

Higher Surveying
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Module - 8
Radar (Radar Grammetry)
Lecture-30
Radargrammetry

Hello everyone. Welcome back in the course of Higher Surveying. Today we are in 4th lecture of module 8 ok. In the last lecture of Radargrammetry, we have really talked about the how to obtain the topographic information using the RADAR images. So, in the lecture 1 and 2 we have learnt that so, what are the geometric aspects and radiometric aspects of the RADAR image ok. In the third lecture we learned that how to use it to create the 3D model and that is why we said the name of the particular lecture as radargrammetry.

Today, we are going to talk about interferometry or RADAR interferometry. Yes, what is the connection of the interferometry and RADAR ok? You can recall here, the simple concept of inference right. So, if you remember the 11th or 12th standard knowledge, there we say that if there is a constructive interference; that means, if there are two waves in a same phase, you will have constructive interference ok. If they are apart from π or the phase difference is apart from the π or half of the wavelength, then we will have destructive interference. Also in our experimental classes of 11th and 12th standard, we have seen these interference phenomena for the optical waves or optical light. I hope that you can recall those things again.

Well, here the context is similar, but not the same ok. First of all just imagine just recall again the concept of RADAR that RADAR image is a complex image; that means, it has quadrature component at it has phase component and finally, using quadrature and the phase component we calculate the magnitude fine. So, that was very clear. Now instead of using the quadrature component, we want to use the phase component of the RADAR image in order to measure the height of a point or maybe a changed detection fine; that means, if the height of a point or the position of a point changes still I can use the interferometry ok. So, in this lecture we are going to talk about all these aspect. First we will discuss about the technical perspective of the interferometry and how is it used in

order to measure the height or the range difference fine. So, let us go ahead and delve into this lecture.

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Module Contents	
□	L-1: Imaging RADAR fundamentals: Geometric aspects
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□	L-4: Imaging RADAR interferometry
□	L-5: Geoscience perspective for RADAR applications

So, this is our fourth lecture and again books are same fine.

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Introduction	
□	Imaging RADARs record <u>complex images</u>
□	Interferometry ▪ Records phase ✓ ▪ Calculates interferogram ✓ ▪ Derives <u>minute differences</u> in the <u>range measurement</u>
□	SAR interferometry delivers] ▪ High resolution topographic maps ✓ ▪ Velocity mapping ✓ ▪ Change detection

So, let us introduce what is the interferometry or RADAR interferometry ok. As I told that RADAR records the complex images. So, in case of interferometry, what do we do? We do record the phase and using the phase we calculate something called interferogram that we are going to discuss in this lecture as we proceed fine. So, that interferogram is

then used to derive the minute differences in the range measurement ok. So, that is a logic here ok.

So, now ultimately, we can find out that the SAR interferometry can deliver high resolution topographic maps. It can be used for velocity mapping and it can also be used for the change detection and we are going to learn all these thing in this lecture.

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Phase of RADAR

- RADAR backscatters are coherent
 - Same phase for a sensor for a point on terrain
 - Two different sensors have different coherent phases for a point on terrain

$\phi_1 = \text{phase recorded by RADAR sensor-1}$
 $\phi_2 = \text{phase recorded by sensor-2}$

$$E_1 = e^{j\phi_1} = e^{-j\frac{4\pi R}{\lambda}}$$

$$E_2 = e^{j\phi_2} = e^{-j\frac{2\pi(2R+\delta R)}{\lambda}}$$

$$\phi_1 = \frac{2\pi}{\lambda}(2R)$$

$$j = \sqrt{-1}$$

$$(2R + \delta R)$$

So, let us consider what is the phase of the RADAR right ok. If you remember we said that synthetic aperture RADAR or the RADAR we are going to discuss in this module is only valid for the coherent backscatter; that means, one RADAR sensor is throwing the some pulses and it is recording the backscatter; the backscatter is also coherent fine. Now you can imagine that there are two RADAR sensors and each is recording the coherent backscatter for one point on the terrain of earth right. What is the game here or what is the logic here to do this thing?

Fine since we know that if there are two RADAR sensor which are in different position right, what will happen? The range from each RADAR sensor will be different for the same point on the earth. And now we can find out so, whatever information they capture about the point, from that information that is a phase information; we will derive some kind of logic and that logic will be equipping us to derive the difference of the range of the two RADAR sensor from the same point on the earth. And by seeing that that since the point has moved a point does not move, I can find out what is the impact or what is

the height. After finding out the range the difference of the range, I can also find out what is the height of the point. So, let us look into this aspect here.

So, now, let us say that we have same phase for a sensor here and then two different sensors have different coherent phase for a point on terrain ok. So, here I can stress say there. Let us represent our phase information by this complex image and here we are representing this ψ_1 which is a phase here. We are using word ψ_1 here, but is coherent phase of recorded by sensor 1 and then we have ψ_2 , I can say the phase recorded by the RADAR sensor 2 or I can say here both sensors are same or both RADAR sensor are having the similar or same properties fine. So, I am saying that my phase ψ_1 here and I am putting this in the form of complex this one. If you remember, the phase can be represented as 2π upon λ into the distance travelled which is $2R$ here fine.

So, I am writing it here like this and j is nothing, but minus 1 and under root here. So, now, E_2 for the other sensor, I am writing; let us if I observing the phase ψ_2 for same point and I know that the distance from another sensor or the sensor number 2 is R plus ΔR right. So, if ΔR is nothing, but the change in the total range $2R$. So, first we have $2R$ range and now we say that from the next sensor we have $2R$ plus some difference in the range right ok. So, we are writing our 2 information like this. So, I am saying that this is my image, complex image and this is another complex image for point on terrain or I can say for 1 pixel that is acquired on the terrain by RADAR right.

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Interferometry Principle

- Phase difference information of two phase images provides range difference estimation for a point

$$\begin{aligned}
 E_1 &= e^{j\varphi_1} = e^{-j\frac{4\pi R}{\lambda}} \\
 E_2^* &= e^{j(-\varphi_2)} = e^{j\frac{2\pi(2R+\delta R)}{\lambda}} \\
 E_1 E_2^* &= e^{j(\varphi_1 - \varphi_2)} = e^{-j\frac{2\pi(2R - (2R + \delta R))}{\lambda}} \\
 E_1 E_2^* &= e^{j(\Delta\varphi)} = e^{j\frac{2\pi(\delta R)}{\lambda}}
 \end{aligned}$$

$$\Delta\varphi = (\varphi_1 - \varphi_2) = \frac{2\pi(\delta R)}{\lambda}$$

$$\text{phase difference} = \frac{2\pi(\text{path difference})}{\lambda}$$

So, now to see this is the one information that is ok. Now take the complex conjugate of E_2 which is nothing that E_2^* and I can say here that it is my $e^{j\varphi_2}$ and I put minus sign over there and it becomes $e^{j\varphi_2}$ and then we have $2R$ plus δR divided by λ ok. So, now, multiply these 2 here this. So, this is my here I can say right I am getting the same expression here. So, now, you multiply the E_1 that is my first image or first complex image of 1 pixel and that is the another complex image. So, if you multiply this pixel to pixel, what will happen? For each pixel, I will get this phase difference by simple algebraic sum for exponential here you see right.

And as a result, I get this expression where I am cancelling the $2R$ with this $2R$ and then we will get this expression you can do it yourself. So, now, you can see here that $\Delta\varphi$ which is phase difference here is equal to this quantity and I am writing it here. And now if I measure the phase difference of the 2 complex images of RADAR where each image is acquired by the same sensor or sensors can be two, but both sensor should be same fine. So, then it will be equal to this and so, using this expression I can estimate what is the change in the range. And now by this range difference, I can also estimate what is the height difference of the point or that is the height of the point on the terrain. And so, that is the application of the interferometry. So, that is the basic fundamental logic of the interferometry ok.

So, let us go ahead I hope that you understand what is the phase principal and so, I am writing the words that phase difference is nothing, but 2π divided by λ and path difference. So, ΔR is my path difference between the paths of 2 RADAR sensor. So, one is having $2R$ path and another having $2R$ slightly different from $2R$. So, that is $2R + \Delta R$, I hope you agree with me ok.

