if you recollect in axis translation technique we have a high pressure chamber. So, in that we have a high air entry disk and you have water up to the nearly high air entry disk level.

So, now the high air entry disk can have different capacities like if it is having a 15 bar capacity. So, then it can allow suction difference or the pressure difference between water and air up to 1500 kilo Pascal. Before on this high air entry disk we put a soil sample initially probably saturated sample when you put. There is a hydraulic equilibrium between pore water in the soil, this is a soil sample, in the pore water in the soil and the free water here will have a hydraulic equilibrium. Then if you increase the air pressure beyond atmospheric pressure and you maintain a water pressure equal to the atmospheric pressure you can generate a suction value of u a minus u w.

So, when air pressure becomes more than atmospheric pressure and water pressure is 0 then you can develop suction value this is psi of u a minus u w you can exert on the soil sample. So, when you initially on a saturated soil sample when you provide this much of suction or u a minus u w then water contents starts decreasing, the water in the soil sample will come out through this and reaches in the water reservoir. If this is connected to a burette, so then we can monitor the water levels that are changing with time.

So, as we have seen earlier in axis translation technique as the suction pressure increases on the soil the water content decreases which establishes soil water characteristic curve there is relationship between volumetric water content and soil suction for different soils. We utilize the same concept for understanding the flow rates. When a constant suction is applied on the soil, when water contents starts decreasing if we can increase a small suction very small suction at any given initial suction value, so then the change in the water content with time if you can monitor that is flow rate if you can monitor. So, then we should be able to establish the hydraulic conductivity. Let us understand how it works.

When a suction pressure is applied on a soil sample water content decreases this phenomena is similar to consolidation. In the consolidation when soil sample is subjected to a given loading mechanical loading the water content decreases, water content decreases because pore water pressure increases to the same value to the applied mechanical pressure that is called total stress. And because the sample is connected to two boundaries in the consolidation odometer test there is a gradient that is established across the soil sample because at the boundaries, you have 0 water pressure and at the middle of the soil sample you have pore water pressure equal to the applied normal stress.

So, because of this gradient the water flows through the soil sample and at equilibrium the pore water pressure is close to 0 or when we say 90 percent consolidation is achieved the pore water pressure is a very small value. So, then at this particular point we estimate the void ratio and corresponding effective stress because pore water pressure is nearly 0 the applied mechanical stress becomes effective stress. So, we establish void ratio and sigma dash relationship.

Similarly, here when the soil sample is subjected to suction the water content in the soil decreases because there is a negative pressure that is applied slowly the water starts coming out of the soil sample. If we can establish a governing equation for this transient case, we can estimate the hydraulic conductivity function. So, this is the flow equation which is similar to the terzaghis one dimensional consolidation equation which is the dou by dou z of k times dou h by dou z dou z is equals to dou theta by dou t. k here is the hydraulic conductivity which is the function of h or theta h is the total head in this case, it is a matric suction head and theta is the volumetric water content. So, here h and theta both are the state variables dependent variables and z and t are the independent variables, z is the spatial variable and t is the time temporal variable.

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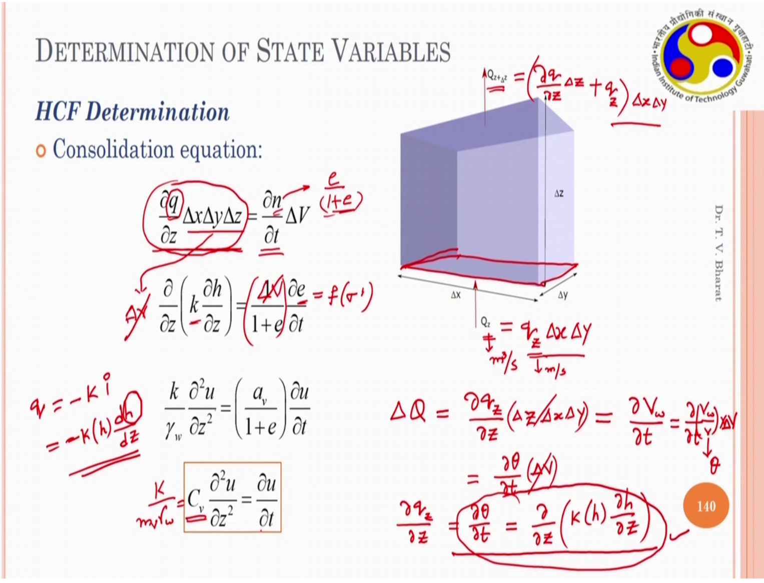

