Unsaturated Soil Mechanics
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Week - 01
Lecture – 01
Introduction
Fundamental aspects of Unsaturated Soil Mechanics and its Basic Principles

Hello everyone, welcome to the course Unsaturated Soil Mechanics. Now, in this first lecture I will take your attention to the scope of this particular course and possible applications in the field of geotechnical engineering.

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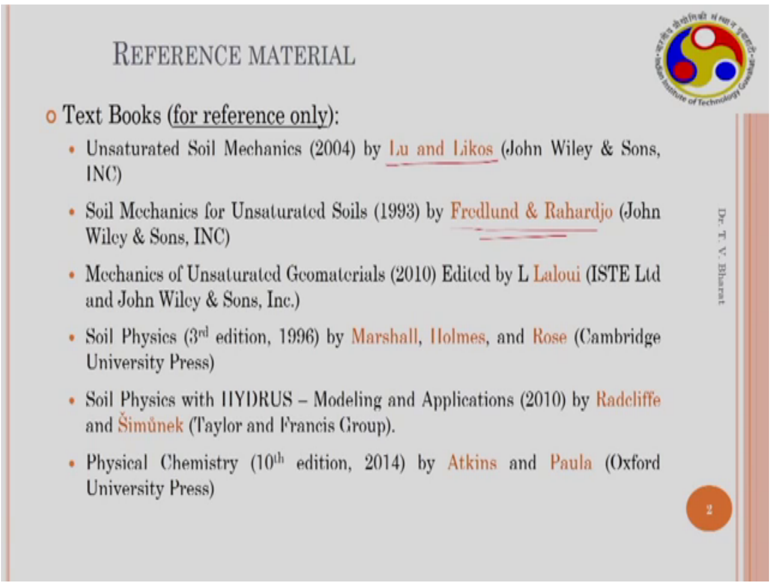

Let us look at some reference textbooks available on this topic. The first one is Lu and Likos, these are the two authors Ningaloo and William Likos. They have developed a book on this particular topic called Unsaturated Soil Mechanics in 2004, which is very popular book.

And earlier book was Soil Mechanics for Unsaturated Soils by Fredlund and Rahardjo in 1993. And this is the probably the first book available on this particular topic. And Fredlund's contribution in unsaturated soil mechanics is significant and often Fredlund is considered to be the father of unsaturated soil mechanics for his contribution in this particular topic. During his tenures as professor at the University of Saskatchewan, Canada, he has developed many concepts on unsaturated soils. And especially the concepts taken from soil physics and soil science are brought into engineering and he has developed many concepts. And those are all compiled into one textbook that is Soil Mechanics for Unsaturated Soils in 1993. And most of the concepts that are given in Lu and Likos are extended from the earlier topics that were earlier concepts developed by Fredlund and Rahardjo. And for this particular course most of the concepts are taken from this first two books.

And you have another book by Laloui on Mechanics of Unsaturated Geomaterials in 2010 which contains most advanced topics of unsaturated soils. Some concepts like osmotic suction, matrix suction and osmotic component of matrix suction such concepts are considered from this textbook for this good course.

And the other book is Soil Physics by Marshall, Holmes, and Rose. This is a wonderful book on soil physics many physical chemistry concepts of soils are taken from this textbook and then considered in this course. And the next book is Soil Physics with Hydrus contains many concepts on modeling, the flow behavior in unsaturated soils. And the application of hydra software which is a freebie and those are discussed in this particular textbook. And some modeling aspects of unsaturated flow behavior are taken from this book. And the last book is on Physical Chemistry by Atkins and Paula. This is a excellent book on physical chemistry. Many fundamental concepts like Kelvin's equations, the cavitations, condensation, vapor pressure lowering, Raoult's law and many other concepts are taken from this textbook and explained in this course.

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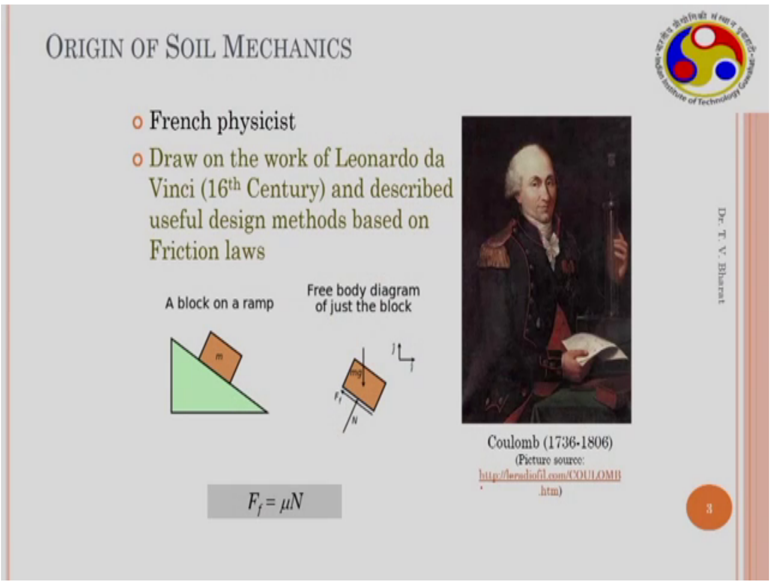

Let us begin in understanding the shortcomings of our soil mechanics for understanding the soil behavior it all starts with coulomb who proposed the friction loss which can be described by this simple mechanism where a simple block which is sliding on a ramp, which has a weight component and there is a frictional weight component. And there is a resistance that is offered, that is a frictional component, and there is a normal component which is acting on this block this is a free body diagram of the block. According to the friction law, the frictional force is proportional to the normal force. And this is the material constant which is independent of the normal force or weight of the block.

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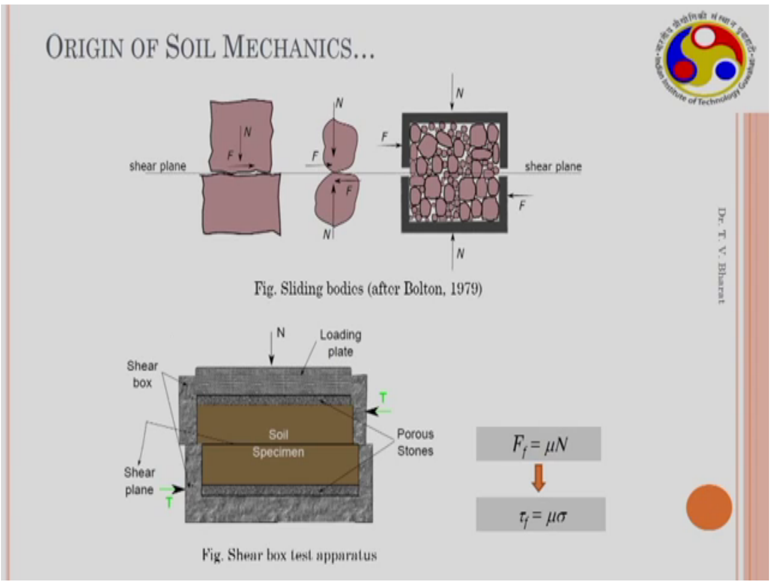

Either a block sliding on a ramp or a block is moving against a surface are two individual grains shearing along this plane are lump of particles which are sheared along this shear plane when there is a normal load. All this would represent a failure along the shear plane and the Coulomb's principle is applied applicable. So, this is a motivation for developing the shear box test operators where you have a soil sample which is sheared along this predefined shear plane by applying some normal load. And the same equation is valid. In terms of stresses, we represent this using tau f equal to mu sigma; sigma is normal stress, and tau f is shear stress at failure.

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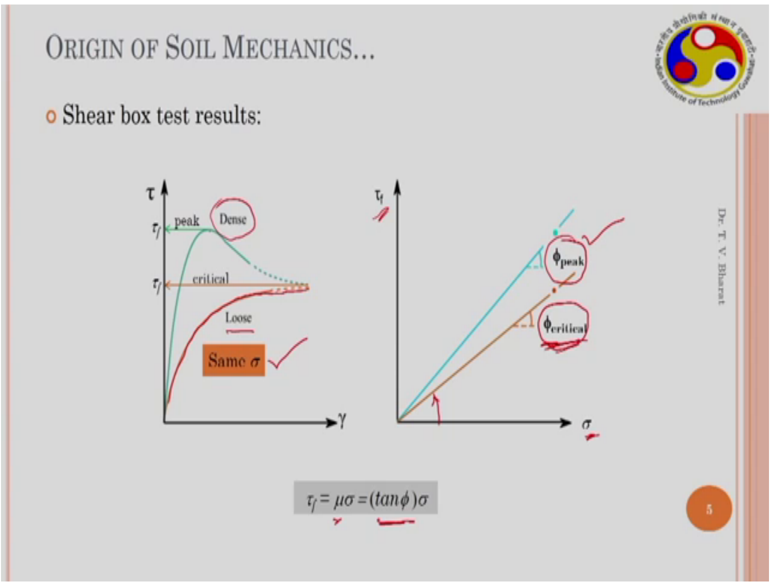

These are the shear box test results. For the same normal stress as given soil behaves in this manner, either in this manner or in this manner. Soil shear stress gradually increases with increase in the shear strain, and reaches a critical value or the shear stresses shear stress approaches a peak value and decreases and approaches the critical value. Generally, the dense soils are over consolidated clays where OCR is more than 2, they exhibit this behavior are loose sands are normally consolidated clays exhibit this behavior.

If when you conduct tests with different sigma by changing the normal stress, you will get the relationship between tau f versus sigma. So, therefore, using the Coulomb's principle you will get a material constant called angle of internal friction that is tan phi here either mu or it could be represented with tan phi, this is a angle phi, this is a angle of internal friction at critical state. If you take peak data, the shear stress at peak values and plot you may get phi peak - the angle of internal friction using peak values, as this value can change with initial condition this does not represent the material constant.

But phi critical would represent the material constant because for whatever the densities you use whatever be the initial condition, this is constant. So, therefore, phi critical represents a material constant which is often used in the design, so all this knowledge is from Coulomb's friction law. However, here we utilize you have a two phase system you have a soil solids and air in the pore space.