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Interferometry for Mapping

□ Interferometry SAR: InSAR or IfSAR

▪ ✓ Single Pass Type

- Two antennas (transmitter and two receivers) are mounted at the same along track position, but one above other
- Temporal baseline is zero

▪ ✓ Two Pass Type

- Two antennas are placed in different tracks

□ Differential interferometry SAR (Three Pass Type): DInSAR

- Three Pass Type: three antennas in three different tracks
- Measure two spatial base lines

So, now we have interferometry and that is called InSAR that is interferometric synthetic aperture RADAR is also called InSAR or IfSAR right; that means, we are using the interferometry for the mapping and for that purpose the we use the word InSAR or IfSAR; now InSAR is of 2 type single pass type and two pass type ok. So, we will look into this thing one by one.

And then we have another mode called differential interferometrt SAR that is three pass type and it is also called differential interferometric synthetic aperture RADAR or DInSAR or sometimes its call also called DInSAR. So, let us look one by one.

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Important Terms

- ❑ Spatial baselines $\delta R = R_2 - R_1$
 - Target area is imaged in two different SAR tracks simultaneously
 - We measure target elevation from the known platform positions
- ❑ Temporal baselines
 - A pure case is that the SAR measurement acquired from exactly identical tracks
 - The temporal baseline is used to measure the radial velocity
- ❑ Mixed baselines
 - Combination of both baselines

So, before we go into the single pass, two pass and the three pass type or the differential DInSAR IfSAR or InSAR right. So, let us first look into the important definitions or important terms here first is a spatial baseline. So, let us assume that there are two RADAR sensor indicated by my fingers here right ok. So, one RADAR sensor is located here and another here and both are observing the same point on the terrain of earth at the same time; that means, there is no time difference in their observation. So, we call the temporal baseline is 0; that means, the time at which both are observing same. So, there is no time difference in their observation and that is called the 0 temporal baseline

Now here if you see that there is a clear difference of the distance between the two positions of my RADAR sensor and this difference is called the spatial baseline right. Now I hope that you agree, what is the idea of a spatial baseline and temporal baseline. Now imagine the other situation, let us say there is a RADAR sensor which is revolving around earth in maybe in air bound or space bond, but it is basically observing a point at different different times; that means, let us say at a certain time interval what we call the temporal resolution of the sensor. So, after one temporal resolution it is again observing the same point. So, I can say here that this is my baseline related with time or it is called a temporal baseline fine ok.

There could buy another situation also; that means, if I have two RADAR sensor they are moving in the different different tracks like this and now this particular sensor here, it

observes a particular point here. Now after sometime this sensor comes in the position and observe the same point here; so, in this case I can say that the baseline the spatial baseline also exist at the same time since they are observing at different different times. So, the temporal baseline also exist right. So, temporal baseline is not 0, similarly the spatial baseline is not 0. So, what could be a possibility where my spatial baseline is 0 ok? Is it possible that a same RADAR sensor is observing a point multiple times from the same position? Not possible.

So, at always in every case we have the spatial baseline, but sometimes temporal baseline could be 0 or could not be 0 fine. So, we should understand these concepts very nicely because we are going to use this terms again and again in different different cases. For example, single pass double pass or two pass and the triple pass or three pass fine ok So, what is my mixed baseline now? The mixed baseline when both the spatial baseline and temporal baseline exist together it is called mixed baseline ok. Now imagine one case that the RADAR sensor tracks are identical exactly the same ok.

What happens is let us say this is RADAR sensor it is moving like this and again coming back to the same point. Well if the point is exactly the same, what we can say about that that there is no change in the distance between the two RADAR sensor or the two locations of the RADAR sensor for the acquisition. Now you can see here that temporal baseline is positive or non-zero, but since the point is same I can say that a spatial baseline is 0 fine. I hope you agree with the idea of spatial baseline temporal baseline and the mixed baseline fine.

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Single Pass Interferometry

- In this, we record phase of the echo from the target in the two channels mounted on a platform

Phase difference:

$$\Delta\varphi = \varphi_2 - \varphi_1 = \frac{2\pi}{\lambda} p(R_2 - R_1) = \frac{2\pi}{\lambda} (R_2) - \frac{2\pi}{\lambda} (R_1)$$

only Return path

$$= \frac{2\pi}{\lambda} p(R_2 - R_1)$$

$\rightarrow p=1$

$p = 1$, if channels share the transmit antenna

$p = 2$, each channel is transmitting and receiving on its own antenna

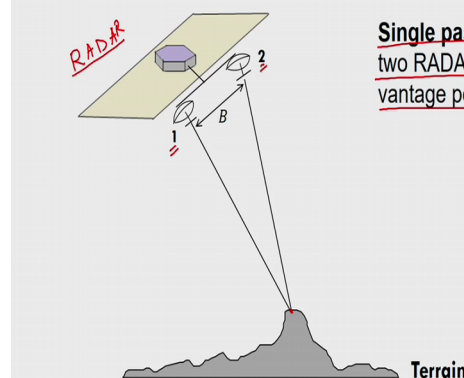
$$\varphi_M = \text{mod}(\Delta\varphi, 2\pi)$$

φ_M - phase difference shown by the interferogram

Now, consider the single pass interferometry logic here. What we do here we record the phase of the echo from the target that is the terrain point in two channels mounted on a platform. What does it mean basically ok.

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Single Pass Interferometry



Single pass or simultaneous baseline:
two RADARs acquire data from different vantage points at the same time.

So, this is the idea here that I have a sensor RADAR sensor here, this is my RADAR sensor where I am using the one transmitter, but there are two receiver antenna fine. So, this let us say 1 receiver antenna and 2 receiver antenna. So, the difference between the 2 receiver antenna is called the baseline or a spatial baseline fine. This is my terrain point

on the terrain surface ok. So, what happened here is when there is a transmitter, it is transmitting the RADAR pulse and RADAR pulse is travelling for a distance. Let us say R and after that it is reflected in the form of backscatter and that backscatter is received by the 2 antennas or 2 receivers fine. You can see here, there simple logic that while transmitter has transmitted the RADAR pulse. It is travelling the same distance for both the receivers.

However after backscatter or after reflection the reflected backscatter which are received by the two sensors have different path and try to find out the change in the ranges or the slant ranges for the return path only because transmitter is same and we have the two different receivers fine. So, this is my logic of single pass or simultaneous baselines and is called two RADARs acquired data from different vantage points at the same time ok. So, point receiver 1 and 2 acquiring the data at same time, but at different location. So, we have the spatial baseline existing ok. Now I can easily write this relationship here, $\Delta \phi$ equal to ϕ_2 minus ϕ_1 which is nothing, but if I write let us say, this is a range R_2 for the sensor 2 and R_1 range for the sensor 1. So, I am writing the difference of the two ranges.

Now, you can see here that since the range is R_2 , remember that there is a common path between the two receiver antenna. So, we ignore that and hence I can write here that R_1 and it is my return path only both are my return path only return path right. Here we look into one point that we introduce a factor called p such that I can write very generic expression for all the cases right. Whether it is a two travel path that is E and again R or it is one path from the target point to the receiver fine. So, I can write two things. So, I am saying that p here is equal to 1 fine, but in case if I consider the two path that is from transmitter to target and target back to the receiver. So, that will be two path travel; so, in that case I will take p equal to 2 fine.

So, there are cases if I let us assume that if I have 2 RADAR sensors; that means, each RADAR sensor have the different antenna or different receiver. So, what will happen here? Both I need to consider the complete path from transmission to reflection back to the sensor. In that case p will be equal to 2 fine because at the time we have travelled double distance from transmitter to a target point and again target point to the receiver. So, that is for the RADAR sensor 1. Similarly same thing happen for RADAR sensor 2 in that case, my p will be equal to 2 fine. Now it is written here very clearly ok. Once we

determine this phase difference of the 2, what we do we calculate? This quantity ϕ_M and ψ_M is nothing, but the mod of the $\Delta\psi$ over 2π ; that means, I am reducing the $\Delta\psi$ in the range 0 to 2π and that is finally, we call the ψ_M .

Now, for each pixel if I plot this value on my RADAR image, what will happen? This image is called interferogram and that is nothing, but the difference of the phases. I hope you got the idea now what is my interferogram and now we say that using the interferogram, I can estimate the height of a point and that is the logic of interferometry.

