Higher Surveying

Dr. Ajay Dashora
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Module – 02
Coordinate System and Reference Frame
Lecture – 06
Applications of concepts of astronomy

Hello, everyone. Welcome, back in the course on Higher Surveying and we are in module – 3 again, and this is lecture – 2. Well, in the last lecture we have discussed about reference system for astronomy and we say that in order to measure the various angles or various positions or the 2-dimensional position of a star we need to have some reference systems, ok. Why do we have different reference system because in one difference system I can measure some angles in another reference system I can make them globalized; that means, everyone can refer to those angles.

For example, if I say that I want to do a measurement in azimuth and let us say alleviation angle or altitude angle at a celestial horizon then I will say I will use some instrument let us say total station or theodolite or may be similar instrument like that and using that I can do some measurements, and for that reason we have defined some concepts like celestial sphere, celestial horizon and zenith, nadir, north celestial pole, south celestial pole and fundamental of basic astronomic idea we have defined.

But, we then defined second reference system, because we realize that in the first reference system which consist of measurement of altitude and azimuth. This altitude and azimuth both are dependent upon the observer’s position and they are also dependent on the time that is measured on the observer’s position. We will see this thing in the coming lecture at this lecture as well that what do we mean by time measured at the celestial meridian or observer’s position and then you are all already aware of the term celestial meridian, that we defined as a circle passing through zenith, nadir NCP and SCP.

So, celestial meridian if defining the observer’s position, similarly celestial horizon is another circle that is defining the horizon position of an observer. And the intersection of the two circles will be you know what is that, that will be the meridian line north south what we call on the celestial horizon, well you have some idea like that, but then we realize, let us make out self little free from the dependencies. So, we defined an another reference system that is declination and hour angle.

We realize that declination was measured from the celestial equator above and below. So, below is negative declination above is positive declination, and since the equator position or the celestial equator position is not going to change even with the rotation of the earth. So, we said that declination is free of observer's position as well as the rotation time or the rotation speed or the a spin of the earth, but what about the hour angle?

Hour angle we have measured from the celestial meridian as celestial meridian itself is defined with respect to the observer's position. So, no doubt, hour angle has to be dependent on the observer's position, but more specifically if a look deeply we realize that the even on the celestial meridian, if I vary in the latitude position; that means, on the celestial meridian I am changing the position of the observer and I am changing the latitude basically, but I am not changing this plain of celestial meridian I know that hour angle is for a star is going to remain same for all latitude position on a celestial meridian. That is why we say that hour angle depends on the observer's positions although, yet it is free of the latitude of observer. It is only a function of longitude.

And, then we say that which latitude and longitude we are talking about? We are talking about reduced astronomic latitude and reduced astronomic longitude that we derived from the terrestrial reference frames. Well, now you can understand where we are connecting the astronomic reference system and the terrestrial reference frames. You can easily understand what is connection now, you can imagine that things and these are very very simple logic they are very simple idea, ok.

What next? Then, we realize, since hour angle is dependent upon the observer's position as well as the time; time at the celestial meridian, again as I said we will come to know, what is the meaning of that that the time reckoned or the time understood at the celestial meridian. Then we said that let us derive one more coordinate and that we said right ascension and for that we defined a point called vernal equinox, which itself is a free from the rotation of the earth. Why? Because vernal equinox is the intersection point of ecliptic, that is, a annual path of the sun around the earth surprising basically it is the relative path of the sun around the earth and that is annual path.

So, then we have realize that there is a point called vernal equinox, which we which lies exactly at the equator, but it is free of the motion of equator or observer's position, so that we can use it in order to define an another angle called right ascension. Then we measure the right ascension with respect to the vernal equinox in the plane of equator. So, we also saw that, what is the relationship between the right ascension and hour angle because both are measured in the equatorial plane.

Now, in this lecture we are going to establish the relationship between the 3 reference system. In the last lecture also we talked about the fourth reference system that is celestial longitude and celestial latitude, then we rejected it in a sense it is not very useful for our purpose, ok, but still I can say that system is completely independent of observer's position and the time.

So, now today let us discuss how to transform or how to develop relationship between these three reference system. First is horizon reference system, second is dependent equatorial reference system third is independent equatorial reference system.

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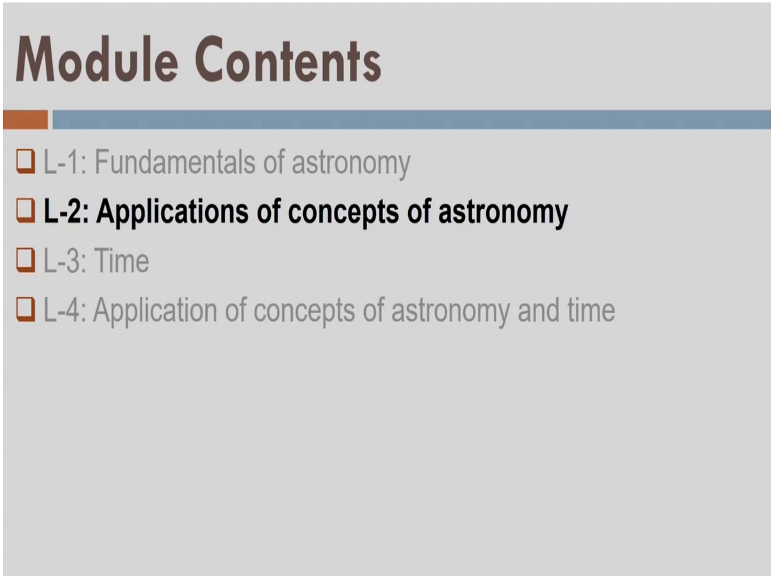

So, this is the lecture – 2, where we say that applications of concepts of astronomy that is whatever we have learned in the last lecture L – 1, in the fundamentals of astronomy we are going to implement here to develop the relationship between the three reference system.

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Here books are there.

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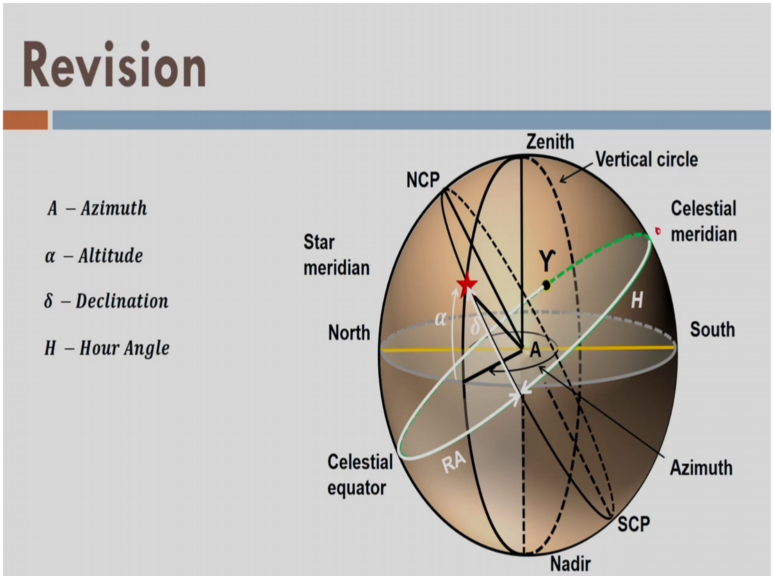

Now let us do small revision this is my celestial sphere. So, this is my zenith, nadir, ok. So, that is celestial equator. So, that is a NCP – North Celestial Pole and south celestial pole, ok, that is my celestial meridian; if you remember the circle that is passing through zenith, nadir, NCP and SCP and also it is a great circle because it is containing the center of the earth or center of the celestial sphere.

Then, we define a north a point which is closer to the NCP on celestial horizon. So, this is automatically south, and then let us see there is a star which is already projected on the celestial sphere. Then, this is the vertical circle passing through the star and be call it vertical circle of star. Now, then we defined this circle as circle that is passing through star and NCP and SCP and we call it stars meridian, ok.

Now, this is the north south line. So, I have already connected the center point of the star of to the center point of the celestial sphere. So, this is the meridian line shown here as a yellow color and this line is joining the center of the earth and the intersection of the vertical circle and the celestial horizon, well. So, this angle will be called as azimuth here. Remember, we have measured the azimuth from the astronomic north and this azimuth is astronomic azimuth not be magnetic azimuth or the magnetic bearing that we measure in the classical survey.

So, this is azimuth, now I have measured the azimuth. This is my altitude angle. So, this is I measure the altitude angle here then this is my horizon reference system, ok. What next? So, this is the angle I call declination delta, well and this angle I called the horizon or hour angle not horizon it is hour angle that, I measured from the point in equator that is above the south point of celestial horizon, right.

Hence we have taken this point here you see to measure the hour angle and then we said we measure the hour angle in the westward direction, positive in the third system we defined. So, this is this system is dependent equatorial system because hour angle is dependent then independent equatorial reference system where we have said declination and right ascension.

In order to measure the right ascension we defined a point called vernal equinox like this here and then we said, measure the right ascension opposite to hour angle, that is right ascension is measured in the equatorial plane, but in the opposite to hour angle or measured positive in east wise direction. So, this is my right ascension that is what we have discussed. And, then we wrote a relationship if I want to measure the hour angle of vernal equinox then it should be equal to the hour angle of the star and right ascension of the star. So, I can measure it that way.