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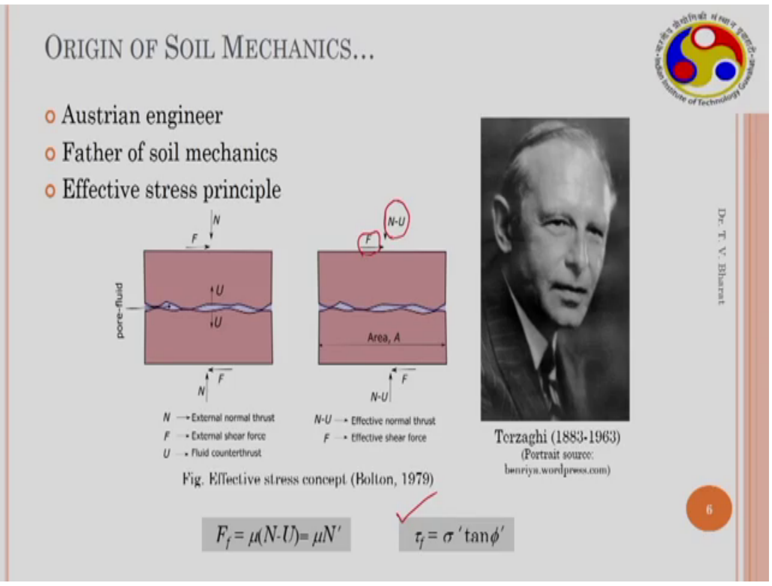

Then comes this effective stress principle; where in case if you have a pore fluid, in the pore space of the system, when you apply normal force and frictional force there is a counter thrust that is acted acting from the pore fluid. So, therefore, this is the force that is acting at equilibrium; and this is the force act acting at the equilibrium, this is given by Terzaghi who has given this effective stress principle. And then we modify the coulomb spring coulombs law by changing the normal force changing the normal force to effective value N minus U we add this counter thrust.

So, essentially you get a tau f that is a shear stress at failure is equals to sigma dash an effective stress times tan phi dash. So, because of this soil exhibits different strength during so soil exhibits different strength depending on the drainage condition; whether you allow the drainage to take place during the shearing, or drainage is not allowed during the shearing would influence the shear strength property of the soil.

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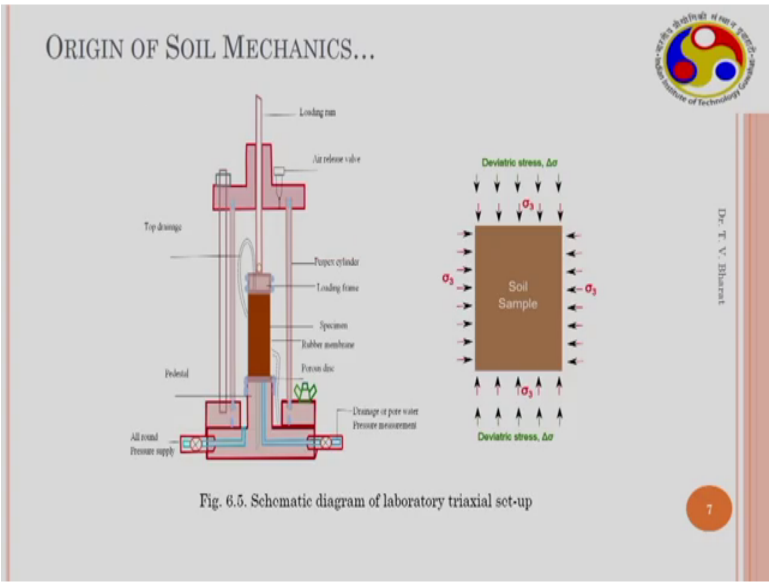

So, we get different strengths therefore, and in triaxial tests we are able to control the drainage; and we conduct consolidated un drained and consolidated drain test where wherein we get strength parameters in drain condition and un drained condition.

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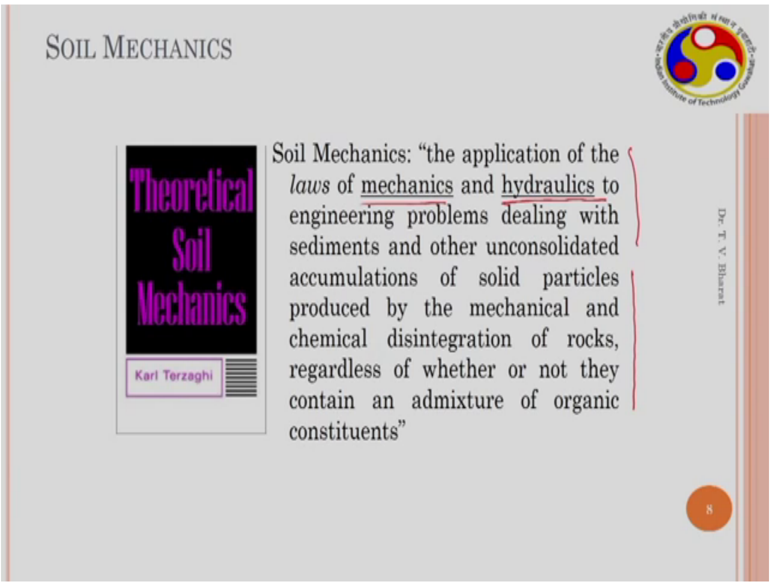

So far we have utilized the soils, in considering soils to be a two phase system; where either you have soil solids and water, or soil solids and air. So, according to Terzaghi in his infamous theoretical soil mechanics textbook; he says that the soil mechanics, he is defined as the application of the laws of mechanics and hydraulics to engineering problems dealing with sediments and other unconsolidated accumulations of solid particles produced by the mechanical and chemical disintegration of rocks, regardless of whether or not they contain an admixture of organic constituents. So, in this description he is defining what is soil; and here he is defining what is a soil mechanics, essentially the application of mechanics and hydraulics for understanding the soil mechanics that is what is soil mechanics.

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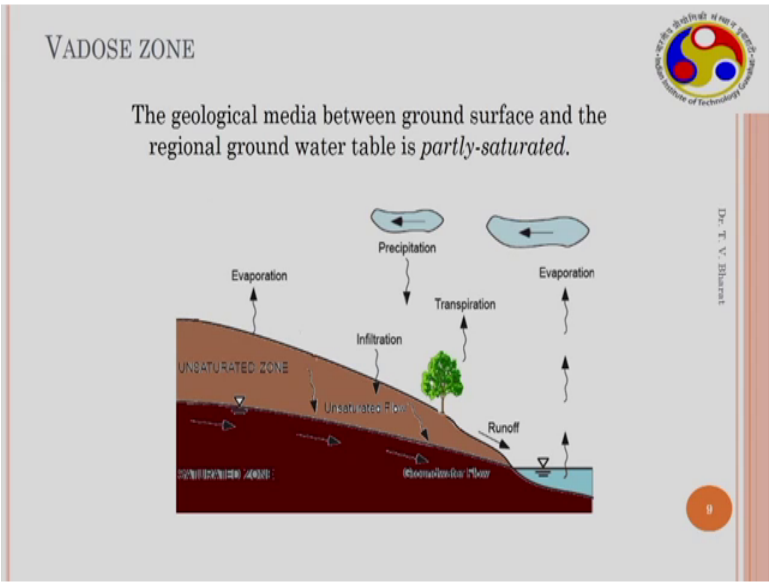

But however, in nature you have unsaturated soil, the geological media between ground surface and regional ground water table. You have a thick or depth varying geological media which is partly-saturated, essentially you will have three-phase system.

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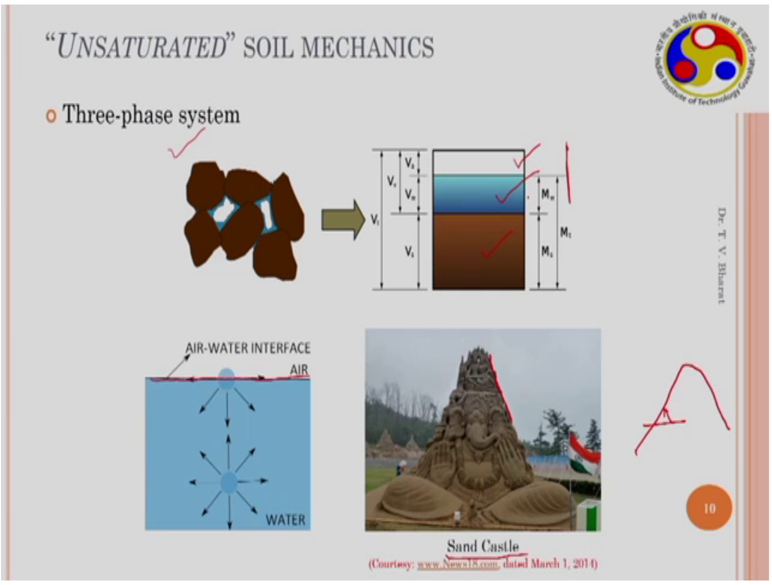

So, you will have you have pore air space, pore water and soil solids. So, this is a volume of air, volume of water and volume of solids. So, this is volume of voids within the void space you have two phases, one is air; one is generally water, or sometimes you may have oil or any other thing. So, this is most natural case. So if this is the case, then soil within the soil at the air water interface, you may have surface tension developed at the interface this provides additional strength to the soil, and this can change with change in the water content, change in the air content in the system.

So, for example, we make sand castles at beachside on the on the shore, we make sand castles on the shore. Here with sand you can achieve steep angles, angle which is much steeper than the angle of internal friction, this is because of additional strength that is provided from the surface tension; or the negative pore water pressure which is present in the pore space, due to the presence of two phase system, two additional due to presence of air and water.

If you go to the sand castle, sand castles where you find that the angles are steep enough to make it, to be mould into different shapes; and this angles are much steeper than the angle of internal friction. If you dump a sand, dump sand from a truck; it assumes an angle which is equal to the angle of repose, steeper than this you cannot make unless until there is an additional strength that is coming from the surface tension or negative pore water pressure.

So, in the sand castles we are able to make structures with angles steeper than the angle of internal friction of the soil because of the surface tension effects. So, because you have two phases in the pore space, you are able to get additional strength.

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Therefore in natural slopes, they may be steeper than the natural slopes; may be steeper than natural slopes are much steeper. And you may see vertical cuts in many a times that will be withstood that would which stood. Therefore, in natural slopes if you see, often you see a vertical cut which can withstand the slope for very long time without failure, this is all because it has additional strength due to negative pore water pressure in the system that is because you have two phases in the pore space, so these all fine.

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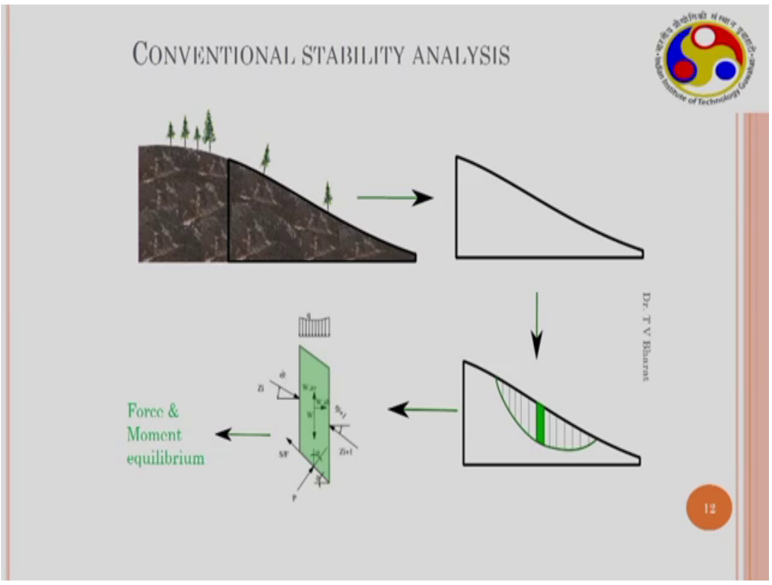

So, this we analyzed using conventional stability analysis where each slice is analyzed for force and moment equilibrium. And then we find out the factor of safety.