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Interferogram ✓

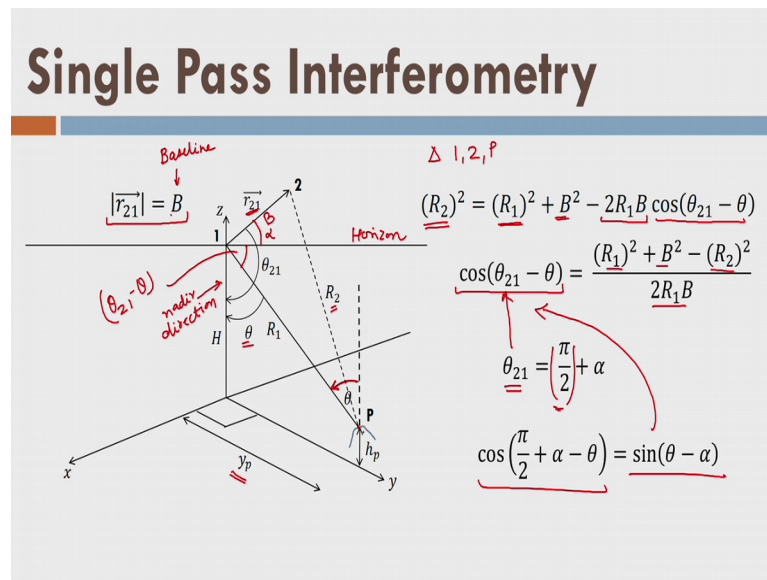
- RADAR interferogram contains only phase difference information of two phase images for same points

$$\Delta\varphi = \frac{2\pi(\delta R)}{\lambda}$$

$$\varphi_M = \text{mod}(\Delta\varphi, 2\pi)$$

So, that is the idea here first we estimate the range difference of the two points from two different locations of sensor and using this range defined δR , I can find out the height of a point. So, this is the logic of the interferogram; that means, interferogram records my ψ_M information which is nothing, but modulus of the observed $\Delta\psi$. Now that is the idea we have already explained about a single pass interferometry.

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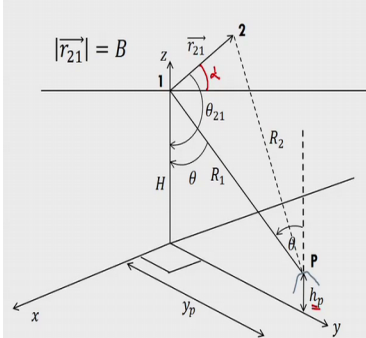
So, let us go for some kind of derivation fine. So, let us see there are 2 antennas: 1 and 2 here and the baseline between the 2 is indicated by R_{21} and nothing. So, R_{21} the length of the R_{21} vector is equal to the baseline here. So, now you can imagine that there is a sensor like this and there is a antenna. So, one antenna is there at this location and another antenna at this location and sensor is moving in this direction like this and both sensors or a both the receivers are observing a point on the surface of earth let us say this is the point. So, it is being observed by this receiver antenna and this receiver antenna right. So, now, you can imagine that this is the way we are observing different different points and this is length is called baseline ok

Now, we can see this is the across the track direction y_p and this is my along track direction which is not shown here. Now since I know that point, point 2 and point pl they are in one plane as you can see here like this is my range R_1 this is my range R_2 fine. So, now, this is the angle θ_{21} and this is angle θ they are nothing, but the look angles θ_{21} and θ are look angles for point p by the different different receiver antenna here ok. Now I can write from this triangle 12 and P that R_2^2 which is this distance equal to R_1^2 plus B^2 right. This is my B length here minus $2 R_1 B \cos$ of θ_{21} minus θ fine. So, basically I am saying that this angle is my θ_{21} one minus θ and I am using the cosine rule to write the R_2 in the form of R_1 and B . I hope you agree with that ok

So, now if I do some kind of small transfer on left and right side I will get this expression, $R_1^2 - R_2^2 + B^2$ divided by this one equal to this angle ok. Let us see that we know here that θ_{21} is $\pi/2$ because this angle is my $\pi/2$ here; from horizon, this is the horizon here and this is the nadir angle here. So, I say that nadir direction. So, I know that this angle is ninety degree here between the horizon and nadir. So, I write $\pi/2 + \alpha$. So, this angle is my α here ok. So, now, if I replace this thing here, what will I get? I will get this expression and from that I will get this one. So, you can look here that angle this is my θ angle here because it is equal to this angle and the point P is having height h_p here that I want to estimate ok.

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Single Pass Interferometry



$$|\vec{r}_{21}| = B$$

$$\sin(\theta - \alpha) = \frac{[(R_1)^2 - (R_2)^2] + B^2}{2R_1 B}$$

$$\stackrel{2R_1}{=} \frac{(R_1 + R_2)(R_1 - R_2) + B^2}{(2R_1)B}$$

$$R_1 \approx R_2 \Rightarrow R_1 + R_2 \approx 2R_1$$

$$\sin(\theta - \alpha) = \frac{R_1 - R_2}{B} + \left(\frac{B^2}{2R_1 B} \right) \left\{ \frac{B}{2R_1} \approx 0 \right.$$

$$\sin(\theta - \alpha) = \frac{R_1 - R_2}{B}$$

$$\sin(\theta - \alpha) = \frac{R_1 - R_2}{B}$$

Now, let us write this one there like this and here again on the right hand side. There is some approximation ok. Let me $R_1^2 - R_2^2 + B^2$ as $R_1^2 - R_2^2 + B^2$ divided by $2R_1 B$. Now here we make an assumption that R_1 and R_2 are approximately equal as a result $R_1 + R_2$ equal to I can say approximately equal to $2R_1$ fine and as a result the expression here comes $\sin \theta - \alpha$ equals to this is my $2R_1$ and this will be cancelled by this. So, I have $R_1 - R_2$ divided by B plus B^2 upon $2R_1 B$. So, it is cancelled here. So, this is nothing, but B upon $2R_1$.

Now if you see that baseline is in order of 2 to 3 meter or maybe 3 meter. So, the R_1 distance is very high in kilometers maybe some 100 kilometers or so, so this B by $2R_1$

here is approximately equal to 0 I can say it is very small value. Finally, I have this expression $R_1 - R_2$ divided by B which is nothing, but $\sin(\theta - \alpha)$ and which is written here I hope you got this idea ok. So, what is my θ here? θ is my look angle fine. What is the α ? α is the angle, this angle which I already know by my design criteria; that means, when we design our sensor; we know this α angle ok. Now, I can find out the exact value of the look angle θ ok.

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Single Pass Interferometry

□ Interferometric phase, look angle and the baseline angle from horizontal

$$\sin(\theta - \alpha) = \frac{R_1 - R_2}{B} = \left(\frac{1}{B}\right) \left(\frac{-\lambda(\Delta\varphi)}{2\pi p}\right)$$

$$\theta = \alpha - \sin^{-1} \left(\frac{\lambda(\Delta\varphi)}{2\pi p B} \right)$$

$$h = H - R_1 \cos \theta$$

$$y = R_1 \sin \theta$$

$(R_2 - R_1) = \Delta R$
 $\Delta\varphi = \frac{2\pi}{\lambda} (\Delta R)$
 $\Delta R = R_2 - R_1 = \frac{(\Delta\varphi) \lambda}{2\pi}$
 $R_1 - R_2 = \left(-\frac{(\Delta\varphi) \lambda}{2\pi} \right)$

α – Baseline angle from horizontal plane
 h – Height of the terrain
 y – Ground range of target
 θ – Look angle

And so, the θ is written like this. Before that remember the phase difference is what we written there that R_2 minus R_1 is equal to ΔR . Here we know that $\Delta\varphi$ is equal to in case of single pass interferometry 2π upon λ into ΔR fine. So, I replace this expression here; that means, ΔR which is R_2 minus R_1 . So, I say that ΔR which is R_2 minus R_1 is nothing, but $\Delta\varphi$ into λ by 2π and if I take R_1 minus R_2 ; then it becomes minus $\Delta\varphi$ into λ upon 2π . If I replace this value over there and then, I will get this value you can see here very easily fine ok.