So, somehow now you can understand that since the hour angle is measured from the celestial meridian it is giving me some sense of time, you can understand why because is my hour angle is 360 degree it will be completing one circle at the equator and so, this will be the indication of complete 24 hour because earth rotates by 360 degree in one rotation.

So, if I want to measure a time a local time or time with respect to the celestial meridian or observer's position I should say that let us use hour angle. Oh, now you understand, right what is the meaning of hour angle and what is the importance of hour angle. So, let us see I have measured right ascension also. So, now, you have understood that how did you measure these five variables and different different reference system for astronomy.

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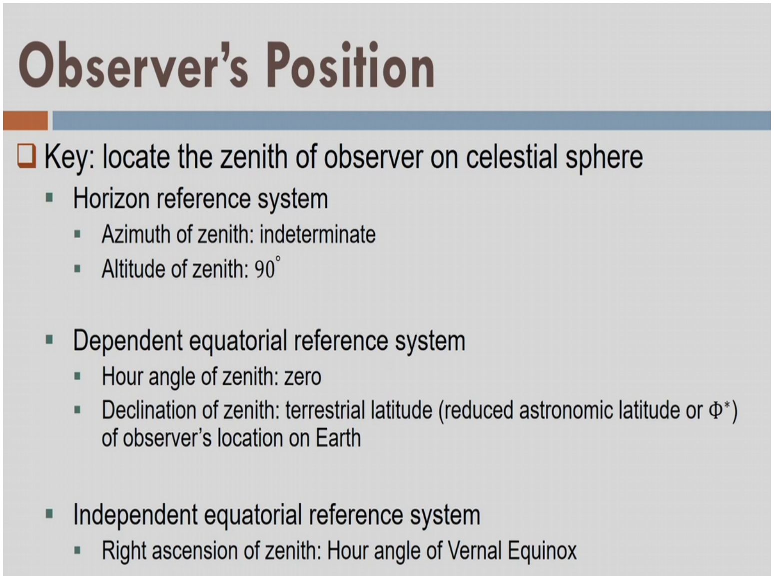

So, now try to locate the observer's position in this free system go back to the slide in the last slide and try to understand how can you locate the position of observer, that is a zenith of the observer.

So, now I am giving the answers here. First thing in horizon reference system, azimuth of zenith is indeterminate because. Azimuth is measured at the zenith itself. So, zenith is a point and I need two vertical circle in order to measure the azimuth. So, there are no vertical circle passing through the zenith for zenith itself and as a result it is indeterminate the azimuth of the zenith is indeterminate, ok. What about the altitude of zenith? It is obviously, 90 degree because it is making 90 degree angle with the celestial horizon for a given observer's position.

Now, dependent equatorial reference system, that is right that is hour angle and the declination. Hour angle of the zenith, you can say it is 0 because celestial meridian itself it is passing through zenith. So, there is no angle between zenith a circled passing through zenith and the circle passing through zenith. So, what about the declination of zenith, it is the terrestrial attitude and more specifically since we are using the word terrestrial latitude I should bring the term reduced astronomic latitude or phi star that we already discuss in detail in the last module we will also discuss here briefly, ok.

So, if you know the phi star of place that is reduced astronomic latitude of your place you can find out the declination of the zenith. Remember, it is declination of the zenith not of the star, ok. Let us go ahead in case of independent equatorial reference system. As I told the right ascension of zenith it is nothing, but the hour angle of vernal equinox. So, try to answer, try to imagine or try to go back to the previous slide and try to see whether these answers are correct or not, whether I have really responded correctly to you or whether we are I really shared the correct information with you or not.

So, we have defined now the observer's position that is the zenith of observer in three reference systems, fine.

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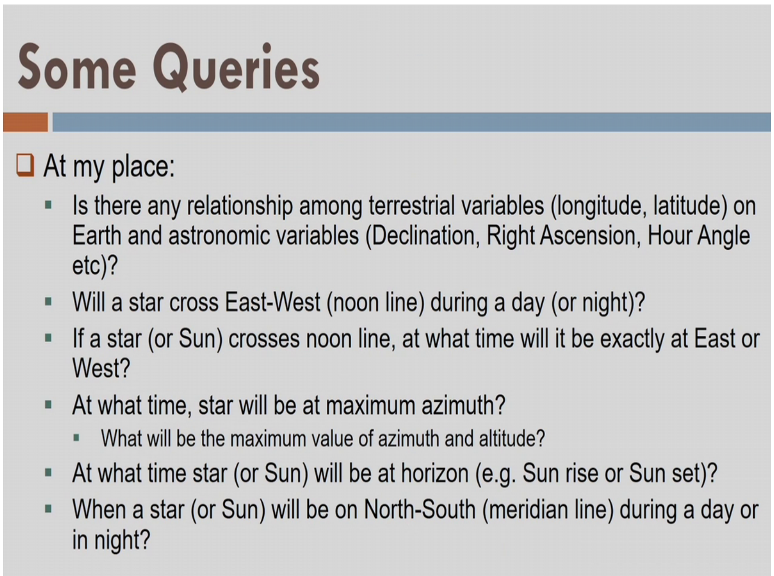

Now, I want to raise some queries to you. Let us see at my place; that means, at my celestial meridian or at my celestial horizon is any relationship among terrestrial variables that is longitude, latitude and astronomic variables like declination right ascension hour angle etcetera?

Second question is, will a star cross East-West or the noon line during a day or may be night? Because, remember all the stars are available around us just because of the position of the sun which is closer to the earth compared to other star. So, we have day and night. Secondly, the intensity of light of sun it is so high, that we are not able to see the star in day time, but in fact, they are all around us every time moment and there apparently moving around the earth because of the rotation of the earth as well as the revolution of the earth.

So, do not forget those fundamental things, that is why do not thing that a stars cannot be appear in day time and best example is the solar eclipse. During the solar eclipse when sun is covered by moon partly or fully between earth and the sun moon comes and it covers the or rather shadow the sun in that case we can see the stars also around us. So, it is a best example that yes a stars are always around us, well.

So, third question is if a star or sun crosses noon line at what time will it be exactly at east or west? Because, I have some conception about east, because if I say east is the direction where sun rises so, my meaning is does sun really rises at the east exactly or is it slightly here or there? You will find out sun basically rises on a certain day in a year at exact east at a given celestial horizon, but rather and other days it rises little side ways to north or little side ways to south. We will see all this thing.

So, at what time star will be at maximum azimuth? Further, at what time star will be at horizon that is sun rise or sun set? Similarly any star can set or rise above the horizon and below the horizon. Moreover like other stars or like sun other stars also moves from east to west, all the stars all the it is different matter that they are not visible during the day time, but day also move, ok.

When a star or sun will be on North-South or the meridian line during a day or in night? Ok. So, let us try to respond to be queries. I hope in that case we will be able to connect our reference systems. Let us see how.

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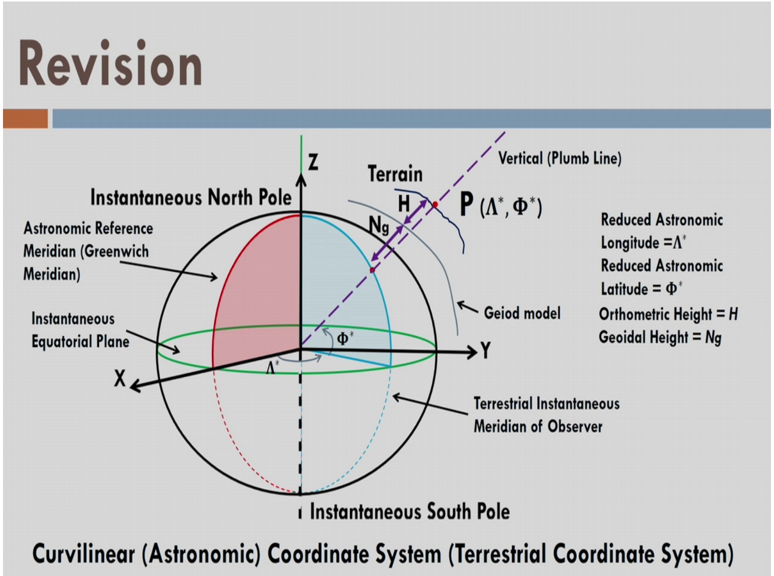

So, before that once again there is slight revision, I would like to do here about the astronomic coordinate system there and explaining it here with astronomic reference that is Greenwich meridian and that is astronomic reference now, it is not the Greenwich meridian of the ellipsoid system. It is for the astronomic system that we have defined with the help of vernal equinox. So, we can imagine that vernal equinox is on the celestial sphere.

Now, you are bringing that vernal equinox location on the earth surface. Just the same plain is there and your bringing it into; that means, the meridian of the vernal equinox will be cutting the surface of the earth also and the intersection of the celestial meridian of vernal equinox and the earth surface will be the plain of vernal equinox and the intersection of terrestrial equator with the celestial meridian vernal equinox will be the point vernal equinox on the surface of earth where Greenwich meridian passes and that is astronomic reference meridian.

Then, we have instantaneous equatorial plane, because what do we mean by instantaneous? So, right now whatever the position of earth is there I am taking it as it is or rather I am measuring with respect to that. So, that is I am using instantaneous equatorial plane. Secondly, automatically I will have my instantaneous north pole and instantaneous south pole as the rotation axis; that means, these two points are connecting my rotation axis or at the end of the rotation axis these to point will lie. Well, now you understand what is my coordinate system that if meant astronomic coordinate system.

