

Higher Surveying
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Module - 10
Lecture - 35
Navigation

Welcome back in the course of Higher Surveying. And today we are in our last module, module 10; that is Navigation ok. We have used this term navigation many times. For example, we say air navigation, sea navigation, rocket navigation or we say that space navigation, satellite navigation, ship navigation, road navigation and so on.

Well, what is the meaning of the navigation? Remember I gave you an example in module 2, where we discussed about a particular case of moving from one point to another point. You wanted to celebrate some event with your friends in a restaurant and you have realized that there is a very famous restaurant in your city.

However, you do not know how to reach there. In order to reach at the restaurant place what do you do? You use your Smartphone and in the Smartphone you have an application or app called map. So, in the map what do you do? You basically put the address of your place or rather the GPS is showing your location where you are currently located ok.

The next is you try to put a query where you put the name of the restaurant and you finally locate it. And then we try to say navigate. So, that by putting the navigation option, what do you have? You have the complete path of the restaurant from your home through the city right. And then you start to move on that path ok. What happens during that time? GPS is continuously marking your position on the background map of the app ok.

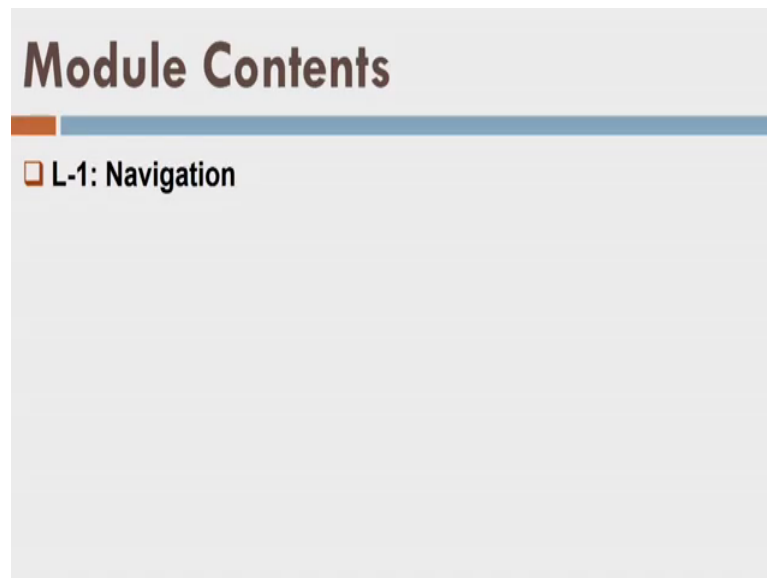
In that position marking suppose you get deviated from the course on the track, so GPS shows you where you are exactly at a given moment ok. Realizing that you are deviating away from the course that has been shown by the app to reach the restaurant, so what do you do? To use your brain or you use a control, your brain is like a control system here.

So, it controls you to go back to the course ok. So, once you get on the course, again you will start following a certain direction so that, you remain on the course. So, using this logic you reach to the restaurant fine. So, we use the word navigation here. So, what is the exact meaning of the navigation now? Is it different from the mapping or traversing?

So, in this lecture, we are going to discuss about the navigation. However, I would like to confirm here that the navigation is the science that uses most of the technologies that we have discussed before. Because, by those technologies that we have discussed, we prepared a map and ultimately navigation uses the map at the background and some kind of sensing device like GPS in order to look at your current position.

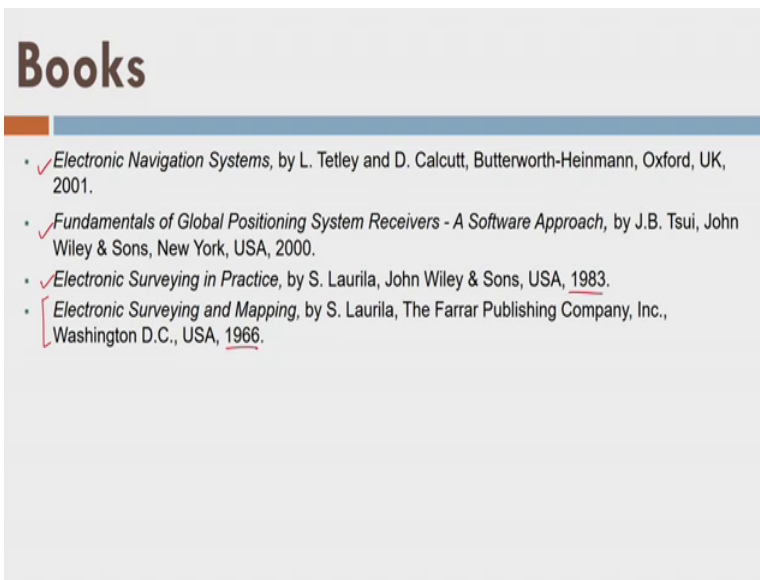
I hope I have given you enough hint now. So, let us go to the lecture on navigation. Today we have our module on navigation and it has only one lecture.

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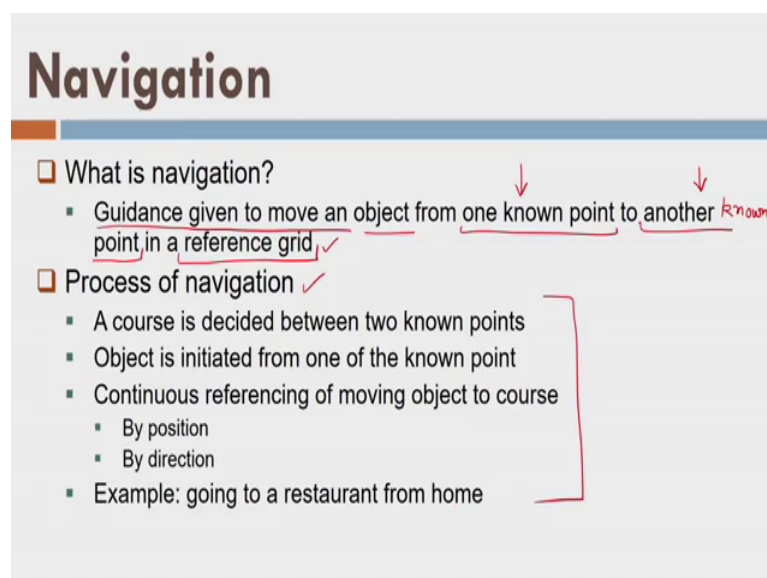
So, like this, this is my one lecture here of navigation.

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And these are the books. So, I would like to highlight here that the first book is easily available here, and the second book is also fine, it is about GPS, the third book, and the fourth book written by the same author here, there is a fourth book is little old. So, you may get this third book here, which is published in 1983. So, again I request you to go to some library; perhaps books are a little expensive here also ok.

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So, what is the navigation? I have written a formal definition over here. If you read it carefully, so it is guidance given to be move an object from one known point to another

known point right. I should say here another known point in a reference grid here right. So, what is the meaning of each and every point?

Guidance means, you are giving a guidance or controlling the object or from one known point. So, the first point is known to me as well as the next point is also known to me right. So, both points are known. Now I am giving the guidance to the object that how to move on a certain path such that, it will reach to another known point right.