Again I am putting the factor p here for different different cases fine. So, p factor we assumed here one, but still from the general case I am writing p here also fine. Ultimately this is my look angle value that is nothing, but if I take the $\sin$ inverse here. This my flying height h and minus this $h p$. So, this is, but R , this distance is R_1 it is $R_1 \sin \theta$. So, this is the expression here and y is my another coordinate across the track direction coordinate fine that is $R_1 \sin \theta$. So, now, I have find out the coordinate of

point p on the terrain fine using the single pass RADAR interferometry ok. So, let us look into the performance limitations. So, what are the limits of the performance of single pass RADAR interferometry fine?

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Performance Limitations

$$\sigma_{\Delta\phi} = \sqrt{\frac{1}{\text{SNR}}}$$

$$\sigma_{\theta/\Delta\phi} = \left(\frac{\lambda}{2\pi p B} \right) \left(\frac{\sigma_{\Delta\phi}}{\cos(\theta - \alpha)} \right)$$

$$\sigma_{\theta/\alpha} = \sigma_{\alpha}$$

$$\sigma_{h/\theta} = R \sin \theta \sigma_{\theta}$$

$$\sigma_{y/\theta} = R \cos \theta \sigma_{\theta}$$

$\sin(\theta - \alpha) = \frac{\lambda (\Delta\phi)}{2\pi p B}$

$\sigma_{\theta} = \dots$

$\theta = \alpha - \sin^{-1}(\dots)$

$\sigma_{\theta} = \sigma_{\alpha}$

$\sigma_{\Delta\phi}$ – Phase noise level

$\sigma_{\theta/\Delta\phi}$ – Look angle error due to phase noise

σ_{α} – Base line angle error

$\sigma_{h/\theta}$ – Height error in elevation of terrain point

$\sigma_{y/\theta}$ – Ground range error (across track error)

σ_{θ} – Error in the look angle

So, let us see that there is a signal to noise ratio and we measure the RADAR image using the signal. So, we have signal to noise ratio and that is giving me one sigma precision value of my phase difference measurement or the interferogram measurement fine. From there we can write this expression that given the sigma delta phi, what is the value of sigma theta? That is the one sigma precision of my theta value fine that is the error in the look angle fine. So, we get this expression here ok. You can do it yourself because we have sin theta minus alpha and by using the error propagation or using any systematic error logic, we can find out this thing. We have this lambda delta psi divided by 2 pi p B right from there if you do; that means, you try to find what is my sigma theta? You will get this expression ok.

Now, we can see here that the theta is equal to here alpha minus sin inverse some expression was there ok. So, if I just assume that all this information is given I can write that sigma theta is equal to sigma alpha; that means, given the values of alpha what is my sigma theta that is nothing, but sigma alpha which is error in the base line angle error; base line angle error means the alpha angle here. Then we write using the same expression of the r h and y. So, what is the error here? Given this error sigma theta or for

given value of theta I get R sin theta into sigma theta and sigma y theta is equal to R cos theta sigma theta fine. So, here I can estimate the sigma theta by 2 logic; this is one and this is another ok. So, which one I will take for the worst case accuracy, I should take the higher of the 2 values fine.

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Performance Limitations

$\begin{aligned} \text{SNR} &= 20 \text{ dB} \\ B &= 2.5 \text{ m} \\ \lambda &= 5 \text{ cm (6 GHz)} \\ \theta &= 45^\circ \\ H &= 13 \text{ km} \\ \sigma_\theta &= \pm 0.018^\circ \end{aligned}$	$\begin{aligned} R &= \frac{H}{\cos \theta} = \frac{13 \times 10^3}{(\sqrt{2})} = 18384.78 \text{ m} \\ \sigma_{h/\theta} &= R \sin \theta \cdot \sigma_\theta \\ &= \frac{18384.78 \times (\sin 45^\circ) + \left(\frac{0.018^\circ}{180} \pi \right)}{1} \\ &= 4.08 \text{ m} \quad \rightarrow \text{radians} \\ \sigma_{y/\theta} &= R \cos \theta \cdot \sigma_\theta = 4.08 \text{ m} \end{aligned}$
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So, let us look into some kind of a simple exercise of the numerical here signal to noise ratio is let us say that 20 decibel fine and my baseline is equal to 2.5 meter fine, lambda wavelength is equal to 5 centimeter or I can say 6 gigahertz ok. Then we have theta is equal to 45 degrees that is my look angle and we have the flying height equal to let us say 13 kilometers up the above the earth surface. And then we find out sigma theta is equal to this value. Let us say, this is the value given to us and ok

So, what is the value of range? First of all that is my h upon cos theta and so, I write one upon root 2 here cos 45 and here h is nothing, but 13 kilometers. So, here I get 18384.78 meter and then we have let us calculate the error in the height given the value of theta R sin theta into sigma theta right. So, here R is 18384.78 into sin 45 into sigma theta is in the radian form I can write 0.018 degrees divided by 180 into pi in the radians fine. So, this remain radians ok, Now I can find out that this value comes out to be 4.08 metre. At the same time, what about the sigma y theta? This value comes out to be R cos theta sigma theta and you put the same theta equal to 45 degrees. We have this also equal to 4.08 metre.

So, that is kind of a realistic estimate of my height estimate here right; that is the error in the height will be around 4 metres ok. So, we should not expect very higher level of accuracy ok; so, where this accuracy is useful when the height accuracy itself is 4 metres ok. Now imagine there is some global phenomenon for example, there is some landslide has occurred right and there was so, severe landslide that few metres land has passed. Now if I use this data which is 4 metre accurate fine, in height measurement and the height measurement is say 20 metres. So, this 4 metre accuracy is good enough for that purpose fine.

So, that is the idea here or imagine something a building has demolished completely which is of 100 metre height. Now if I use the 4 metre data that is absolutely fine. One more thing I would like to tell you ok. So, that is the idea here, we should know about the what are the realistic aspects of my resolution here ok. So, this is be some idea about the performance limitation.

One more respect we can address, the aspect is very simple that and this aspect is related to the baseline ok. If you can imagine that if there is a baseline between the 2 receivers; if it is increased, what will happen? Since the term B comes into the denominator of the one sigma variance of the one sigma value of my accuracy of θ what happens here. You can imagine that that accuracy is inversely proportional to the baseline length. So, higher the baseline, I will get superior accuracy or better accuracy fine or I can say that I will get higher the accuracy.

With this aspect now, there is a limitation to which we can increase the baseline that is the one aspect ok. Now what could be the most optimal value of the baseline and that is called a critical baseline length; that means, we can increase the baseline length up to that limit beyond which there is no point ok. So, let us look into this aspect. What is that ok? So, first of all let us say that we know if across a pixel or the range resolution or what we call a pixel on the ground surface? If my ψ value or my phase difference changes by minus π to π or 0 to 2π right that is the maximum possible value I can have fine and for that corresponding to that one, what we call is the relevant baseline and it is my critical baseline because a baseline that can create that much of difference could be the maximum possible value of the baseline. So that is what we have to now consider.

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Performance Limitations

- ❑ Multi look filtering is used to reduce the phase noise by $\sqrt{N_{eq}}$
- ❑ Critical baseline
 - Baseline corresponding to random phase between $-\pi$ to π

$$\frac{\partial \varphi}{\partial R} = \left[\frac{2\pi p B}{\lambda} \cos(\theta - \alpha) \frac{1}{R \tan \theta} \right] = \text{fringe rate}$$

For fringe rate of 2π per range resolution

$$\left(\frac{\partial \varphi}{\partial R} \right) \Delta R = 2\pi \Rightarrow \left[\frac{2\pi p B}{\lambda} \frac{\cos(\theta - \alpha)}{R \tan \theta} \right] \Delta R = 2\pi$$

$$\Rightarrow B_c = \frac{R \tan \theta \lambda}{p \cos(\theta - \alpha) \Delta R} = \frac{R \sin \theta \lambda}{p \cos(\theta - \alpha) \cos \theta \Delta R}$$

$$\Delta y = \frac{\Delta R}{\sin \theta}$$

$$B_c = \left[\frac{R \lambda}{p \cos(\theta - \alpha) \cos \theta \Delta y} \right]$$

So, let us define something called the fringe rate; that means, what should be the range for which what should be the rate of change of phase angle with respect to the range such that it should create a phase angle by angle 2π ok. So, what I am saying is let us say if there is a range resolution or right over this resolution; if the phase changes by 2π angle which is nothing, but 2π minus 0 difference ok; that means, that is the best possible range for which I will have maximum change in the phase.