Let us say there is terrain and point P is there. So, I am trying to locate point P in astronomic coordinate system. So, let us say this is the plumb line and if I explain the plumb line I called it to vertical line, right and so, sometimes it is also called vertical fine. So, now, I am assuming the plumb line it is passing through the center of the mass of the earth and what that assumption I say that and measuring the reduced astronomic latitude and a reduced astronomic longitude and not be other astronomic latitude and astronomic longitude. No, we are measuring reduced that is geocentric.

So, let us see this is vertical and so, this is a point, not on the ellipsoid. Well it is clearly shown on the ellipsoid, but no problem ellipsoid is just place for the purpose of reference so, that you can imagine the earlier story and the module – 2 what we have discussed. So, that is my terrestrial instantaneous meridian of observer. Well, you can understand there is a celestial meridian of observer and again that is intersecting with the earth and wherever the intersection of earth surface is there with the celestial meridian observer I will have this instantaneous meridian of observer. So, I am making this terrestrial instantaneous meridian of observer.

Now, this angle, we call it reduced astronomic longitude and this angle as reduced astronomic latitude for the in complete for the sake of completion I am just showing this could be a figure that you can find in some book. Now, the point P is shown by 2-dimension position lambda star and phi star both are capital lambda and phi.. So, let us see it was the geiod model and for the sake of completion we had putting this figure it was orthometrified H and it was geiodal undulation N g.

Now, we have defined reduced astronomic longitude lambda star, reduced astronomic latitude phi star and orthometric height H and geoidol height N g. So, now, since we have said in astronomy that there is no meaning of the height so, let us ignore this.

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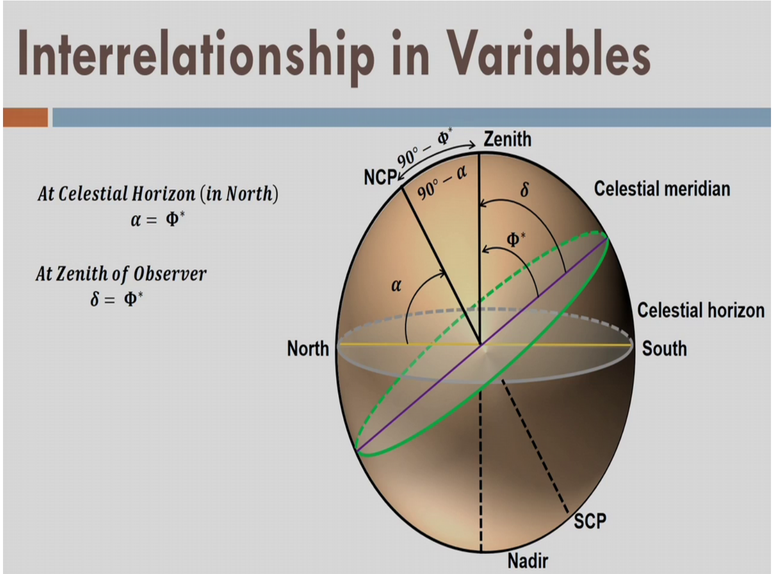

So, now let us go for be interrelationships in variables. So, we have made our make round very clear now, about the terrestrial latitude and longitude which are reduced astronomic longitude and latitude.

Now, this is my celestial horizon this is zenith and nadir, this is celestial equator. So, this is my NCP and SCP. So, this is celestial meridian no problem tell this point we have done it many many times, ok, that is north and that is south and the line connecting north and south it is meridian line, fine. This angle if you just read it carefully this is nothing, but equal to the, I will not say it is exactly, but it is equal to what? Now, you can imagine that the instantaneous terrestrial equator of the earth and instantaneous celestial equator are same and as a result I measure that terrestrial latitude with respect to be equate of the earth.

So, this angle, which is just shown here, it is equal to the reduced astronomic latitude, fine. I hope you can comprehend this thing and so, I am saying that there were relationship between terrestrial latitude of a place and which is reduced astronomic latitude remember and this angle reduced astronomic latitude is same if I measure from the in the celestial sphere also with respect to celestial equator to the zenith of observer.

So, now, this is my 90 minus phi star, ok. Here this angle which I measure in celestial horizon with respect to celestial horizon or above the celestial horizon. So, of NCP it can be the alpha angle the altitude of north celestial pole with respect to celestial horizon that is your position could be my position whatever. So, I will have we will have different values of alpha, but no doubt at a given position let us my position it will be some value alpha.

So, I am trying to measure the elevation or the altitude angle of north celestial pole at my celestial horizon, fine. Now, can you find out some relationship here this is 90 degree minus alpha we can see from the geometry. So, I can write straight away that 90 degree minus phi star is equal to 90 degree minus alpha and in other words, I can say at celestial horizon in north alpha equal to phi star, what is a meaning? If I direct myself at the north direction and the celestial horizon which is nothing, but simple horizon and then if I tried to elevate my eyes like this by an angle equal to phi star let us see the phi star angle or the reduced astronomic latitude of this place is 25 degree for example.

So, if I a stand in the north direction and try to raise my eyes or try to measure the alpha angle equal to phi star I will get NCP and that is nothing, but the pole star Polaris or north star. Is it not surprising that you got the location of pole star? Ok, let us do some more exercise what is you define the declination of zenith? Again, try to do just you stop the video for one minute and try to mark in this screen. Well, I will do it for you this is my declination because declination is measured from be celestial equator above positive below negative. So, for the zenith this is a declination, delta and hence I can write delta is equal to the phi star for an observer that is delta of the zenith, I have declination of zenith which is equal to the terrestrial latitude or reduced astronomic latitude.

Let me share you by clearly here, in this coming lecture in this coming lecture or this lecture of this module we will be using the word longitude and latitude for the two terms that is reduced astronomic longitude. That means, I am going to use term longitude for a reduced astronomic longitude and I am going to use latitude for the reduced astronomic latitude. If I want use longitude and latitude for the earth, I will say terrestrial longitude and terrestrial latitude, but for astronomic terms I will use directly longitude and latitude in this module.

So, let us make this as a clear understanding now, let us move ahead.

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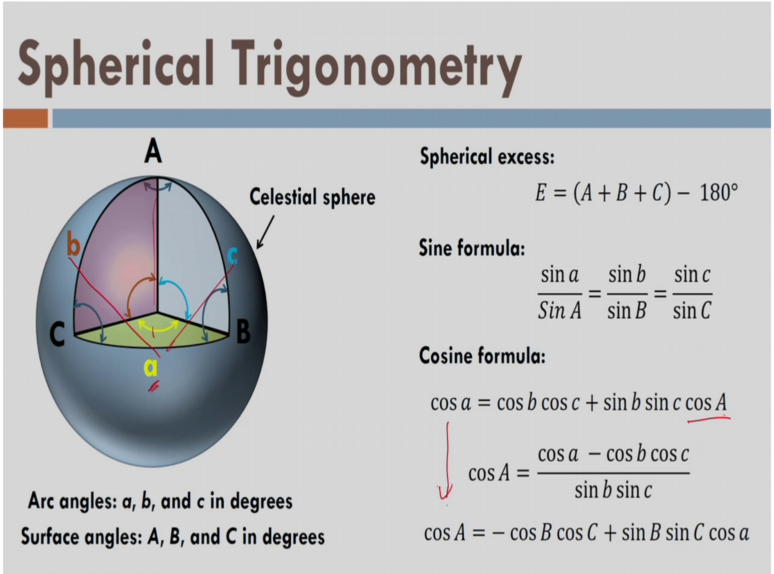

In order to respond to some of the queries, now we have learned how to define observer's position and we also learned what is the relationship between the variables, that is declination and latitude of a place also what is the altitude angle and that and latitude of a place we have all recently we have done. Now, in order to respond to the other queries we need to first understand a spherical trigonometry.

And, let us say this is my celestial sphere and this is spherical triangle made by A, B and C points on the surface of sphere. So, you can say this is angle C which is opposite to capital C. So, this is angle C, this is angle B which is opposite to B and this is angle A which is opposite to point A, ok.

I would like to say very clearly here spherical trigonometry is not like plain trigonometry where if you have a triangle like capital ABC the some of the angles of three angles at form let ABC will be equal to 180. No, in spherical trigonometry the some of the three angles on the surface of these spherical surface or on the celestial sphere it will be the summation will be more than 180 and the excess amount of more than 180 we called it a spherical excess. Well, all this terms we are going to define here now.

So, let us see I call arc angles small a, b, c because their creating arc and basically their measured at the center of the celestial sphere or center of the earth small a, b, c right and that is why their appearing as arc on the surface of celestial sphere, but on the surface we have three angles a, b, c that is measure in the surface of the celestial sphere. So, I call it this angle, this angle, these angle surface angles A capital A, capital B and capital C.

Remember that as I told a capital A plus capital B plus capital C is more than 180. Moreover, in this figure you may feel that angle capital C and capital B are 90 degree, yes, this is not the proper figure or rather this figure is used to demonstrate the spherical trigonometry concept. So, I can write this is my spherical access. So, it also follows the sin rule the spherical trigonometry follows the sin rule for this way that is small a, b, c and capital A, B, C angles are connected this way if we are defined the way they are shown in this figure.