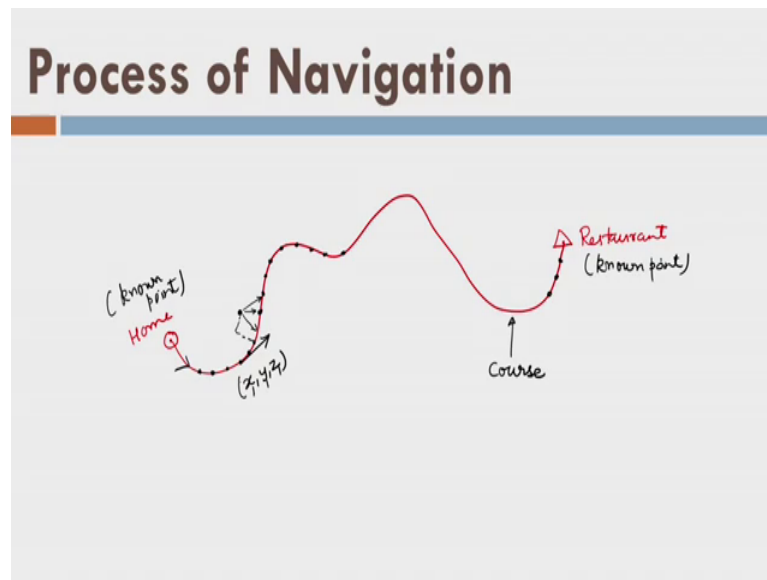
So, basically I am initiating the point on a track or on a course between the two known points. And again we are using a word reference grid. Remember the example of the module 2, where we talked about the map projection and we said that we are going to use the 2D map projection of my spherical earth; why because, the accuracies are not that critical for me here in case of navigation. However, the process of navigation is very very critical, when the accuracies are not that critical. Because, in case of for example, aircraft which is landing at the airport some error of 5 10 meters in its position can be acceptable to the pilot; so that, because pilot is using some visual devices in order to land here.

Similarly if you are reaching to a restaurant 545 meter errors also find because, you are very close to the restaurant and again you do not want a pin pointed accuracy or the pinpointed location. So, your purpose is to reach very close to that, not exactly at a certain point with centimeter level accuracy.

Similarly, if there is a ship, that is navigating from one country to another country across the sea, when it comes to the next country if the land is visible from let us say, couple of meters or a couple of kilometers it is absolutely fine right. So, this is a kind of understanding we should have that; there is a relevance of the reference grid or the 2D map projection. Then there is a one known point and there is another known point. And the moving object has been given the guidance and that is called the navigation.

So, what is the process of navigation? Here I have written the process of navigation in these steps. You can read it yourself I am going to explain it in the next slide.

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Let us say the same example of your home and you want to go to a restaurant. In the grid system or I can say the 2D map projection that is a restaurant is located here in your city ok. So, once you have the application that gives you the path from map to restaurant. Let us say it has given you some path like this ok. GPS is showing that you are somewhere here. And now, you want to reach here ok. So, what happens is, if basically starts in certain direction, let us say this direction on the path ok.

Now, GPS is showing you your correct path like, this, like this ok, if you are moving on the course. Now the moment for example, you get deviated let us say, from somewhere like this and you have somewhere here now. Let us say, so what happens is your brain works as a control system and that says you right, that immediately reach to the course here right somewhere on the course. So, you may go here or you may go here, but anyhow you will try to reach to the course ok.

Let us say again you have reached here. So, GPS is again showing you this position and you keep on continuing like this; so all these positions shown by the GPS dynamically with time. And finally, by following this path, you reach to the restaurant like this. So, all these are GPS positions, so I can say here that, your position is changing with the time ok. Some of the few important terms, this term we call the course. Remember, the restaurant is my known point and home is also the known point. So, the course can be known or can be unknown does not matter right.

But generally when you use some applications like mobile application, what happens is it gives you some path which could be shortest or maybe slightly longer than the shortest one, but it gives the some path here. And that is why I should know; what is the exact path; when I know the known points. If path is not known, well we can do it on the fly also.

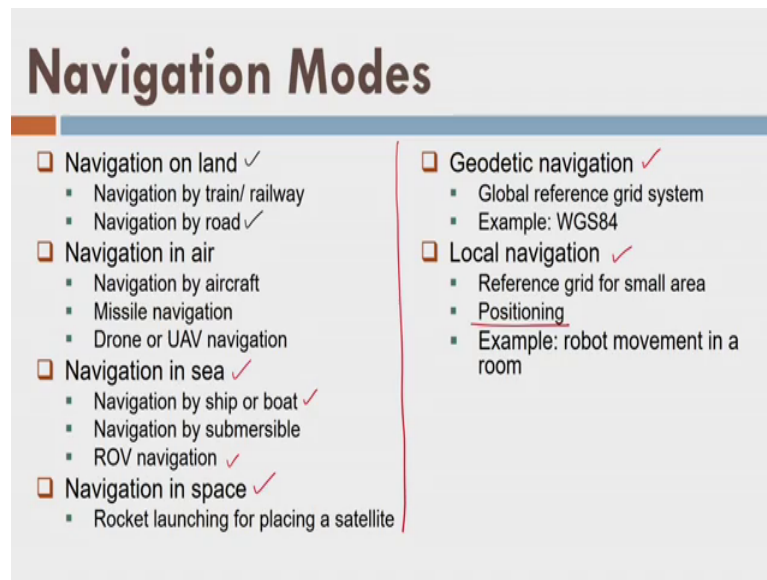
And that kind of options are also called navigation; however, we generally use the term on the fly navigation. So, there is a process here and therefore that, so in the course if you look at now what do you do basically? We place an object on the course somewhere at a known position; let us say if I place at this location let us in this position is x , y and z or call it x_1 , z_1 .

Now, from this point we are started directing or guiding or the moving object in certain direction right. Let us say could be this direction ok. So, in the next point again the direction changes. So, we keep on continuously guiding the object right. In case of human being, in the example of home to restaurant, what is the control system, your brain is control system who is guiding you ok. So, in place of human being; so in spite of brain, we are using some kind of artificial brain or we use the artificial intelligence in order to control the vehicle or the moving object fine: I hope you got the idea.

So, in the process of navigation is you place an object on a course; that means, already 2 points are known to you, you have decided a course, you place the object on a course, and at the known position. And from that known position you start guiding the object in a known direction. And from that you try to estimate the position, the next position of the object and based on that information again you guide it.

So, the process of control is also there and that is why you can see here that the navigation process requires the control system in some way. It could be an artificial control of machine or it could be a natural brain control of a human being but we need some kind of control system here. And that is why this topic of navigation is also considered in a detail by the control system subjects or the courses. Well, you can also read about navigation in the control system books.

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Now, let us go ahead. So, what are the modes of the navigation? The first mode is on the land. That means we travel by train from one known point to another known point, let us say from the Patna to Banaras. Both points are known to us and there is a track between the two places and under track, the train is moving. Well, that is a perfect example of the navigation. So, train is navigating from Patna to Banaras. Similarly, the navigation by road I have given you example of the restaurant. Then navigation in the air; Navigation in the air can be by aircraft or we are trying to send missile from one point to another point.