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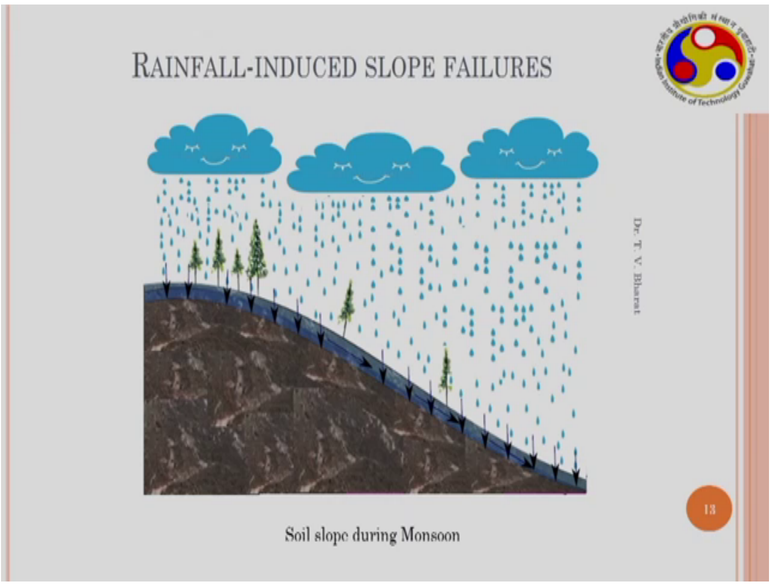

However when during the rainfall, water may seep into the ground surface or the slope. And then the air water interface may disappear, because the pore space will be replaced, pour air space will be replaced with water; and you may have only essentially two phase system that is water and soil solids. In that particular case, the strength may drop and which causes a decrease in the shear strength and which causes a landslide.

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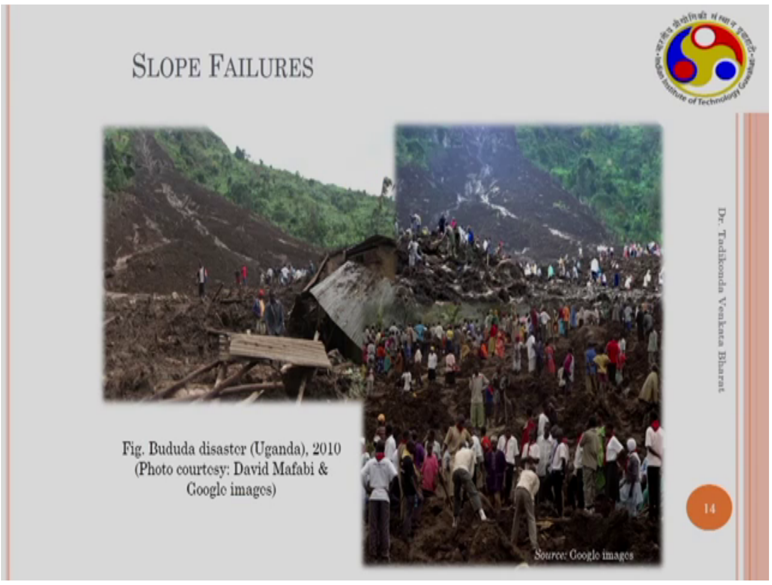

And we have seen several disasters recently, in the recent past like Bududa disaster in Uganda this is due to rainfall induces slope failure.

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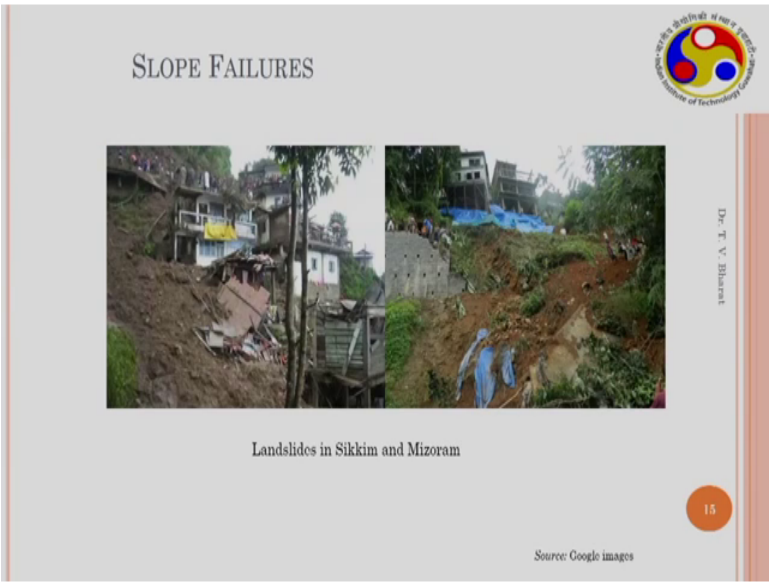

And this is another landslide in Sikkim and Mizoram, where during the monsoons season you often see these landslides, this is because soil has sufficient strength when it is partly saturated or due to the negative pore water pressure, but this additional strength disappears has water imbibes into the soil system.

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This is another disaster we had seen recently in India.

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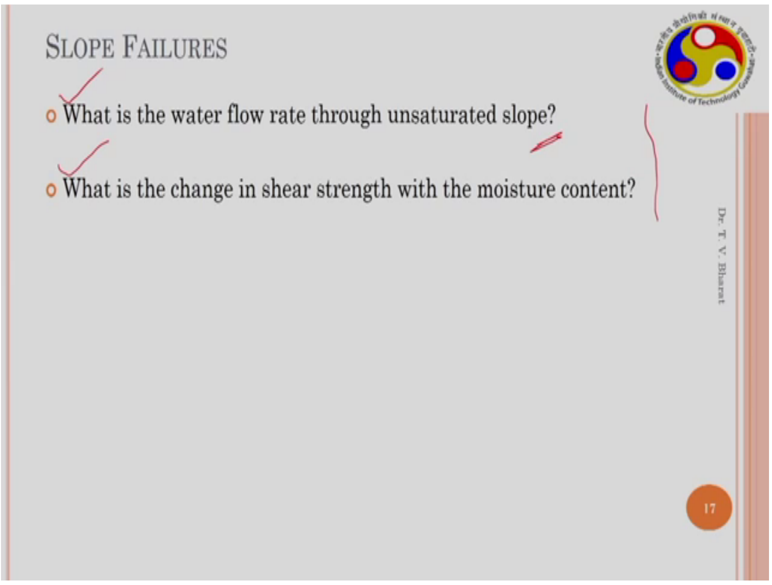

So, essentially in these cases, we need to answer these two questions, what is the water flow rate through unsaturated slope? In our traditional soil mechanics, we only learned the flow behavior through saturated soil system using Darcy's law; whether Darcy's law is valid or not here is one question, because you have air space through which water will not pass through, but it may saturate eventually at time with time. So, what exactly the flow behavior through unsaturated soils is one question.

Another question, the shear strength definitely changes with change in the moisture content, but how does it change? These two questions we need to address, if we are analyzing the slope behavior, due to rainfall infiltration.

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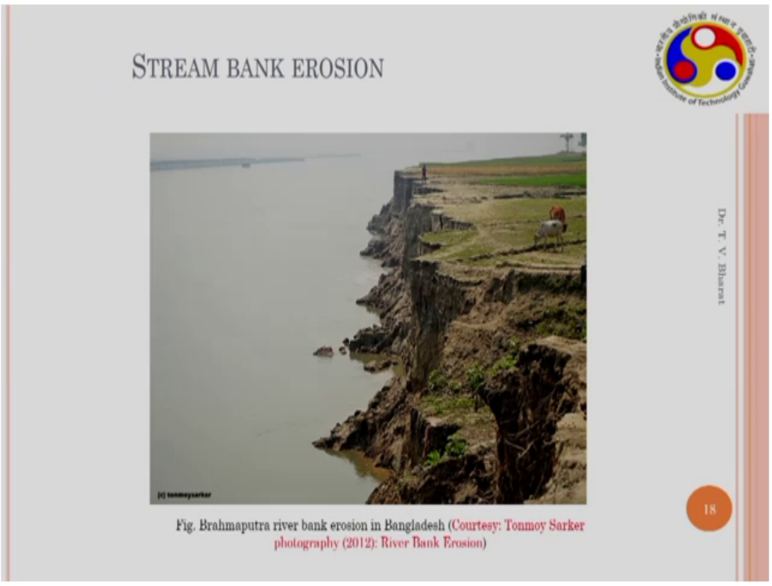

This is another situation, where in the unsaturated soil mechanics is very useful; this is a Brahmaputra bank in Bangladesh. Here, the banks got eroded, and here you could see the vertical cut, vertical cut in the bank this is all because the soil is not fully saturated, it is partly saturated. Therefore, it is able to maintain nearly vertical cut. Here the slope stability plays an important role in controlling the erosion of this stream bank.

So, the unsaturated soil mechanics principles are again very important, and applicable for addressing such problems.

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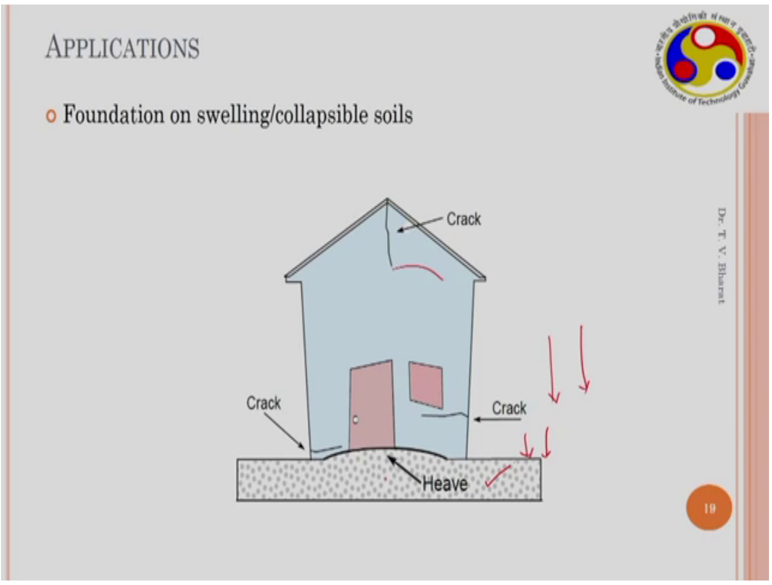

And most frequently we encounter such issues, where in the soil mass beneath foundation either swells which we call heave, or collapses depending on the type of soil during saturation. So, initially in dry state the foundation soil will have sufficient strength to carry the structure, but when rainfall occurs the water seeps into the ground, which causes decrease in the shear strength. And either building fails in shear; or depending if you have an expansive soil, the soil may expand. Or if you have a collapsible soil, soil will collapse due to wetting. This causes collapse of the entire structure. The whole structure may sink into the ground or the entire structure will be uplifted; or there will be cracks that may be appear; or this whole thing will fail in shear, all these problems are due to the change in the moisture content of the foundation soil due to seasonal effects, which causes changing in the shear strength and volume change.