Now, we have also cosine formula and this is written like that cos of small a equal to cos of small b cos of small c plus sin of small b sin c and cos a. Here you can imagine that their angle a is there here and now I am writing cos a in terms of small b small c which is opposite to this like this here and this. Now, sin b sin c again I am using sin b and sin c and then cos a here which is opposite to the small a here this, these angle here.

Now, you can develop can some kind of simile to write other formulas for other angles, ok. So, let us see this is my cos a I can write from here ok. One more formula I am writing for cos a similar to this line like this. So, now, you can replace the capital A by small a or you can replace the small a small capital you can replace small a by capital A. So, now, you can write this formula with this minus sin be careful on this minus sin where I am trying to show something here.

So, that is the simple formula for spherical trigonometry.

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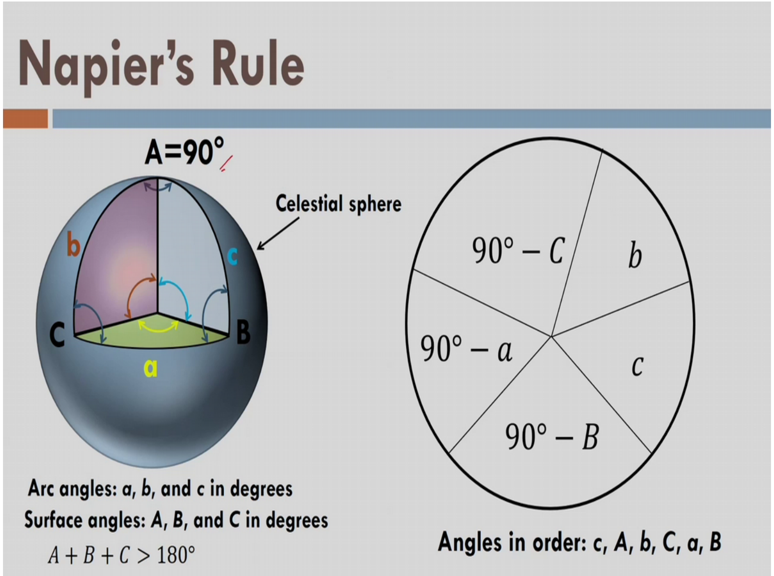

Now, let us come to the Napier's rule which is an especial case of spherical triangle. So, let us again define angle c, b and a arc angles angle capital A, capital B and capital C surface angles and A plus B plus C is more than 180 degree as we see in the previous slide. Now, we have a special case that is A becomes 90 degree like this. So, one of the angles surface angles is 90 degree, then if you consider remaining five angles which is small b, small c, small A and capital B and capital C this five angles are here and if you write this five angles in order I can draw that in a compartment five compartment of a circle. So, what does it mean let us tried at.

So, first consider A is equal to 90 degree and try to put the adjacent angles. So, adjacent angle are this angle here you see where my circle is moving and this angle where this b is there. So, let us put them this b and c. Now, put the remaining three angles in order, but put with complimentary of those; that means, we will put instead of c I will put 90 degree minus C. So, close to b here you see in the screen you close to b this is capital C. So, I will put 90 minus capital C, then close to 90 degree minus C we have 90 minus a and then we have 90 degree minus B.

Remember, for remaining three angles I will put the complimentary angle or 90 degree minus the value of angle, ok. This is very magical circle because using this circle I can write two fundamental rules, mathematical rules. Let us see that.

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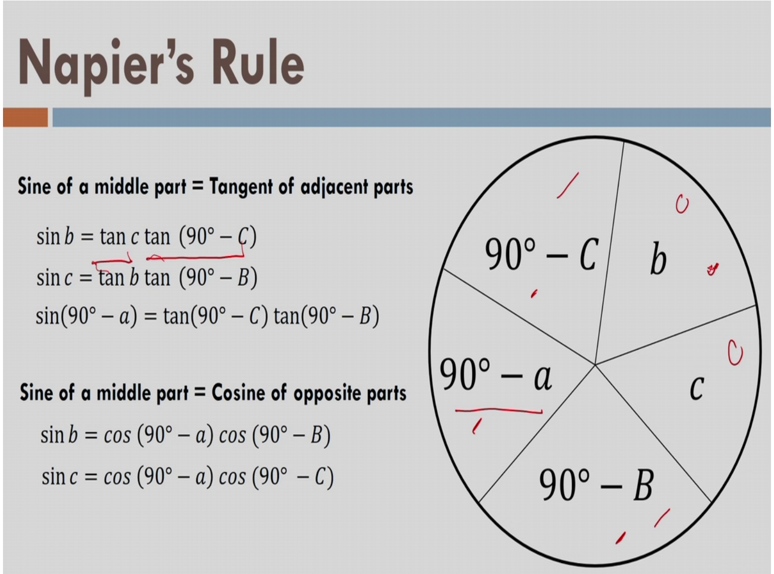

Now, I will define one angle as my middle part, any one angle I can say is my middle part and with respect to this middle part there are two adjacent part and two opposite parts. So, let say we were respect to b I have these is my adjacent part, this is my adjacent part ok. I write then a here and now remaining two are opposite parts I write O, O.

Now, we can say the first rule is the sine of middle part is equal to tangent of adjacent part; that means, sin b is equal to tangent c and tangent 90 minus C, you got it? Ok another rule says sine of the middle part is equal to cos of 90 degree minus a and cos of 90 degree minus B, that is cos of opposite parts. Well, this is simple rule and now using these rules we are trying to solve some of the problems, where I find at least one angle the one surface angle is equal to 90 degree.

So, let us see those cases, ok. Before that let us write the other angles also for example, if I create this angle c as my middle part so, what will happen? This angle and this angle is my adjacent part. So, I can write this relationship sin c equal to tangent of b and tangent of 90 degree minus B, right. Similarly, I can write this becomes my opposite part this and this. So, I can write sin c equals to cos of 90 degree minus a and cos of 90 degree minus C. This is not magical write try to write yourself, ok.

Let us try for one more angle let us see make this angle as middle part and then these two becomes adjacent part and this becomes opposite part. Try to write yourself I am writing it here for you can check it your answers try to do yourself.

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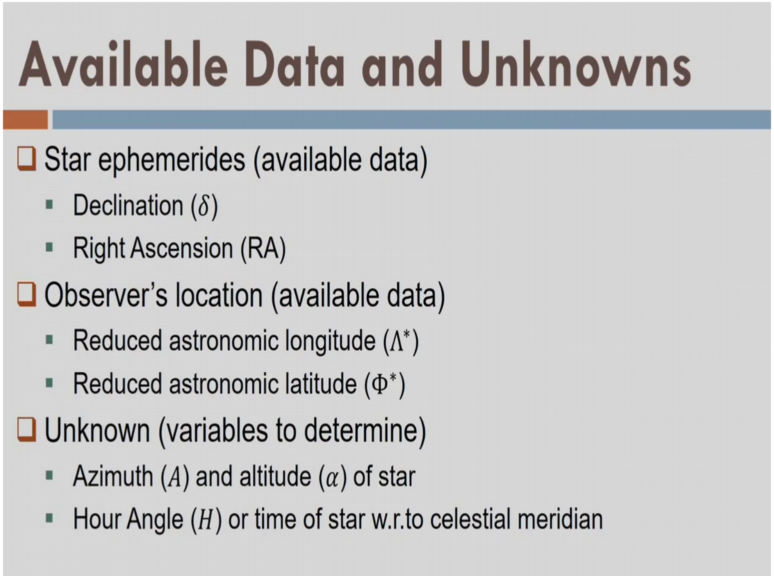

Now, before solving any question we have very clear idea about available data and unknowns. What are the unknowns that we want to determine and what are the available data? So, the available data is the a star ephemerides or star almanac or star a catalog. From there we have declination and write ascension available to us according to the given day of the year and given time on the day.

So, now we have observer's location that if reduced astronomic latitude and longitude as phi star and lambda star that is available to me. I already know about my place, you might be in about your place at least in a questions it is assumed to be known, then we have azimuth hour angle and altitude of the star as unknown. Why? Because I want to find out what is the hour angle or what is the time because hour angle written as the time at which a star will be at horizon.

So, horizon means if alpha is equal to 0 my altitude angle is 0 and I want to find out at what time it will be as I already told the sun is sometimes rises slightly north or slightly south in the year. So, I want to know what is the exact value of the azimuth of sun where it is raising it is coming above horizon or it is having alpha is equal to 0 degree and it is have a tendency to come above the horizon. So, for that reason azimuth is also unknown.

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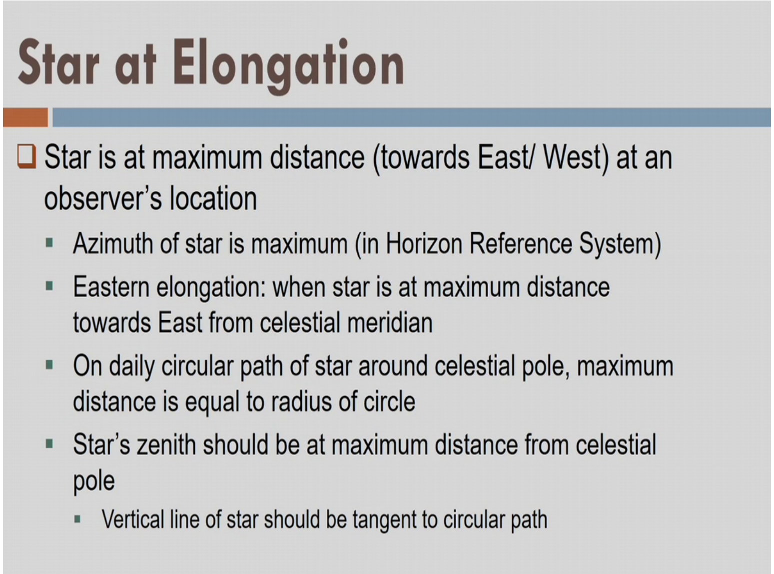

So, let us try to find out for some of the problems. So, star at the elongation. So, instead of defining what is elongation in words you can read the slide stop the video. So, what I can say a very simple thing here, that when azimuth of star is maximum in horizon reference system we call it a star is that elongation.