Remember, both points are known to me. Sometimes when we launched at the missile, you may say that one point is known from where we launched. However, another point is not known no. In fact, we know beforehand that; where is the exact point where we want to bin or we want to put the missile on. That is why again we used the track between the two points and we want a certain track that missile should follow. And that is why the word missile navigation came.

Then we have drone or UAV navigation ok. Again when we fly the UAV, when we do really do the mapping what happens? A drone is an air it moves from this way, it comes back on another track, it moves on the parallel tracks and finally, it reaches to a some known point that is a exit point. Same thing happened in case of air bond mapping. We

have an aircraft and it also moves on the parallel lines in case of LiDAR photogrammetry. Similarly we have the space navigation or the satellite navigation ok.

So, we can also see here the navigation in the space here that means, here when we launch a rocket, what do we want? We want to place 1 satellite in certain orbit So, what happens is rocket is launched that he is having the satellite ok. So once it goes in certain orbit, the rocket touches the satellite. And satellite is placed in that orbit and satellite starts moving on certain track ok. You can see here that satellite is moving or satellite is navigating.

First the rocket was navigating from point on the earth to the certain orbit point and later after the satellite starts navigating in certain orbit because, the orbit is known. So, even when we know the orbit or when we know the elliptical orbit of satellite; that means, I know each and every point. So, if I measuring the position of the satellite on the orbit and then I am trying to track it, in case of let us say GPS, the master control station tracks those GPS satellites whether, they are in moving in the design trajectory or not.

So, once they away from the trajectory, there is a control system that brings them to the trajectory. Again it allows it to move, if again they move away from the trajectory it is again brought back to the trajectory. And so, we have some kind of a is it s a path and around the design trajectory. This is another good example of the navigation right.

Now, you can understand that how control system is playing a big role here ok. Again we have the navigation in the sea, where we are having a ship or boat. In another case, we may be having a submersible that is moving under the water from one point to another point ok. What about the divers, we have some you know divers they do the underwater shooting do they know the points. In fact, they know the point from where they start and from there they will end their work.

However, if they do not really know what is the end point they taking a big risk ok. So, rarely divers do this job. They generally know from where to start and where to end all right. Then ROV navigation it is called ROV is just like UAV, UAV unmanned aerial vehicle. So, it moves in the air and ROV is remotely operated vehicle, which moves under the water ok. So, they are basically automatic vehicle, that is used in order to detect or some kind of you know search operations under the water; maybe sea or river whatever ok.

Now, there is another classification. So, other idea is the geodetic navigation and the local navigation. What is the geodetic navigation if I have a global reference grid system; that means, let us say WGS84 or I am using some map system may be some LCC projection or UTM projection, having a some global grid like WGS84 or Everest, it is called global navigation or the on the contrary we have local navigation. What is local navigation? When suppose in this room I am having a robot and I want to test my robot whether it works the way I want or not.

So what will I do? I give instructions to my robot throughout that it should move from this point to this point on a certain course or track. So once robot starts then we keep on controlling it and try to see whether it performs well or not. So in this case the map of my room is enough for me for this purpose because, I just want to test my robot.

So that is kind of local navigation or I can say in a certain area for example in your locality or colony where you staying suppose you celebrate an event. For example some kind of party or some kind of you know public event like festivals like Diwali or maybe Christmas or 31st December new year evening; so, what do you do there? You try to bring some people to say that from this point you go to this point ok.

So, in that case also your concern is all about your local place and that will be the example of an local navigation fine. I hope you got the idea ok.

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Navigation: Classification

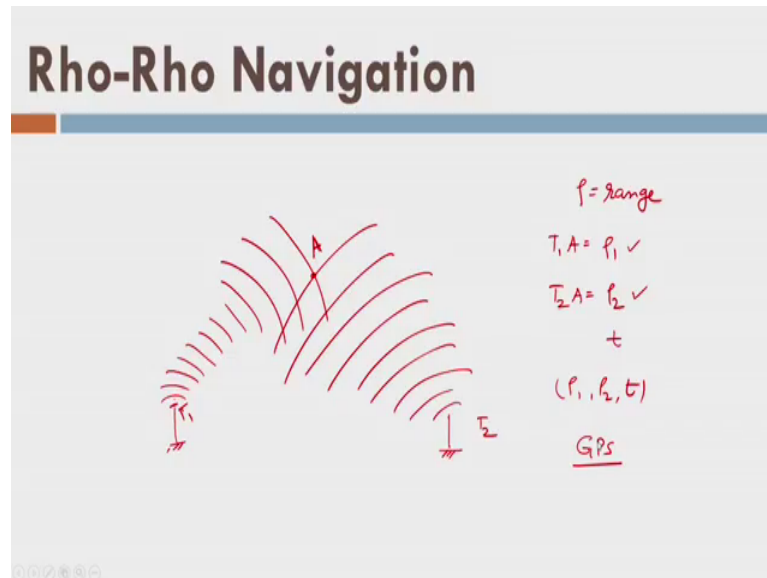
- ☐ Rho-Rho navigation
- ☐ Rho-Rho-Rho navigation
- ☐ Hyperbolic navigation

circulate navigation

✓

So, let us classify the navigation systems. So, basically we are classified in two categories; one is Rho-Rho navigation or Rho-Rho-Rho navigation or sometimes we are called the circular navigation. And then we have other mode is hyperbolic navigation all right ok. So, let us look into what is circular navigation and what is hyperbolic navigation.

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In case of Rho-Rho navigation or Rho-Rho-Rho navigation Rho basically represent their range. Remember, we use the word Rho to represent the range in the survey engineering and that is why it is called Rho-Rho navigation. So, what is the meaning here? Let us say there is a transmitter here and it is basically transmitting the signals like this, the wave fronts are like this. You can there is another transmitter here let us say T 2 and it is also sending the wave fronts or pulses.

So, now I can find out the intersection of these two pulses or the wave fronts. So, if I say that distance here from here to here from point let us say T 1 to the point A is equal to R 1 or Rho 1 and from T 2 to A is equal to Rho 2 right. And at certain time T, I am measuring it. So, using this combination of Rho 1 Rho 2 and T, I can find out the position of point A. And that is why, since I am measuring two ranges, it is called the Rho-Rho navigation. In case of 3D systems, where I am measuring 3 ranges with time, so I have 4 measurements time, 3 distance is Rho 1 Rho 2 Rho 3. So, that is called the Rho-Rho-Rho navigation all right fine.

So, now we can see here that in case of Rho-Rho navigation, we need 2 transmitter or 2 devices ok. On the other hand, I can also say that the point A where my object is there, it could also send the signals. And their signals are received by these receivers instead of transmitter now replace the receivers ok. So, both things are same, only the location of the transmitter is changing or the way we are you know finding out the location is same. However, is only the arrangement of transmission and receiving is changing.

So, both are example of Rho-Rho navigation. So, you now you can imagine that GPS is an example of the circular navigation or it is a Rho-Rho-Rho navigation fine because, we measure the distance of the receiver on the ground surface from couple of satellites all right ok.

Now, let us look into the hyperbolic navigation ok.