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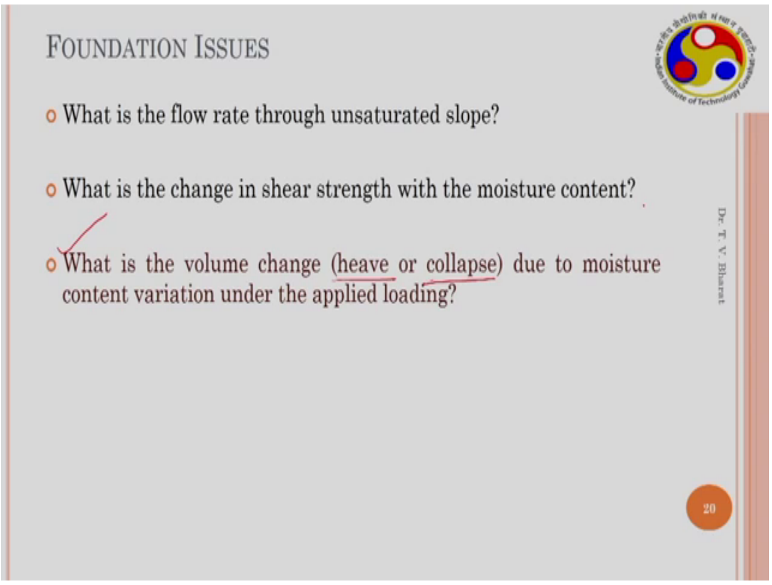

So, addressing in addressing such issues we need to address another issue that is what is the volume change that is either heave or collapse, that may take place due to moisture content variation under the applied loading. Apart from these two questions which we addressed for slope stability.

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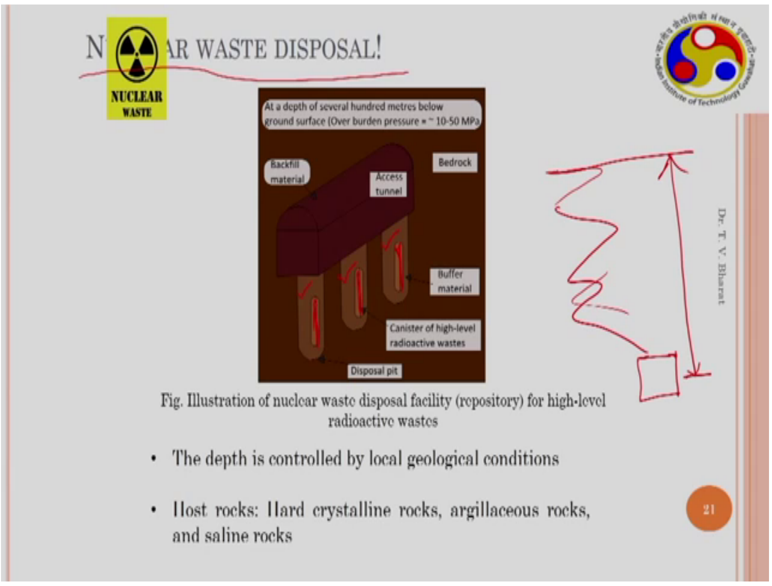

There is an important issue these days is a nuclear waste disposal even though it is at present India does not require this facility, but then in long term we require such facilities to address the disposal of nuclear waste high level nuclear waste. The Swiss, according to the Swiss design, the nuclear waste is kept in a canister copper canister and which will be dumped or placed at several hundreds to thousand meters below the ground surface by making tunneling.

And reaching the reaching the prescribed location where which is much below the ground surface which is a nearly several hundred meters below the ground surface where the canisters are these are the canisters which contains the radioactive waste. This is placed these canisters are placed and which is back filled with bentonite material. The bentonite it is a highly expensive clay high plastic clay, so which is back filled with.

So, essentially the bentonite which is available around the canister would contain the radioactive waste will element by making the permeability very low. The permeability of the bedrock in unsaturate in unsaturated condition are as low as 10 power minus 16, 10 power minus 18 meter per second such a low permeability they create. So, essentially if there is a leakage that happens from the canister radioactive elements would diffuse through the bedrock that would take nearly very long time. And which is surrounded by a rock mass saturated rock mass. And you have access tunnels which are again back filled by the bedrock here this is called buffer and here you have a bentonite backfill.

So, back filling and bin buffering requires the bentonite material to be used. And the properties of these bentonite materials or the characteristics of these bentonites are important. And moreover the mechanical behavior of this bentonite in this particular condition is very important. In this particular situation, when it is buff when the bentonite is used as a buffer material here and back filled elsewhere.

Now, there could be a water which may be penetrating from the surrounding saturated rock mass into the buffer material. So, in that particular case, the unsaturated flow through bentonite material is important. And secondly, there may be thermal gradients across the bentonite layer. So, thermal flow thermal conduction is important and the mechanical behavior. When the bentonite is saturated from the saturated rock mass, water which is coming from the saturated rock mass bentonite applies huge pressure on the surroundings.

So, this mechanical pressure could be as high as 40 mega Pascals depending it depends on the type of soil. However, so this swelling pressure, this is called swelling pressure that the bentonite exerts on the surroundings when it is confined and when it is not allowed to swell due to saturation will be nearly equal to the hydrostatic pressure that is acting at that particular depth from the ground surface.

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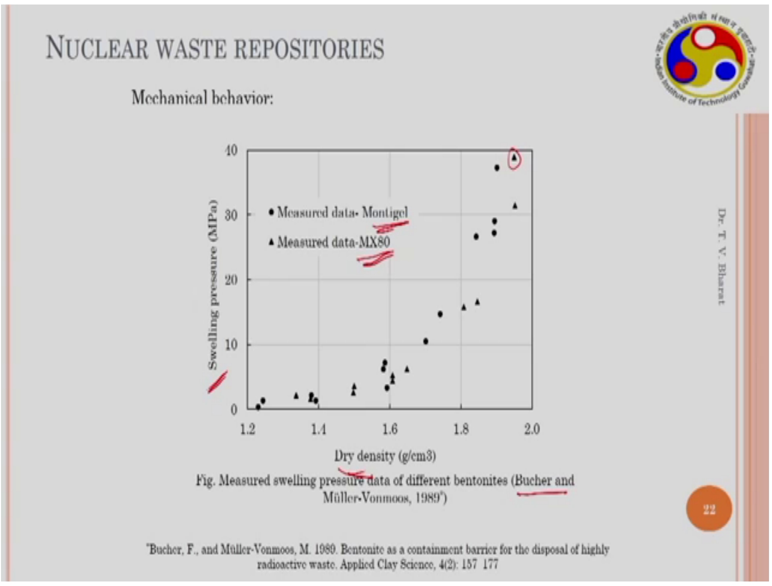

So, the mechanical behavior such as this swelling pressure with dry density. And similarly the swelling pressure with water content are important for different soils for understanding the mechanical behavior. So, this is a typical data from butcher and Muller Vonmoos here for MX 80 bentonite and Montigel bentonite these are highly plastic clays, where the soil is compacted to very high densities nearly 2 gram per centimeter cube dry density and it exhibits a swelling pressure of nearly 40 mega Pascals 40,000 kilo Pascals.

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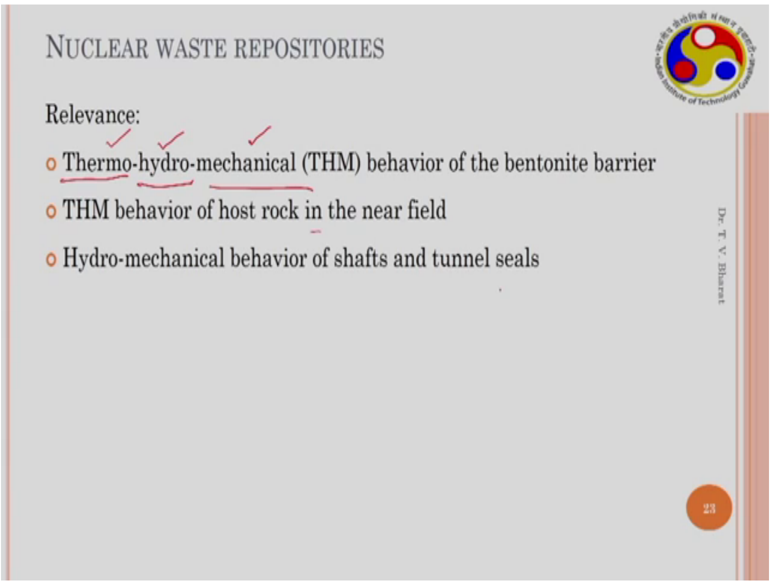

So, the thermo hydro mechanical behavior because you have a thermal gradient across the bentonite, you have a hydraulic gradient across the bentonite. And the pressure which is generated due to the saturation because of which the coupled analysis of thermo-hydro-mechanical behavior of the bentonite barrier is important. And also THM behavior of host rock is important near field; and hydro-mechanical behavior of shafts and tunnel seals are also required.

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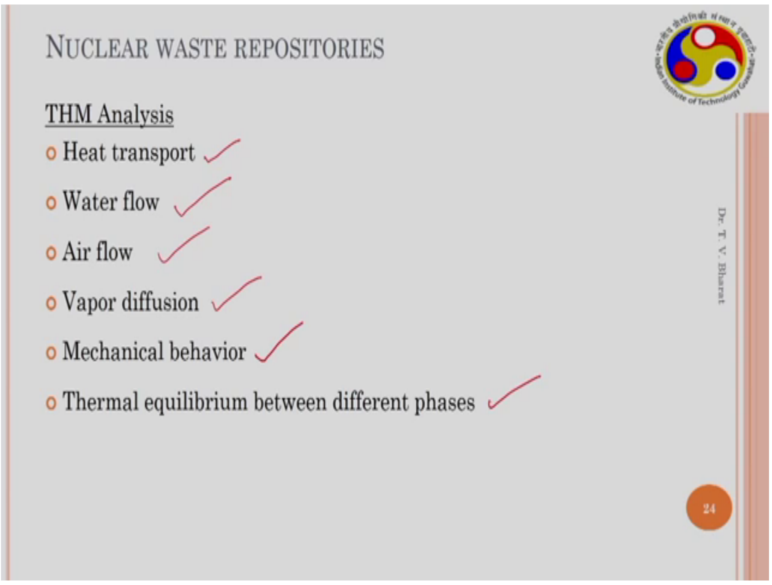

So, THM analysis contains heat transport, water flow, air flow, vapor diffusion, mechanical behavior, thermal equilibrium between different phases, all these need to be coupled and then solved the expressions for understanding the bentonite behavior in that particular situation. There is another situation where you have buried pipelines buried pipe lines are used for carrying the natural gas or any other material from one point another point.