So, you might be surprising that for a sun we have maximum azimuth at east or west or north or south at north it is 0, at south it is 180 degree, at east it is 90 degree and at the west it is 270 degree. So, when it is maximum. So, it is not applicable for those stars like sun which are rising and setting star. So, we are talking about non setting star which will never set at a given position. So, let us see what is meaning here ok.

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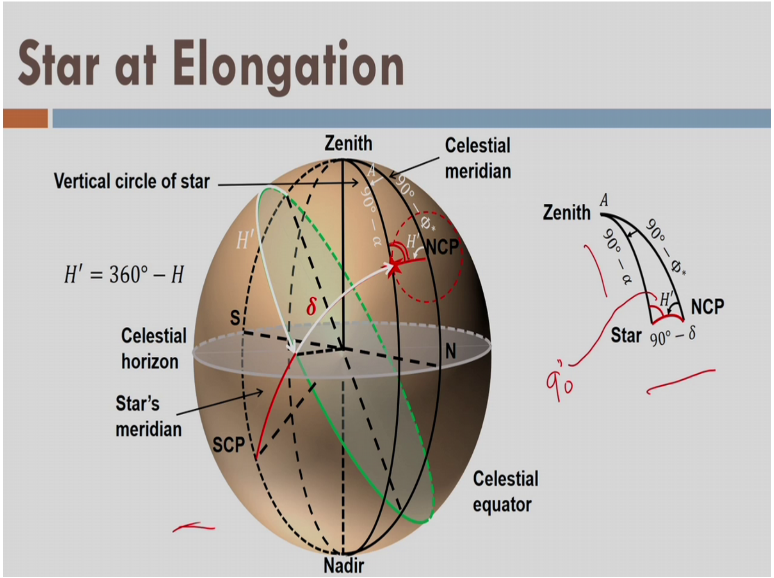

I have already drawn celestial horizon and celestial equator. So, this is my NCP, SCP. So, this is the celestial meridian. So, this is my north and south and celestial horizon the star is encircling the north celestial pole in this form every day because of the rotation of the earth apparently it is moving in this circular track.

Now, you can say when the maximum azimuth is there this is the position. Why? Because if I draw a vertical circle of in for zenith and nadir, then it is the maximum possible value of azimuth ok so, that is the vertical circle of the star. Now, you can see that vertical circle is tangent to the a star trajectory and hence it has at maximum distance from the north celestial pole and so, it is having maximum azimuth.

Now, I want to find out what could be possible value. So, before that see this is the meridian of the star. So, it is stars meridian, right. So, let us see this is my astronomic triangle where this is my zenith NCP and star. Now, as I told that this angle is 90 degree here like that fine, because you can understand the vertical circle is tangential to the trajectory of the star and secondly, it is at the maximum distance. So, any point which is like this joining the NCP and the star is maximum distance and equal to radius of the circle and radius of circle with the tangent of the circle makes an angle of 90 degree. So, this angle is my 90 degree shown here this is 90 degree, ok.

Now, what about other angles? So, this is my see the hour angle, how do I measure the hour angle? This is the way I measure hour angle and since it is difficult for meat to show this hour angle in this figure. So, I am showing other angle which is 360 degree minus H and I am writing it H dash. So, this is my H dash here, right I removed this thing. So, I mark H dash here at NCP right you can connect yourself and so, in the this figure I am also marking the angle it is H dash.

Now, this is my altitude angle of the star. So, this my 90 degree minus alpha. So, this is my 90 degree minus alpha here, ok. Similarly, try to see all this measurement and I am putting in the spherical triangle fine. So, let us see this is my latitude of the place phi star which is equal to the latitude of the NCP at my celestial horizon from direction of north and we have already concluded it in the previous slide look at this slides.

So, this is my 90 degree minus phi. So, now, this is my 90 degrees minus phi, ok. This is my azimuth angle this is the azimuth angle fine, A now this is angle A here, ok. Ok, what about last angle, which is this angle? Ok, let us see look in this figure here, this is my declination angle, which I just drawn again I will draw this angle is my declination measured from the celestial equator. So, this is the declination delta of the star. So, automatically this angle becomes 90 minus delta. You see, why because the a star to NCP distance is 90 degree minus delta.

Now, we have realize that where is one surface angle which is equal to 90 degree and where is that 90 degree? It is at the star this angle as I already wrote on the screen, ok.

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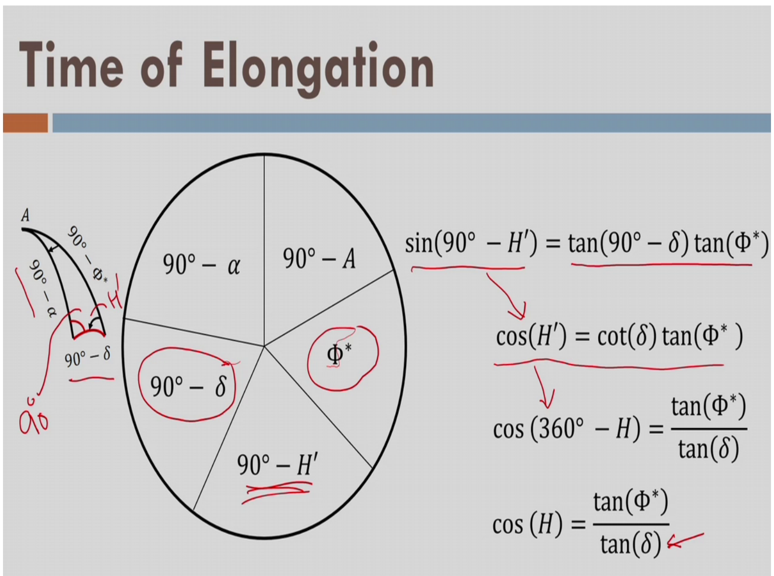

Let us go ahead. So, this the situation where this angle is 90 degree. Now, you try to write yourself, stop the video for 1 minute and once you write your answer try to check with my answer. So, first of all I will draw two angles which are adjacent to 90 degree. So, that is one angle is this and one angle is this. So, you can also write let us say 90 degree minus alpha and 90 degree minus delta.

Now, remaining three angles I will write the complimentary of those, that is for a angle 90 minus A for 90 minus phi let us see for H dash I have this angle, this is my H dash angle here. So, I have 90 degree minus H dash here and then we have phi star because it is already 90 degree minus phi star. So, 90 degree minus 90 minus phi, so, it becomes positives phi star well.

Now, can you write the Napier's rule for any angle? First of all what do I want to know that is most important, ok. What is known to me, this delta is known to me and this phi star is known to me this and this. So, this quantity is known to me. So, these two quantity are known to me. So, using these two quantity I want to find out other three quantities and I want to do all these quantities independently, so, try to write.

So, let us see first thing write this angle as a middle part. So, if you write as a middle part sin of middle part equal to tangent of the adjacent part. Well, if you can find out this or not right cos of this one that is coming from here, there I am writing cos of 360 degree minus H H H dash and so, I will get this as at answer.

So, you can see that for a given value of tangent delta or the delta declination at higher altitude or at higher latitude places whatever fact of H. So, H higher the value of H; that means, it will take star will take more time for elongation, lower the value of H star will take less time. Now, try to find out figure it out when cos of the angle will be less when this phi star is more or what. Just try to do this analysis yourself.

We will go to another case now; that is I want to find out the other angles may be alpha or azimuth A.

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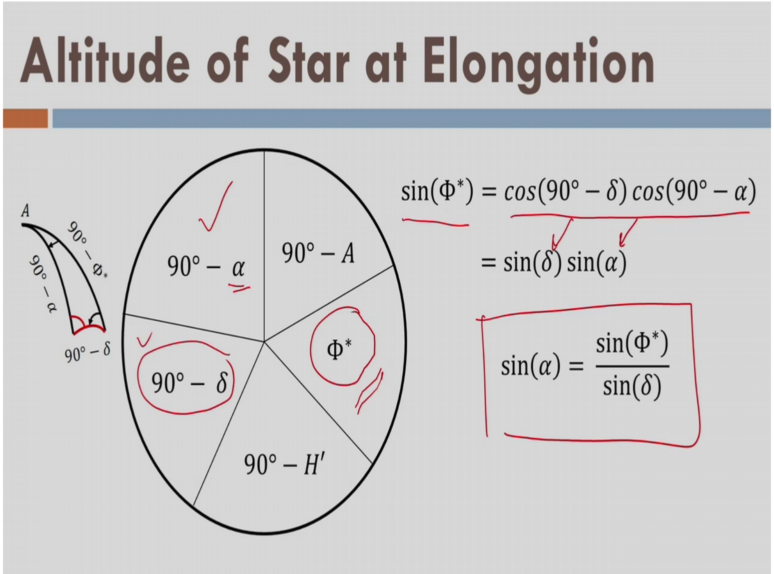

Altitude of the star at elongation so, gain I am writing let us see alpha this complete once again. Now, I want to find out this angle alpha altitude of star at elongation. How can I write? I again know this quantity and this quantity. So, how to can I include this quantities, right? Are they opposite angles or are they adjacent angles let us see. So, I can write this angle sin equation for this angle because let us see this is my middle part and so, these two part becomes opposite part here and here.