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Rho-Rho Navigation

- ❑ Rho: distance between transmitter and moving object (aircraft)
- ❑ Time of flight (between each of the two transmitters and airborne receiver) are measured
- ❑ Two distances from moving body to two transmitters are estimated
- ❑ Intersection of two ranges (range circles) gives the position
- ❑ Minimum two transmitters are needed
- ❑ Flexibility of selection of two transmitters
- ❑ Multiple transmitter allows for selection of station-user geometry
- ❑ Disadvantage: stable time clock, clock drift affects accuracy
- ❑ Accuracy: geometry of station and user, clock drift

So, before we go here, so the Rho is nothing, but the distance or the range between transmitter and the moving object is there or I can say that between the receiving object and the transmitting object is my distance Rho ok. So, times of flight as well as phase measurement both are possible here I have not written here. Please note the time of flight or it could be the phase difference measurement both are used in case of Rho-Rho navigation in order to look at the point ok.

So, 2 distances from moving body to 2 transmitters or 2 receivers are estimated. So, intersection of the 2 ranges basically the range circles give the position ok. Here I would like to tell you one important thing about the local navigation right. Sometimes, we also use the word positioning instead of local navigation right. Positioning does not mean the we want to determine the position only and that is why for local area navigation where position is much more important than the hiding and other things, we say that it is positioning ok.

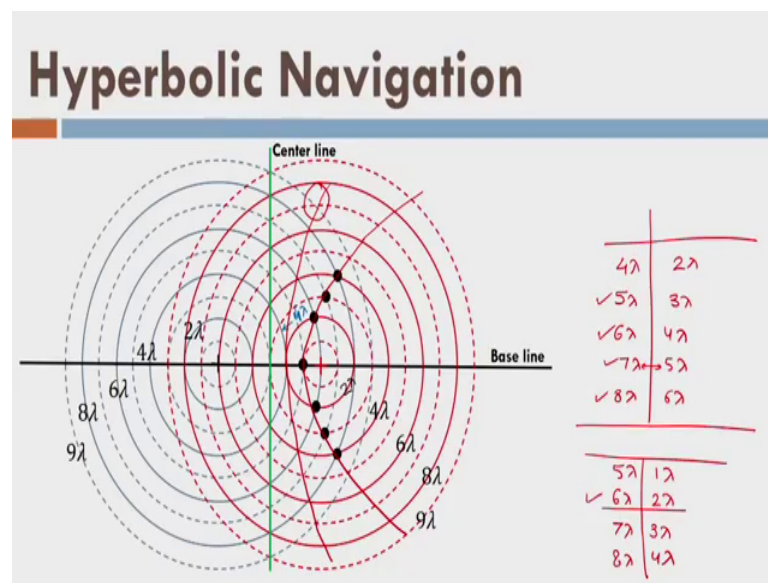
Now, as we have already said that the sometimes that along with the direction of movement, position is equally important in navigation. That is why we are determining the position of the moving object and with time ok. So now, we can see here, minimum 2 transmitters are needed or we can say minimum 2 receivers are important ok.

Now, let us say that if there are more than 2 receiving objects or which are fixed on the ground surface or there are 2 transmitters, they are fixed in the ground surface right and update is moving ok. Now imagine situation there are more than two fix objects are there which can receive or transmit on the surface of ground. What will happen? The moving object has a choice which two objects from which the signal interaction should be done and that gives the flexibility. That means, whether I take the a b or a b c or b c or c and a out of 3.

So, a moment if I take all the such objects into consideration what will happen, that accuracy of the moving object will increase. So, that is advantage here. Now you see; that what is the disadvantage that I need to have a stable time clock because, time is very very relevant if I want to measure the distances or ranges. Because, the range is measured with the help of time difference only or I can say the time difference from the transmitted signal to the received signal. So, the time of travel is basically converted to the distance or ρ . And that is why the stable clock is very much important here ok.

So, any drift in the clock affects my accuracy. Moreover accuracy is a function of geometry, geometry of the network that we are making the object on the ground surface and the object that is moving. The geometric dilution of precision concept comes from here only all right.

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Now, let us look into the hyperbolic navigation. Let us say, there are 2 transmitters shown by the black cross and the red cross or shown by the red plus sign and the black plus sign here fine. What happens is as we can see here that transmitter at black plus sign is radiating some energy or some power or some sending some signals. So, these are the wave fronts like this. So, these wave fronts are you know moving in the space and with different-different you know the multiples of wavelength. So, as it increases the distance or other it reaches to a larger distance it has covered some multiples of your wavelength right So, let us 2λ , 3λ , 4λ , 5λ , 6λ and so on ok.

Now, we can see here that these are the wave fronts created by the transmitter shown by the black plus color ok. Now similarly this is a red plus sign transmitter, so it is also sending the pulses like this right, so we can see that a complete net ok. By the interaction of these pulses, we can find out the position or the intersection point. That means, if my object is moving on the certain point I can find out this location by the solution of the two such circles but should I call it the circular method. No because, there is small gap here what is the gap, that a gap of the understanding. So, let us look into that.

So, let us see there is a line here which is connecting 2 points and now if you see this line is called the base line ok. Similarly if there is a line here, that is a central line where I can say that from the center line each of the transmitter are at same distance, which means if the 2 transmitters are sending the pulses at the same time, what will happen.

They will have equal range of the or equal diameter of the wave fronts at the center line ok.

However, if we place the object other than the center line or base line what will happen. Let us look into this thing. Let us say this point ok. So, what is the point here, if we just look here it is 2 lambda basically here this is 2 lambda So, I have placed the point at the 2 lambda and here if you look at it is basically the 4 lambda. So, this circle is the four lambda here ok. I am using the same color, so as to avoid any confusion.

Now, you can see that there is a difference of 2 lambda and 4 lambda which is equal to 2 lambda ok. So, let me just write it here that on this side we have 2 lambda circle, and on the side we have 4 lambda circle right. Now I can see here one thing. What if I maintain the same difference of this 2 lambda right and if I select the point let us say 5 lambda and 3 lambda, 6 lambda, 4 lambda and let us say 7 lambda 50, lambda you can see the difference between the 2 values is equal to the 2 lambda always.

So, we have that 8 lambda and 6 lambda; so what will I get? Just choose those points. So, this is these 2 points 5 lambda, 3 lambda intersection then we have two points which are this. Then we have two more points, two more points and ultimately this one.

So, we have than right. So, you can see here very carefully that, this path if I join this path like this and it is nothing, but the hyperbola. And that is why if my points in which are shown by the black dots if the moving object is there and it is receiving the signals from both of these transmitter, let us say at this point what will happen or it is on the hyperbolic red curve or hyper red is itself is hyperbola.

So, now if I want to find out the location of that moving object at that instant what happens is I need to solve the hyperbola? And that is why this kind of methods or with the methods which are based on this logic are called hyperbolic navigation methods ok. I hope that you got the idea ok.

Let us say one more example fine. Let us say, we take another point like this ok. So, this is I can say here that 5 lambda from left side and 1 lambda from the second side. So, now, we say that there is a difference of 4 lambda So, let us maintain this difference and write the terms 7 lambda, 3 lambda, 8 lambda and 4 lambda ok. If I try to plot these points these points will be like this. So, these 2 points are corresponding to this one, 6

lambda and 2 lambda circles. These two points are intersection of these 7 lambda and 3 lambda circles and these two points are the intersection of 8 lambda and 4 lambdas circles right. So, if I draw us curve through these points it is another parabola like this.