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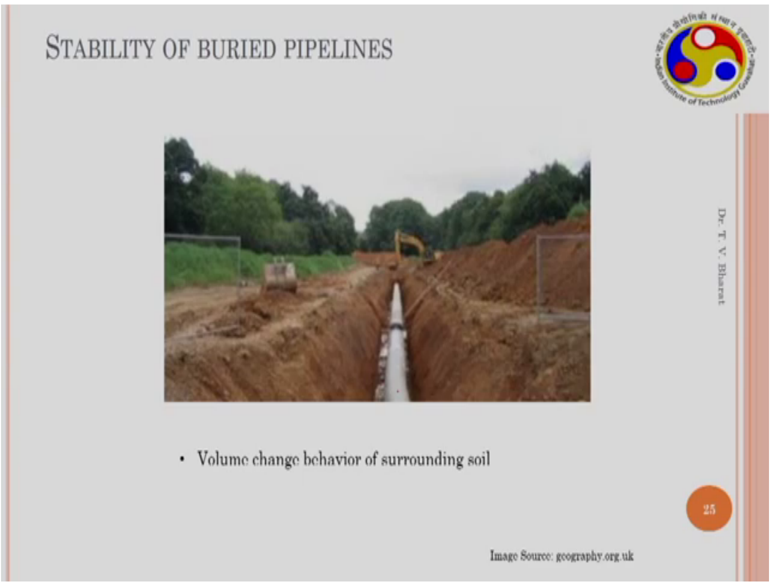

Here the soil which is surrounding the buried pipeline might exert some pressure on the pipes due to change in the moisture content or they may exhibit a volume change behavior due to change in the moisture content which causes additional stresses on the pipes. And if the stability of pipes is not sufficient, then it may break and leakage can occur. So, to address these issues, we require to understand the unsaturated soil mechanics are unsaturated soil behavior.

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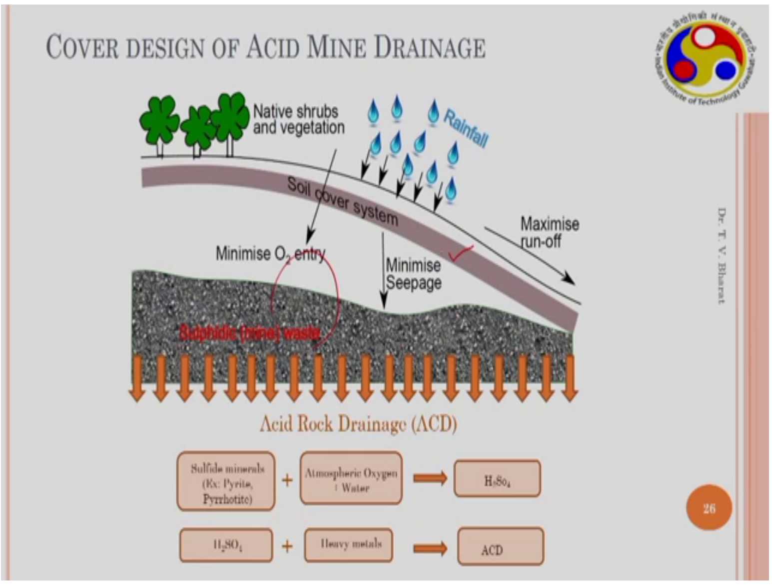

So, another application is a cover design in acid mine drainage, in acid mines and in landfills. Cover designing plays a very important role in this particular situation where the rainfall percolation into the system should be controlled.

Here, the permeability of the cover system should be low enough so that the water generally runs off from the surface and the percolation is minimal. Secondly, if you use directly bentonite type of soils here, it may develop cracks. When there is a excess of drying that takes place or evaporation that is taking place during dry periods, so it should have low permeability, at the same time it should not crack. Otherwise in the next season during monsoon these cracks would prompt the water to enter directly into the system either acid mine tailings or it could be landfill liners landfills.

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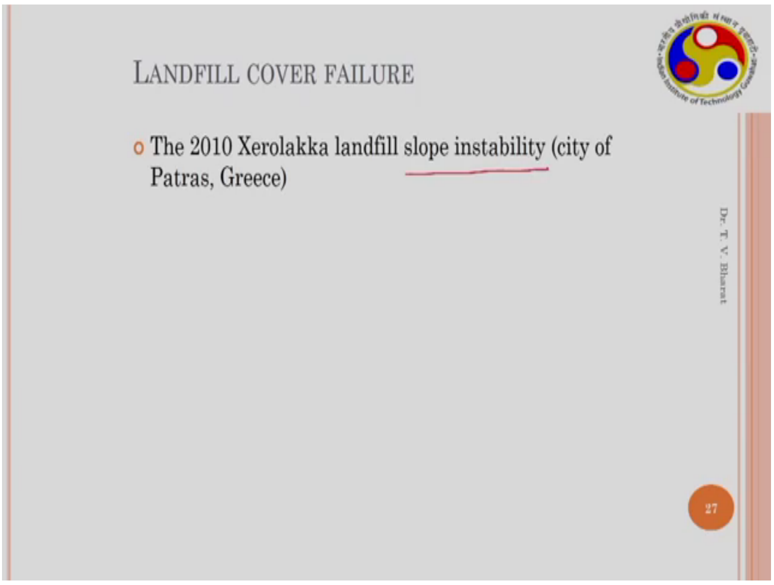

So, the permeability of the soils are very important permeability of the unsaturated soil is important. And the slope stability is also important. This is another aspect which is not addressed in basic soil mechanics. There is osmotic effect.

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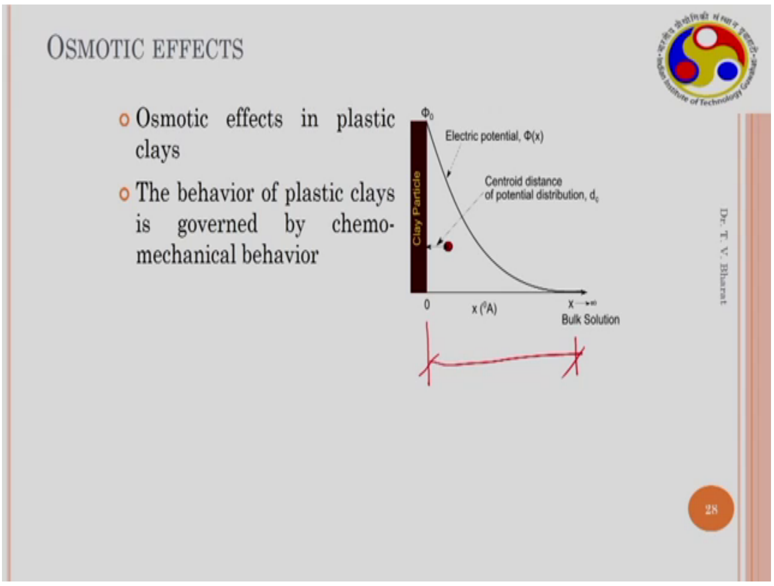

Expansive soils such as bentonites black cotton soils on the clay particle surface they have negative charge surface due to isomorphism substitution, so that attract positive ions under the surface and they had the cation there are exchangeable cations on the surface. When there is a moisture that is available, the exchangeable cations get hydrated and the external surface and internal surface of the clay particles also would get hydrated. And if there is additional water that is available free water available, then there is a formation of diffuse double layer where the electrostatic potential varies from the clay surface to one particular distance called diffuse double layer thickness.

So, clay particles exist along with the diffuse double layer, and the this is due to the osmotic effect. And when two different particles come close to each other, there is a osmotic potential that is developed at the interpolate distance. These osmotic effects are important for controlling the flow behavior, for volume change behavior and shear strength behavior of fine grained soils or clays. So, such osmotic effects are not addressed in basic soil mechanics. The beauty of unsaturated soil mechanics is that the osmotic head and the negative pore water pressure that we call a matrix suction head. All these heads are considered in the same head and we considered for the flow behavior volume change behavior and shear change behavior.

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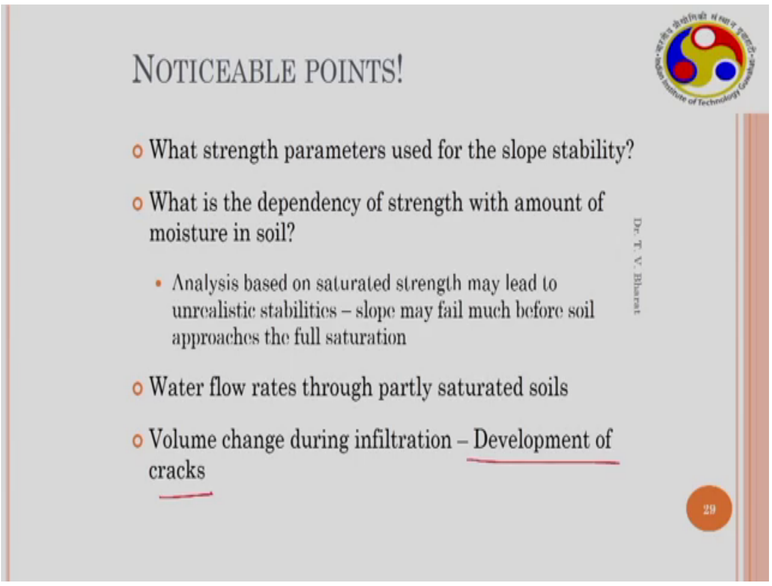

So, these are the noticeable points, what shear strength parameters we use for the slope stability analysis, because the shear strength depends on the amount of moisture that is available. If you analyze the soil slope using saturated strength our strength obtained at the saturated state, then it may lead to unrealistic stability analysis. And slope may fail much before for the soil approaches the full saturation if it is going from the dry drying to full saturation state.

And at what rate the water flows through the soil, so that you can couple the flow behavior and the strength or mechanical behavior so that one can analyze rainfall induced slopes slope instability. And the third aspect is the volume change behavior during infiltration of water under given normal stress normal load, and development of cracks due to volume changes. These issues can be well addressed using the unsaturated soil mechanics.