So how can I write it? So, first of all one should know that $d\psi$ by dR is nothing, but $d\psi$ by $d\theta$ and $d\theta$ by dR ok. So, now, you put the values of these variables from the last previous slides; that means, you can differentiate it partially and then you can put it ok. What happens is once you put it you will get this value and that is called the fringe rate here.

Now, we say that what we want? We want a change in the phase such that over one pixel resolution, it should be maximum which means if I multiply that $d\psi$ by dR it is fringe rate with one range resolution ok, what will happen? I should get it equal to 2π which means if I put this value here then, what will I get? I will get $2\pi p B$ into divided by λ into $\cos$ of θ minus α divide by $R \tan \theta$ into the ΔR which is this here should be equal to 2π .

So since I said that this is what we, this is the way we define the critical baseline. So, let me write B is equal to a critical baseline here right. So, what will I get here, the critical

baseline? B C is equal to first of all lets remove this factor 2π common one $R \tan \theta$ lambda divided by $p \cos$ of θ minus α and ΔR over here alright. I can write here like $R \sin \theta$ lambda divided by $p \cos \theta$ minus α into $\cos \theta$ into ΔR here fine.

Now, we know that the Δy that is the ground resolution in acrossic direction is given by ΔR upon $\sin \theta$ alright. So, what can I write here? That this term will be equal to $R \lambda$ upon p here ok; so, we can replace this term here this and this term by the Δy here. So, I am just writing it here B C is equal to we can see that $R \lambda$ divided by $p \cos$ of θ minus α into $\cos$ of θ and then we write here Δy alright.

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Critical Baseline

$\frac{\partial \phi}{\partial R}$ – Interferometric fringe rate
 B_c – Critical base line
 $B_{c\perp}$ – Critical base line orthogonal to look direction

$$B_{c\perp} = B_c \cos(\theta - \alpha) = \frac{R \lambda \tan \theta}{p \Delta R} = \frac{R \lambda}{p \Delta y \cos \theta}$$

$$B_c = \frac{R \lambda}{p \Delta y \cos(\theta - \alpha) \cdot \cos \theta}$$

Now let us come to the next stage well.

(Refer Slide Time: 38:06)

Performance Limitations

$$B_c = \frac{R \lambda}{p \cos(\theta - \alpha) \cos \theta \cdot \Delta y}$$

$$B_c [\cos(\theta - \alpha)] = \frac{R \lambda}{p \cos \theta \cdot \Delta y}$$

$$\text{here } B_{c \perp} = B_c \cos(\theta - \alpha) = \frac{R \lambda}{p \cos \theta \cdot \Delta y}$$

So, from that expression of B C, so we have written like B C is equal to R lambda divided by p cos theta minus alpha here written sin theta divided by cos theta delta R alright. Now we know one thing ok.

(Refer Slide Time: 38:59)

Performance Limitations

□ Multi look filtering is used to reduce the phase noise by $\sqrt{N_{eq}}$

□ Critical baseline

- Baseline corresponding to random phase between $-\pi$ to π

$$\frac{\partial \varphi}{\partial R} = \left[\frac{2\pi p B}{\lambda} \cos(\theta - \alpha) \frac{1}{R \tan \theta} \right] = \text{fringe rate} \quad \text{For fringe rate of } 2\pi \text{ per range resolution}$$

$$\left(\frac{\partial \varphi}{\partial R} \right) \Delta R = 2\pi \Rightarrow \left[\frac{2\pi p B}{\lambda} \frac{\cos(\theta - \alpha)}{R \tan \theta} \right] \Delta R = 2\pi$$

$$\Rightarrow B_c = \frac{R \tan \theta \cdot \lambda}{p \cos(\theta - \alpha) \Delta R} = \frac{R \sin \theta \cdot \lambda}{p \cos(\theta - \alpha) \Delta R \cdot \cos \theta}$$

$$\frac{\partial \varphi}{\partial R} = \left(\frac{\partial \varphi}{\partial \theta} \right) \left(\frac{\partial \theta}{\partial R} \right) \quad \boxed{(2\pi - 0) = 2\pi}$$

So, I can also write the same term by like this R sin theta lambda divided by p cos theta minus alpha delta R and cos of theta here in the acoustic direction. So, by replacing the value we can write here B C is equal to R lambda divided by p cos of theta minus alpha and cos theta and then here delta y alright ok. Very nicely one nice expression I can write

here that let us multiply this B into $\cos$ of $\theta - \alpha$ fine and here it is equal to R λ divided by $p \cos$ of θ Δy . And this term is nothing, but the projection of my baseline BC in the horizontal plane or I can say it is normal to the track direction right or the as my direction right.

So, here so, you write here that BC , we write here BC perpendicular equal to or orthogonal to the loop direction is equal to $BC \cos$ of $\theta - \alpha$ is equal to $R \lambda$ divided by $p \cos$ θ Δy alright.

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Phase Determination

- It includes
 - Phase unwrapping
 - Negative and positive residues
 - Absolute phase determination

Absolute
phase
diff

$\Delta \varphi = \varphi_m$

Now, we want to determine the phase also because using the phase information also we have determine the height. So, we have basically converted our phase $\Delta \varphi$ $\Delta \psi$ into ψ_m . Now I want to recollect this information from this. So, that is called phase unwrapping.

And for phase unwrapping there are many many algorithms available someone who is really interested in phase unwrapping should refer to those algorithms fine. So, we are not touching it here. So, that is my absolute phase here phase difference ok.

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Two Pass Interferometry

□ Important issues

- A point on terrain is observed by two RADAR sensors in two tracks at different times
- Temporal base line ^{exists} but no change is terrain point
- Unknown spatial baseline (length and orientation)
- Non parallel tracks leads to difficulty in finding baseline
- Change in target area (temporal decorrelation)
- Motion compensation is difficult (aircraft case)
- **So estimation of baseline is difficult**

Now, let us come to the next concept called two pass interferometry, we have discussed the single pass interferometry in detail. Now let us consider the two pass interferometry. What are the important issues here fine; the idea here is two pass interferometry that means, there is RADAR sensor which is moving on a certain orbit or track and there is another RADAR sensor which is moving like this fine. So, they are observing a point entering earth at different time automatically since they are moving on a different tracks. They are the baseline a special baseline exist as well as temporal baseline also exist fine. So, that is the important issue we have highlighted.

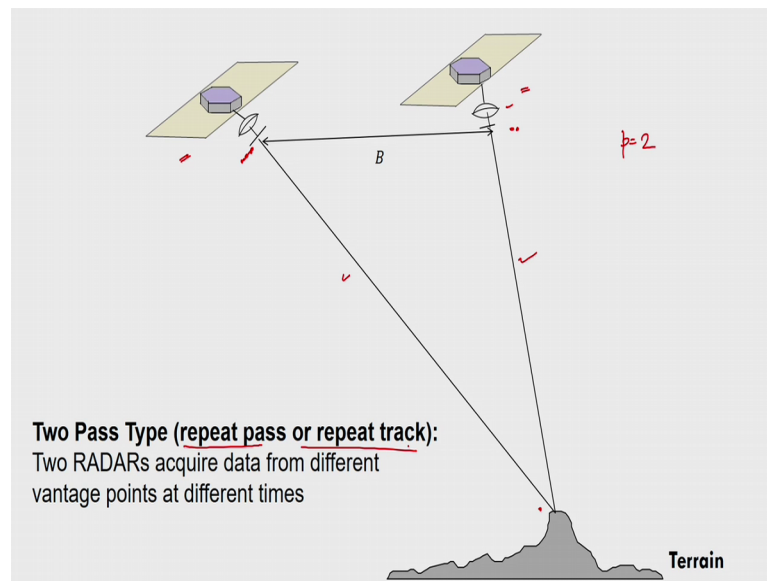
So, a point on terrain is observed by 2 RADAR sensors; that means, even I can have one RADAR sensor coming in different paths or there are 2 RADAR sensors in different path, but both are observing. So, the mechanism will be same even I have 2 RADAR sensor in different paths or ere is one RADAR sensor which is having some temporal resolution and which is coming in the next part at different path fine.