If we recollect the consolidation equation which is derived based on the mass conservation principle where if you assume representative volume of the soil sample there is a volumetric flow rate of q z which is going into the soil sample, and q z plus delta z which is coming out this is the volumetric flow rate that is coming out. So, this can be written as flux q z times delta x and delta y.

This is a cross section area. So, the units of volumetric flow rate is meter cube per second similarly here the units for q z flux or this is Darcy's velocity which has units of meter per second times meter square, so the units satisfy. Similarly here the flux is or the volumetric flow rate is equals to the dou q by dou z times delta z, this is the plus q because this value is initial value of q z and variation in q z, because there is change in the flux; it is a transient condition that there is a change in the flux that is because of this into delta x and delta y.

So, change in the flow rate is nothing but the dou q by dou z into delta x times delta z, delta x flow rate is equals to dou q the difference between this one and this one which is nothing but dou q by dou z times delta x delta y delta z, which is equals to the change in the flow rate is because of change in the volume of the soil sample, because in a two phase system where the soil is fully saturated when you apply mechanical load whatever the amount of water decreases in the system the same amount of deformation takes place.

So, here change in the porosity is dou n by dou t times delta v rate of change of flow is equals to change in the volume. So, from this the n can be written in terms of e by 1 plus e and because that when the n is written as e by 1 plus e. So, the 1 plus e represents the total volume. So therefore, change in the volume by total volume is strain; for a small strain cases you can take the 1 plus 1 by 1 plus c times delta v out. So, you can take this out, this is a strain term into dou e by dou t. So, here also you have delta v, so these two get cancelled. So, you have this expression. Here the q is substituted with the k times I assuming the Darcy's law is valid.

So, k times the gradient is dou h by dou z. Here the hydraulic head can be written in terms of pore water pressure by dividing with gamma w. Then assuming the hydraulic conductivity is constant you can take this out of this derivation then this becomes k by gamma w times dou square u by dou z square. And here void ratio again if it can be written in terms of sigma dash, because e versus sigma dash, this can be related to sigma dash a simple linear relationship is assumed. So, then it can be converted in terms of sigma dash and again sigma dash is related to the excess pore pressure by effective stress principle. So, this can be written in terms of u. So, you get this expression here C v is k by m v gamma w. So, this is consolidation equation.

So, why discuss the consolidation equation here is; your flow equation is similar to your consolidation equation. Let us see how. The flow equation derivation, I am not going to discuss here but I will try to derive the flow equation from your mass conservation expression; how similar this is with your consolidation expression I am going to show. So, here the change in the flow rate is equals to dou q z by dou z times delta z into delta x times delta y.

So, this is the change in the flow rate which is equals to the change in the volume in consolidation expression, but here the flow rate is changing; initially the flow rate was 0 when there is no suction applied, but as soon as the suction is applied there is a flow rate; flow rate starts increasing because the water content in the sample is decreasing. So, the water content of the sample is decreasing therefore, change in the volume of water with time this causes change in the volume of water with time.

So, this can be written in terms of theta by dividing and multiplying with capital V or total volume. So, if this is divided with capital V and multiplied with capital V, so this ratio is nothing but theta volumetric water content so, this is equals to dou by dou t of theta times so delta v. So, total volume. So, approximately you know if you have show that the volume does not change, so this volume get cancelled which results dou q z by dou z is equals to dou theta by dou t.