So, I am writing cos of these two angle and accordingly I will get these two values here and so, this is the final answer here given declination and given latitude of the place I can find out what is the elevation of or altitude angle of the star for elongation.

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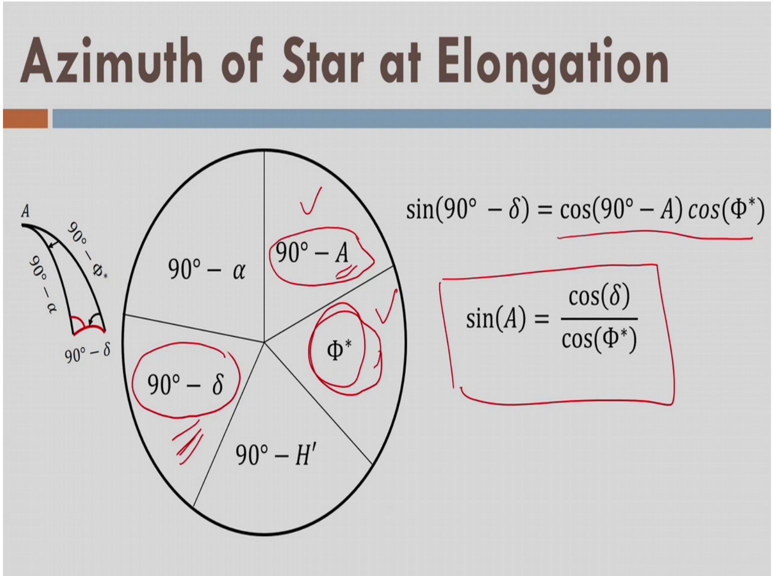

Further coming again a star of the a star at elongation what is azimuth now? Third angle again I write the same thing here. Now, I want to find out this angle A. So, what can I do again? Let us see these two stars these two values are known to me. So, let us imagine what can I do can I use adjacent part or can I use opposite part I will write sin of 90 degree minus delta like this equal to their opposite part these two angles are opposite for this angle.

So, I can write this thing ok. So, I will get this as my final equation. Now I can find out azimuth. So, if I already found altitude and hour angle; that means, time altitude and azimuth at which elongation of the star will have; that means, it will have maximum azimuth, ok.

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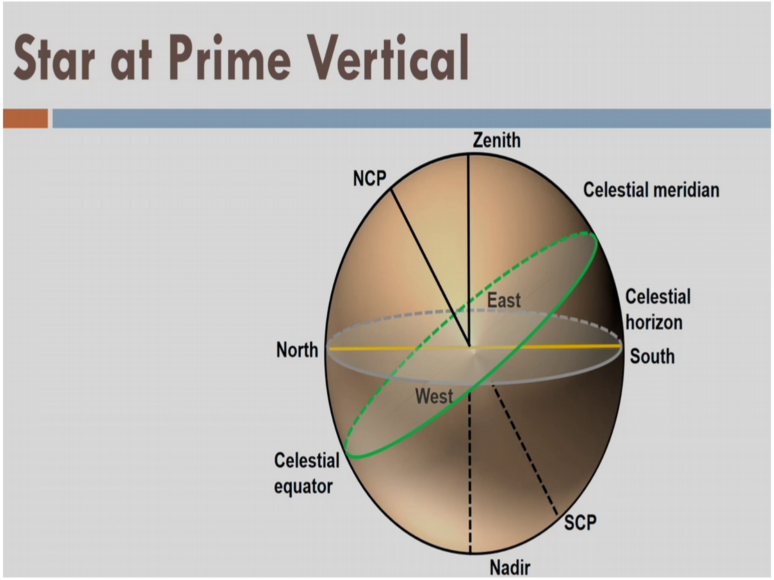

So, now let us take a star at prime vertical. So, what is the meaning of that? The meaning is a star comes 90 degree to the vertical circle ok, what is the meaning let us see. This is celestial horizon that is zenith and nadir. So, that is my celestial equator that is NCP, SCP, so, this is my celestial meridian. Now, if a star comes at some vertical circle which is perpendicular to the this celestial meridian, but vertical circle ok, the vertical circle that passes through zenith, nadir and great circle.

So, let us see East-West. So, I can see that my star will passed through East-West line, right. Now, you can imagine whatever vertical circle or what is time vertical?

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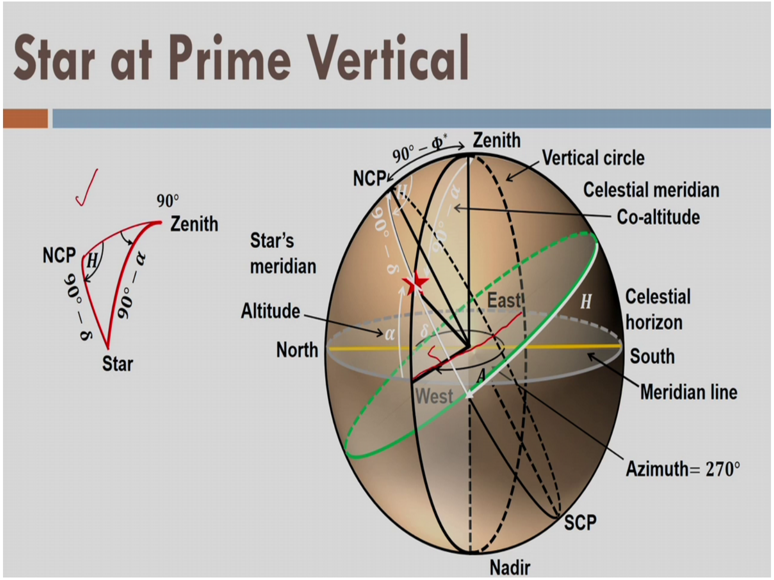

This is star and that is at the East-West, fine. So, now, imagine that these two points are east and west, where this line is joining here. This is my east and west point, fine and that is why this vertical circle is called time vertical because this angle is 90 degree now. You see, it is very easy, ok. Now, we have realize that that is one angle that is 90 degree I can use in Napier's rule.

So, what about the other angles? So, this is my a spherical triangle from at the star using stars meridian. So, now, this is the NCP, zenith and star making the triangle, where one angle is 90 degree which is this angle, you see I have already drawn this thing because this angle is 90 degree here. So, let us see this angle plays alpha or altitude. So, this is my 90 degree minus altitude. So, go altitude I am I am writing it here this spherical triangle.

Further, this is my azimuth angle which is equal to 270 degree. You see here as I easy to understand measuring the azimuth from the north in the celestial horizon. So, it is equal to 270 degree at time vertical. Now, this is my declination angle of the star and this is co-declination. So, this is my 90 degree minus delta and see this is hour angle here H, right. So, this is my H angle here in the figure big figure. So, now, I am writing it H here in the triangle.

Before that one more angle is missing which is this angle. So, what is this angle by this angle between the NCP and the zenith and we already know it is co latitude of a place. So, I have written 90 minus phi degree, fine.

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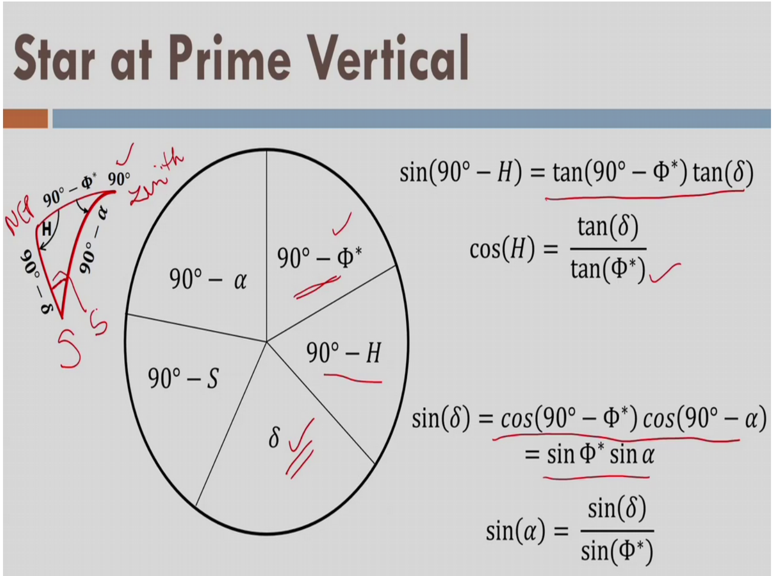

Let us go ahead in this thing we know this is my 90 degree angle here, ok. What are the adjacent angles that I should first write? So, this is 90 degree minus alpha, this is 90 degree minus phi, now rest of the three angles I should take in order, so that they should be complimentary angle, ok. So, 90 degree minus S, S means this is a star here S, ok. If you go back here we have NCP and there we have zenith, this point at zenith we have 90 degree angle.