Now, I hope you can take absolutely correct idea what is the hyperbolic navigation. And it is very very relevant in many terms. First of all it is not necessary that we will always have the position of the object on the center line. And hence whenever the object is away from the center line which is used to happen we have the hyperbolic navigation very very relevant and we can use it ok.

So, now we can see here that we said that both transmitters are sending the signals. Now we can see one thing here. First of all both transmitters are sending the signal at the same time, which means the signals will meet at the center line. However, if one transmitters is slightly delayed and it is sending the signal with slight delay; what will happen? It will meet somewhere on the hyperbolic path got it. So, you I hope that you got the idea, what is the meaning of the sending the signal at the same time or maybe at different time. So, at the time of different time automatically they will be meeting at the hyperbola.

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Hyperbolic Navigation

- ❑ Position is determined by difference measurements of signals
 - Minimum 3 transmitter stations
 - If signals are transmitted at same time, equidistant intersection at centreline
- ❑ If signals are received at off center point – *hyperbolic navigation*
 - Measured difference in time (or phase) ✓
 - Points of same difference defines a hyperbola
 - Each hyperbola is a line of position (LOP)
 - Each hyperbola represents a constant difference of range from transmitters
 - Gradient increases from baseline
- ❑ Accuracy: inferior away from baseline (higher grid spacing)
- ❑ Disadvantage: minimum two LOPs (three stations) required

In case of hyperbolic navigation position is determined by the difference measurement of the signals right. So, minimum we need 3 transmitters stations why because: because of the two transmitters we will not get the correct position, we need to have one more transmitter right. Although, the demonstration was good enough to show you but, in fact,

when we do the calculations, we need to have minimum 3 transmitters. If signals are transmitted at the same time, equidistance intersection at the center line are possible. What is the signals are received at of the center point? So we have the hyperbolic here solutions here in this case ok.

Now, we can see that the we can measure the difference in time or phase points on these same difference defines a hyperbola as we said at and each hyperbola is line of position or lop right, because we are determining the position here. So, each hyperbola represents a constant difference of range from the transmitters. So, gradient increases from the baseline, as we go away on the hyperbola from baseline the gradient of the hyperbola increases. As we have seen the 2 examples, the 2 hyperbolas are having different gradients. And as we go away from the central line accuracy degrades that is the reason, because the spacing between the circles or the we can say points on the hyperbola increasing right.

So, that is somehow a disadvantage of the hyperbolic navigation. So, we can say here that we need to have the minimum 2 LOP's. That means, if you have 3 transmitters I can construct minimum 2A line of positions and by solving these 2 line of positions I can find out the position of A point.

So, in case of Rho-Rho navigation if you look at we need only two transmitters or two reference objects on the surface of earth, here we need minimum 3, alright. Now you understand; what are the major idea about the navigation methodologies, which is one is hyperbolic and one is circular. So, in case of circular, we call it Rho-Rho or Rho-Rho-Rho depending on the dimensions we use.

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Position Update in Navigation

- Concept of updating a position
- Observe a constant distance n times ($x_1, x_2, x_3, \dots, x_n$)
- You observe one more data: x_{n+1}

$$\begin{aligned}\bar{x}_{old} &= \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \\ \bar{x}_{new} &= \left[\frac{x_1 + x_2 + x_3 + \dots + x_n + x_{n+1}}{n+1} \right] = \left[\left(\frac{x_1 + x_2 + \dots + x_n}{n} \right) + \frac{x_{n+1}}{n+1} \right] \\ \bar{x}_{new} &= \left[\left(\frac{x_1 + x_2 + \dots + x_n}{n} \right) \left(\frac{n}{n+1} \right) + \left(\frac{x_{n+1}}{n+1} \right) \right] = \bar{x}_{old} \left(\frac{n}{n+1} \right) + \left(\frac{x_{n+1}}{n+1} \right)\end{aligned}$$

Now, as I told that what is happening here, the position of the vehicle or the moving object is changing. And I need to measure it why because, if it is away from the design trajectory, I need to put a correction. So, what is required here? I need to measure the position with some sensor ok. At the same time I need to calculate the errors also that about which what is the error likely error in the position, and that will tell me whether the moving object is away from the designed course or designed trajectory or not.

And if it is away, I need to make some corrections and the amount of correction will be depending on the amount of error or amount of the ambiguity in the position of the moving object. And that is a logic here about the position update in navigation ok. So, let us look into this aspect, it is very very important here and I will give you some simple example.

Let us say you have a constant and you want to measure that constant. Let us say distance is a constant beta 2 points and you are measuring that distance using a device. Let us say tape or total station whatever and you are getting some readings like x_1, x_2 and so on. So, n number of observations you have. And you know that, the constant distance or the constant value should be constant. And hence, we can develop an idea that each and every observation is having some error right. And if you remember, we take the mean $\bar{x}$ is equal to x_1 plus x_2 plus x_3 and so on up to x_n divided by n .

Now, let us imagine that you have one more data, let us say x_{n+1} . So, what is the new mean ok, so we can write here that let us say the new mean will be again x_1 plus x_2 plus x_3 and so on up to x_n plus x_{n+1} divided by $n+1$. Ok, here I would like to rewrite this thing, let us say that x_1 to x_n divided by $n+1$ plus x_{n+1} divided by $n+1$ here ok.

Now, we can see here that this term can again be rewritten as I am rewriting this term as let us say x_1 plus x_2 plus x_n divided by n . Again I am multiplying with n and divided by $n+1$ here. So, I hope that this term here and this term are same and so I am writing here the next term x_{n+1} divided by $n+1$ ok. What is this term here you see? This term is nothing but my $\bar{x}$ or I can say that $\bar{x}_{old}$ or my this is my $\bar{x}_{old}$ here let us say ok. So, I can $\bar{x}_{old}$ into n divided by $n+1$ plus x_{n+1} divided by $n+1$ here. You see what is the advantage here; what is the magic here.

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Position Update in Navigation

$$\bar{x}_{new} = \left[\bar{x}_{old} \right] \left(\frac{n}{n+1} \right) + \left[\frac{x_{n+1}}{n+1} \right] \checkmark$$

$$\bar{x}_{new} = \bar{x}_{old} + \left[\frac{x_{n+1} - \bar{x}_{old}}{n+1} \right]$$

observe n data $\Rightarrow \bar{x}_{old}$,
 you observe 1 more number of more data $\Rightarrow \bar{x}_{new}$

Let us say that $\bar{x}_{new}$ or rather $\bar{x}$ new is equal to $\bar{x}$ old, that is my old mean into n divided by $n+1$ plus x_{n+1} divided by $n+1$.

So, why what I am doing here? I am just doing 1 job here. I am trying to update my old mean by this multiplying factor and by this additional term. That means, I am not doing any new calculation anymore, I am just putting this factor multiplying by this one and some additional factor here, I am adding it here right. So, this is my completely new observation and I am updating my old mean and I am getting my new mean. And that is

what is we call the position update. Here remember, in case of the position of a point or the distance between the 2 points is not changing dynamically, but it is a good example to understand ok.