So, then we have temporal baseline exist I am sorry it should be exist, but no change in the terrain point. Now we have unknown spatial baseline tracks are not parallel of the 2 RADAR sensor. It will lead to some difficulty in finding baseline ok. So, change in the target area that is we have assumed that there is no change in the target area. Suppose there is a change for example, we are trying to study the glacier and glacier is on melting, what will happen? If there is a big rock mass is there on the glacier surface and glacier is

moving the point that is rock mass will also move and that we assume that there is no change in the position of the point and that is wrong assumption.

So, in this in this case we have to consider the temporal decorrelation fine. What about the motion compensation because of the in case of aircraft it is slightly difficult. So, the estimation of the baseline is very very difficult here, in case of two pass interferometry. But let us look into that how do we do it ok.

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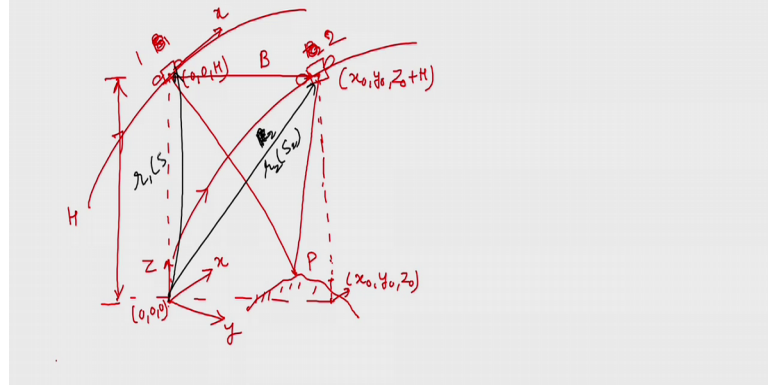


This is my two pass interferometry and we have RADAR sensor 1 here and RADAR sensor 2 here and both have different receivers fine. So, now, we should consider p equal to 2 if we are considering the 2 paths here fine. This is path is different and this path is different fine.

So, there will be starting point here, half point here and again total range should be considered from this point to this point here. Similarly for this antenna from this point to this point again total range of travel should be considered ok. So, it is called repeat pass or the repeat track fine. So, I hope you got the idea that this point is my under observation and this is the baseline now ok.

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Two Pass Interferometry



So, here I can show you that let us say there is a one track here and this is my satellite over there and it is observing a point here p on terrain and there is another track here. From here again this another RADAR sensor, let us say R_2 or it could be R_1 also no problem right. So, let us say sensor one or sensor 2 here and sensor 1 here and it is also observing the point here fine.

So, now it could be different path or even one RADAR sensor can come into the different path fine. Now I want to estimate the baseline ok, what can I do here? Fine the idea is very simple. Again if I go back here let us assume that there is a point here and which has some reference frame here x , y , z . This is my z here and this is my x which is along the track direction here this is my x and this is my y fine. Now you can see here that if it is 000 and if this is my flying height h . In this coordinate system I say that this point has a coordinated called x_0 , y_0 sorry if I draw this point here. So, the nadir point here the nadir point here; if I draw up here like this. So, this point nadir point has some coordinate, let us say x_0 y_0 and z_0 .

And as a result because of this flying height which is here up to so, this is point here, what will happen? This point is having coordinates like x_0 y_0 z_0 plus H fine and this is my baseline now and since the tracks are having different orientation I have to match my tracks first and then I will estimate the baseline by estimating the difference between the 2 parallel tracks. So, that is logic here.

So let us see the idea here is there are 2 tracks which are not parallel. So, what will I do? There is a this track which is not parallel to this track. So, I will bring this unparallel track. So, this is my track 1 here and is another track which is not parallel. So, what will I do? I will orient this track in a way that it becomes parallel to this track and after that I will just find out the distance between the 2 parallel tracks right. So, that is the way we determine the baseline in case of two pass interferometry.

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Estimation of Baseline

□ Assumption is coordinate system is aligned with the track of pass one

$$\underline{\vec{r}_1(S_1)} = \begin{pmatrix} 0 \\ 0 \\ H \end{pmatrix} + \underline{S_1 \Delta x} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\underline{\vec{r}_2(S_2)} = \begin{pmatrix} x_{02} \\ y_{02} \\ z_{02} + H \end{pmatrix} + \underline{S_2 \Delta x} \begin{pmatrix} 1 + \gamma_2 \\ \frac{\partial y}{\partial x} \\ \frac{\partial z}{\partial x} \end{pmatrix}$$

$\left(\frac{\partial y}{\partial x}\right) = \text{track divergence}$
 $S_2 \Delta x \left(\frac{\partial y}{\partial x}\right)$
 $S_2 \Delta x \left(\frac{\partial z}{\partial x}\right)$

So this logic is there I have written the position of R 1 which is nothing, but this is my R 1 here to here this vector and this vector is my R 2 here, this is my R 2 I write it r 2 S 2 and here this is my r 1. So, now, this is my r1s1. So, I am writing 00 h plus this is nothing, but S1 is my pulse number and delta x access the along track pulse spacing. So, for one pulse spacing I have delta x for many pulse, S1 number of pulse spacing I will have this distance fine. So, then I am writing one 00 here fine that is my r1 vector.

Now, what is r 2 here ok? r 2 is my this coordinate of point of the second sensor and then this is the distance here ok. This distance is first calculated to the dy by dx that is the track change or track divergence in y direction. So, if I consider this track divergence, I have put the correction by this amount that is S 2 delta x into dy by dx. Similarly I have put corrections for the track divergence in z direction like this. So, this is my correction.

And then I know that both tracks are parallel ok. So, is there is only a difference of the distance. Now I say that the track 2 is having some kind of scale factor and as a result

this scale factor is my gamma 2 which is less than one. So, that is 1 plus gamma 2 is my total differential scale factor ok. So, by this way I have estimated my r 2 that is the position of the RADAR sensor 2 in x y z system. Now both tracks are in same coordinate system and if I take the difference of 2 vectors I can find out the baseline length right. So, that is the idea here about the two pass interferometry. So now let us look in one application of two pass interferometry and that is the velocity mapping.

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Velocity Mapping

- Application of two pass interferometry
 - Two different look angles
 - Zero spatial baseline, not sensitive to topography
 - The interferogram used to extract information about the scene dynamics

$$\varphi = \frac{2\pi p \underline{u} \delta t}{\lambda} = \frac{2\pi p \underline{u} d}{V\lambda}$$

$$u = u_y \sin \theta - u_z \cos \theta$$

$$\underline{u}_y = \left(\frac{u}{\sin \theta} \right)$$

δt – time delay

u – radial target velocity

u_y – across track velocity

u_z – vertical velocity

V – vehicle velocity

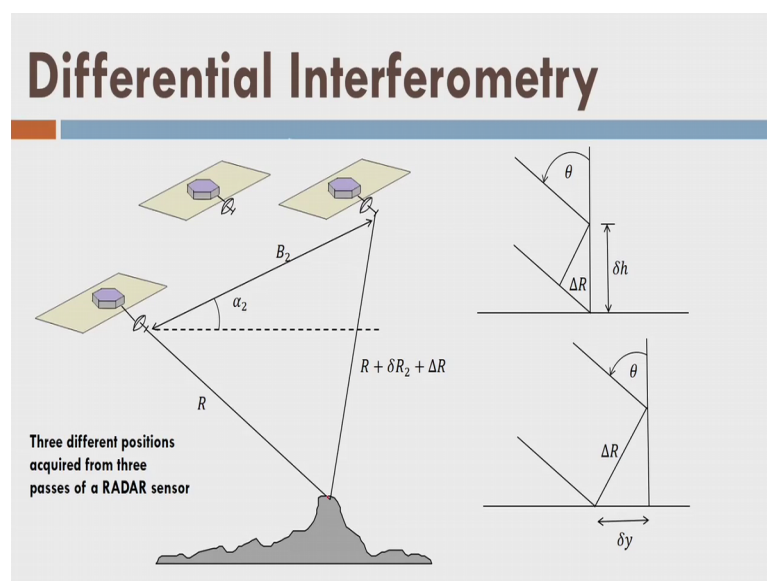
d – movement of vehicle during δt

Remember I said that during these 2 passes, we assume that the point is on the terrain is not moving. Now assume that there is a glacier and glacier is moving and because of that the point is also moving. What will happen? In that case when the RADAR will come on the next pass during that time, what you call as the delta t time, what will happen? The point has moved. So, both the RADAR sensor will be observing different different points for the same feature and by observing these two paths, I can find out what is the velocity of the movement of the point. So, that is the one application of the RADAR interferometry fine.