So, now 90 degree minus S which is this angle is S you see this angle is S. So, I am writing 90 degree minus S. So, delta, fine ok, then we have 90 degree minus H. Now, you can also constitute this kind of Napier's circle. Now, write Napier's rule what do we want to what do we want to find out you already know azimuth is 270 degree altitude what is altitude at which this is happening remember one and what about the hour angle, do you really want to find out? Yes, we want to find out.

So, let us see at what time a star will come at my primary line that is East-West line or be noon line, right. So, let us write for hour angle, ok. If you make this angle is the middle angle what will happen these are two known angles to me I can write adjacent rule and then I will find out the value of H, that is the hour angle or the time at which a star will be at time meridian and time is measured with respect to my celestial meridian, right.

So, now you can feel that whether my sun is exactly on east or north you put the value of this delta for given a day and given time of the sun and then you put the value of this phi star in this formula, you will come to know what is the value of H and then you can see whether H is exactly equal to 90 degree or not? If it is 90 degree then sun is exactly at the east, otherwise it is somewhere near east. If it is less than, then it is towards north or south you calculate yourself, ok.

What about the next quantity, that is alpha. Let us see I know again these two quantities this and this can I use it here see here sin of delta then these two parts becomes opposite part. So, I can write cos formula here and then it will get this one so, I can write sin of alpha. Now, you can find out what is the altitude of the sun when it is it exactly east or it is exactly at west you know it is 0 degree or it is 0 degree again, but there is a problem.

As we said that it is not necessary the sun will rise exactly at the east. So, if you use this formula you will get some other answer. So, alpha will not the 0 if you use this formula, let us try to understand then sun will be at horizon or any star will be at horizon.

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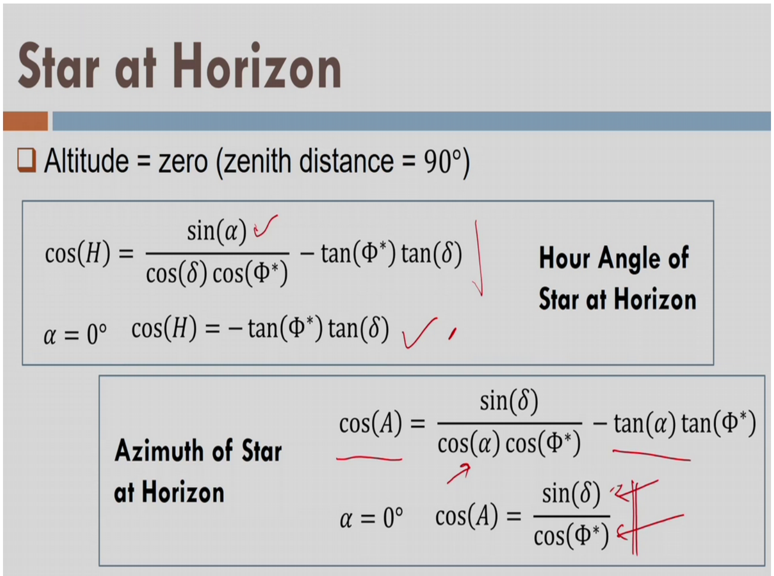

So, for the horizon I am using another formula which is completely independent of Napier's rule because I cannot guarantee that at the time of horizon when sun is at horizon at any star is at horizon I will have 90 degree or the 0 degree elevation angle or altitude angle of the star. So, I am not using Napier's formula rather I am using very few formula that you we have shown in the first slide.

So, here now I will use this formula, where if I put the alpha equal to 0 degree here I will get this formula, check yourself. So, you can find out at what time sun will be rising that is a time of sun rise or sun set may be. So, you add 12 after that or something like that or you add the delta according to that position of sun, where sun is on the west. So, whatever delta value of the sun at that point of west you will get the correct value of sun set also. The sun rise and sun set both has alpha equal to 0 altitude equal to 0, right.

Similarly, what is the azimuth of star; that means sun is aggressively at east at the time of sun rise or not. So, you find out this is the time now you put same time or you can find out using this angle this formula. If you put alpha equal to 0 here you will get cos A equal to this thing. So, for a given value of sun declination at a given time or at a given location you can find out by putting this values what is the value of azimuth.

Now, you can confirm whether azimuth is 90 degree or not? If it is 90 degree; that means, on the day sun is exactly rise the on the east. What do you mean by either day; that means, for let us say June, 21st, if you put the value of delta declination of sun then you will get this value what is the value of azimuth 90 degree or not? Try to do try to go to internet find out the delta value for this date and try to find out whether tomorrow sun is going to rise in morning east or not? Do it yourself.

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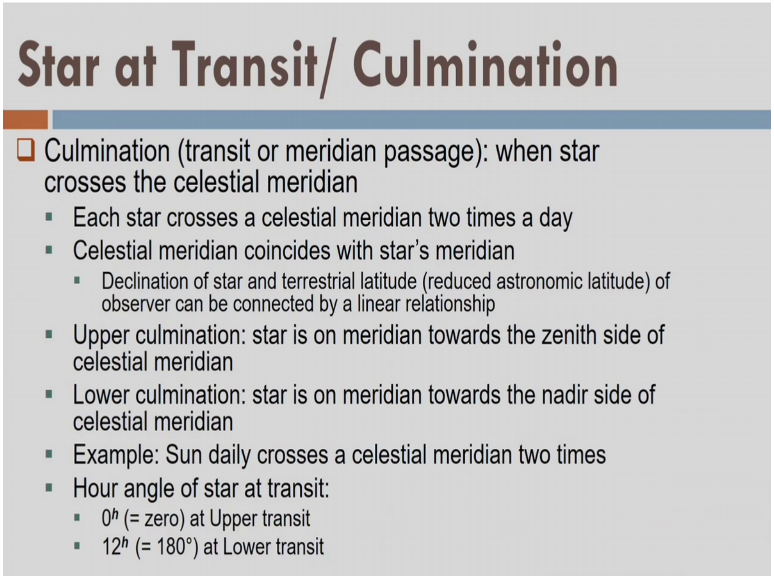

Now, sun at the transit. What is the meaning of transit? Transit is when sun is exactly at the meridian celestial meridian. So, what is the meaning of that? Ok so, that is each let us see in the figure.

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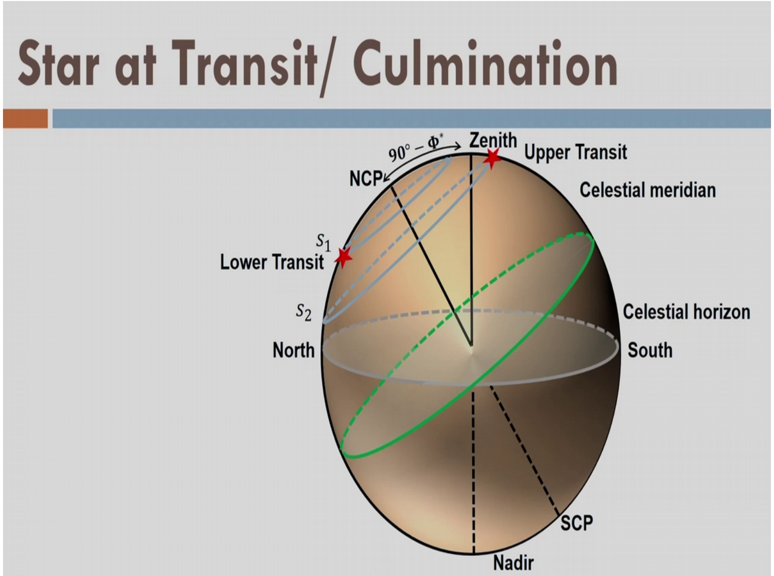

This is my celestial horizon and this is zenith, nadir, again this is NCP and SCP, fine. So, this is celestial meridian. Now, star is on the celestial meridian there are two possibilities because a stars are going to move around the NCP. So, it will cut two times meridian. So, this is North-South, ok, that is my this one.

Let us see there are two stars like this. So, path of the two stars I am drawing here. This location is called upper transit then a star is near to the zenith, ok. When it is away from the zenith and is still on the meridian I call it lower transit, fine. Now, what about a star S 2, fine. Now, you got what is lower transit and upper transit for a star. They will have lower transit as well as upper transit ones in a day.

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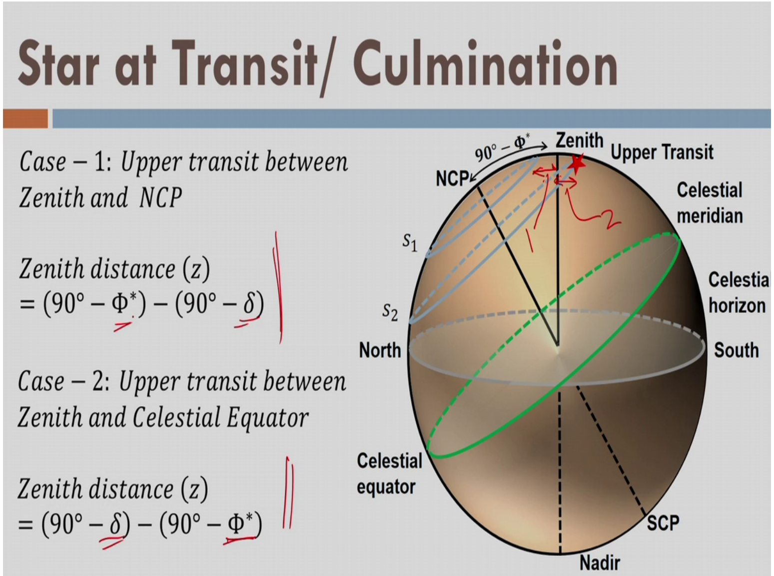

So, there are two possibilities here that we are going to consider now let us see star S 1. So, that upper transit is between zenith and NCP. So, now, you see that 90 degree minus 5, this angle zenith distance which is nothing, but this distance here, right. When case – 2, so, this is my zenith distance here. Now, in case – 2, you can find out yourself it is very easy to find out.