Now, imagine that the distance between the 2 points is also changing dynamically with time. So, I need to measure the time also in order to estimate how my distance is changing or I can say one thing here; as my moving object is moving in the space or air or maybe in the water or maybe in the road, I need to measure the time also. In order to find out its updated position using this logic; that means, at every position for the new position update, what will I do? I will not consider the complete data set rather, I will consider only the conclusion or they mean of the old data set. And I will try to update that mean in order to estimate my new position. That is the beauty of this position update logic ok.

So, what is advantage here? First of all whatever data you have collected before and once you find out the mean, you can ignore this complete data; that means, you are saving the memory of your system. Number 2: in the next time when you do the calculation of your updated mean, once you get one more data. What will happen here, you are just considering the mean value and your new data only.

Let us say this is the equation ok, you can see the equation. Here we are using a new position x_{n+1} and you are using only the mean not the old data. So, you are saving your calculations also; that means, your calculation time also. So, that is the big advantage here and somehow based on this logic only the Kalman filter is developed. Well, which is slightly advanced topic here but still we try to get the gist of what is Kalman filter here.

So, now you got the idea. What is the position update here? Now I can rewrite the whole thing as this complete term I can also write rewrite the same equation here once again like, $\bar{x}_{new}$ as $\bar{x}_{old}$ plus I have just rewriting the things here $x_{n+1} - \bar{x}_{old}$ divided by $n+1$. So, I have just again rearrange the same equation here and I find write this one.

Now, we can see here very clearly that, you are updating your old mean by this factor ok. The concept of updation is like that and here you see that you are putting some kind of

correction to the old mean using the new data right and you are using some kind of multiplying factor or dividing factor and you are trying to find out your new mean right.

Now, imagine that you have a situation, where you have observed n data and find out your $\bar{x}$ old ok. Now, you observe p more number of data or p number of more data right. Try to find out what is your $\bar{x}$ new. Now, imagine a dynamic system. In case of dynamic system let us say there is a point here that my position is changing dynamically with time ok.

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Position Update in Navigation

$$\vec{x}_k = \begin{bmatrix} p_k \\ v_k \end{bmatrix} \Rightarrow \Sigma = \begin{bmatrix} \sigma_p^2 & \sigma_{pv} \\ \sigma_{vp} & \sigma_v^2 \end{bmatrix}$$

$$\vec{x}_{k+1} = \begin{bmatrix} p_{k+1} \\ v_{k+1} \end{bmatrix} \quad \begin{aligned} p_{k+1} &= p_k + \Delta t v_k \\ v_{k+1} &= 0 + v_k \end{aligned}$$

$$\Rightarrow \vec{x}_{k+1} = \begin{bmatrix} p_{k+1} \\ v_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}}_{\Phi_k} \begin{bmatrix} p_k \\ v_k \end{bmatrix} \Rightarrow \vec{x}_{k+1} = \Phi_k \vec{x}_k$$

Error propagation
 $\Sigma_{k+1} = \Phi_k \Sigma_k \Phi_k^T +$

So, let me write in state vector which consists of both position and the velocity. So, I write like this. My state vector at k-th position or k-th timestamp is the position p_k and the velocity v_k here, got it ok.

Now, what is my error in this position and the velocity? Let us say they are coming from this covariance variance matrix, which I say is σ_p^2 σ_{pv} σ_v^2 here. So, these are the errors or we can say covariance variance matrix of my p_k and v_k . And now I am measuring also am right. So, using the measurement process I am introducing some error here. That is the idea here ok.

So, what next? I want to estimate my x_{k+1} . So, that is something about p_{k+1} and v_{k+1} ok. The thing here is I can write from the standard dynamic equations or the kinematic equations, the position is updated from my old position plus Δt time

speed V_k fine. That is a Δt is nothing, but the time difference. So, I can write this thing. Similarly what about the V_{k+1} right, there is no update as such, so I am saying that V_k or my speed is constant fine; so how can I write this thing in the matrix form? I can write thing here like this, my x_{k+1} , the state vector.

So, this is my state vector at time $t+1$ or $k+1$ and this is also state vector at time stamp k or position k ok. So, x_{k+1} is nothing, but $p_{k+1} V_{k+1}$ here and it is equal to the: we can say $1 \Delta t 0 1 p_k V_k$. I hope you got the idea, how what do I write it. Now can I write it like this, that my x_{k+1} state vector is some function called ϕ into x_k vector and this is nothing, but the ϕ .

Now, what is the error that I expect in my x_{k+1} right. So, I am saying that the error is calculated by law of error propagation. We know that the error in the k -th position is this and so I expect the error in the k -th position as σ , I can write let us say $k+1$ is equal to $\phi \sigma \phi^T$ ok. So, if I that it is define at the k -th state, so that is fine here right. So, this is the idea here ok. So, this is my ϕ which is defined at k -th step then it becomes my t_k here, that is the time difference between t_{k+1} and t_k right.

So, using this logic we can find out what will be the error in my $k+1$ position or the position at $k+1$ time stamp ok. Now this is the way we keep on updating the position ok. Now if errors are higher what will I do? I will take this small step of the movement. Now if errors are smaller, I can take a big step in the direction of movement, in the direction of velocity right. So, this is the way we do the updation in my position and somehow this logic is related to the Kalman filter ok.

So, if someone is more interested in Kalman filter they can read it themselves. One more thing I would like to tell you here. If you try to control your system now what will happen because of the control of your moving object, some errors from the control parameters will also be introduced. Error will be added to this thing somewhere here right and that is what we call the process noise ok. So, that process noise will also decide your overall error of the position. And based on that, you will try to update your position. So, some higher the error, small update in the position smaller the error you try to update your position faster or at the higher rate.

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Navigation Systems

- ❑ Dead reckoning system
 - Speed sensor
 - Heading sensor: magnetic compass, gyroscope, gyrocompass
- ❑ Acoustic navigation system (used under sea)
 - Long base line system, short base line system
- ❑ Inertial navigation system
 - Accelerometer and gyroscope
- ❑ Radio navigation system
 - Circular and hyperbolic systems (examples: LORAN, Omega, GEE)
 - Instrument landing system (ILS)
 - Satellite navigation system (GNSS)

Now, let us consider different-different navigation systems ok. So, first of all there is a dead reckoning system. It consists of speed sensor and heading sensor. Heading sensor could be a compass like magnetic compass it could not be gyroscope or gyrocompasses.

Now, acoustic navigation system ok, it is used under the water because acoustic waves are travelling in the water. Now we have long based line system and short baseline system. So, I will explain all these systems very briefly fine. And then we have the inertial navigation system, which is having accelerometer and gyroscope. Then we have the radio navigation systems, where we have circular as well as hyperbolic systems and instrument landing systems are there. And then we have satellite navigation system, which are radio navigation system only and what we call GNSS or GPS.

So, I am not going to consider the GPS here, rest of the things I will consider. Moreover I would like to tell you in the radio navigation system, all the systems now are superseded by the GPS So, that LORAN, Omega are no more relevant today. Still in order to understand the philosophy I will discuss ok.