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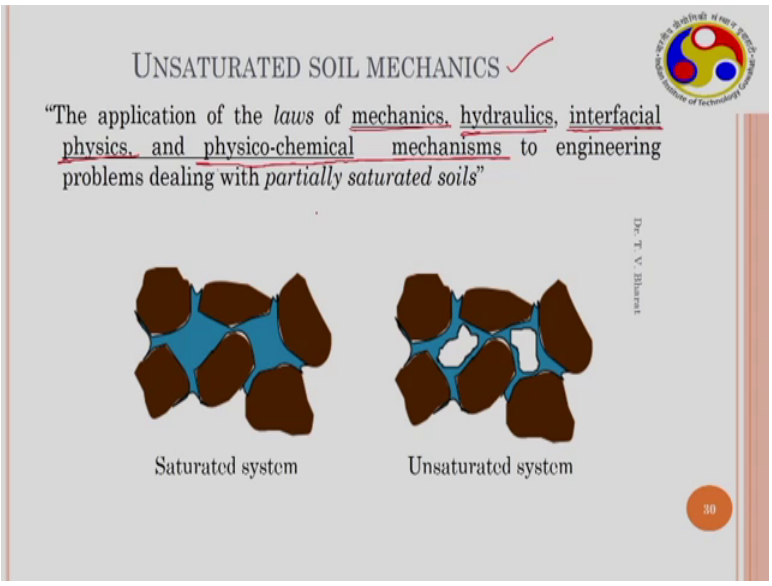

So, therefore, unsaturated soil mechanics should be defined as the application of the laws of mechanics, hydraulics, these are any ways used in basic soil mechanics the additionally interfacial physics. So, you can allow the surface tension forces to come in and then you can explain many things. And physico-chemical mechanisms so that you can address the fine grained soil behavior to engineering problems dealing with partly saturated soils.

With this I will stop here. And I am sure this would provide enough motivation for studying this subject.

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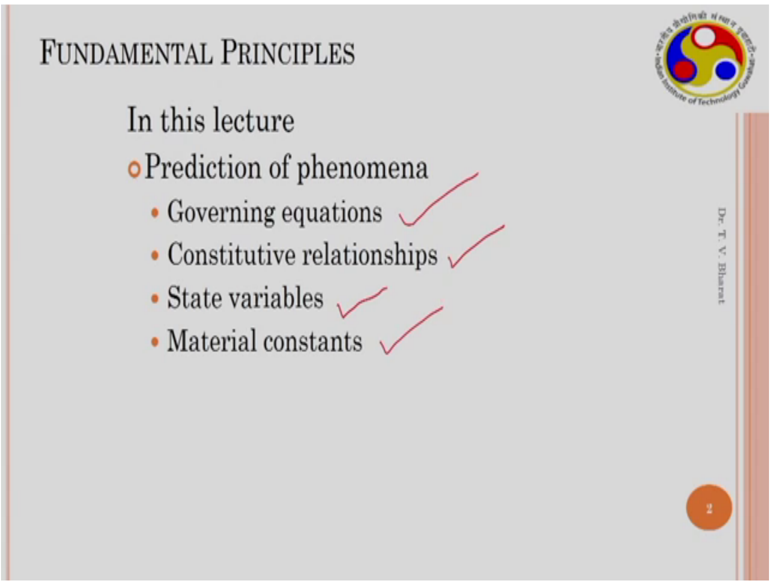

Now, we will discuss some fundamental principles that are essential for understanding the unsaturated soil mechanic.

In this lecture, we will discuss about the prediction of a phenomenon, how a phenomenon can be prediction predicted. For that it is important to recognize what are the governing equations need to be invoked, the constitutive relationships. For understanding the constitutive relationship you need to identify what are the state variables, and build the constitutive relationships based on some experimental observations, and determine the material constants and we predict the phenomenon.

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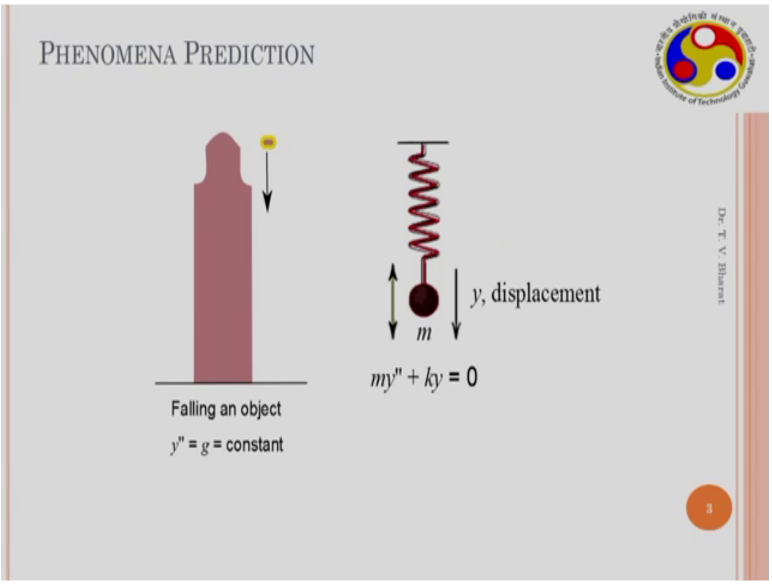

We will try to understand today for any prediction of a phenomena, for phenomena prediction requires mathematical representation of a given problem. So, for example, a simple problem you have, where in a object is falling an object is falling from a certain height from a tower. And at what rate how much time it takes to reach the ground is one problem. And or a mass of sphere which is oscillating which is attached to a spring and which is oscillating which is having a displacement of y; and prediction of this phenomenon requires a mathematical representation of this problem.

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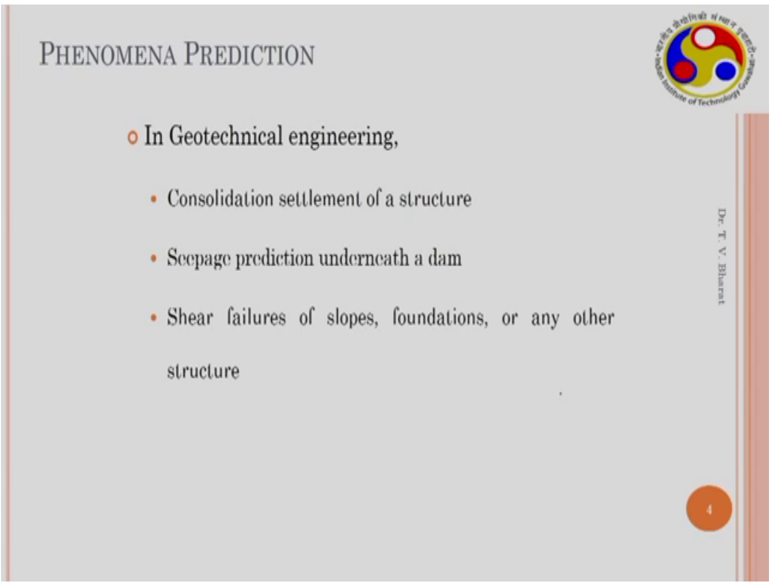

When it comes to the geotechnical engineering, we are interested in understanding the consolidation settlements of structure which is resting on soil. And seepage prediction underneath a dam, or it could be a shear failures of slopes, foundations, or any other structures.

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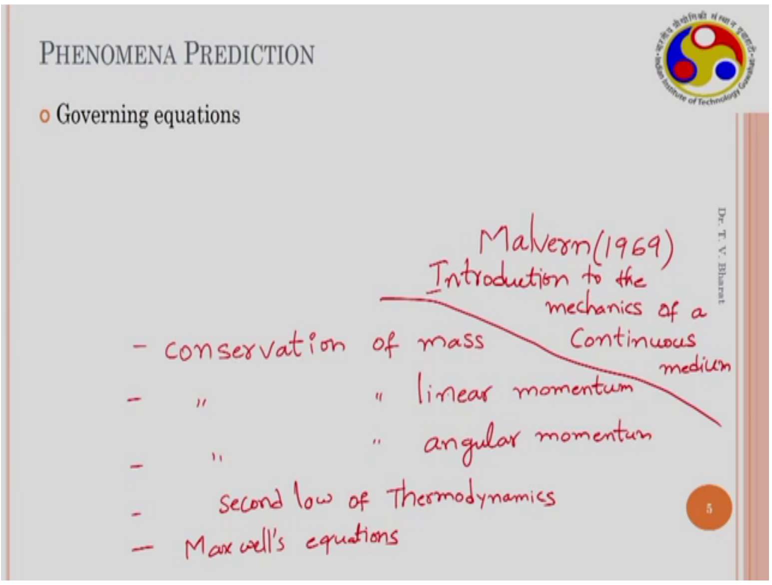

Let us try to understand we have governing equations such as conservation of mass, we have conservation of linear momentum and conservation of angular momentum, second law of thermodynamics and Maxwell's equations. These are fundamental governing equations we need to invoke for understanding any phenomenon mathematically, mathematically you represent mathematically to represent and for understanding any phenomenon, we need to invoke these governing equations.

Now, readers are advised to refer, the audience are advised to refer any standard textbook on continuum mechanics for more explanation on these physical laws. These are physical laws; the audiences are advised to refer any standard textbook on continued mechanics for more explanation on these physical laws. So, these physical laws need to be invoked for mathematically representing any phenomenon and prediction of phenomena. One of the best textbook could be Malvern on Introduction to the Mechanics of a Continuum Medium Continuous Medium.

Apart from this physical loss we also require additional equations to solve the problem uniquely for satisfying the constituents, it is material dependent equations we require.

Constituent's equations therefore, explain the interdependency of different state variables. It is stress, strain, void ratio, effective stress etcetera. The proportionality constants of the constitutive relationships are called material constraints that represent the fundamental constituents of the system. Material constants may also depend on the state variables. The dependency of material constraints on the state variables and one state variable on another state variable through constituent equations is required for understanding the behavior.

Let us identify the governing equations, state variables and material constants in the basic soil mechanics. Such identification exercise is important for the prediction of physical phenomena or simply the engineering behavior of soils.

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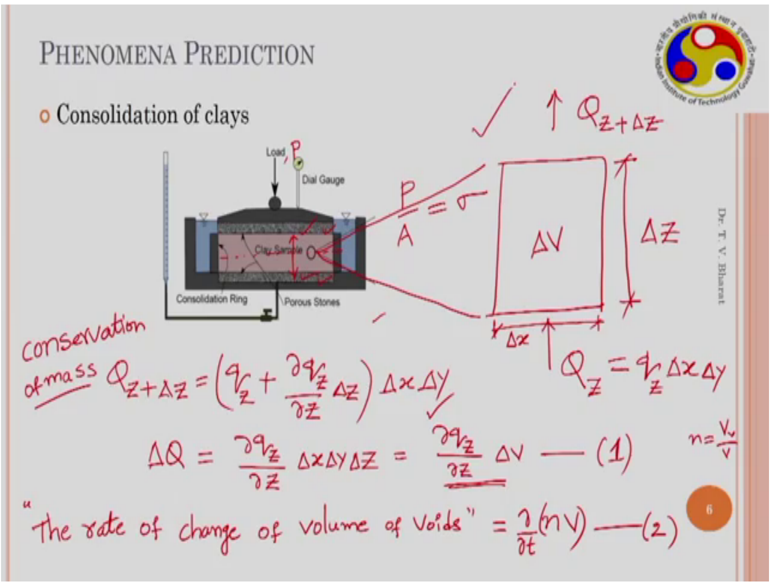

Let us take the problem of consolidation of clays is a very important subject area in solid mechanics and also which is very important for prediction of settlements and settlement rates, ultimate settlements and rate of settlement of any structure which is existing on soil. Actually the soil samples from the field are brought to the laboratory and then studied in studied in the laboratory on in odometer cells odometer cells illustration is given here.