The dou q z by dou z can be further simplified if we assume the general Darcy's law is valid. The general Darcy's law for unsaturated soils is the head represents: the total head which includes matric suction head, and if you have solutes then osmotic suction head as well then that is total head is a hydraulic conductivity is a functional form, hydraulic conductivity varies with moisture content or suction head.

So therefore, if you consider that here q is equals to k i or minus k i, we use here if k can be written in terms of you know functional form and i is represents the total head, here h includes matric suction head osmotic suction head everything, and then this is generalized Darcy's law; if generalized Darcy's law is valid. So, then you get an expression for flow in unsaturated soils which is nothing but dou by dou z of k times h and dou h by dou z. So, this is the expression we have seen here.

So, this is flow equation. Here we used the same mass conservation equation which is used for consolidation and which can be further simplified and also shown very close to the consolidation equation.

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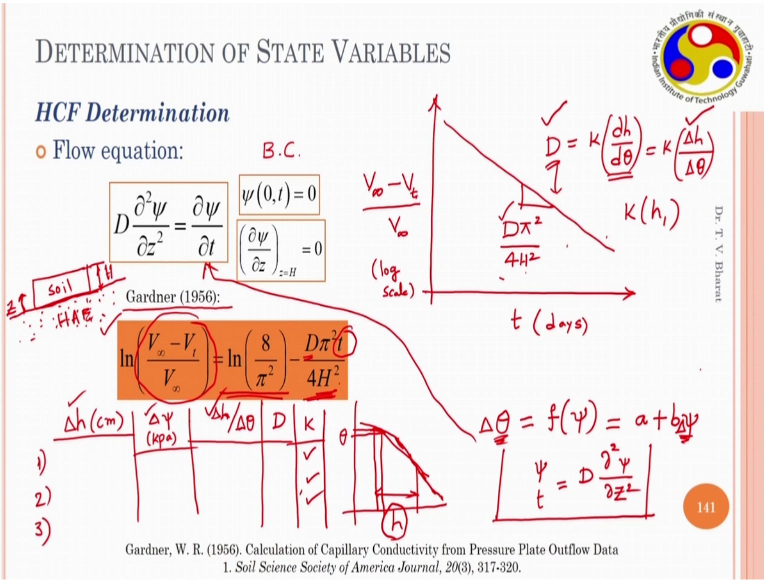

Let me show it here. So, the expression which we have written is: d dou by dou z of k times dou h by dou z is equals to dou theta by dou t. Here, if I write this as k times d h by d theta times dou theta by dou z this is dou theta by dou t, so then this is slope of soil water characteristic curve and multiplied by hydraulic conductivity function.

So, this whole thing together which has units of meter square per second, because hydraulic conductivity has units of meter per second and this has units of meter, so this whole thing together has units of meter square per second which represents diffusivity. So, therefore, I can call it as; I can write this expression as diffusivity which is again a function of either h head or theta times dou theta by dou z is equal to dou theta by dou t.

If I can approximate the d variation with z is ignorable if my sample size is very small or something, and if I can ignore the variation in diffusivity with depth then this equation becomes D times dou square theta by dou z square. This expression is similar to the consolidation equation. So, here D has D is diffusivity which has units of meter square per second similar to the coefficient of consolidation; coefficient of consolidation also has units of meter square per second. Only difference with consolidation equation is, in consolidation equation the dependent variable is u the pore water excess; pour water pressure here the dependent variable is moisture content.

In the consolidation process when you apply a mechanical load the water content decreases with increase in the load or for a given load the water content decreases with time and with space also it varies. So in but however, the soil is completely saturated at any given time. However, in this particular case with the change in the at any given suction there is a change in the volumetric water content with time and space due to decrease in the water content. The soil becomes unsaturated or soil is in unsaturated state soil need not be fully saturated. There may be air space also available in this particular case. However, this expression is similar to the consolidation equation.