So, let us say if I measure the phase here the psi, what will happen here? This is my delta t time here and this formula is given right. Now this is the delta t time that is during which the time the point has moved. So, once there is RADAR sensor here and in the next pass or the in the next time when RADAR sensor comes, it finds that the point has moved during this delta t time.

Now I can see that that RADAR sensor is moved by a distance d on a trajectory during this time Δt right. Now what happens is I can write this Δt is equal to d upon V here in the same expression where u is the speed or I can say radial target velocity on the surface of earth. More over we assume here that the vertical one the velocity in the vertical direction of terrain point is 0. So, I get this expression here I find out the u from this equation and by putting the value of u here, I can find out what is the value of the velocity of the terrain point in y direction or in across track direction fine. So, that is application called velocity mapping here.

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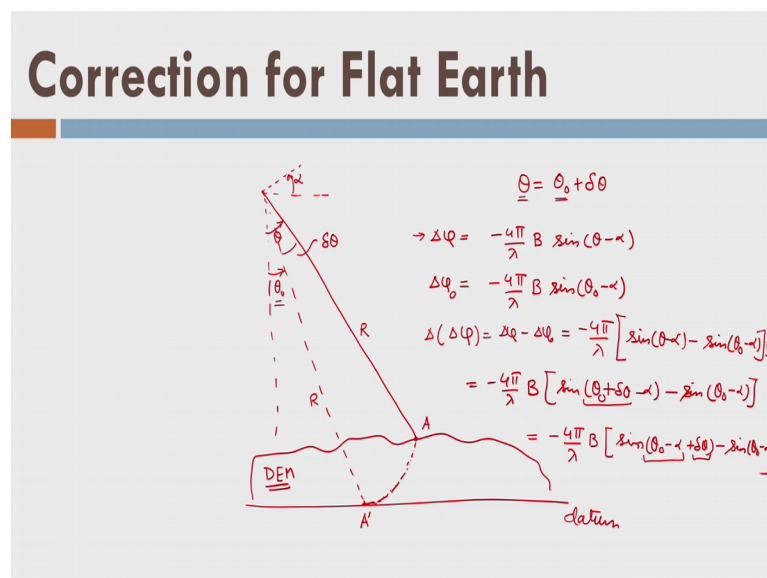
So in case of differential interferometry, now, what we want to do here? We want to find out the change in the position of the point. Let us say, there is a glacier as we have already discussed we have already shown that how to use the two pass interferometry in order to measure the velocity of the glacier, but we are measuring the velocity of the glacier on the some datum surface where we assume that there is no height of the point alright. But we are saying that we want to find out the topographic changes in the surface point on the surface of earth.

So, this point has some elevation alright. So, it is not on the datum, it is situated somewhere up more over. We see that once we observe that point from sensor location 1 and 2, there were no change, but; however, after observation from point 2 the time duration between the observation sensor location 2 and the sensor location three the point

has changed its position. So, this duration between the second observation and the third observation could be of any magnitude, it could be one month, it could be one week, it could be one year or maybe it depends on the phenomenon alright.

So, in case of landslide it could be very very small duration or in case of glacier melting it could be one year also alright. Now we want to do this thing. So, what do we do basically? We create the one baseline between the sensor 1 and 2 and another baseline between the sensor 1 and 3 because we know that change has happened after the observation from sensor 2 or the location of sensor 2 alright and between the 3. So, let us look first that first we need to understand, what is the contribution of the topographic changes with respect to the datum ok.

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So, what is the correction we need for the earth and we have a datum here, this is my datum and its real terrain is something like this alright. Now this is the nadir position here of the sensor and let us look this is the range which has been observed here. Let us call the range as R and so, this was my angle alpha as before and so, this was my look angle theta.

Now, because of this range and because of the elevation of the point, what will happen here because of the topography? If I image this point with a same range R here; so, it will be meets here like this alright. So, this is also R, but this angle is my theta 0; that means, if point this point A if it is situated on the datum I could have got the angle theta 0. So, let

us see the angle difference is nothing, but my delta theta. So, these are my look angles basically theta and theta 0 alright.

Now, I can write here that theta is equal to theta 0 plus delta theta. So, theta 0 corresponds to the point which is on the datum and delta theta is a variation and theta is the angle look angle correspond to the point a on the real terrain surface alright. Now if you remember we have written the delta psi as minus 4 pi by lambda baseline B sin of theta minus alpha. I want to know what is the correction in my delta phi that I should apply if the point lies a on the datum let us say A dash alright.

So, if point lies at A dash ok, what can I write here? I can write here very simply that let us say delta psi in that case let us say delta psi 0, what will happen ? 4 pi by lambda B sin of theta 0 minus alpha alright ok. Now if I take the difference of the 2; that means, delta of my delta psi which is nothing, but difference of my delta psi minus delta psi 0, what will happen? I will have four pi by lambda sin of theta minus alpha minus sin of theta 0 minus alpha into B here alright.

Now, I can straight away write it 4 pi by lambda B into sin of theta 0 plus delta minus alpha minus sin of theta 0 minus alpha here ok. Now we can expand this term by sin formula where this is my one angle alright and this is ok. So, I can also write it B sin of theta 0 minus alpha plus delta theta minus sin of theta 0 minus alpha alright. So, I can expand this term by sin formula where this is my one angle and this another angle alright. What will happen? So, let us look into the next slide ok.

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Correction for Flat Earth

$$\Delta(\Delta\varphi) = -\frac{4\pi}{\lambda} B \left[\sin(\theta_0 - \alpha) \cos \delta\theta + \sin(\delta\theta) \cdot \cos(\theta_0 - \alpha) - \sin(\theta_0 - \alpha) \right]$$

$$\delta\theta \approx 0 \Rightarrow \cos \delta\theta = 1 \quad \& \quad \sin \delta\theta = \delta\theta$$

$$\Rightarrow \Delta(\Delta\varphi) = -\frac{4\pi}{\lambda} B \left[\cos(\theta_0 - \alpha) \delta\theta \right] = \text{correction term}$$

$$\sin(\theta - \alpha) = \sin(\theta_0 - \alpha) + \cos(\theta_0 - \alpha) \delta\theta$$

How to find θ_0
measure θ

So, I write here my let us say delta term which is nothing, but delta of delta psi is equal to minus 4 pi by lambda B into sin of A into cos B plus sin B into cos of a minus this original term. Now you can see that delta theta is very small or approximately equal to 0 as a result I have cos delta theta is equal to 1 and sin delta theta is equal to delta theta.

As a result we can write that this term which is differential interferogram is nothing, but B is the whole term. This will become sin theta and this will cancel with this term and now I will have only this term where this becomes my delta theta. So, I can write here that cos of theta 0 minus alpha into delta theta. Now you can see here in spite of this term and it is nothing, but a correction term for the flat earth, I can basically write my sin theta minus alpha is equal to plus correction term alright. So, which is nothing, but cos of theta 0 minus alpha into delta theta; I hope you agree with that and as a result I can write.

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Differential Interferometry

- Three pass (repeat track) interferometry
 - Two base lines, two interferograms
 - The incidence angle and the absolute range are approximately same

- Interferometric phase between the first and the second acquisition

$$\Delta\varphi_1 = -\frac{4\pi}{\lambda} B_1 \sin(\theta_0 - \alpha_1) + \left[-\frac{4\pi}{\lambda} B_1 \cos(\theta_0 - \alpha_1) \delta\theta \right]$$

flat
topography

- Interferometric phase between the first and the third acquisition

$$\Delta\varphi_2 = -\frac{4\pi}{\lambda} B_2 \sin(\theta_0 - \alpha_2) + \left[-\frac{4\pi}{\lambda} B_2 \cos(\theta_0 - \alpha_2) \delta\theta \right] - \left(\frac{4\pi}{\lambda} \Delta R \right)$$

flat
topography
change

Now, the phase for the first and second acquisition that is my base line is B 1 and so, I have $\sin \theta_0 - \alpha_1$ plus this correction factor here alright. I hope you agree with that. Similarly for the first and the third acquisition I can write this term with base line B 2 and this one α_2 angle here and this one. One should note here that this term is coming because there is a difference in the range and that happens because the topographic position of the point of interest has changed between the second and the third acquisition alright.