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Acoustic Navigation System

- ❑ Acoustic frequency (5-20 KHz)
 - Speed sensor ✓
 - Heading sensor: magnetic compass, gyroscope, gyrocompass
- ❑ Position of determined with respect to transducers
 - Transducers are mounted to a ~~fixed structure~~ *moving structure*
- ❑ TOF or phase based measurements
 - Speed of sound (and sound profile) is known
- ❑ Local navigation system
 - Area of coverage is defined around transducers
- ❑ Accuracy of positioning
 - Depends upon vehicle net geometry
 - Errors are small, statistically constant, smaller temporal component, larger spatial component

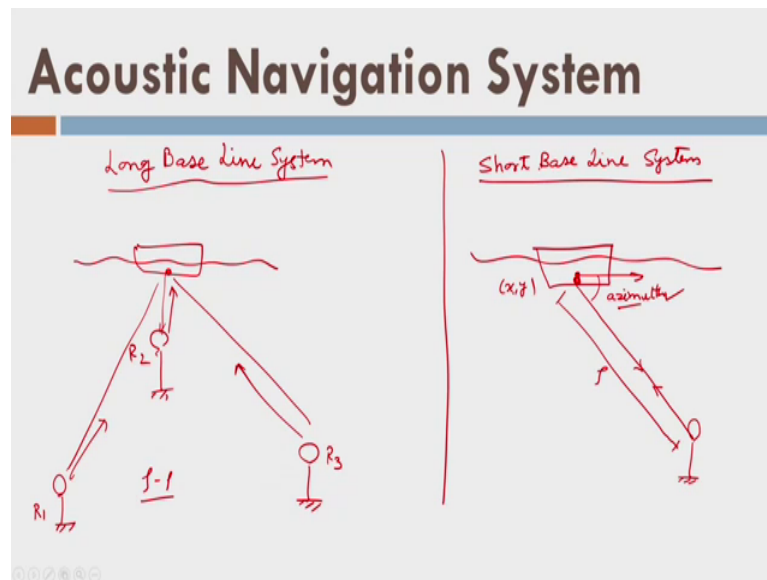
So, let us look into the acoustic navigation system. Generally, we use the frequency in this range 5 to 20 kilohertz of the sound waves. And then it has a speed sensor as well as heading sensor ok. Position is determined with respect to the transducers here or transducers are basically mounted on a moving structure, I should say not here it is a on a moving structure here ok.

And then the time of flight or the phase based measurements are performed. That means, using the speed of the sound, we try to measure the time of travel or sound profile right. That means we assume here that sound profile in the sea water is known to me ok.

So, basically it is example of local navigation system. We will explain you why. Area of coverage is defined around the transducers or basically the reflectors also. So, what is the accuracy of positioning? Well it depends on the vehicle net geometry or the GDOP geometric dilution precision at a given instant. Errors are small, statistically constant smaller temporal component but larger spatial component; what is the meaning here? Errors are here very very small, but their spatial component is bigger compared to the temporal component, bits with time errors are not increasing that much, but at a given moment errors in the space are much larger compare to the temporal component.

Moreover, as we see over the time, the errors are consistent they are not changing much and that is why we say that statistically constant errors are there ok.

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So, here we can see here that we have 2 system, one is called the long baseline system. So, in case of long baseline system; what happens, let us say this is a sea water here and there is a moving vehicle and moving vehicle have a transducer over here and that is sending the pulses in all the directions.

Now let us say there are 3 receivers are there, let us say R_1 , R_2 and R_3 , which are firmly based on the sea bed. So, the as soon as this moving vehicle comes near to that, they will start interacting with this thing and they will reflect back the energy like this and this, so like this and like this fine.

Again the ship hull or the ship is having receiver also that will receive the reflected energy. And using that time of travel or the phase measurement we can find out the position of the moving vehicle right. And that is the long baseline system or now we can see here it is nothing but Rho-Rho system because, it is measuring the distances between the receiving object and the transducer ok.

Now in we have another system called short baseline system. In case of short baseline system, we can say the ship hull is there, ship hull is having a transducer here and when we send the pulse that the azimuth of the pulse is known fine. So, that is a moving direction here.

And now let us see there is a reflecting surface over there. So, it will reflect back the energy. So, based on this range and this azimuth angle, we try to find out the location x y at a given time stamp. And that is called the short baseline system.

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Inertial Navigation System

- ❑ INS measures
 - ✓ Position: longitude, latitude, and depth
 - ✓ Velocity: velocity relative to Earth
 - ✓ Attitude: attitude around three axes
- ❑ Uses gyros and accelerometers to sense motion in inertial reference frame
- ❑ Initial alignment should be performed
- ❑ Errors
 - Accumulate, if vehicle moves in a straight line for long distance

Now, we have inertial navigation system. In case of inertial navigation system which is completely independent of any transmitter or receiver right. So, it measures the position, it measures the velocity and it also measures the attitude or ω ν κ with respect to 3 axes ok.

So, now in case of position we say that longitude, latitude and the depth along the gravity direction. Remember, we have said all this thing in our LiDAR module ok. So, it basically uses the gyros and accelerometers to sense the motion in inertial reference frame. Remember, it measures in the inertial reference frame, which is quite independent from any other non inertial reference frame and that is why INSS is today underused very frequently ok.

So, the initial element is should be performed and then errors basically errors accumulate over time. So, what is the needed here? We need to change the direction ok. Suppose it is mounted on an aircraft, so aircraft turns automatically directions change. And as a result error does not accumulate right. These are the things one should keep in mind when you use the INS.

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LORAN – C

- ❑ Hyperbolic navigation system
- ❑ Uses ground waves at low frequencies (90-110 KHz)
- ❑ Signals are transmitted by permanent transmitter network
- ❑ Operating range: 600-1500 nautical miles (nm)
- ❑ Absolute accuracy: 463 m (0.25 nm)
- ❑ Repeatable and relative accuracy: 18-90 m

Let us see another system LORAN C. Remember LORAN C is a system that is used in order to locate the ship vessels or any other object ok. What happens here is the transmitters are basically based on the ground surface, not only object ok. So, it is example of an hyperbolic navigation system and it uses the ground waves. That means transmitted pulses are interacting very close to the ground ok.

And at very low frequencies that higher wavelength ok. The signals which are electromagnetic signals here remember are transmitted by the permanent transmitter network and the operating range is in this range, 600 to 1500 nautical miles or we will also write it nm ok. So, that absolute accuracy is 463 meter or you can say close to 500 meters, but fine. These accuracies are absolutely fine for the navigation of the big vessels or big aircrafts ok.

Now, we can see that repeatable and relative accuracy. If I try to take the repeated observations like accuracy will be improved up to this level 18 to 90 meter ok. Now you see that there is one more system called omega here and again it is an example of hyperbolic navigation system. It uses the sky waves. Remember, the purpose of sky waves is to increase the operating range or the operating distance.

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OMEGA

- ❑ Hyperbolic navigation system
- ❑ Uses sky waves at low frequencies (10-14 KHz)
- ❑ Operating range: 5000 nautical miles (nm)
- ❑ Accuracy: 2 to 4 nm
- ❑ Differential Omega
 - ❑ Standard signal received by airborne unit with differential corrections
 - ❑ Accuracy: 0.2 to 0.3 nm

So, again it is using in the very low frequency range, the electromagnetic signals ok. In operating range is 5000 nautical miles and accuracy is 2 to 4 nautical miles ok. Now we can say that there are some differential omega is also there; just like differential GPS ok. So, standard signal received by the airborne unit with differential corrections and then accuracy goes to the almost 0.2 to 0.3 nanometers here all right.