So, in this slide, it is shown the clay sample which is sandwiched between a porous stone top porous stone and bottom porous stone. And this is a fully submerged in water. So, the sample is in fully saturated condition. And you apply a normal load P and generally the cross sectional area the diameter of the sample would be 6 centimeter and the thickness of the sample is 2 centimeter. So, the load is p applied then the stress existing on a sample is P by cross section area which is the load by the cross section area which is the total stress that is acting is sigma.

So, under this particular condition as there are two boundaries where you have highly permeable porous stones are sitting which replicates the situation of clay sample which is sandwiched between sand layers in the field. So, there is a hydraulic gradient that is developed because of excess pore water pressure at the boundaries are 0. And at the middle of the sample, it has a maximum pore water pressure excess pore water pressure. So, there is a gradient that is developed within the sample and water excess pore water pressure dissipates through the boundaries. So, in turn the clay sample consolidates with time.

So, this phenomenon is theoretically studied using Terzaghi's one-dimensional consolidation theory. This consists of considering one elemental volume a representative volume a representative volume and the flow through this volume could be considered. Volume of the sample is delta v; and the thickness is delta z. If you consider the volumetric flow rate Q, which is in the flow direction; and the volumetric flow rate which is coming out which is Q Z plus delta Z.

So, the change in the volumetric flow rate can be understood the Q Z in terms of unit discharge or flux is small q z times the cross section area. If the length of this elemental cross section area is delta x, then this is delta x times delta y in the in plane direction delta y. Similarly, Q z plus delta Z is small q z there is a flux plus change in flux with distance into delta z into delta x into delta y. So, the change in the discharge volumetric flow rate is nothing but delta q is dou q by dou z times delta x delta y delta z which is nothing but dou q z by dou z times delta v. So, this is the volume of the element.

So, this quantity is nonzero for transient flows. If it is a steady state flow this quantities is 0, but because here a transient flow that is taking place or time variant that is taking place flow is dependent on time here that is a transient flow that is taking place. This quantity is equal to the rate of change of volume of voids because the flux is changing the flux is changing because the volume of the element is getting compressed.

So, therefore, this is nothing but the volume of voids is nothing but n that is that is porosity which is porosity is equal to volume of void spread total volume. So, therefore, porosity times total volume is your volume of voids and rate of change dou by dou t of n v. So therefore, so these two quantities should be equal this is first equation; this is second equation.

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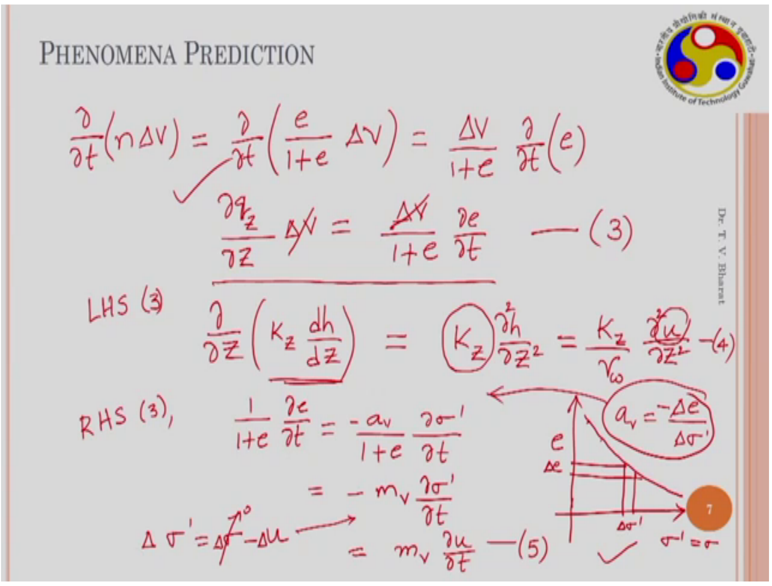

And this can be written as so the dou by dou t of n into delta v can be written in terms of void ratio has dou by dou t of void ratio by 1 plus void ratio times delta v. So, delta v is a small elemental volume and 1 plus c represents total volume. So, for small strains, one can assume one can approximate this as delta v by 1 plus e times dou by dou t of e.

So, if when you equate this expression the second expression and first expression, you get dou q z by dou z times delta v is equal to delta v times delta v by 1 plus e times dou u by dou t. This expression is a mass conservation expression, conservation of mass is considered here. Here the conservation of mass is considered. So, whatever the change in the flux is due to the change in the pore volume with respect to time. So, this is conservation of mass, mass conservation principle.

So, if the conservation of mass is satisfied, then this equation should be satisfied. So, here physical law is applied for deriving next expression. Now, we can simplify these expressions in terms of known state variables. This left hand side expression of this expression left hand side of the equation 3 can be written as dou by dou z of if Darcy's law is valid then q z can be represented as k z into d h by d z. So, because when the load is applied the gradient is available only on one side in one direction that is either up upward direction or downward direction.

So, you have gradient that is available only in vertical direction that is z direction. So, which is equal to K z times dou square h by dou z square. If we assume that the hydraulic conductivity does not change with space that is an assumption. For small strains and consolidation operators this could be assumed for low plastic soils etcetera, and then this assumption should be valid assumption is valid. So, then representing the head in terms of this is a hydraulic head representing in terms of pressure, this becomes K z by gamma w into dou square u by dou z square because u is equal to h into gamma w. So, this is equation number 4.

If you consider RHS - right hand side of the equation 3, here we require constitutive relationship, because it is with respect to void ratio 1 plus c into dou e by dou t. Here we require one constitutive relationship need to be invoked. Here we use the constitutive relationship between constitutive relationship between void ratio and effective stress. So, for normal scale, this plots somewhat like this.

So, the slope of this if you consider, a small change in delta sigma das h and what is the change in delta e. This can be obtained from different loadings, when you load the sample load the sample, then change in the void ratio at equilibrium when sigma is equal to sigma dash can be obtained when excess pore water pressure is nearly 0. Then sigma is equal to sigma dash and you get the change in void ratio with respect to sigma dash, this is a one constitutive relationship which is a relationship between one state variable called void ratio; and another state variable is effective stress.

So, we assume that the slope is constant. We assume that this is linear for a small increment of delta sigma. Then the coefficient of compressibility a v is written as delta e over delta sigma dash with negative sign; because increase in the effective stress, decreases the void ratio.

And if you divide so now this expression if you substitute a v here, so you get a v by minus a v by 1 plus e into delta dou sigma dash by dou t. So, this is nothing but delta e by 1 plus c which is strain divided by stress. So, this is a 1 over bulk modulus called coefficient of volume compressibility times dou sigma dash by dou t. As the left hand side expression is in terms of excess pore water pressure excess pore water pressure, here also we try to represent in terms of excess pore water pressure. For that we need to invoke the effective stress principle that is a Terzaghi's effective stress principle says that sigma dash is sigma minus u effective stress is sigma total stress minus excess pore water pressure or pore water pressure.

So, here the change in delta sigma is change in delta sigma sorry change in delta sigma dash if you take consider, then it is equal to minus delta u, because the consolidation is taking place under the applied normal stress, the total stress is not changing only the pore water pressure is getting dissipated and which is replaced the pore excess pore water pressure becomes equal to the effective stress. Therefore, delta sigma dash is equal to minus delta u. If you substitute in this expression, you get mv times dou u by dou t.

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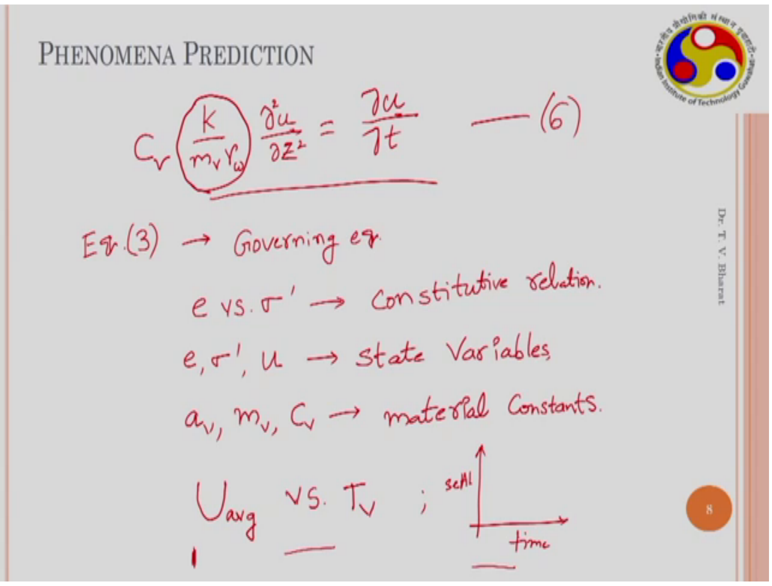

So, if you equate the expressions 4 and 5, you get the Terzaghi's one-dimensional consolidation equation that is a k by m v gamma w into dou square u by dou z square is equals to dou u by dou t. So, this quantity we write it as c v. So, c v times dou square u by dou z square is equal to dou u by dou t, this is terzaghis one-dimensional consolidation equation. So, this is a governing equation which is derived using a physical law that is mass conservation principle. So, this is the mass conservation principle that is used.

And we invoked the effective stress principle and we use Darcy's law here to represent this expression in terms of state variable excess pore water pressure. So, the governing equations are generally represented in terms of the governing equations are generally in terms of state variables. So, this to derive this governing equation, we require additional equations such as the constitutive relationship that is u versus sigma dash which has a relationship between two state variables.

So, here now identify the constitutive relationship state variables and metal constants in this derivation. The equation 3, the equation 3 is a governing equation based on the mass conservation principle. Equation 3 is the physical law which is mass conservation using that we got the governing x governing equation. We utilized e versus sigma dash relationship which is a constitutive relationship the state variables are here void ratio and sigma dash even pore water pressure excess pore water pressure is also a state variable. So, these are the state variables.

The material constants are a v coefficient of compressibility, coefficient of volume compressibility which is nothing but 1 over the bulk modulus even c v these are material constants. As we do not measure the excess pore water pressure in the odometer test, we represent the solution of equation as we do not measure the excess pore water pressure in the odometer test, we represent equation 6 to estimate the material constant one requires to solve this equation 6. As we do not measure the pore water pressure in the odometer test, we represent the solution of equation 6 in terms of average degree of consolidation and versus time factor T v.