I can further simplify by assuming that the change in the volumetric water content with suction is linear. If I assume that the volumetric and water content varies are; the volumetric water content varies with suction in a linear manner. So, I can assume that this is a plus b psi. So, in this linear manner if it varies, then if you substitute here for theta you get expression for dou psi by dou t is equals to d times dou square psi by dou z square. So, this expression is valid provided theta variation with psi is linear. This is valid.

Let us see the assumption after little more time. Let us revisit this assumption when we discuss the axis translation technique or multi step outflow technique. So now, this expression is same as this expression. For this expression this is a flow equation and psi these are the boundary conditions. The boundary condition at the bottom of the soil sample, you have a soil sample which was placed on a high air entry disk a fully saturated high air entry disk. So, the water level is up to the high air entry disk and this is the soil sample. So, variation in depth is z is upward positive.

Now, at z equal to 0 at this boundary the suction is 0 at any given time ok. These are the boundary conditions, when you utilize this boundary conditions Gardner has given an analytical solution. Gardner in 1956 has given an analytical solution for this particular set of equations which is log V infinity minus V t by V infinity equals to log 8 by pi square minus d times pi square t by 4 h square.

H is the height of the soil sample so this is height, height or thickness of the soil sample. So, this particular work was published in Soil Science Society of America Journal the title of the work is Calculation of Capillary Conduction Conductivity from Pressure Plate Outflow Data. This particular technique is also called pressure plate operators or multi step outflow technique.

So, once we have this expression we can estimate diffusivity and if you have diffusivity and soil water characteristic curve data we can establish the hydraulic conductivity function. So, let us see how we do it.

So, as I said in this multi step outflow technique initially you take the fully saturated soil sample which is which has a very good hydraulic conductivity with the high air entry disk. Then you apply very small increment in the suction. So, small increment in the air pressure can create small increment in the suction. When you gives small increment in the suction then the volumetric water content in the soil sample should decrease, so slowly the water will start coming out from the soil and it will come to your burette or water collector.

So, you can monitor the volume of water which is coming out from the soil. At any given time volume of water which is coming out, the volume of water is V t and at equilibrium the total volume which has come out is V infinity. And beyond that there is no change in the water, there is no change in the water content, so then it has achieved the equilibrium. So, this is at equilibrium and this is transient; volume of water can be recorded with change in the time at different time intervals, so you get V t and you get V infinity.

Now, this data you will have for a given change in a suction or change in the h head you know small centimeter; this is a small change in the head and you know corresponding delta h delta psi. So, this is in kilo Pascal. And you know change in the head to change in the total moisture content. This data also you have, because when you apply a small increment the change in the volumetric water content is known this ratio can be constant if the applied head is very small. That means, in the soil water characteristic curve this is theta versus h, this is expression. For any small increment in the head this variation is can be approximated as linear.

So, at any small interval the any value the change in the volumetric water content can be approximated as linear, but if you change a huge variation in the head then this cannot be linear; that will be erroneous. So, it should be very small increment in h. So, the same assumption we also made here. The theta is linearly dependent on psi. So, essentially the variation in volumetric water content with change in the delta sigma is linear. So, you choose very small increment in the head then this is linearly varying. So therefore, this is satisfied.

So, you have suction variation or delta h and this is in your hand, and you know what is the slope that is delta h by delta theta, because theta is the initial water content minus the water content at the end of the test that is delta theta.

So, we have that data as well. And we have how volume is changing this is for one particular set of data. For this particular data we know how volume is changing volume of; volume of water which is changing with time and volume at infinity time that is at equilibrium also we know. So, when I plot V infinity minus V t by V infinity on y axis and time on x axis in probably days it takes very long time for water to come out. Then when you plot this you got a straight line. So, for a small increment in the suction or small increment in the suction head the variation in V infinity minus V t by V infinity and other is this quantity. And t, t on x axis on normal scale and this is on log scale.