So, we can see here that how things are improving. They are basically now very old. So now, the GPS is there I am not going to discuss GPS anymore.

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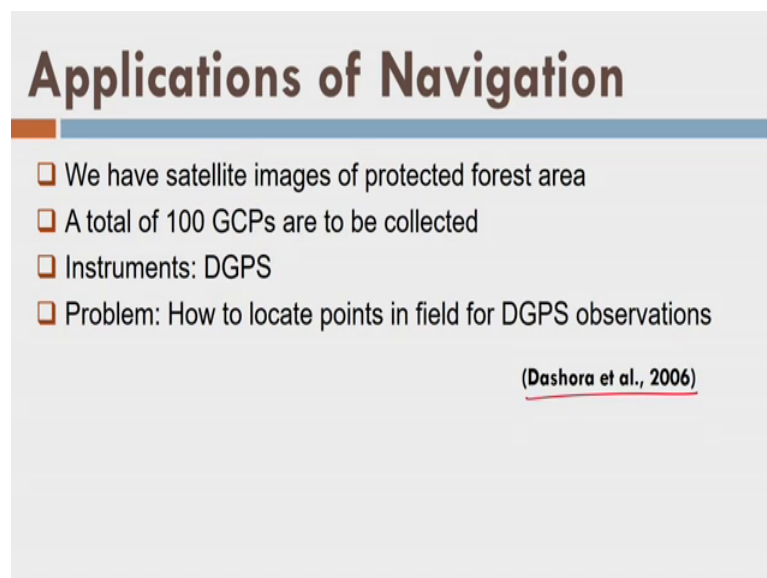
Mobile Navigation Software



What about the mobile navigation software? Now I would like to tell you something here. How is it different from the standard map system, but in case of mobile system you have the map at the background and map is showing you some path from the home to a restaurant, alright.

If you have an palmtop devices of the for the navigation, they will also show you where to turn, where to stop, whether traffic is too line or one line whether traffic is not allowed to take the overtake or not. So, all these informations are available and those informations are to be first collected and then your navigation software has a provision to upload those information on the background map. And that is why the mobile navigation softwares are completely different from the standard maps ok. So, that is the difference.

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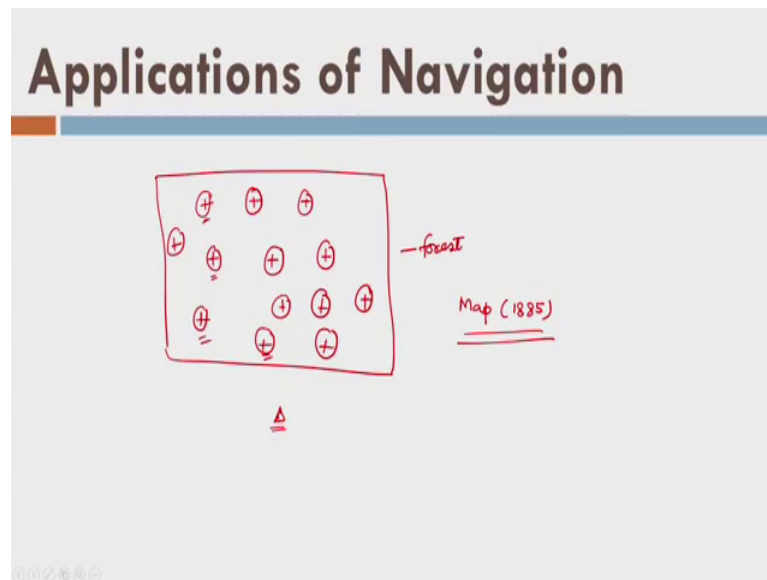
Applications of Navigation

- ❑ We have satellite images of protected forest area
- ❑ A total of 100 GCPs are to be collected
- ❑ Instruments: DGPS
- ❑ Problem: How to locate points in field for DGPS observations

(Dashora et al., 2006)

If you see what are the applications of navigation, I would like to request you please go through this paper here. And it is well explain that how can use the GPS navigation in order to find out some completely unknown places that you want to go.

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That means, in this study what has been done here. Let us say you have an satellite image like this of a certain area. And then you find out there are few points that you want to go and collect for your photogrammetry right. So, that you can correct the satellite image.

Now, you know that it is a forest area. And in this forest area, you would like to go to those points. Problem is there you are not never being to this forest area and completely unknown place. You try to arrange a map and map is very very old; for example, let us say 1885 very old a map. Now by the map we will come to know various villages around that forest and various other places right.

So, using the 1 to 1 correlation between the features drawn in the map as well as the features in this image; so you have to do a lot of homework here and then during the planning you find out So, this is the particular village and that is visible in the satellite image also because, there are few settlements right ok. And those settlements are also visible in the map as well as image.

Now, we can see we can correlate it slightly fine. On the image you have also identified your GCP's that these are the locations I want to acquire from my GPS device let us say GGPS ok. But the problem is how to reach there ok. So, if you take the map, you will not understand the map that easily, but if you take the image you have clear understanding of the features also right. So, what you do? You first which register your image in the map

coordinate system using geo referencing process and in that geo referencing process what will happen, you will get the coordinates of these points ok.

So, since these points are visible both in the image as well as map, so first you collect the data of these points from the map. Try to put on the image and try to geo reference your image ok. These points could be having some error, which are given by the map. No problem; let us say you have a error of for example, 50 meters no problem.

Now, you know that this is the map, now you have converted this map from the let us say map projection system to WGS84 projection system all right. So now, you know that you have the coordinates of all these ground control points in WGS84 system ok. Now you take your GPS device ok, feed those coordinates here and try to navigate from some location, let us say this location which is near to the forest area ok.

Let us say this could be a railway station or this could be a bus stop where you try to reach near to the forest and from this point you will start navigating to reach to these points. The moment you are very very close to the point let us say 50 meters or so because, you have the image what will happen? Using the image identification or the features you collect from image you will try to see when you are very close to the point. And GPS device is telling you how close you are to that point. And you know that the error of this GCP's that we estimate it from the map. Let us say it is 50 meters ok.

So, GPS is bringing you very close to the point around 50 meter range. And from that 50 meter range, now you use your perception and try to find out where the point is exactly located. And that is done with the help of image, as well as map because, image has some certain features showing you that the particular point is having some features around that in the field. And using that logic your perception, you will reach to the point and using the GPS and that is very beautiful application of the navigation ok.

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References and Bibliography

- Dashora, A., Sreenivas, B., Lohani, B. Malik, J.N. and Shah. A.A., 2006. GCP collection for CORONA satellite photographs: Issues and methodology, *Journal of Indian Society of Remote Sensing*, 34(2) , pp 153-160.

So, here you can read this paper. The complete details are there and you can get benefited from this approach ok. So, that is an excellent example of how to use the GPS for the navigation to some unknown places ok. I am saying unknown places physically, but in fact, beforehand we know; what is the place, where I am right now and what is the GCP location. So, using these 2 locations, we are deciding the navigation path all right.

So, here we stop our lecture. And in the next lecture, we are going to conclude all our 35 lectures or the 10 modules that what we have learned in this course. Moreover, we will also try to find out whether I have solved your problems that we have raised in the lecture 1 module 1, alright.

Thank you.