Now, let us go ahead that this component is caused by the flat datum and this is the component by topography in the both the terms alright and this term is because of the change alright. So, this is again because of the flat topography or flat datum and it is because of the topography alright ok. Here we can do one thing now to understand the change in the topography as we know that only this factor is contributing here. Similarly only these 2 factors are contributing here alright. These factors can be ignored alright because they are coming from the flat datum of flat topography alright. So, let us consider these three factors 1, 2 and 3 in order to measure the change in the range order change detection.

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Differential Interferometry

$$\begin{aligned}\Delta\phi_{1_{topo}} &= \left[-\frac{4\pi}{\lambda} B_1 \cos(\theta_0 - \alpha_1) \delta\theta \right] \\ \Delta\phi_{2_{topo+change}} &= \left[-\frac{4\pi}{\lambda} B_2 \cos(\theta_0 - \alpha_2) \delta\theta - \frac{4\pi}{\lambda} \Delta R \right] \\ \Delta\phi_{2_{topo}} &= \left[-\frac{4\pi}{\lambda} B_2 \cos(\theta_0 - \alpha_2) \delta\theta \right] \quad \Delta\phi_{2_{change}} = \left[-\frac{4\pi}{\lambda} \Delta R \right]\end{aligned}$$

So, now we can write only the topographic term here for first term and here for the second term, we can write topographic term as well as the change term together like this alright. But the problem is we have the different different baselines. Now because of the different baseline, what happens? Because of different baseline, it happens that there are 2 sensors; they are of different characteristics. Now we want to bring their measurements in one reference system or in one comparison frame; so, that I can compare their values.

What will I do? I will rescale the one with respect to the another. Generally we have the term delta R in the second first and third equation and that is why, we will rescale the psi 1 term. Note down that this is the topographic term here which is nothing, but this term and here this is my changed term which is here alright. So, we will compare this two. What you want to do? You want to make this and these comparable; so, that we can determine this one alright. So, we do the rescaling like this.

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Differential Interferometry

- Rescaling the first interferogram as if it was acquired with the second interferogram baseline

$$\Delta\varphi_{1_{topo-new}} = \left(\Delta\varphi_{1_{topo}} \frac{B_2 \cos(\theta_0 - \alpha_2)}{B_1 \cos(\theta_0 - \alpha_1)} \right)$$
$$\delta\varphi = \left[\Delta\varphi_{2_{topo+change}} - \Delta\varphi_{1_{topo-new}} \right] = \left[-\frac{4\pi}{\lambda} \Delta R \right] \Rightarrow$$

This is the concept of differential interferometry

So, we find out what is the value of ψ_1 new as with the old value and then, we do the rescaling with this factor alright. We know the θ_0 alright. Now what happens here is we have done this derivation very clearly, but how to find the θ_0 ? So, we say here that we measure the θ , but we want to find out θ_0 . So, what do we do here? Basically we use a DEN that is already available to me right which is an approximate DEN alright.

So, then using this range R , we project this point here and we find out what is the point A dash and accordingly find out what is the angle θ_0 . So, this is the way determine the angle θ_0 using the DEN which is already available to us. We know already θ_0 , we already know α_2 from the sensor mechanism α_1 from sensor mechanism and θ_0 from my calculation B_1 B_2 we already know from the sensor information alright. So, now we have done the rescaling ok. So, let us find out the phase difference between the 2 values here like this and that will be equal to this value alright.

So, any residual here right they will be different values after re scaling also right. So, any residual will be telling me this value here alright. So, that is the idea of the differential interferometry and this is the concept of differential interferometry alright. So, I hope you got the idea what does it mean by the differential interferometry. Now if you measure the ΔR by measuring the residuals between the differential interferogram

because there are 2 interferograms and now I am calculating the differential interferogram by this logic delta psi here or we can say delta of the delta psi alright.

So, by this I calculate the value of delta R on the right hand side alright so; that means, I measuring the residual on the differential interferogram that is the final value that is given by the differential interferogram. And using that value I am calculating delta R and that delta R will be the change in the range of the point on topography because there was a change between the second equation in the third acquisition.

That means, let us imagine that there is some changes happen within 1 year. So, my second acquisition happened before 1 year and after 1 year of that third acquisition happened and during that time whatever changes are there that will be reflected in the range delta R or the change in the range delta R. And using that difference I can find out how my point has shifted in the 3D position from original place to another place. So, this is the concept of the differential interferometry and this is the use of the differential interferometry. So, that is the application of the differential interferometry here.

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Change Mapping

$$\delta R = R_2 - R_1 = \frac{-2R_1 B}{(R_1 + R_2)} \sin(\theta - \alpha) + \frac{B^2}{(R_1 + R_2)}$$

$$\delta R = -B \sin(\theta - \alpha) + \frac{B^2}{2R_1} \rightarrow 0$$

$$\delta R = -B_{\parallel}$$

$$(R_2^2 - R_1^2) = (R_2 - R_1)(R_2 + R_1) \quad R_2 + R_1 \approx 2R_1 \quad (\because R_1 \approx R_2)$$

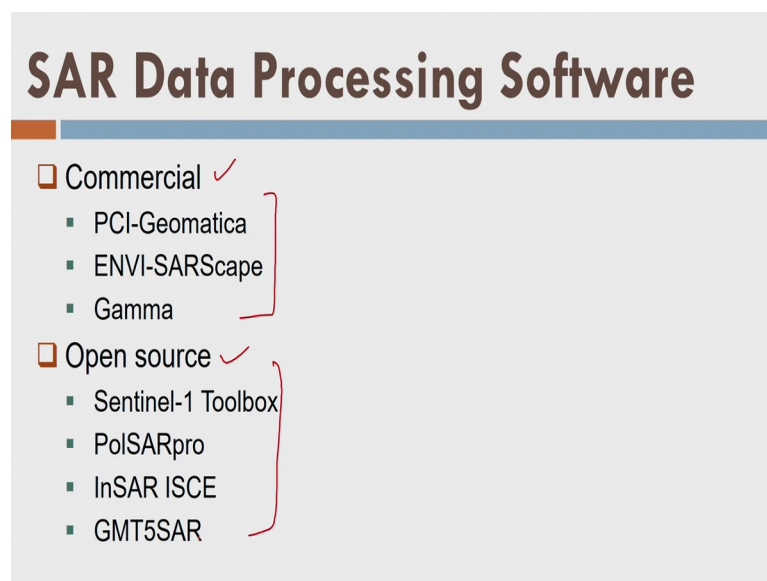
We say if you remember the same expression there of R 2 one is R 1 square fine. So, from here I can write let us say that R 2 minus R 1 square is can be written as R 2 minus R 1 into R 2 plus R 1 fine. So, this part is divided here for that remaining expression and we have R 1 plus R 2 here. Now as we said already that R 2 plus R 1 can be approximately 2 R 1 because R 1 is approximately equal to R 2. Now what happens here

is this will be cancel out with this factor here and finally, I will have B here; so, B into sine of theta minus alpha right.

What about this factor? Again it becomes $2 R_1$ and again we see that B^2 by $2 R_1$ is going to be 0 here and as a result now ΔR is equal to minus B into $\sin \theta \alpha$ and this is nothing, but the baseline length parallel to the loop direction. And there by this logic, we estimate the ΔR for change detection. So, what is my change mapping? Change mapping is from well I take the data for 1 and 2 pair; there is no change.

But while I take the data for pair 1 and 3; that means, in the third pass doing that time change has occurred. So, if I take the baseline B 2 between 1 and 3 sensor location, what will happen? I can observe this change and this change is indicated by this. So, that is the application of the three pass interferometry and three pass interferometry is basically need for change detection only ok; I hope you got the idea.

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Now, at last we would like to tell you that in case of SAR data processing there many softwares are available for commercial as well as open source. So, these are the lists here. So, you can use anyone in order to do some kind of interferometry practices from the available data right. So, with this thing, I would like to finish this lecture and in the next lecture. So, far we have seen different different aspects of the RADAR; geometric aspect, radiometric aspect, RADAR grammetry and the interferometry aspect ok. Now finally, in the last lecture of this module, we will have the applications of my RADAR

for the geoscience perspective or the geoscience applications of the RADAR; we will look into the last lecture.

Thank you.