When you plot this is linear which has an intercept of log 8 by pi square and which has a slope of d times pi square by 4 h square; this is a slope. Slope is d times pi square by 4 h square. So, we can determine the slope. So, knowing the slope knowing the h value and a pi universally it is a constant then we obtain the D value. Once D is known, so as D is equals to k times dh by d theta. So, the slope of soil water characteristic curve or this particular quantity we already know, because for a change in for a change in head what is the change in the volumetric we already know.

So therefore, this quantity is known and this quantity is known which is determined from the graph. So, we can obtain k value at the increment of h 1. So, corresponding D and corresponding K can also be obtained. So, for different values of delta h variations we can obtain k values. So, this is how one can establish the hydraulic conductivity variation with volumetric water content or hydraulic head.

So, this is a popular technique which is widely used the same technique can be used for the determination of soil water characteristic curve and hydraulic conductivity function at the same time. So, this using the single method, you can established both SWCC and HCF. So, most of the available literature data for different soils; data of HCF and SWCC are based on the multi step outflow method only. Laboratory determination is based on this particular technique because this is widely used and this is most reliable compared to all other available techniques.

So however, I want to bring your attention to the some of the limitations exists in this particular technique. First limitation: we all know that the maximum suction range in this particular technique is influenced by the high air entry disk capacity. So, mostly the in available market the maximum capacity of high air entry disk is 15 bar. So, maximum suction we can go up to is 1500 kilo Pascal. So, beyond that it is not possible to determine the hydraulic conductivity. So, knowing the fact that the maximum suction range is 0 or maximum suction range is 1500 kilo Pascal this technique is not useful for establishing the entire hydraulic conductivity function as well as soil water characteristic curve on clays.

Generally, clays will have very wide range of suction for SWCC and HCF. So, generally I discussed probably one case of bentonite in the earlier lectures where we have seen that the hydraulic the air entry value of the bentonite, in powdered state is more than around 5000 or 10000 kilo Pascal high air entry sorry bentonite sample. So, on a compacted state it will be more than that.

So therefore, has a higher the air entry value of the soil is so high we cannot determine we cannot we cannot establish the entire SWCC and HCF using this particular technique. Moreover, this technique can be therefore used for determination of SWCC and HCFs for sands: sand, silt, where the maximum suction range falls below 1500 kilo Pascal.

So therefore, generally the available data is for such soils. For pure clays, like the bentonites in a compact state or in a powdered form or loose state the entire the SWCC cannot be established using this particular technique. And also we have seen that using any single technique we cannot establish the entire SWCC and as well as HCF.

So, this technique is very useful for sandy soils or some silts. Moreover it requires lot of maintenance, because the high air entry disk should be properly cleaned and it should be fully saturated in this particular case. Secondly, the increment in the suction should be very small, because we are making an assumption that volumetric water content is linearly dependent on the suction. That validity is possible if you consider only small increment in the suction.

If you satisfy that condition then this particular analytical solution is valid and the estimated hydraulic conductivity function is also valid. However, when the suction increment is very small there are other issues. The issue is measurement of water; water content or measurement of volume of water.

So for example: if a small suction value is increased in this particular case, so the amount of water which is coming out is nearly unnoticeable. So, therefore, estimating the volume of water that is coming out with time is very very difficult. So, in that such case we cannot accurately determine the volume of water at the end of the test that is at equilibrium minus V t by V infinity with respect to time cannot be accurately estimated. So therefore, the estimated diffusivity is wrong, so the estimated hydraulic conductivity function is wrong.

So, there should be a balance in choosing the increment in suction head. If you choose very huge variation in the delta h wrong approximation of the Gardner's expressions therefore the estimation of D is wrong. And choosing very small delta h value results in poor estimation of volume of water which is coming out from the soil sample.

Similarly at the residual state also: it is very difficult to account for even when there is a significant change in the suction the amount of water that is decreasing is very very small it is very difficult to monitor the water which is coming out. So, generally when the soil reaches the residual state the increase in the suction results in very small amount of water leaving from the soil.

So, these are the some of the assumptions involved in multi step outflow technique. So, there are other techniques that we will discuss in the next class.

Thank you.