So, this is case – 2 when upper transit occurs between zenith and celestial equator. So, this is my case – 2 and this is my case – 1 and so, this is the answer here for zenith distance. Try to find out yourself it is very easy, right or go back to the previous slide try to see what are the values how to we define phi star delta delta and phi star, well, ok. Let us go ahead.

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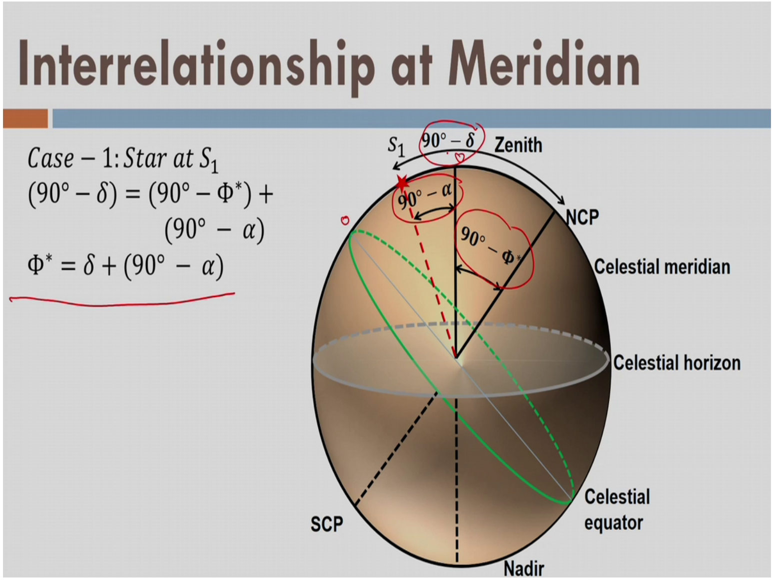

So, I am doing it for you now. So, this is the star position let us see S 1. So, now, this angle is delta as you know it is measured from the celestial equator. This angle is 90 degree minus delta as you see from the figure, ok, fine. So, what else, what other angles? This is my alpha angle measured from the celestial horizon and so, this is my 90 degree minus alpha. You can see from the figure.

Now, what about the one more angle I need. This angle is measured from the equator. So, it is nothing, but phi star because this point is zenith here. So, yes, this angle is 90 degree minus phi and yes, I can write now that 90 degree minus delta this angle is equal to summation of this angle and this angle. So, I am writing this thing there and then I am proving this relationship here. So, a star is at meridian, it could be lower or upper transit I do not know. So, let us see what is that, but I am showing you different different cases; that means, right now that is S 1 which is between the zenith and the equator here. This point is equator and this is my zenith here. So, between the two stars there so, transit to occurring here.

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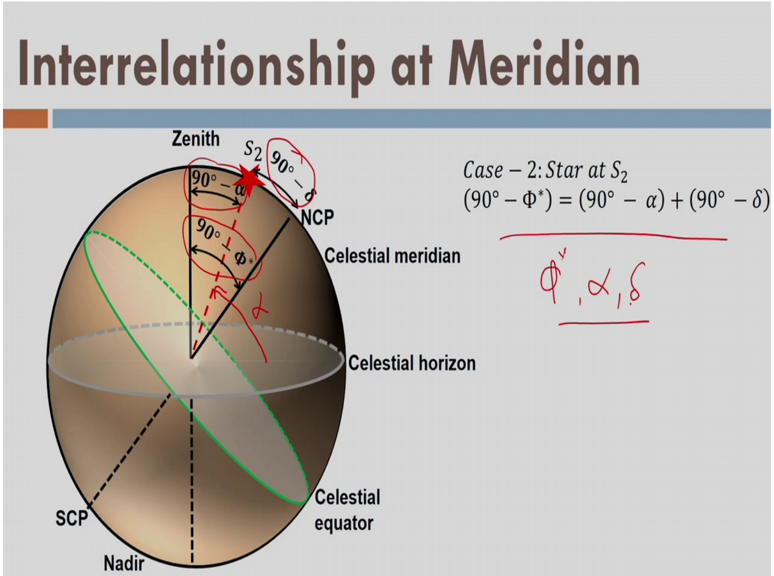

What about next one? Let us see that star is here somewhere that is between zenith and NCP. In the last slide it was between zenith and equator somewhere here now this was S 1 and now, this is S 2 fine this another location for another star. So, let us see this angle is my phi star as before, this angle is 90 degree minus phi star; this angle is my delta declination of the star, ok.

So, automatically this angle is my 90 degree minus delta. Now, one more angle we need and CBC, ok. This angle is 90 degree minus alpha why because this angle is my alpha measured from the celestial horizon the when. So, I am saying that we have different position of the star possible at meridian. So, we have taking different different stars and we are trying to predict what could be possible value of its zenith distance.

So, let us see now I can see just this angle and this angle if for their added it will be equal to this angle. So, let us write this thing, right. So, from here you can find out what is the relationship between phi star, alpha and delta. Try to find out yourself, ok.

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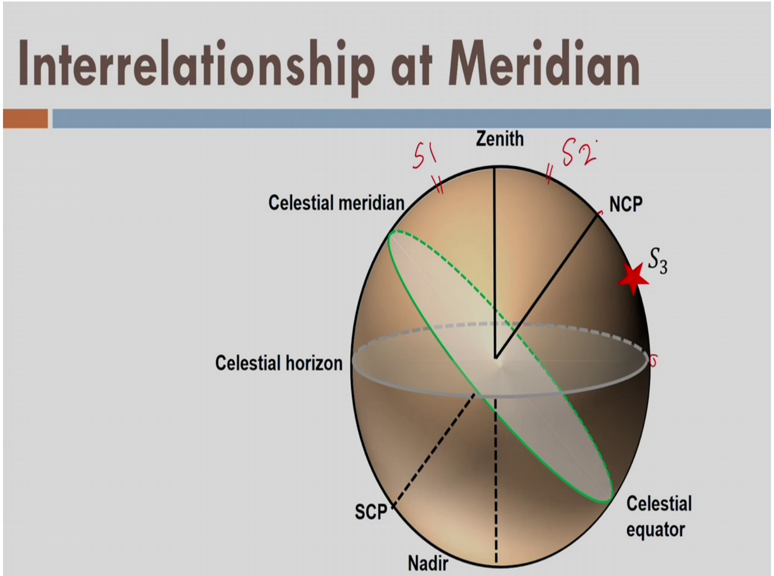

The next one is the and this is the last case we have, fine. So, let us see star is between the NCP and the horizon. So, this my horizon here and this my NCP here and star is here. So, earlier we have discussed somewhere star here and here. So, that was my S 1 case, this was my S 2 case and this is my S 3 here. So, I remove the earlier cases now, fine.

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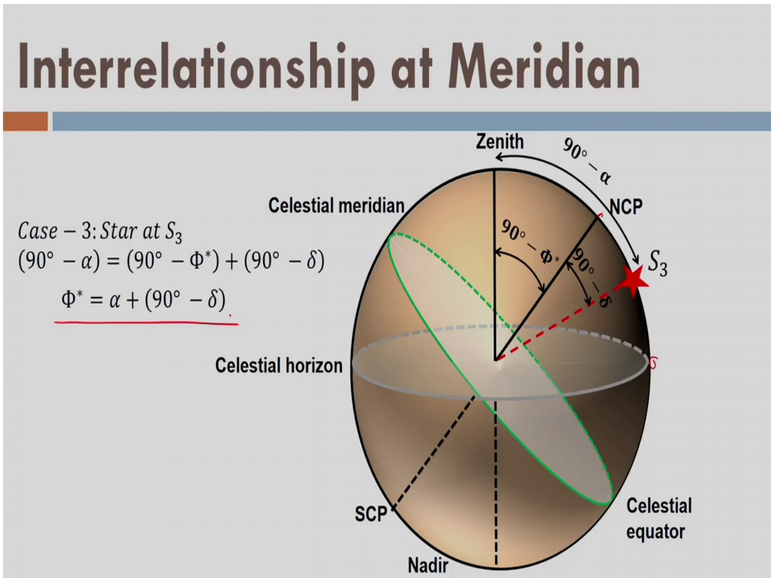

So, for the S 3 case, let us see this is the angle alpha, this is 90 minus alpha, this is delta angle, so, this is my 90 minutes delta. Now, this is my phi star again as before. So, this is my 90 degree minus phi, then you can write it that 90 degree minus alpha is equal to 90 degree minus phi star plus 90 degree minus delta. So, let us see like that. So, I can write this relationship here, we have seen all the possible relationship between alpha that is altitude azimuth capital A, hour angle H and the declination delta. We have not included the right ascension for some reason.

So, in some of the lecture in this module only we will talk about that also, but now we have realize that what are the possible cases that we can solve using spherical triangle especially in Napier's rule, in order to develop the relationship between the three reference system for astronomy, ok. In the next lecture we will talk more about the time using hour angle and till then thank you and I again will I will welcome you in the next lecture.

Thank you.