And due to unique nature of this relationship and similarity between measured settlement versus time the settlement versus time relationship, because these two are similar. Using this similarity, due to the similarity between U average versus T v and settlement versus time we use we utilize either Taylor's method that is square root of time fitting method or logarithm of fitting method to estimate the time factors T v 90 and T v 50 respectively.

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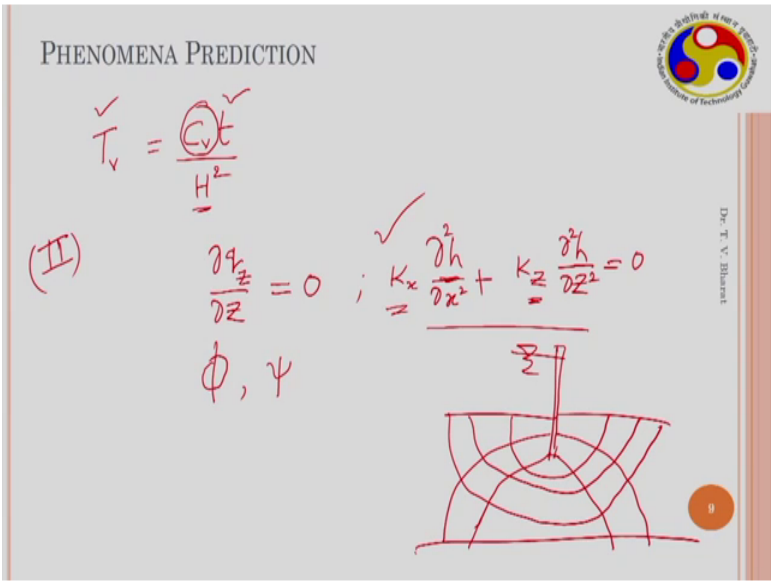

And we estimate the c v from the expression, we use expression T v equals to C v times t by H square, where T v is estimated from the graphical technique. So, time is known at what time you achieve a given T v is known and drainage path length is known. So, one estimate C v indirectly. So, essentially we indirectly estimate C v from equation 6 using graphical techniques.

So, here the phenomena is predicted. So, once C v is known, one can estimate the rate of settlement in the field and one can estimate the total settlement ultimate settlement from this expressions. This is possible because we used physical law that is mass conservation principle. And we identify the state variables and we represented a cost relationship that is e versus sigma dash and we utilized that. And we estimated the material constant which only depends on the constituents or the material property. So, therefore, we can predict any given phenomena.

We see another example, where similarly for steady state flows the equation 1 that is a dou q z by dou z equation 1 this expression should be equal to 0 for steady state flows. If it is for two-dimensional flows, this is nothing but k x dou square h by dou x s square plus k z dou square h by dou z square is equals to 0. For isotropics for isotropic case the kx and kz is same this is simply dou square h by dou x square plus dou square h by dou z square. This we solved by invoking velocity potential and stream function. And knowing that these two functions satisfy the Laplace expression, Laplace equation, we graphically solve this for estimating the seepage rates underneath a dam.

If you have a dam or sheet pile wall, so this is the water which is behind the sheet pile wall. And what is the flow that is taking place can be estimated, they confined environment very easily graphically we can estimate. And this one we have already learned in basic soil mechanics. So, here also we utilized mass conservation principle. So, there is a physical law. So, the head here is the state variable. All the governing equations are always represented in terms of state variables.

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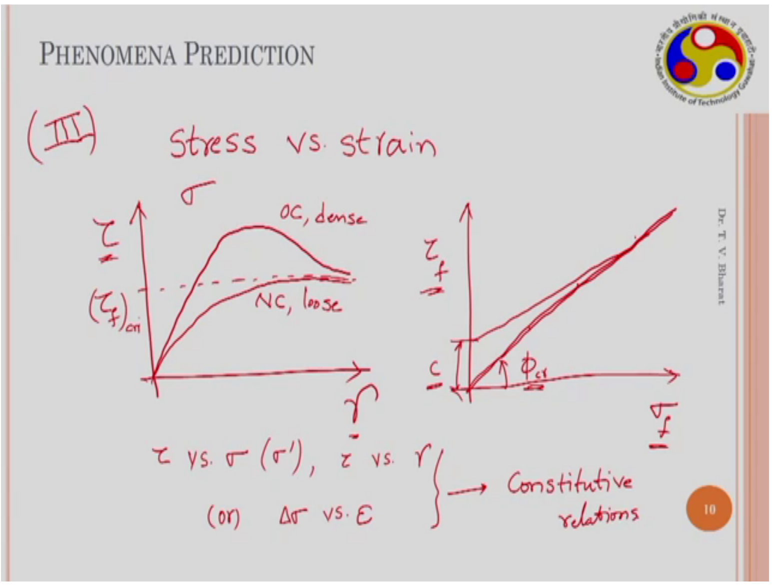

Similarly, for estimating the shear strain characteristics of soils, we invoke stress-strain relationship, stress versus strain, stress versus strain relationship. Indirect shear, we get the shear stress versus shear strain relationship, where for normally consorted where normally consolidated clays, they exhibit this behavior normally consolidated clays are loose sands exhibit this behavior. And over consolidated clays and dense OC soils are dense sands exhibit a peak behavior and after that the stress decreases. And NC loose sands.

So, here we obtain the failure shear stress at failure for critical state. And from this is a stress-strain relationship. And this is a shear stress versus shear stress at failure versus sigma at failure. This whole test is conducted at 1 sigma. And for different sigmas if you conduct different shear stresses, you get. And from that you can obtain either straight line joining from origin or with some intercept. So, these are the material constants phi critical and cohesion, these are the metal constants and sigma f, tau f are the state variables. Similarly tau and shear strain these are state variables. So, the constitutive relationships are tau versus sigma. If it is a drain test in triaxial test, then it could be sigma dash also. And tau versus gamma or in triaxial test, you get delta sigma versus strain. So, these are the constitutive relationships we have.

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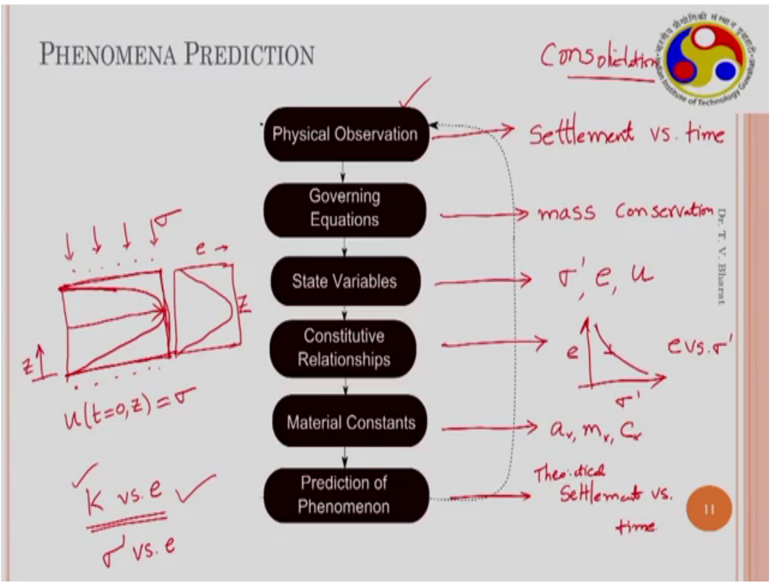

So therefore, for the phenomenal prediction, you require first physical observation. For example, in case of consolidation this is a settlement versus time settlement versus time. And you have governing equations which are derived from the physical laws. Here it is a mass conservation. This is for consolidation problem. Then after getting the observation settlement versus time, you invoke the mass conservation principle. And to derive the governing equation and you identify the state variables, here the state variables are sigma dash, void ratio, u etcetera. You identify the state variables then construct a constitutive relationship using experimentation.

So, you build void ratio versus sigma dash. So, this is a constitutive relationship which is obtained from the experimental observations e versus sigma dash. Then identify the metal constants such as a v or m v, and finally c v. Then you predict the phenomenon because one C v is known, C v is obtained you predict at what rate the settlement is taking place you get a theoretical settlement versus theoretical settlement versus time.

So, once you theoretically once you theoretically obtain the settlement versus time, you compare with the measured or observed physical observation that is settlement versus time.

If that does not match, so then you need to refine whether you have considered all the physical laws for building the governing equation or not you need to check, or are there any anymore state variables need to be recognized and constitutive relationships need to be built that you need to check. For example, in one case that is a consolidation settlement of soft soils which is of concern in many applications like mine tailing applications and other applications, and phosphate clays or tailings deposition in all these situations, the soft soil sedimentation or soft soil settlement is very important.

In such situations, the Terzaghi's fundamental consolidation theory or estimates the settlement rates that is because the soft soils would settle even because of its own weight. Here the conservation of momentum that is force equilibrium is not invoked here, because we are assuming that sample size is very small 2 centimeter thick and we are ignoring the effect of self weight. So, when we invoke the self weight, when we invoke the conservation of momentum principle that is force equilibrium then we can predict the behavior very well that is done by Gibson.

And also in this particular derivation one also need to express another constitutive relationship that is k versus e the hydraulic conductivity in the sample. So, if this is the consolidation sample, you have a porous stones at the top and bottom or sand which is at the top and bottom. Now, as soon as the load is applied from the soil sample, you have pore pressure at any given point at time t equal to 0. at time t equal to 0, pore pressure at any given depth is equal to the applied stress. But just after time t equal to 0, the pore pressure at z equal to 0, and z equal to l it becomes 0, and this is a pore water pressure distribution you get. Pore pressure is excess pore water pressure is maximum at the center and they are 0 at the boundaries.

So, this is the condition. If the excess pore water pressure is 0 at the boundaries two boundaries, then the sigma dash sigma dash should be maximum here. And from the expression of sigma dash versus e relationship and sigma dash is maximum, the void ratio is minimum. So, if the void ratio plot is made with the depth here, so the void ratio should be smaller here and larger here and smaller here. So, this is how the void ratio changes. If the void ratio changes in this manner, the hydraulic conductivity also changes similarly; hydraulic conductivity way should be smaller, when the void ratio is small; and should be higher when the void ratio is large. So, k also changes accordingly. So therefore, this expression is also invoked. This constitutive relationship is also built sorry this relationship is also obtained from the consolidation tests. And this is also used in the analysis.

And finally, the Gibson's model which considers the Lagrangian coordinate system, where the element also changes the element the element we initially be considered this also changes with time that is also considered by considering the Lagrangian coordinates. And it considers the another physical law that is conservation of linear conservation of momentum that is considered, so that means force equilibrium is considered and the expression for sigma dash versus e and K versus e both are invoked. And Gibson's expression predicts very well the consolidation behavior of soft soils.

Thank you.